Try to avoid calculation mistake

Read question carefully

MADE EASY

Leading Institute for ESE, GATE & PSUs

ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

Electrical Engineering

Test-2 : Digital Electronics + Microprocessors + Systems and Signal Processing

Name : ....

Roll No :

Test Centres			Student's Signature
Delhi ☐	Bhopal ☐	Jaipur ☐	
Pune ☐	Kolkata ☐	Hyderabad ☐	

Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	24
Q.2	45
Q.3	
Q.4	
Section-B	
Q.5	49
Q.6	41
Q.7	32
Q.8	
Total Marks Obtained	191

Signature of Evaluator: Saurabh kumar

Cross Checked by: ....

IMPORTANT INSTRUCTIONS

CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.

DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

Section A : Digital Electronics + Microprocessors

Q.1 (a) Perform the BCD operation on the following number system:

(i) (010001101001.0110) + (100000010111.1000)

(ii) (011001010100.0101) – (100101111001.0110)

(iii) (011001100110.0110) – (010010011001.1001)

[4 + 4 + 4 marks]

Sol:- (i) (010001101001.0110)

= −1129.075

& (100000010111.1000) = 23.50.

⇒ (010001101001.0110) + (100000010111.1000)

−1129.75 + 23.50

= −1106.25.

⇒ (10101 0101110 .01).

ii) (011001010100.0101) − (1001 0111 1001.0110)

→ 0110 0101 0100 . 0101

+0110 1000 0110 . 1010

1100 1101 1010 . 1101

Actual result = 0011 00100101 .0011

(2)

Go through the made easy solution

iii) 0110 0110 0110 · 0110

10 · 0101

Do not write in this margin

.1 (b) The given below decimal numbers are stored in 6 digit register in sign-magnitude form. Convert them to signed 10's complement form and perform the following operations. Also convert the result into equivalent hexadecimal number.

(i) (+9286) + (+801) **(ii)** (+9286) + (–801)

(iii) (–9286) + (+801) **(iv)** (–9286) + (–801)

[12 marks]

.1 (c) Determine the minimized expression for the given below Boolean function in SOP form using K-map:

$F(A, B, C, D) = \Pi(0, 1, 4, 5, 6, 7, 9, 13)$.

Implement the minimized function using NAND gate only.

[12 marks]

1(c) $F(A,B,C,D) = \Pi(0,1,4,5,6,7,9,13)$

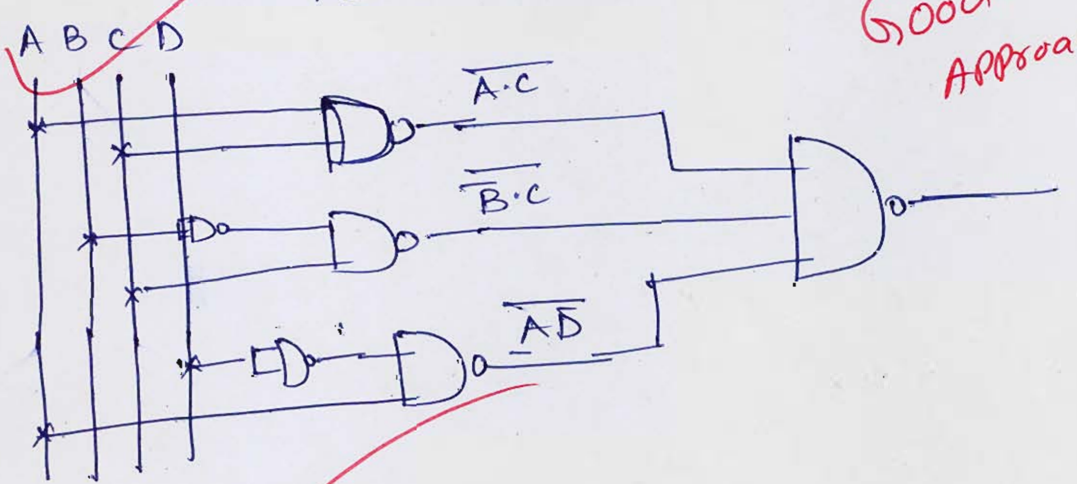
$F(A,B,C,D) = \Sigma(2,3,8,10,11,12,14,15)$.

	$\bar C\bar D$	$\bar C D$	CD	$C\bar D$
$\bar A\bar B$	(0)	(1)	1 (3)	1 (2)
$\bar A B$	(4)	(5)	(7)	(6)
AB	1 (12)	(13)	1 (15)	1 (14)
$A\bar B$	1 (8)	(9)	1 (11)	1 (10)

$F(ABCD) = AC + \bar B C + A\bar D$

$= \overline{\overline{AC + \bar B C + A\bar D}}$

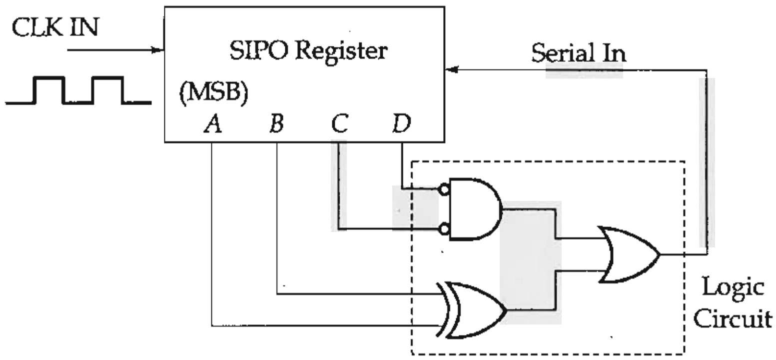
$= \overline{\overline{AC} \cdot \overline{\bar B C} + \overline{A\bar D}}$



11

Good Approach

.1 (d) A shift register with associated combinational logic circuit is shown in figure below:



Explain its operation by drawing waveforms for the output of shift register ($ABCD$). Assume that initially shift register is in state $ABCD$ = 1111.

[12 marks]

Q.1 (e) For an 8085 microprocessor write an assembly language program to move a block of 10 bytes of data from one section of memory to another section of memory. Write comments for selected instructions.

[12 marks]

Sol:- Consider the 10 Byte data is stored at location C300H and Need to be transffered to D300H.

~~LXI H, C300H → initialed the HL with Address.~~
~~LXI D, D300H → initiated DE pair with Address where I~~
~~MVI B 0AH → counter for 10 Byte to transfer data~~

~~MOV A, M → data is copied from Memory location to accumulator.~~

~~INX H~~

~~DE~~ STA D

Try to avoid

LXI H C300H :- Initialed the HL with Address where data is

LXI D D300H :- initialized DE pair with Address where data need to be stored

MVI B 0A H :- Initial counter for 10Byte of data

loop MOV A, M. :- Content of Memory is moved to Accumulator

STAX D :- Content of Accumulator is stored to DE.

INX H :- Increment HL for next data

~~DE~~ INX D :- Increment DE to store data into new location

DCR B :- Counter is decrease

JNZ loop

HLT Halt.

