



**MADE EASY**

Leading Institute for ESE, GATE & PSUs

## ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Mechanical Engineering

#### Test-2 : Strength of Materials + Machine Design + Engineering Mechanics

Name : .....

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Kolkata <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

#### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

#### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	39
Q.2	-
Q.3	40
Q.4	-
Section-B	
Q.5	48
Q.6	07
Q.7	-
Q.8	15
<b>Total Marks Obtained</b>	<b>149</b>

Signature of Evaluator

Cross Checked by

*Caan Shoon*

## IMPORTANT INSTRUCTIONS

**CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.**

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

### DO'S

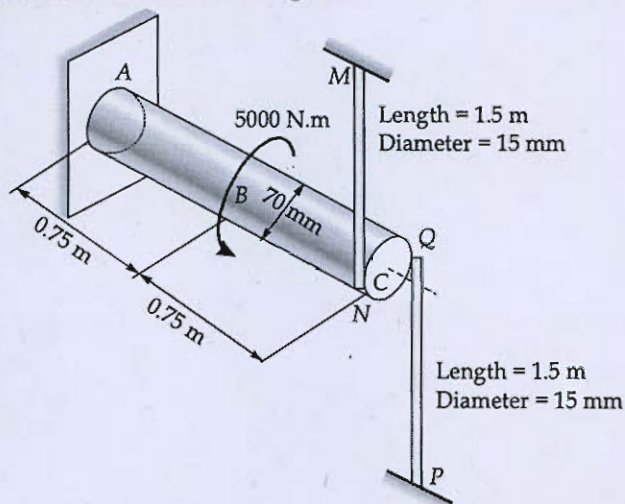
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.



## Section A : Strength of Materials + Machine Design + Engineering Mechanics

- Q.1 (a) A steel shaft ABC, of constant circular cross-section and of diameter 70 mm, is clamped at the left end A, loaded by a twisting moment of 5000 N.m at its midpoint B, and elastically restrained against twisting at the right end C as shown in the figure.

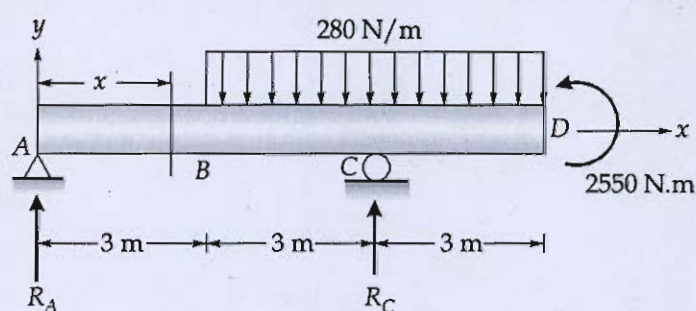
At end C the bar ABC is attached to vertical steel bars each of 15 mm diameter. The upper bar MN is attached to the end N of a horizontal diameter of the 70 mm bar ABC and the lower bar PQ is attached to the other end Q of this same horizontal diameter as shown in the figure. For all materials  $E = 200 \text{ GPa}$  and  $G = 80 \text{ GPa}$ . Determine the peak shearing stress in bar ABC as well as the tensile stress in the bar MN.



[12 marks]

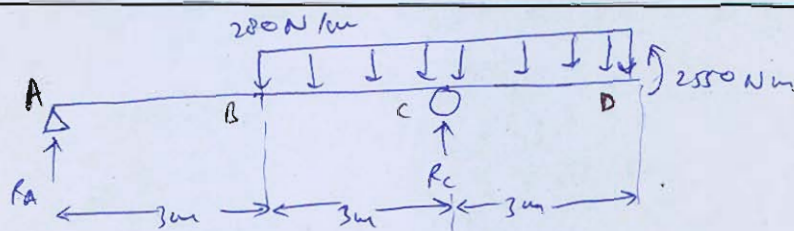


- Q.1 (b) The beam AC is simply supported at A and C and subjected to the uniformly distributed load of  $280 \text{ N/m}$  and the couple of magnitude  $2550 \text{ Nm}$  as shown in the figure. Write the equations for shearing force and bending moment and make sketches of these equations.



[12 marks]



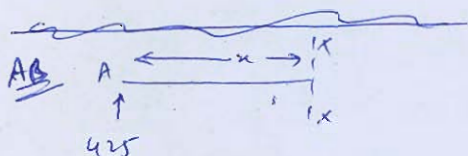


$$\sum M_A = 0 \Rightarrow (R_C \times 6) + 2550 - (280 \times 6 \times 6) = 0$$

$$R_C = 1255 \text{ N}$$

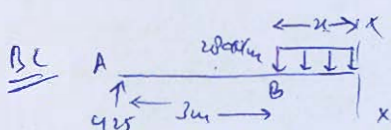
$$\sum F_V = 0 \Rightarrow R_A + R_C - 280 \times 6 = 0$$