11

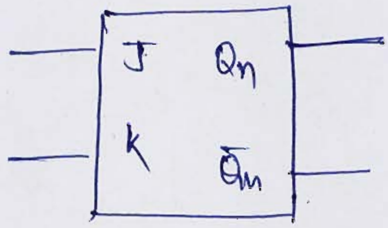
Do not write in this margin

2 (a) (i) Write truth table and logic symbol of J-K flip-flop and also draw J-K flip-flop using NAND and NOR latch.

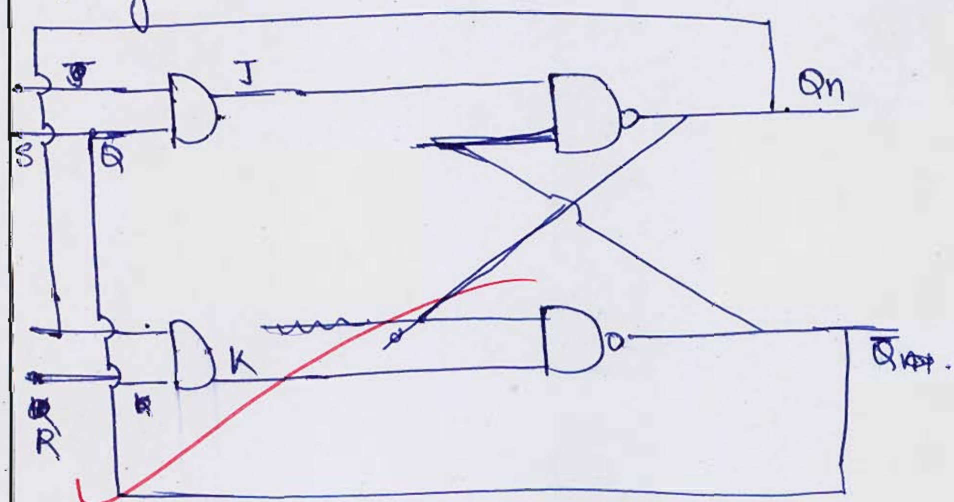
[10 marks]

Sol:-

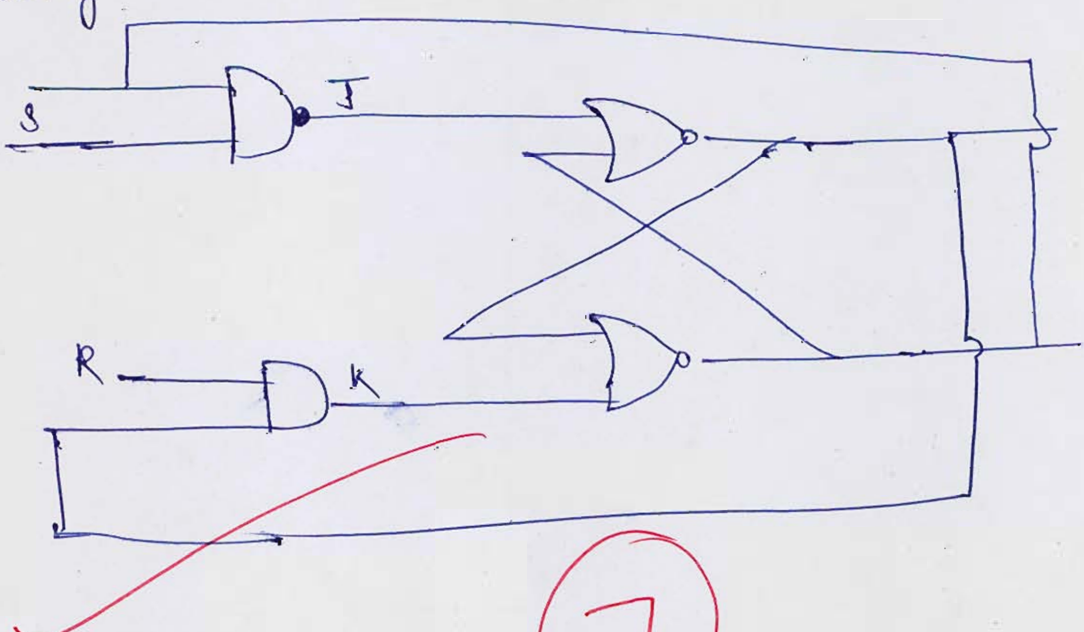
J	K	Q_{n+1}
0	0	Q_n
0	1	0
1	0	0
1	1	$\bar{Q}_n$



Using NAND Latch.



Using NOR Latch



7

2 (a) **(ii)** Convert SR flip-flop into JK flip-flop and explain race around condition in J-K flip-flop.

[10 marks]

Sol:- ~~Excitation~~ Truth Table of SR.

S	R	Q_{n+1}
0	0	Q
0	1	0
1	0	1
1	1	X

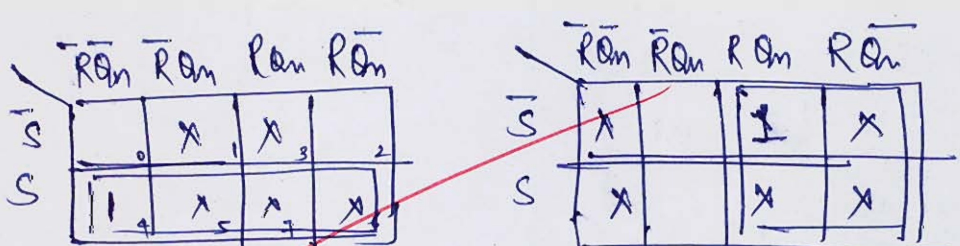
Excitation table for J-K flipflop is given by.

Q	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Read question carefully

Now

S	R	Q_n	Q_{n+1}	J	K
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	X	1
1	0	0	1	1	X
1	0	1	1	X	0
1	1	0	1	1	X
1	1	1	0	X	1



J = S.   K = R.

	$\bar{R}\bar{Q}_n$	$\bar{R}Q_n$	RQ_n	$R\bar{Q}_n$
$\bar{S}$	0	X 1	X 3	2
S	1 4	5	X 7	1 6

$J = S\bar{Q}_n$

	$\bar{R}\bar{Q}_n$	$\bar{R}Q_n$	RQ_n	$R\bar{Q}_n$
$\bar{S}$	X 0	1	1 3	X 2
S	X 4	5	1 7	6

$K = RQ_n$

⇒ The Race around condition in JK flip flop occur in the JK flip when Both the input are high. The state toggle is present state multiple time if the width of CLK pulse is greater than input pulse. To avoid race around condition.

1) Edge trigging is used.
2) Pulse width must be $t_{d} < t_{CLK} <$ ~~T_{CLK}~~ input
3) Master-Salve flip configuration is used.

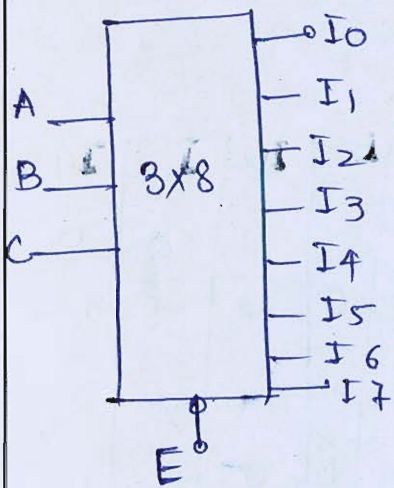
5

2 (b) Design a 3 × 8 decoder with active low enable pin and also draw its logic circuit.