$$\therefore R_A = 425 \text{ N}$$



$$S_x = +425 \text{ N}$$

$$S_A(x=0) = S_B(x=3) = 425 \text{ N}$$



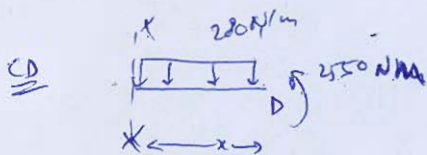
$$S_x = +425 - 280x$$

$$S_B(x=0) = 425 \text{ N}$$

$$S_C(x=3) = -415 \text{ N}$$

$$425 - 280x = 0$$

$$x = 1.5178 \text{ m}$$



$$S_x = +280x$$

$$S_D(x=0) = 0$$

$$S_C(x=3) = 840 \text{ N}$$

$$M_x = +425x$$

$$M_A(x=0) = 0$$

$$M_B(x=3) = 1275 \text{ N-m}$$

$$M_x = +425x - 280 \cdot x \cdot \frac{x}{2}$$

$$= 425x - 140x^2$$

$$M_B(x=0) = 0$$

$$M_x = 425(3+x) - 280 \cdot x \cdot \frac{x}{2}$$

$$= 1275 + 425x - 140x^2$$

$$M_B(x=0) = 1275 \text{ N-m}$$

$$M_C(x=3) = 1290 \text{ N-m}$$

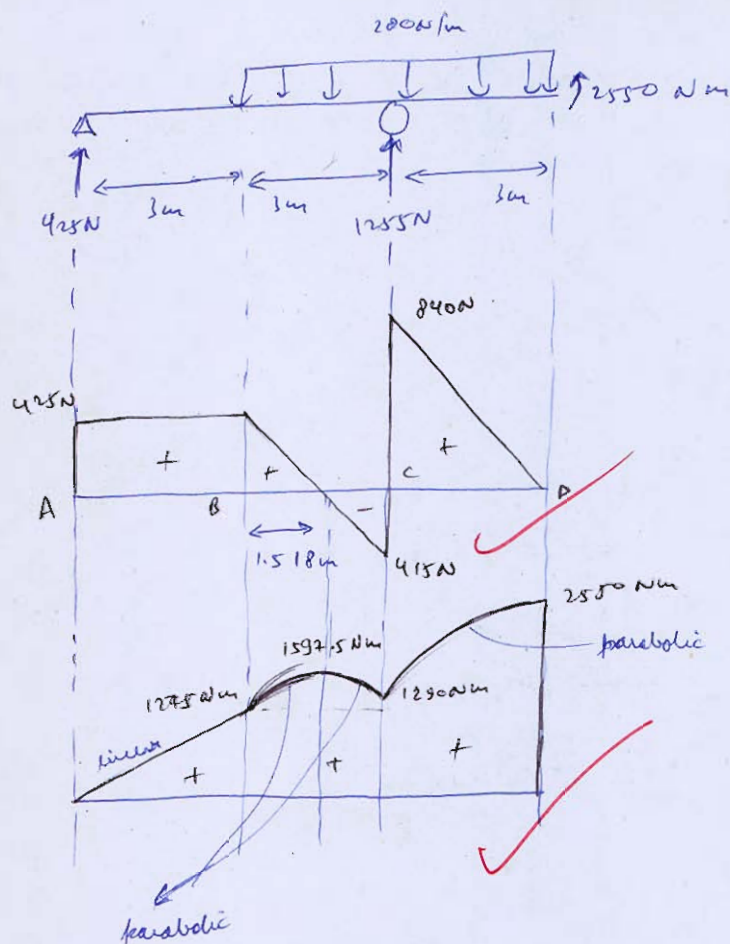
$$M_x = 1.5178 = 1597.546 \text{ N-m}$$

$$M_x = +2550 - 280 \cdot x \cdot \frac{x}{2}$$

$$= 2550 - 140x^2$$

$$M_D(x=0) = 2550 \text{ N-m}$$

$$M_C(x=3) = 1290 \text{ N-m}$$



12

- Q.1 (c) A plate clutch consists of one pair of contacting surface and transmits 30 kW power at 900 rpm. The ratio of outer diameter to inner diameter is 2. The coefficient of friction is 0.3 and the permissible intensity of pressure is  $1.5 \text{ N/mm}^2$ . Assuming uniform wear, calculate the inner and outer diameters.

[12 marks]

$$\frac{D_o}{D_i} = 2 \Rightarrow \frac{R_o}{R_i} = 2$$

$$\mu = 0.3$$

Uniform wear,  $p \cdot r = \text{const.}$

$$\int dW = \underbrace{p \cdot 2\pi r}_{c} dr = 2\pi \cdot c \cdot \int dr$$

$$W = \frac{p \times 2\pi R_i (R_o - R_i)}{\text{per}}$$

$$T = \int r \cdot dW \cdot \mu = \mu \cdot 2\pi c \int r dr$$

$$T = \mu \pi \times p_{\text{per}} R_i (R_o^2 - R_i^2)$$

$$\text{Power} = T \cdot \omega$$

$$30000 = T \times \frac{900 \times 2\pi}{60} \Rightarrow T = 318.3098 \text{ Nm}$$

$$318.3098 \times 10^3 = 0.3 \times \pi \times 1.5 \times R_i [(2R_i)^2 - R_i^2]$$

N/mm

$$R_i = 42.1815 \text{ mm}$$

$$\therefore D_i = 84.363 \text{ mm}$$

$$D_o = 168.726 \text{ mm}$$

12



- Q.1 (d) A pair of spur gears with  $20^\circ$  full depth involute teeth consists of a 22 teeth pinion meshing with a 44 teeth gear. The module is 3 mm while the face width is 45 mm. The material for pinion as well as gear is steel with an ultimate tensile strength of  $600 \text{ N/mm}^2$ . The gears are heat treated to a surface hardness of 400 BHN. The pinion rotates at 1500 rpm and the service factor for the application is 1.75. Assume that velocity factor accounts for the dynamic load and the factor of safety is 2. Determine the rated power that the gears can transmit. Take Lewis form factor ( $Y$ ) = 0.33 for  $20^\circ$  full depth involute system and  $\sigma_b = 0.33 s_{ut}$ .

[12 marks]

$$N_p = 22, \phi = 20^\circ, N_g = 44, m = 3 \text{ mm}, b = 45 \text{ mm}, S_{ut} = 600 \text{ N/mm}^2$$

$$\text{BHN} = 400, N = 1500 \text{ rpm}, C_s = 1.75, F_s = 2, Y = 0.33$$

$\therefore$  material is same for both pinion & gear.

designing for pinion

$$\text{beam strength} = b \cdot m \cdot Y \cdot \sigma_{\text{per}} \quad \{\text{Lewis Eq.}\}$$

$$= 45 \times 3 \times 0.33 \times \left( \frac{0.33 \times 600}{2} \right)$$

$$= 4410.45 \text{ N}$$

$$\text{wear strength} = D_p \cdot b \cdot Q \cdot K \quad \left\{ Q = \frac{44}{22} = 2 \right\}$$

$$= (3 \times 22) \times 45 \times \left( \frac{2 \times 2}{2+1} \right) \times \left\{ 0.16 \left( \frac{400}{100} \right)^2 \right\} \times \frac{1}{2} \rightarrow N$$

$$Q = \frac{24}{4+1} \quad K = 0.16 \left( \frac{\text{BHN}}{100} \right)^2$$

(external gear)

$$= 5068.8 \text{ N}$$

$\therefore$  beam strength < wear strength

Using this.

$$V = \pi \cdot \omega = \left( \frac{3 \times 22}{2} \right) \times \frac{1500 \times 2\pi}{60} = 5183.627 \text{ mm/s}$$

$$= 5183.627$$

$$\therefore C_v = \frac{3+V}{3} = 2.72787$$

$$\text{Now, } F_t \times C_v \times C_s = 4410.45$$

$$F_t = 923.889 \text{ N}$$

$$P = T \cdot \omega$$

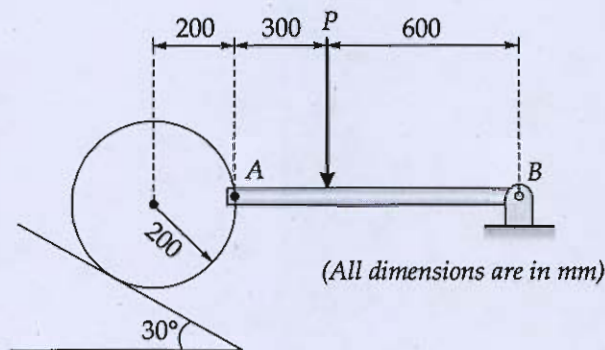
$$= F_t \cdot \pi \cdot \omega = 923.889 \times \left( \frac{3}{1000} \times \frac{22}{2} \right) \times \frac{1500 \times 2\pi}{60}$$

$$= 4789.1016 \text{ W}$$

$$\text{Power} = 4.789 \text{ kW}$$



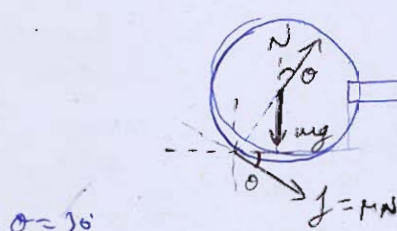
Q.1 (e) A 40 kg disc rests on an inclined surface for which  $\mu_s = 0.3$  as shown in the figure. Determine the maximum vertical force  $P$  that may be applied to link AB without causing the disc to slip at C.



$m = 40 \text{ kg}$

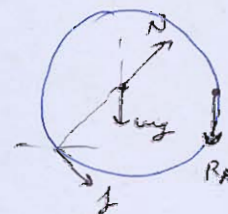
[12 marks]

For max.  $P$ , roller tends to rotate in CW dir.



$\theta = 30^\circ$

$\sum f_{\text{incline}} = 0$   
 $N \sin \theta = P \cos \theta$



$\sum M_A = 0$

$(mg \times r) - (N \cos \theta \times r) + f \cos \theta (r \cos \theta) + f \sin \theta (r \sin \theta) = 0$

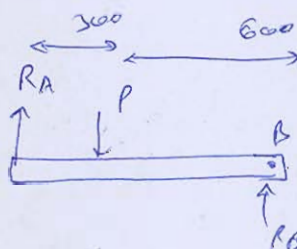
$mg = N (\cos \theta - \mu \cos^2 \theta - \mu \sin^2 \theta) (1 + \sin \theta)$

$N = 943.211 \text{ Newton}$

$\sum F_v = 0$

$N \cos \theta = R_A + mg + \mu N \sin \theta$

$R_A = 282.963 \text{ N}$



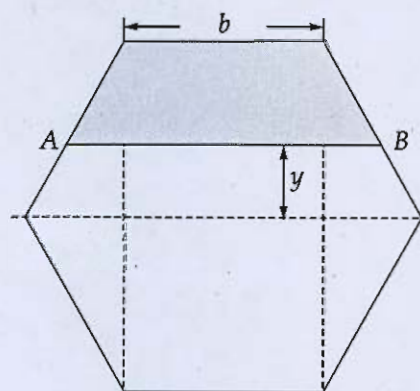
$\sum M_B = 0 \Rightarrow P \times 600 = R_A \times 300$

$P = 141.482 \text{ N}$





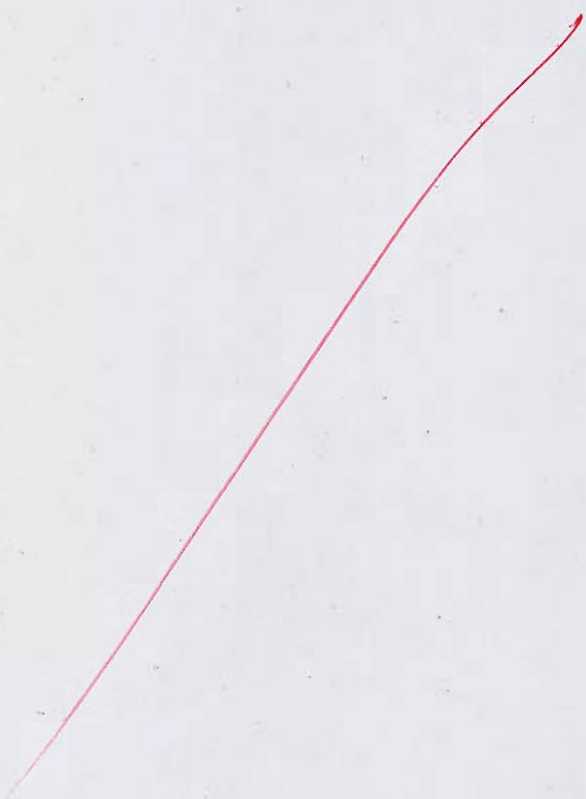
- Q.2 (a) A bar of hexagonal cross-section of side length  $b$  mm is used as a cantilever with one of its diagonals being horizontal. Derive an expression for the shear stress  $\tau$  at the fibre  $AB$  in terms of  $b$  and  $y$ . Determine the shear stress when  $y = 10$  mm,  $b = 30$  mm and shear force applied is 6 kN. Also plot the shear stress distribution plot across the depth of the hexagonal section.



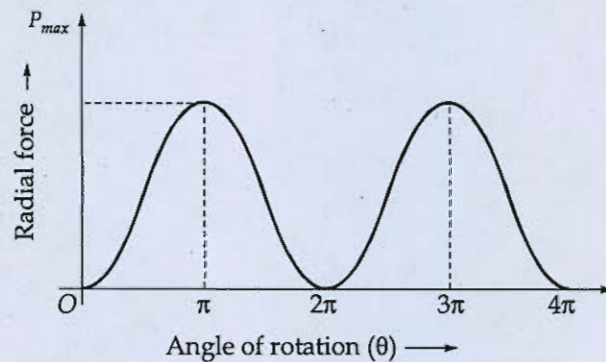
[20 marks]







- Q.2 (b) A ball bearing is subjected to a radial force which varies in sinusoidal way as shown in the figure. The direction of force remains fixed. The amplitude of the force is 2000 N and the speed of rotation is 750 rpm. Determine the dynamic load capacity of the bearing for the expected life of 9000 hr.