[20 marks]

1. Truth Table for 3×8 Decorder.

E	A	B	C	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	0	0	0	1	0
0	1	1	1	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0		0	0	0
1	0	1	0	0	0	0	0		0	0	0
1	0	1	1	0	0	0	0		0	0	0
1	1	0	0	0	0	0	0		0	0	0
1	1	0	1	0	0	0	0		0	0	0
1	1	1	0	0	0	0	0		0	0	0
1	1	1	1	0	0	0	0		0	0	0



$Y_0 = \bar{E}\,\bar{A}\,\bar{B}\,\bar{C}$

$Y_1 = \bar{E}\,\bar{A}\,\bar{B}\,C$

$Y_2 = \bar{E}\,\bar{A}\,B\,\bar{C}$

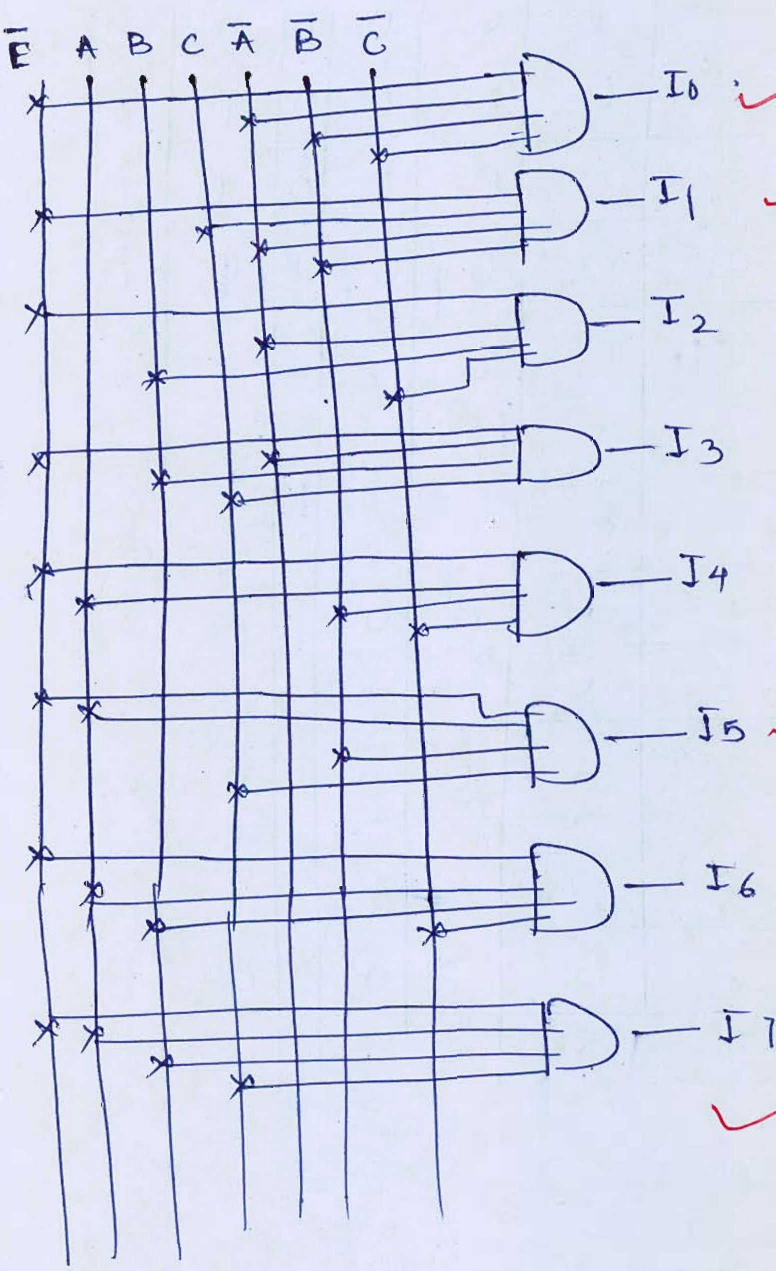
$Y_3 = \bar{E}\,\bar{A}\,B\,C$

$Y_4 = \bar{E} \cdot A\,\bar{B}\,\bar{C}$

$Y_5 = \bar{\bar{E}} \cdot A\,\bar{B}\,C$

$Y_6 = \bar{E} \cdot A\,B\,\bar{C}$

$Y_7 = \bar{E} \cdot ABC$



17

Do not write in this margin

Q.2 (c) Explain the following instructions of 8085 microprocessor:

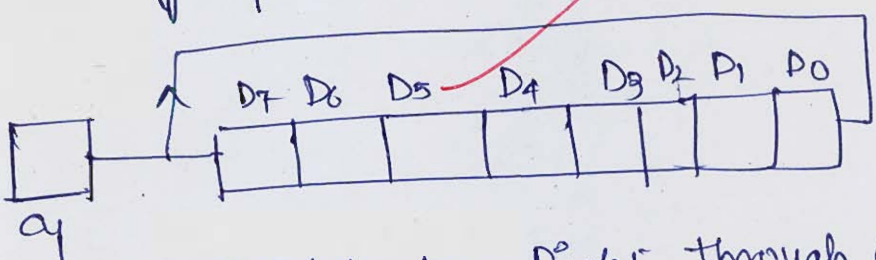
(i) RLC (ii) RAR (iii) CALL 16-bit address

(iv) RET (v) CMP

[20 marks]

1) RLC :- Rotate Accumulator Left without Carry

→ Content of accumulator is rotate to with carry bit being update



1) RAR :- Rotate Accumulator Right through Carry bit

In this Accumulator is rotate as through Carry bit

ii) CALL - 16 bit Address

This is the branch instruction which is used to call the sub-rountine program ~~return~~ written in programme.

When call is excuted following is the sequence

1) The ~~prese~~ next address stored in PC counter is push to stack register.

ii) The call address is loaded to PC counter so as to desired location where sub routine is written

It take 5 M/C and 18 T state as 6T-opcode feetch 2 Read & 2 write cycle.

iv) RET :- It is return statement which is used to take programme to that point from wher call instruction is being called. when RET is excuted ~~seq~~ The orginal Address stored in stack counter is pushed to PC counter and program return. ~~with to~~

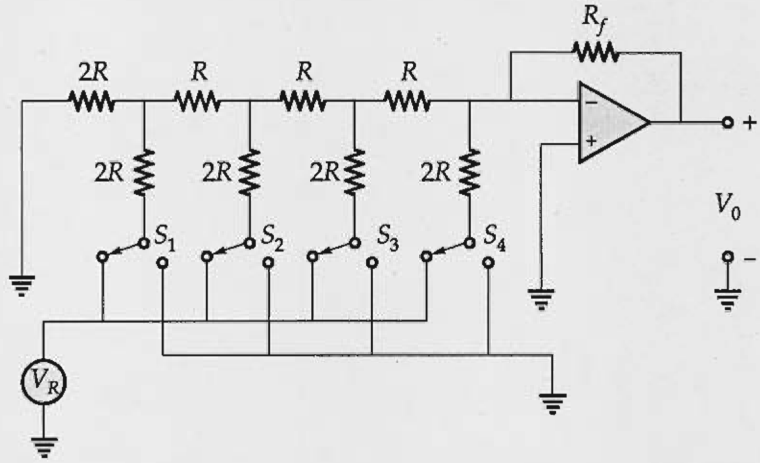
v) CMP :- It is logical instruction know as COMPARE It compare the Accumulator content with registr or memeory content and affect the CY and ZD.