[20 marks]

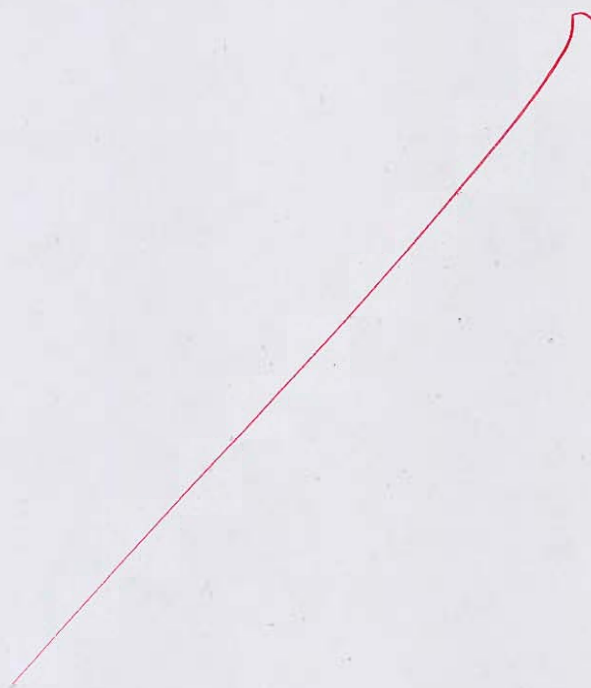


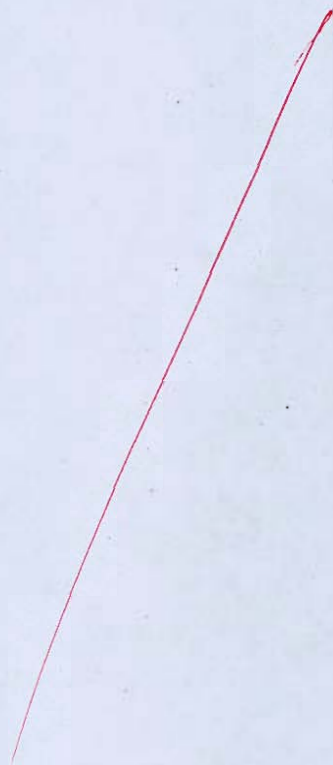




- Q.2 (c) If the density of a hemisphere varies as the distance from the bounding plane, show that the distance of the centre of gravity from that plane is  $\frac{8}{15}$  of its radius.

[20 marks]



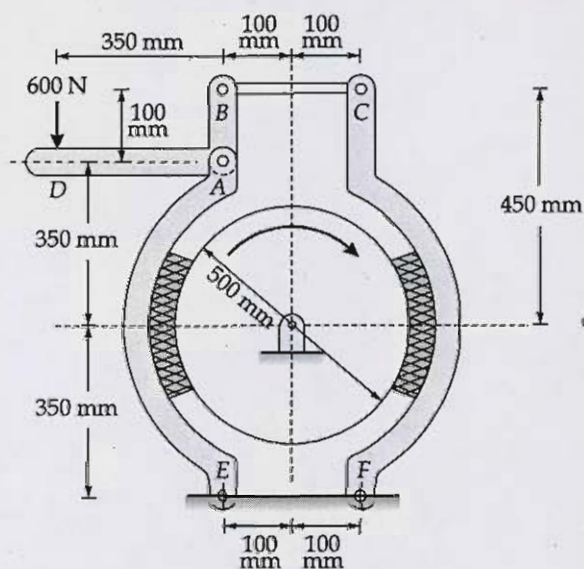




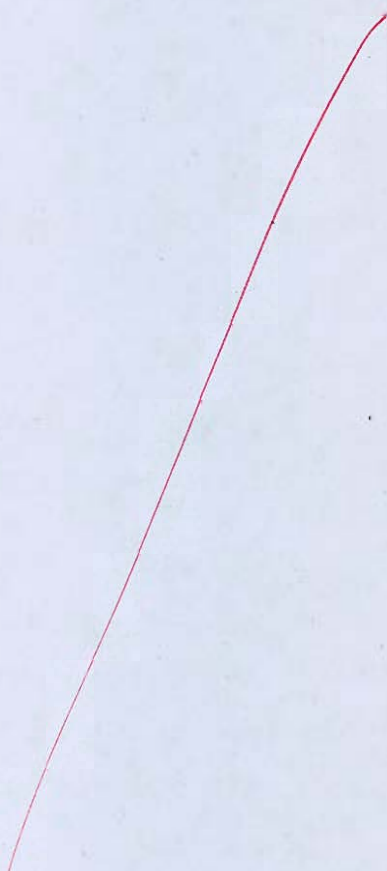
2.3 (a) A double block brake is as shown in the figure. The brake drum rotates in clockwise direction and the actuating force is 600 N. The coefficient of friction between the blocks and the drum is 0.3 Calculate.

- The torque absorbing capacity of the brake.
- The dimensions of the blocks, if the intensity of pressure between the blocks and brake drum is  $1.2 \text{ N/mm}^2$ .

Assume that the blocks are identical and the length of each block is twice its width.

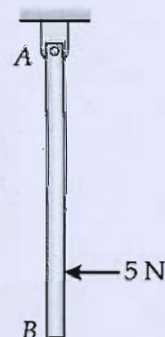


[20 marks]





- Q.3 (b) A wooden metre stick AB of 500 grams mass and length 1 m hangs vertically as shown in figure. If a horizontal force of 5 N is applied at a point that is 30 cm from the bottom end B, determine (a) the angular acceleration of the stick, (ii) the components of reaction at the hinge at A. In addition, determine the point of application of the horizontal force at which the horizontal component of the reaction at A is zero.