A >	B	CY = 0	Z = 0
A <	B	CY = 1	Z = 0
A =	B	CY = 0	Z = 1

16

Do not write in this ma

.3 (a) (i) Consider a 4-bit R-2R DAC shown in figure. Assume the feedback resistance R_f of the op-amp is variable, the resistance $R = 10\ k\Omega$ and $V_R = 10$ V.



Determine the values of R_f that should be connected to achieve the following output conditions:

1. The value of 1 LSB at the output is 0.5 V.
2. An analog output of 6 V for a binary input of 1000.
3. The full scale output voltage of 12 V.
4. The actual maximum output voltage of 10 V.

(ii) Explain the working of flash type *ADC* with the help of truth table.

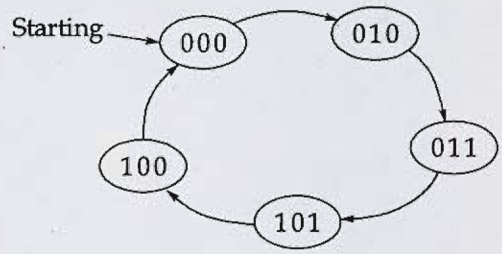
[10 + 10 marks]

Do not write in this mar

.3 (b) Design a sequential circuit using J-K flip flop followed by T-FF followed by S-R flip flop to count a sequence $1 \rightarrow 3 \rightarrow 4 \rightarrow 2 \rightarrow 0 \rightarrow 1$ (Use J-K FF for MSB).

[20 marks]

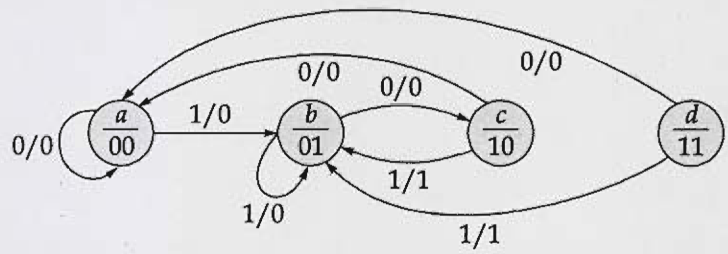
Q.3 (c) **(i)** The state transition diagram of a synchronous counter is given in figure. Design the counter circuit using S-R flip flops.



(ii) Draw EX-OR gate using transmission gates.

[14 + 6 marks]

4 (a) Consider the state diagram shown below:

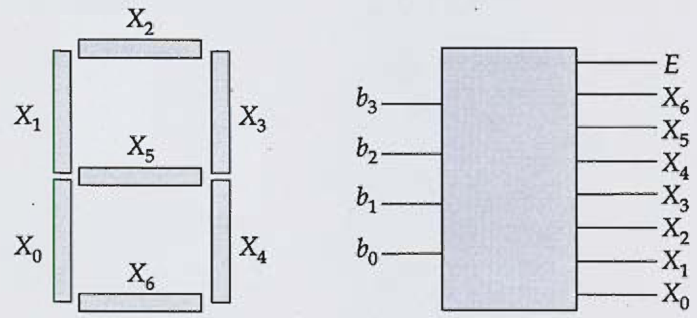


(i) Obtain the state reduction diagram.

(ii) Design a clocked sequential machine using JK FFs for the given sequential circuit.

[8 + 12 marks]

Q.4 (b) A seven segment display consists of seven light-emitting diodes (LEDs) as shown below:



7-Segment display

where, $E = 0$ for valid digit with the display showing the corresponding digit.

$E = 1$ for invalid digit with the display showing the symbol 'E'.

Implement the above given seven segment display using a ROM.

[20 marks]

Do not write in this margin

Do not write in this margin

.4 (c) For an 8085 microprocessor, assume a sensor connected at port 30H. Read 10 bytes of data from port and if the value is +ve then store the corresponding bytes from memory location 4000H. If the data is –ve then switch 'ON' LED by sending FFH to port 50H. Write an Assembly Language Program for above condition.

[20 marks]

Section B : Systems and Signal Processing

Q.5 (a) A stable system with zero initial conditions is described by the difference equation $y(n) = x(n) - 2x(n-1) + x(n+1)$.

(i) Find the impulse response $h(n)$ of the system.

(ii) Plot the output $y[n]$ for an input, $x[n] = u[n-2]$

(iii) Find the value of $\sum_{n=-\infty}^{\infty} y[n]$ for input $x[n] = u[n-2]$.

[4 + 4 + 4 marks]

i) Taking z transform of difference equation

$Y(z) = X(z) - 2z^{-1} + z^{+1} X(z)$

$H(z) = \frac{Y(z)}{X(z)} = (1 + z - 2z^{-1})$

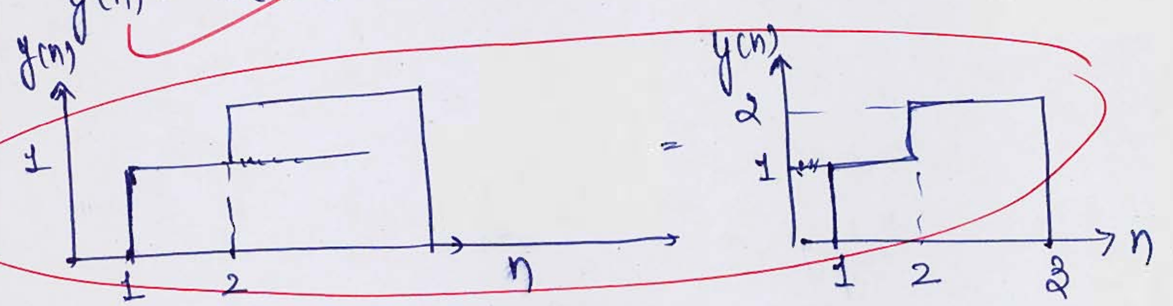
Taking inverse z-transform.

$h(n) = \delta(n) + \delta(n+1) - 2\delta(n-1)$

ii) $y(n) = x(n) * h(n)$

$y(n) = (u[n-2]) * (\delta(n) + \delta(n+1) - 2\delta(n-1))$

$y(n) = u(n-2) + u(n-1) - 2u(n-3)$.



iii) $\sum_{n=-\infty}^{\infty} y(n)$ = Area under the Curve

$\Rightarrow 1\times1 + 1\times2 = 3$ unit.

9

Q.5 (b) Determine the inverse z-transform of the following $X(z)$ by the partial fraction expansion method,

$$X(z) = \frac{z+2}{2z^2 - 7z + 3}$$

if the ROCs are (a) $|z| > 3$, (b) $|z| < 1/2$ and (c) $1/2 < |z| < 3$.

[12 marks]

Sol :-

$$X(z) = \frac{z+2}{(z-3)(z-1/2)}$$

$$X(z) = \frac{2}{(z-3)} - \frac{1}{(z-1/2)}$$

a) ROC $|z| > 3$. R.H. Signal

Inverse ~~z transform~~.

$$X(z) = \frac{2z^{-1}}{(1-3z^{-1})} - \frac{z^{-1}}{(1-1/2\,z^{-1})}$$

Inverse z transform.

$$x(n) = 2(3)^{n-1}u(n-1) - \left(\frac{1}{2}\right)^{n-1}u(n-1).$$

b) For ROC $|z| < 1/2$. Left Hand signal.

$$x(n) = -2(3)^{n-1}u(-n) + \left(\frac{1}{2}\right)^{n-1}u(-n)$$

c) For ROC $1/2 < |z| < 3$.

$\frac{z^{-1}}{(1-1/2\,z^{-1})}$ is R.H signal & $\frac{2}{(z-3)}$ is L.H signal

$$x(n) = -2(3)^{n-1}u(-n) - \left(\frac{1}{2}\right)^{n-1}u(n-1).$$

10

Q.5 (c) Determine the z-transform and the ROC of the signal, $x(n) = a^n u(n) - a^n u(n-1)$.

[12 marks]

Sol:- Z-transform of signal is given by

$x(n) = a^n u(n)$.

$$X(z) = \sum_{n=\infty}^{\infty} x(n)\, a z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (a^n u(n) - a^n u(n-1)) \cdot z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n - \sum_{n=1}^{\infty} (az^{-1})^n$$

$$\Rightarrow \frac{1}{1-az^{-1}} \cdot \frac{1}{1-az^{-1}} - \frac{az^{-1}}{1-az^{-1}}.$$

$$\Rightarrow \frac{1-az^{-1}}{1-az^{-1}} = 1$$

Since it is finite signal Hence it ROC is complete z plane expect $z=0$ $z=\pm\infty$.

11

Do not write in this margin

Q.5 (d) An LTI system has a unit step response given by $s(t) = (1 - e^{-t} - te^{-t})\, u(t)$. For a certain input $x(t)$, the output is observed to be equal to $y(t) = (2 - 3e^{-t} + e^{-3t})\, u(t)$. What is $x(t)$?

[12 marks]

Sol:- $s(t) = (1 - e^{-t} - te^{-t})\, u(t)$

Taking Laplace transform.

$$S(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} = \frac{(s+1)^2 - s(s+1) - s}{s(s+1)^2}$$

$$S(s) = \frac{1}{s(s+1)^2}$$

Now when step input is applied $X(s) = 1/s$.

The transfer function of system $= H(s) = \frac{Y \cdot S(s)}{X(s)} = \frac{1}{(s+1)^2}$

Now, to this system $x(t)$ is applied result in $y(t)$

$$y(t) = (2 - 3e^{-t} + e^{-3t})\, u(t)$$

$$Y(s) = \frac{2}{s} - \frac{3}{(s+1)} + \frac{1}{s+3} = \frac{2(s+1)(s+3) - 3s(s+3) + s(s+1)}{s(s+1)(s+3)}$$

$$Y(s) \Rightarrow \frac{2(s^2+4s+3) - 3s^2 - 9s + s^2 + s}{s(s+1)(s+3)}$$

$$Y(s) = \frac{6}{s(s+1)(s+3)}$$

Now

$$X(s) = \frac{Y(s)}{H(s)} = \frac{6\,(s+1)^2}{s(s+1)(s+3)}$$

$$X(s) = \frac{6(s+1)}{s(s+3)}$$

$$X(s) = \left[\frac{2}{s} + \frac{4}{s+3} \right]$$

Do not write in this margin

Hence $x(t) = (2 + 4e^{-3t})u(t)$

$\therefore$ $\boxed{x(t) = (2 + 4e^{-3t})\,u(t)}$

11

Good Approach

Q.5 (e) Consider the signal $y(t) = e^{-2t}u(t)$ is the output of a causal all-pass system for which the system function is

$$H(s) = \frac{s-1}{s+1}$$

(i) Find and sketch at least two possible inputs $x(t)$ that could produce $y(t)$.

(ii) From the solutions obtained in part (i), what is the input $x(t)$ if it known that a stable system exists that will have $x(t)$ as an output and $y(t)$ as the input? Find the impulse response $h(t)$ for this system.

[12 marks]

Sol e)

$Y(S) = \frac{1}{S+2}$

NOW $Y(S) = H(S) \cdot X(S)$

$X(S) = \frac{Y(S)}{H(S)} = \frac{(S+1)}{(S+2)(S-1)}$

$X(S) = \frac{1/3}{(S+2)} + \frac{2/3}{(S-1)}$.

$X(S) = \frac{2}{3}\frac{1}{(S-1)} + \frac{1}{3}\frac{1}{(S+2)}$

8

For ROC $|\sigma| > 1$

$X(S) = \frac{2}{3}e^{t}u(t) + \frac{1}{3}e^{-2t}$.

$-2 < ROC < 1$

$X(S) = \frac{1}{3}e^{-2t}u(t) - \frac{2}{3}e^{t}u(-t)$

For ROC $\sigma < -2$.

$X(S) = (-)\frac{2}{3}e^{t}u(-t)\ (-)\frac{1}{3}e^{-2t}u(t)$

For stability of ~~system~~, the ~~system~~ ROC must include $j\omega$ axis. ~~ROC < 2~~ include ~~ROC~~ $j\omega$ axis

~~Hence~~

$\therefore$ ~~$X(S) = -2/3\, e^{+t}u(-t) - 1/3\, e^{-2t}u(t)$~~

~~with~~ Hence the input will $-2 < ROC < 1$ include $j\omega$ axis and is stable.