[20 marks]

$$m = 0.5 \text{ kg}$$

$$W = mg = 4.905 \text{ N}$$

About A,

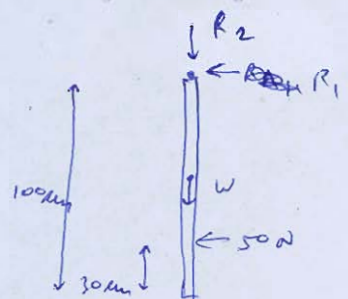
$$(i) \quad T = 5 \times (100 - 30) = 350 \text{ N cm}$$

$$T = 3.5 \text{ N m}$$

$$T = I \cdot \alpha$$

$$3.5 = \frac{0.5 \times 1^2}{3} \times \alpha$$

$$\alpha = 21 \text{ rad/s}^2$$



$$I_{cg} = \frac{1}{12} m l^2$$

$$I_A = I_{cg} + m \left( \frac{l}{2} \right)^2$$

$$I_A = \frac{m l^2}{3}$$

$$(ii) \quad \sum F_v = 0 \Rightarrow R_2 + W = 0$$

$$R_2 = -W$$

$$R_2 = -4.905 \text{ N} \quad \text{(-ve) represents up.}$$

$$a_{cm} = r_{cm} \cdot \alpha = \frac{l}{2} \times \alpha = 10.5 \text{ m/s}^2$$

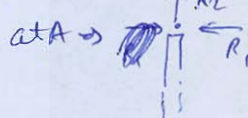
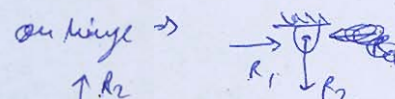
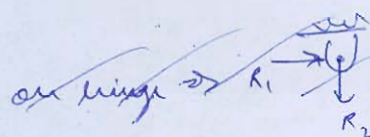
Apply Newton's 2nd law,

$$R_1 + 5 = m \times a_{cm} \quad R_1 + 5 = m \times a_{cm}$$

$$R_1 = 0.25 \text{ N}$$

$$R_1 = 2.5 \text{ N}$$

$$R_2 = -44.75 \text{ N}$$



$$(iii) \quad T = 5 \times x = \frac{m l^2}{3} \times \alpha$$

$$\alpha = 30 \text{ rad/s}^2$$

$$a_{cm} = 0.5 \times 30 = 15 \text{ m/s}^2$$

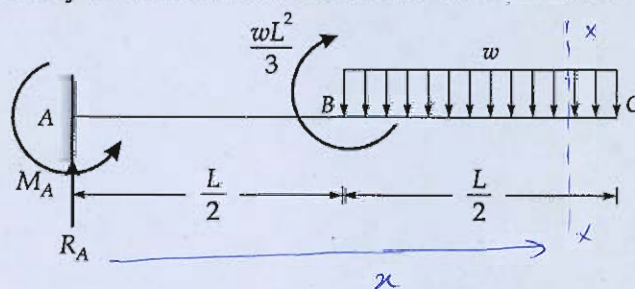
$$R_1 = 0 \Rightarrow 5 = m \times a_{cm} \Rightarrow x = \frac{2}{3} \text{ m from A}$$

20





- Q.3 (c) The cantilever beam ABC as shown below is subjected to a uniform load  $w$  per unit length distributed over its right half, together with a concentrated couple  $\frac{wL^2}{3}$  applied at B. Using Macaulay's method determine the maximum deflection of the beam.



[20 marks]

$$\sum F_v = 0 \Rightarrow R_A = \frac{wL}{2}$$

$$\sum M_A = 0 \Rightarrow -M_A + \frac{wL^2}{3} + \frac{wL}{2} \times \left( \frac{L}{2} + \frac{L}{4} \right) = 0$$

$$M_A = \frac{wL^2}{3} + \frac{3wL^2}{8} = \frac{11wL^2}{24}$$

CW = +ve

$$M_x = -M_A(x)^0 + R_A(x) + \frac{wL^2}{3}(x - \frac{L}{2})^0 = \frac{wL}{2}(x - \frac{L}{2})^2$$

$$M_x = EI \frac{d^2y}{dx^2}$$

$$EI \frac{dy}{dx} = -\frac{11wL^2}{24}(x) + \frac{wL}{2} \frac{(x)^2}{2} + \frac{wL^2}{3} \frac{(x - \frac{L}{2})^2}{2} - \frac{w}{6} (x - \frac{L}{2})^3 + C_1$$

(1)

$$EI \cdot y = -\frac{17\omega L^2}{48} (x)^2 + \frac{\omega L}{12} (x)^3 + \frac{\omega L^2}{6} \left(x - \frac{L}{2}\right)^2 - \frac{\omega}{24} \left(x - \frac{L}{2}\right)^3$$

$$\text{at } x=0, \frac{dy}{dx} = 0$$

$$+ C_1 + C_2$$

(2)

$$\therefore \text{eq. (1),}$$

$$0 = 0 + 0 + 0 + 0 + C_1 \Rightarrow C_1 = 0$$

$$\text{at } x=0, y=0,$$

$$\therefore \text{eq. (2),}$$

$$0 = 0 + C_2 \Rightarrow C_2 = 0$$

$$\therefore y = \frac{1}{EI} \left[ -\frac{17\omega L^2}{48} (x)^2 + \frac{\omega L}{12} (x)^3 + \frac{\omega L^2}{6} \left(x - \frac{L}{2}\right)^2 - \frac{\omega}{24} \left(x - \frac{L}{2}\right)^3 \right]$$

for AB,  $0 \leq x \leq \frac{L}{2}$

$$EI \cdot \frac{dy}{dx} = -\frac{17\omega L^2}{24} x + \frac{\omega L}{4} x^2 + 0 = 0$$

for max. deflection,  $\frac{dy}{dx} = 0$

$$\frac{\omega L}{4} x^2 = \frac{17\omega L^2}{24} x$$

$$\text{at } x=0 \Rightarrow y=0, \quad x = \frac{17L}{6}$$

$x = \frac{17L}{6} \rightarrow$  doesn't lie in range neglected.

$$\therefore \text{for BC,}$$

$$-\frac{17\omega L^2}{24} x + \frac{\omega L}{4} x^2 + \frac{\omega L^2}{3} x - \frac{\omega L^3}{6} - \frac{\omega}{6} \left(x - \frac{L}{2}\right)^3 = 0$$

$$x = L$$

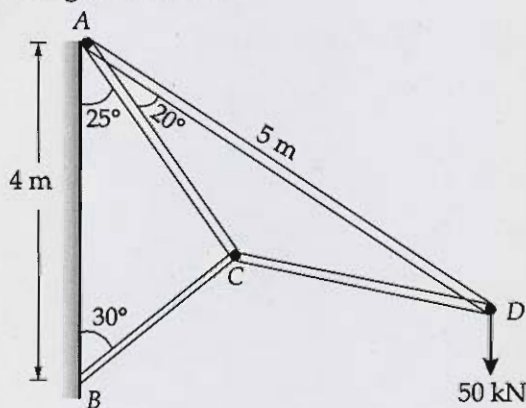
$$\therefore y = -\frac{89}{384} \frac{\omega L^4}{EI}$$

20





- Q.4 (a) Find the force its nature in member  $AD$  and  $BC$  for given cantilever truss loaded by  $50\text{ kN}$  as shown in the figure below.