Q.6 (a) **(i)** Compute: $y(t) = x(t) * h(t)$ where, $x(t) = \text{sinc}\,\alpha t$, $h(t) = \text{sinc}\,\beta t$; $\alpha > \beta$

(ii) Compute : $y(t) = x(t) * h(t)$

$$x(t) = \text{rect}\left(\frac{t-6}{2}\right), h(t) = \text{rect}\left(\frac{t}{2}-3\right)$$

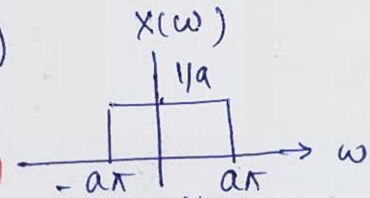
[10 + 10 marks]

Sol :- i) $y(t) = x(t) * h(t)$

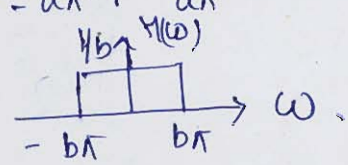
where $x(t) = \text{sinc}\,\alpha t$ or $Sa(a\pi t)$ $\alpha > \beta$

$h(t) = \text{sinc}\,\beta t$ or $Sa(\beta\pi t)$

Take fourier transform of $x(t)$ and $h(t)$



$F\{x(t)\} = X(\omega) = \frac{1}{a} \text{rect}\left(\frac{\omega}{2a\pi}\right)$

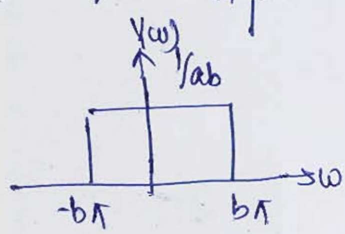


$F\{h(t)\} = H(\omega) = \frac{1}{b} \text{rect}\left(\frac{\omega}{2b\pi}\right)$

Applying convolution Property in fourier transform

$Y(\omega) = H(\omega) \cdot X(\omega)$

$Y(\omega) = \frac{1}{b} \text{rect}\left(\frac{\omega}{2\pi b}\right) \cdot \frac{1}{a} \text{rect}\left(\frac{\omega}{2\pi a}\right)$ since $\alpha > \beta$.



$Y(\omega) = \frac{1}{ab} \text{rect}\left(\frac{\omega}{2\pi b}\right)$

Inverse fourier transform of $Y(\omega)$

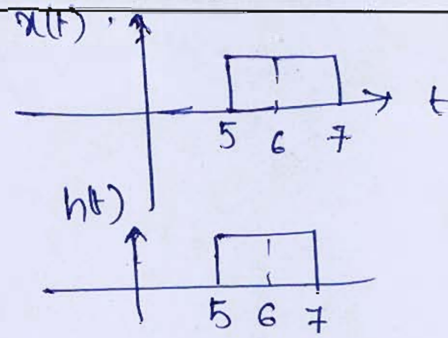
$y(t) = \frac{1}{a} Sa(\pi b t)$ or $\frac{1}{a} \text{sinc}\,\beta t$.

$$\boxed{y(t) = \frac{1}{a} \text{sinc}\,\beta t}$$

9

Good Approach

ii) $x(t) = \text{rect}\left(\frac{t-6}{2}\right)$



$h(t) = \text{rect}\left(\frac{t-6}{2}\right)$

$y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

Case I when ~~$0<t<5$~~ $0 < t-\tau < 5$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = 0.$$

5

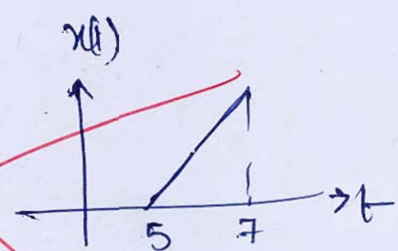
Case II $5 < t-\tau < 7$

$$y(t) = \int_{5}^{\tau+5} 1 \cdot 1\, d\tau = [\tau]_5^{\tau+5}$$

$$\Rightarrow (t+5)-5 = t$$

Case III - $(t-\tau) > 7$

$$y(t) = \int_{5}^{t+\tau} x(\tau)\, h(t-\tau) = 0$$



$\therefore$ $y(t) = \text{tri}\left(\frac{t-6}{2}\right)$

Q.6 (b) Given below are the impulse response of FIR filter. Identify the type of filter implemented using this impulse response.

(i) $h[n] = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$

(ii) $h[n] = 5[\delta[n] - \delta[n-2]]$

[10 + 10 marks]

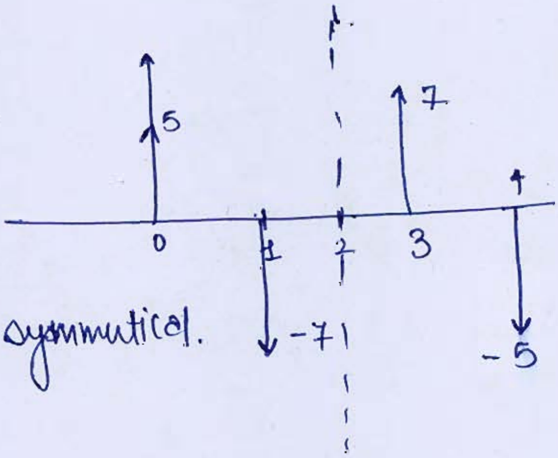
1) $h(n) = 5\delta(n) - 7\delta(n-1) + 7\delta(n-3) - 5\delta(n-4)$

Taking z transform

$h(n) \rightarrow H(z) = 5 - 7z^{-1} + 7z^{-3} - 5z^{-4}$

~~Taking~~

This ~~filter~~ filter is symmetrical about $\delta(n-2)$ and is odd ~~even~~ symmetric.



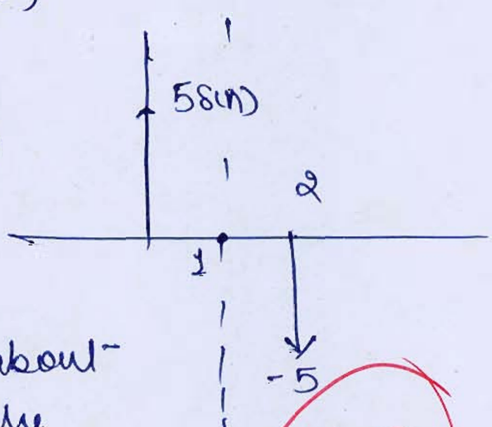
Hence this filter is odd ~~even~~ symmetrical around $\delta(n-2)$.

ii) $h(n) = 5\delta(n) - 5\delta(n-2)$

~~Since this~~

taking z transform of h(n)

$H(z) = 5 - 5z^{-1}$



This filter is symmetrical about $\delta(n-1)$. Theand follow the condition ~~X(z) = X(z*)~~

$X(n) = -X(-n)$

Hence it is odd symmetrical filter.

13

Q.6 (c) (i) Using impulse invariant method, convert the given analog filter transfer function

$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$ to digital transfer function with a sampling period 1 sec.

(ii) Find the Fourier transform of Gaussian modulated signal $x(t) = e^{-at^2} \cos \omega_c t$.

[10 + 10 marks]

Sol 1)

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

The root of this equation $s^2 + \sqrt{2}s + 1 = 0$.

$$s = -\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}}.$$

$$H(s) = \frac{1}{\left(s + 1/\sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$H(s) = \frac{(1/\sqrt{2})}{\left(\frac{1}{\sqrt{2}}\right)\left\{\left(s + 1/\sqrt{2}\right)^2 + \left(1/\sqrt{2}\right)^2\right\}}$$

Taking Laplace transform.