[20 marks]



- Q.4 (b) A long thin bar of length  $L$  and rigidity  $EI$  is pinned at end  $A$ , and at  $B$  rotation is resisted by a restoring moment of magnitude  $\lambda$  per radian of rotation at that end. Derive the equation for the axial buckling load  $P$ . Neither  $A$  nor  $B$  can displace in the  $y$ -direction, but  $A$  is free to approach  $B$ .

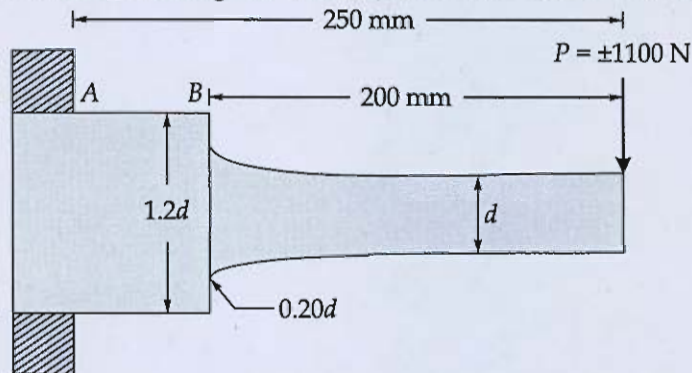
[20 marks]







- Q.4 (c) A cantilever beam made of cold draw steel having surface finish factor ( $k_a$ ) 0.78 and  $S_{ut} = 540 \text{ N/mm}^2$  is subjected to a completely reversed load of 1100 N as shown in the figure. The notch sensitivity factor  $q$  at the fillet can be taken as 0.85 and the expected reliability is 90%. Determining the diameter  $d$  of the beam for a life cycle of 11000 cycles.



Take, reliability factor,  $k_c = 0.897$  for 90% reliability and size factor  $k_b = 0.85$ .

[Use Stress Concentration Factor Chart attached at the end]

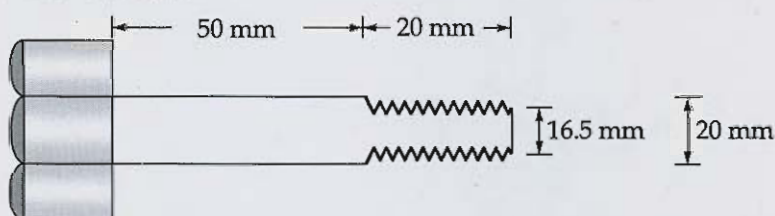
[20 marks]



## Section B : Strength of Materials + Machine Design + Engineering Mechanics

- Q.5 (a) Calculate the strain energy of the bolt as shown in the figure under a tensile load of 20 kN. Show that the strain energy is increased for the same maximum stress, by turning down the shank of the bolt to the root diameter of the thread.

Take  $E = 2 \times 10^5 \text{ N/mm}^2$



Assuming that the load is distributed evenly over the core of screwed portion.

[12 marks]

It can be represented analogous to series of springs :-



$$k_{\text{shank}} = \frac{A_{\text{shank}} \times E_{\text{shank}}}{L_{\text{shank}}} = \frac{\frac{\pi}{4} \times 20^2 \times 2 \times 10^5}{50} = 1.25664 \times 10^6 \text{ N/mm}$$

$$k_{\text{core}} = \frac{A_{\text{core}} \times E_{\text{core}}}{L_{\text{core}}} = \frac{\frac{\pi}{4} \times 16.5^2 \times 2 \times 10^5}{20} = 2.13824 \times 10^6 \text{ N/mm}$$

$\therefore$  in series :-  $\frac{1}{k_{\text{eq}}} = \frac{1}{k_s} + \frac{1}{k_c} \Rightarrow k_{\text{eq}} = 791.4863 \times 10^3 \text{ N/mm}$



$$\text{Load} = 20 \text{ kN}$$

$$\therefore x = \frac{20000}{k_{eq}} \Rightarrow x = 0.025269 \text{ mm}$$

$$\text{strain energy} = \frac{1}{2} k x^2$$

$$= \frac{1}{2} \times 791.4863 \times 10^3 \times 0.025269^2$$

$$= 252.689 \text{ N-mm}$$

$$\boxed{\text{S.E.} = 0.25268 \text{ J}}$$

Now, let  $d = d_c = 16.5 \text{ mm}$

$$\sigma = \frac{F}{A} = \frac{20000}{\frac{\pi}{4} \times 16.5^2} \Rightarrow \sigma = 93.5346 \text{ MPa}$$

$$U_{\text{tot.}} = \left( \frac{\sigma^2}{2E} \right)_{\text{shell}} \times \text{Vol.} + \left( \frac{\sigma^2}{2E} \right)_{\text{rod}} \times \text{Vol.}$$

$$= \left( \frac{\sigma^2}{2E} \times \text{Vol.} \right)_{\text{tot.}} = \frac{93.5346^2}{2 \times 2 \times 10^5} \times \frac{\pi}{4} \times 16.5^2 \times 20$$

$$= 327.37 \text{ N-mm}$$

$$\boxed{(U_{\text{tot.}}) = 0.32737 \text{ J}}$$

after reducing  
shell dia to  
rod dia

$\therefore$  S.E. got increased

$$\text{Increase} = 0.32737 - 0.25268$$

$$= 0.07469 \text{ J}$$

- Q.5 (b) A golf ball is launched with an initial velocity of  $75 \text{ m/s}$  at an angle of  $15^\circ$  with horizontal. Determine the radius of curvature of the trajectory and the time rate of change of the speed of the ball  
(a) just after launch, and (b) at apex  
Neglect aerodynamic drag.