$\sout{H(s)} = h(t) = \frac{1}{\sqrt{2}} e^{-\frac{1}{\sqrt{2}}t} \sin\left(\frac{1}{\sqrt{2}}t\right)$ as $e^{-at}\sin\omega_0 t \rightarrow \frac{\omega_0}{(s+a)^2 + \omega_0^2}$

$$h(t) = \frac{1}{\sqrt{2}} e^{-\frac{1}{\sqrt{2}}t} \sin\left(\frac{1}{\sqrt{2}}t\right) u(t)$$

Now $t = nt_s$. where $t_s = 1$ se.

$$h(n) = \frac{1}{\sqrt{2}} e^{-\frac{1}{\sqrt{2}}n} \sin\left(\frac{1}{\sqrt{2}} n \cdot 1\right) u(n)$$

$$h(n) = \frac{1}{\sqrt{2}} \cdot e^{-\frac{n}{\sqrt{2}}} \sin\left(\frac{n}{\sqrt{2}}\right) u(n)$$

$$H(n) = \frac{1}{\sqrt{2}} e^{-\frac{n}{\sqrt{2}}}$$

Now $\sin(\omega_0 n)u(n) = \dfrac{z \sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$

$\therefore \quad \omega_0 = 1/\sqrt{2}$.

$\sin\left(\frac{1}{\sqrt{2}}n\right) = \dfrac{z\sin(1/\sqrt{2})}{z^2 - 2z\cos(1/\sqrt{2}) + 1} = \dfrac{z(0.650)}{z^2 - z(1.52) + 1}$

Now using $a^n x(n) = X(z/a)$.

~~$(e^{-0.707})^n x(n) = \cdots$~~

Hence.

$H(z) = \dfrac{0.650\cdot(z/0.493)}{\left(\frac{z}{0.493}\right)^2 - 1.52\left(\frac{z}{0.493}\right) + 1} = \dfrac{1.31z}{4.11z^2 - 3.08z + 1}$.

$H(z) = \dfrac{0.32z^{-1}}{(1 - 0.75z^{-1} + 0.24z^{-2})}$

ii) $F(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$.

$F(\omega) = \int_{-\infty}^{\infty} e^{-at^2}\cdot\cos\omega_c t\cdot e^{-j\omega t}\,dt$

$F(\omega) = \int_{-\infty}^{\infty} e^{-at^2}\cdot\left(\dfrac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right)\cdot e^{-j\omega t}\,dt$.

$F(\omega) = \frac{1}{2}\int_{-\infty}^{\infty} (e^{-at^2}\cdot e^{+j(\omega_c-\omega)t} + e^{-j(\omega_c+\omega)t}).\ d$

(15)

(14)

Incomplete solution

Do not write in this margin

Q.7 (a) **(i)** Find the magnitude and phase response for the system characterized by the difference equation

$$y(n) = \frac{1}{6}x(n) + \frac{1}{3}x(n-1) + \frac{1}{6}x(n-2)$$

[8 marks]

Sol:- Taking Discrete fourier transform of function

$$Y(e^{j\omega}) = \frac{1}{6}X(e^{j\omega}) + \frac{1}{3}X(e^{j\omega})\cdot e^{-j\omega} + \frac{1}{6}e^{-j2\omega}X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \left\{\frac{1}{6} + \frac{1}{3}e^{-j\omega} + \frac{1}{6}e^{-j2\omega}\right\}$$

$$H(e^{j\omega}) = \frac{1}{6} + \frac{1}{3}(\cos\omega - j\sin\omega) + \frac{1}{6}(\cos 2\omega - j\sin 2\omega)$$

$$H(e^{j\omega}) = \left(\frac{1}{6} + \frac{1}{3}\cos\omega + \frac{1}{6}\cos 2\omega\right) - j\left(\frac{1}{3}\sin\omega + \frac{1}{6}\sin 2\omega\right)$$

$$|H(e^{j\omega})| = \sqrt{(1/6 + 1/3\cos\omega + 1/6\cos 2\omega)^2 + \left(\frac{1}{3}\sin\omega + \frac{1}{6}\sin 2\omega\right)^2}$$

$$\angle H(e^{j\omega}) = 180° - \tan^{-1}\left\{\frac{1/3\sin\omega + 1/6\sin 2\omega}{1/6 + 1/3\cos\omega + 1/6\cos 2\omega}\right\}$$

6

Q.7 (a) (ii) A filter is to be designed with the following desired frequency response using rectangular window:

$$H_d(e^{j\omega}) = \begin{cases} 0, & -\pi/4 \le \omega \le \pi/4 \\ e^{-j2\omega}, & \pi/4 \le |\omega| \le \pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ if the window function is defined as

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{j\omega})$ of the designed filter.

[12 marks]

Sol:-

$$H(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$$

$$H(n) = \frac{1}{2\pi}\left[\int_{+\pi/4}^{\pi} e^{-j2\omega} \cdot e^{j\omega n}\, d\omega\right]$$

$$h(n) = \frac{1}{2\pi}\left[\int_{\pi/4}^{\pi} e^{j(n-2)\omega} \cdot d\omega\right]$$

$$h(n) = \frac{1}{2\pi}\left[\frac{e^{+j(n-2)\omega}}{j(n-2)}\Bigg|_{\pi/4}^{\pi}\right]$$

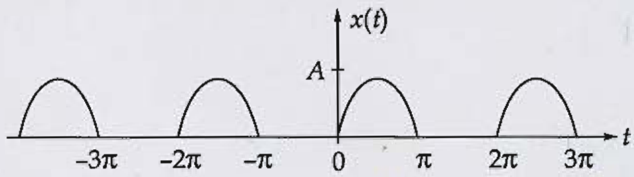
$$h(n) = \frac{1}{2\pi}\left[\frac{e^{j(n-2)\pi} - e^{j(n-2)\pi/4}}{j(n-2)}\right]$$

$$h(n) = \frac{1}{(n-2)\pi}\left[\frac{e^{j(n-2)\pi} - e^{j(n-2)\pi/4}}{2j}\right]$$

$$h(n) = \frac{1}{(n-2)\pi}\left[e^{j(n-2)\pi/2}\left\{e^{j(n-2)\pi/2} - e^{-j(n-2)\pi/2}\right.\right.$$

3

Q.7 (b) Find the exponential Fourier series for the half wave rectified sine wave shown in figure.