[12 marks]



Q.5 (c) Determine the centroid of quadrant of an ellipse, whose equation is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

AC  $\rightarrow$  major axis

BD  $\rightarrow$  minor axis

$$\bar{x} = \frac{\int_0^a y \cdot du \cdot x}{\int_0^a y \cdot du}$$

$$\rightarrow A = \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} \cdot du$$

Let  $x = a \sin \theta$ ,  
 $du = a \cos \theta \cdot d\theta$

$$\bar{x} = \frac{b \int_0^a \sqrt{1 - \frac{x^2}{a^2}} \cdot du \cdot x}{A}$$

$$x = a \sin \theta$$

$$dx = a \cos \theta \cdot d\theta$$

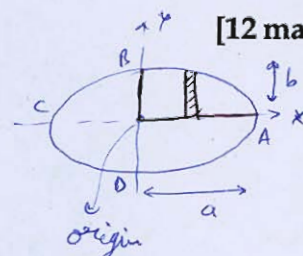
$$\bar{x} = \frac{b \int_0^{\pi/2} \cos \theta \cdot a \cos \theta \cdot d\theta \cdot a \sin \theta}{A}$$

$$= \frac{a^2 b \int_0^{\pi/2} \cos^2 \theta \cdot \sin \theta \cdot d\theta}{A}$$

$$= \frac{a^2 b \times \frac{1}{3}}{\frac{\pi a b}{4}} \rightarrow \boxed{\bar{x} = \frac{4a}{3\pi}}$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y = b \sqrt{1 - \frac{x^2}{a^2}}$$



[12 marks]

$$A = \int_0^{\pi/2} b \sqrt{1 - \frac{a^2 \sin^2 \theta}{a^2}} \cdot a \cos \theta \cdot d\theta$$

$$= ab \int_0^{\pi/2} \cos^2 \theta \cdot d\theta$$

$$A = \frac{\pi ab}{4} \quad \text{--- (1)}$$

$$\begin{aligned}
 \bar{y} &= \frac{\int_0^a y \cdot du \cdot \frac{y}{2}}{A} = \frac{1}{2A} \int_0^a y^2 du \\
 &= \frac{1}{2A} \int_0^a b^2 \left(1 - \frac{u^2}{a^2}\right) du \\
 &= \frac{1}{2A} \left[ b^2 u - \frac{b^2}{a^2} \frac{u^3}{3} \right]_0^a \\
 &= \frac{1}{2A} \left[ b^2 a - \frac{b^2 a}{3} \right] \\
 &= \frac{\cancel{b^2} a}{3 \times \cancel{2} \times \left(\frac{\pi ab}{4}\right)}
 \end{aligned}$$

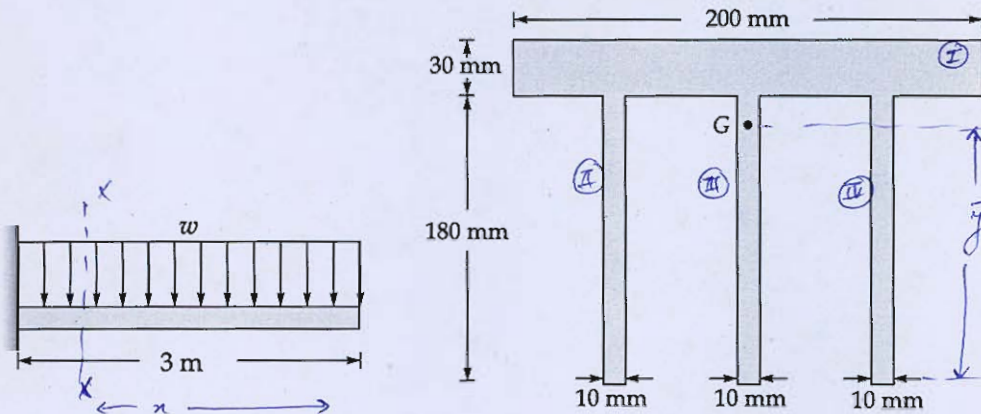
$$\boxed{\bar{y} = \frac{4b}{3\pi}} \quad \checkmark$$

∴ centroid of quadrant  
of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   $= \left( \frac{4a}{3\pi}, \frac{4b}{3\pi} \right)$

12



- Q.5 (d) The extruded beam as shown below is made of aluminium having an allowable working stress in either tension or compression of 90 MPa. The beam is a cantilever, subjected to a uniform vertical load. Determine the allowable intensity of uniform loading.



[12 marks]

$$M_x = -w \cdot x \cdot \frac{x}{2} = -\frac{w x^2}{2}$$

neglecting

M<sub>max</sub> at  $x = 3\text{ m} \Rightarrow M_{\text{max}} = w \times 4.5 \times 10^6 \text{ N-mm}$   
 $x = 3000\text{ mm}$  (magnitude)

$$\bar{y} = \frac{(200 \times 30 \times (180 + \frac{30}{2})) + (3 \times (10 \times 180 \times 90))}{(200 \times 30) + 3(10 \times 180)}$$

$$\bar{y} = 145.263 \text{ mm}$$

$$I_x = \left[ \frac{1}{12} \times 200 \times 30^3 + 200 \times 30 \times 49.736^2 \right] + 3 \left[ \frac{1}{12} \times 10 \times 180^3 + 10 \times 180 \times 55.263^2 \right]$$

$$I_x = 46.3641 \times 10^6 \text{ mm}^4$$

Top  $\rightarrow$  compression  $\Rightarrow \sigma = \frac{M \cdot y}{I}$

$$90 = \frac{w \times 4.5 \times 10^6 \times 64.737}{46.3641 \times 10^6} \quad 2 \times (w) = 14.3240 \text{ N/mm}$$

comp.

Bottom  $\rightarrow$  tension  $\Rightarrow 90 = \frac{w \times 4.5 \times 10^6 \times 145.263}{46.3641 \times 10^6}$

$$(w)_{\text{tension}} = 6.3835 \text{ N/mm}$$

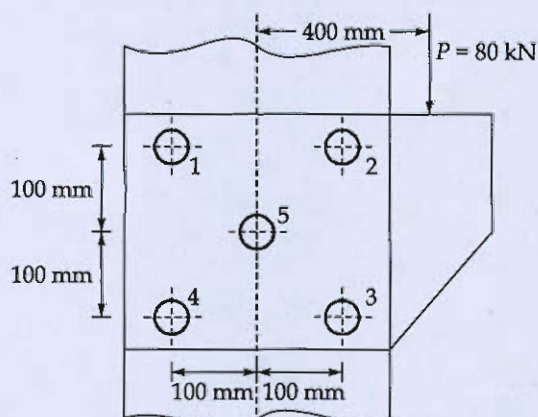
$$\therefore (w)_{\text{tension}} < (w)_{\text{comp.}}$$

$$\therefore \text{allowable intensity of load} = 6.3835 \text{ N/mm}$$

12



- Q.5 (e) A bracket is attached to a steel channel by means five identical rivets as shown in figure. Determine the diameter of rivets, if the permissible shear stress is  $100 \text{ N/mm}^2$ .