[20 marks]

Sol. $x(t) = A \sin t$ with time period of 2π

$\omega_0 = \frac{2\pi}{T_0} = 1$ rad.

$$C_n = \frac{1}{T}\int_{0}^{T} x(t) \cdot e^{-jn\omega_0 t}\, dt$$

$$C_n = \frac{A}{2\pi}\int_0^{\pi} \sin t \cdot e^{-jn\omega_0 t} \cdot dt$$

$$C_n = \frac{A}{2\pi}\int_0^{\pi} \left(\frac{e^{jt} - e^{-jt}}{2j}\right) \cdot e^{-jn\omega_0 t} \cdot dt$$

As. $\omega_0 = 1$

$$C_n = \frac{A}{2\pi}\int_0^{\pi} \left\{\frac{e^{j(1-n)t} - e^{-j(1+n)t}}{2j}\right\} dt$$

$$C_n = \frac{A}{4\pi j}\left[\frac{e^{j(1-n)t}}{j(1-n)} + \frac{e^{j(n+1)t}}{j(n+1)}\right]_0^{\pi}$$

$$C_n = \frac{A}{4\pi j}\left[\left(\frac{e^{j(1-n)\pi}}{j(1-n)} + \frac{e^{j(1+n)\pi}}{j(1+n)}\right) - \left(\frac{1}{j(1-n)} + \frac{1}{j(1+n)}\right)\right]$$

$$C_n = \frac{A}{4\pi j \times j}\left[\left(\frac{\cos(1-n)\pi}{(1-n)} + \frac{\cos(1+n)\pi}{(1+n)}\right) - \left(\frac{2}{1-n^2}\right)\right]$$

$$C_n = -\frac{A}{4\pi}\left[\frac{\cos(\pi-\pi n)}{(1-n)} + \frac{\cos(\pi+\pi n)}{(1+n)} - \frac{2}{1-n^2}\right]$$

$$C_n = -\frac{A}{4\pi}\left[\frac{-\cos(\pi n)}{1-n} + \frac{-\cos(\pi n)}{(1+n)} - \frac{2}{1-n^2}\right]$$

$$C_n = -\frac{A}{4\pi}\left[-(-1)^n\left\{\frac{1}{1-n} + \frac{1}{1+n}\right) - \frac{2}{(1-n^2)}\right)\right]$$

$$C_n = \frac{A}{4\pi}\left[\frac{2}{(1-n^2)}\left\{1 \mp (-1)^n\right\}\right\}$$

5

$$C_n = \frac{A}{2\pi(1-n^2)}\left\{(1 \mp (-1)^n\right\}.$$

Hence For $n =$ ~~odd~~ Even $C_n = 0$.

For $n =$ ~~Even~~ odd $C_n = \frac{A}{\pi(1-n^2)}$.

Hence.

$$x(t) = \sum_{n=0}^{\infty} \frac{A}{\pi(1-n^2)} \cdot e^{jnt}.$$

Q.7 (c) (i) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t)$$

where, $z(t) = e^{-t}u(t) + \delta(t)$.

Determine the impulse response of the system.

[10 marks]

Sol 1)

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)\, z(t-\tau)\, d\tau - x(t)$$

$$\frac{dy(t)}{dt} + 10y(t) = x(t) * z(t) - x(t)$$

Taking Laplace transform.

$$sY(s) + 10Y(s) = X(s) \cdot Z(s) - X(s) \quad -(1)$$

Now,

$$z(t) = e^{-t}u(t) + \delta(t)$$

Taking it Laplace transform.

$$Z(s) = \frac{1}{s+1} + 1 = \frac{s+2}{s+1}$$

Now using equation

$$Y(s)(s+10) = \left(\frac{s+2}{s+1}\right)X(s) - X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+10)}$$

$$H(s) = \frac{1}{9}\left\{\frac{1}{(s+1)} - \frac{1}{s+10}\right\}$$

Taking inverse Laplace.

$$h(t) = \frac{1}{9}e^{-t}u(t) - \frac{1}{9}e^{-10t}u(t)$$

Hence

$$h(t) = \frac{1}{9}\left(e^{-t} - e^{-10t}\right)u(t)$$

9

Good Approach

Do not write in this margin

Q.7 (c) **(ii)** Convert the analog filter with system transfer function $H(s) = \frac{s+0.1}{(s+0.1)^2+9}$ into a digital IIR filter using bilinear transformation. The digital filter should have a resonant frequency of $\frac{\pi}{4}$.

[10 marks]

sol c) ii Given $\omega = \pi/4$ rad/sec.

$\Omega = 3$ rad/sec

$$\Omega = \frac{2}{T_s}\tan\left(\frac{\omega}{2}\right)$$

$$\Rightarrow 3 = \frac{2}{T_s}\tan\left(\frac{\pi}{8}\right). \quad T_s = ~~0.092~~ \text{ sec}$$

$T_s = 0.276$.

$$H(s) = \frac{s+0.1}{s^2+0.01+0.2s+9}$$

$$H(s) = \frac{s+0.1}{s^2+0.2s+9.01} \qquad \text{where } s = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$$

$$H(s) = \frac{7.24\left[\frac{z-1}{z+1}\right]+0.1}{(7.24)^2\left(\frac{z-1}{z+1}\right)^2+0.2\times7.24\left(\frac{z-1}{z+1}\right)+9.01}$$

$$H(z) = \frac{7.24(z-1)(z+1)+0.1(z+1)^2}{(7.24)^2(z-1)^2+1.44(z-1)(z+1)+9.01(z+1)^2}$$

$$H(z) = \frac{7.24(z^2-1)+0.1(z^2+2z+1)}{(7.24)^2(z^2-2z+1)+1.44(z^2-1)+9.01(z^2+2z+1)}$$

$$H(z) = \frac{7.34z^2 + 0.2z - 7.14}{62.8670z^2 - 86.815z + 59.99}$$

~~H(z) = -(+~~

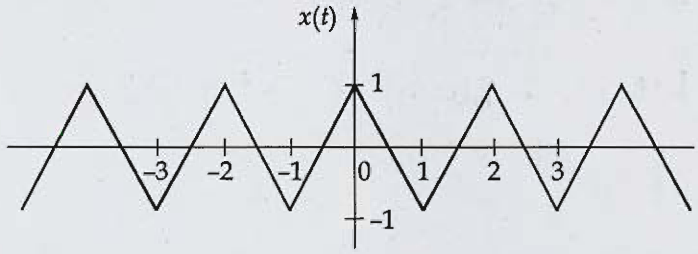
$$H(z) = \frac{0.12 + 8.18\times10^{-3}z + -0.11z^{-2}}{1 - 1.38z^{-1} + 0.95z^{-2}}$$

$$H(z) = \frac{0.12 + 0.0032z^{-1} - 0.11z^{-2}}{1 - 1.38z^{-1} + 0.95z^{-2}}$$

9

Do not write in this margin

Q.8 (a) (i) Write down the exponential Fourier series representation of the signal $x(t)$.



(ii) Find the amount of power (in mW) contained in the 7[th] harmonic of the Fourier series representation of $x(t)$.

[10 + 10 marks]

Do not write in this margin

8 (b) Let $g(t) = x(t)\cos^2 t * \dfrac{\sin t}{\pi t}$, where $*$ is convolution. Assuming that $x(t)$ is real and $X(\omega) = 0$ for $|\omega| \geq 1$, show that there exists an LTI system 'S' such that

$$x(t) \longrightarrow \boxed{S} \longrightarrow g(t)$$

[20 marks]

Do not write in this margin

8 (c) Determine the values of P_x and E_x for each of the following signals:

(i) $x_1(t) = e^{j(2t + \pi/4)}$

(ii) $x_2(n) = \left(\frac{1}{2}\right)^n u(n)$

(iii) $x_3(n) = e^{j\left(\frac{\pi}{2}n + \frac{\pi}{8}\right)}$

(iv) $x_4(n) = \cos\left(\frac{n\pi}{4}\right)$

(Where P_x = Power and E_x = Energy)

[20 marks]

OOOO

Space for Rough Work

Space for Rough Work

Space for Rough Work