[12 marks]

$\rightarrow 80 \text{ kN}$   
 $T = P \times e$   
 $\rightarrow 400 \text{ mm}$   
 $= 80 \times 10^3 \times 400 = 32 \times 10^6 \text{ N-mm}$

80 kN has 2 effects  $\rightarrow$  direct shear  
 $\rightarrow$  torsional shear.

80 kN distributed on 5 rivets

$$\therefore \tau_{ds} = \frac{80 \times 10^3}{5 \times \frac{\pi}{4} \times d^2} = \frac{20371.83}{d^2} \text{ N/mm}^2$$

Torsional load on a rivet =  $\frac{T}{r_1^2 + r_2^2 + r_3^2 + r_4^2} \times r_1$

$\therefore$  symmetry, so taking any rivet

$$P_{tor} = \frac{32 \times 10^6 \times (100 \times 52)}{4 \times (100 \times 52)^2} = 56568.54 \text{ N}$$

$$\tau_{tor} = \frac{56568.54}{\frac{\pi}{4} \times d^2} = \frac{72025.3}{d^2} \text{ N/mm}^2$$

It can be seen, 2 & 3 rivet are critical.

$$\tan \theta = \frac{100}{100} \Rightarrow \theta = 45^\circ$$

$$\tau_{res} = \sqrt{\tau_{ds}^2 + \tau_{tor}^2 + 2 \times \tau_{ds} \times \tau_{tor} \times \cos 45^\circ}$$

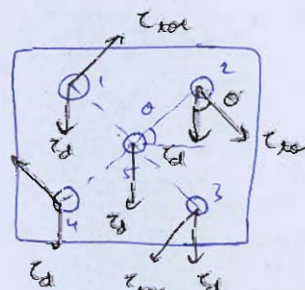
$$= \frac{87622.55}{d^2} \text{ N/mm}^2$$

$$\tau_{res} \leq \tau_{per}$$

$\rightarrow 100 \text{ N/mm}^2$

$$\therefore d^2 \geq 876.225 \Rightarrow d \geq 29.6 \text{ mm}$$

$$\therefore d = 30 \text{ mm}$$



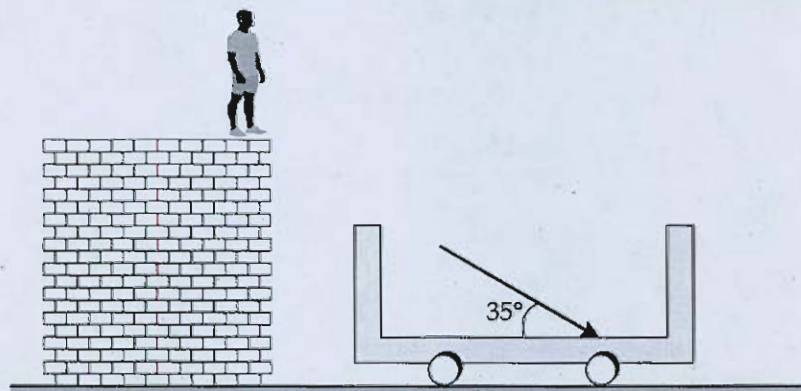
12



Q.6 (a) A man of 60 kg mass standing on a bridge jumps on to a cart below him such that he lands with a velocity of 5 m/s at an angle of  $35^\circ$  to the horizontal direction. If the cart is free to move, determine its velocity after he has jumped in for the following cases : the cart is initially

- (i) at rest
- (ii) moving with a velocity of 1 m/s away from the bridge.
- (iii) moving with a velocity of 1 m/s towards the bridge.

Take the mass of the cart as 130 kg. Also determine the loss in kinetic energy in each case.

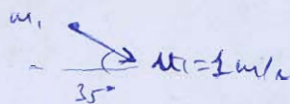


[20 marks]



④

$$m_1 \rightarrow \text{man} \\ = 60 \text{ kg} \\ m_2 \rightarrow \text{cart} \\ = 130 \text{ kg}$$



(i)

By conservation of momentum,

x-dir:-

$$m_1 \cdot u_1 \cos 35^\circ + 0 = (m_1 + m_2) v_x$$

$$\frac{60 \times 1 \times \cos 35^\circ}{60 + 130} = v_x$$

$$v_x = 0.25868 \text{ m/s}$$

$$v_x = 0.25868 \text{ m/s}$$

y-dir:-

$$-m_1 \cdot u_1 \sin 35^\circ + 0 = (m_1 + m_2) v_y$$

$$v_y = -0.18113 \text{ m/s}$$

↳ dispersed as ~~cart~~  
restricted by ground.

$$\therefore v_{\text{after jump}} = 0.25868 \text{ m/s}$$

$$(K.E.)_{\text{lost}} = -\frac{1}{2} (m_1 + m_2) v_x^2 + \frac{1}{2} m_1 \cdot u_1^2$$

$$(K.E.)_{\text{lost}} = 29.365 \text{ J}$$

(ii)

x-dir:-

$$m_1 \cdot u_1 \cos 35^\circ + m_2 \cdot u_2 = (m_1 + m_2) v_x$$

$$(60 \times 1 \times \cos 35^\circ) + (130 \times 1) = 190 \times v_x$$

$$v_x = 0.9429 \text{ m/s}$$

$$v_y \rightarrow 0 \text{ (ground restriction)}$$

$$\therefore v_{\text{after jump}} = 0.9429 \text{ m/s}$$

$$K.E._{\text{lost}} = \frac{1}{2} m_1 \cdot u_1^2 + \frac{1}{2} m_2 \cdot u_2^2 - \frac{1}{2} (m_1 + m_2) v_x^2$$

$$K.E._{\text{lost}} = 10.539 \text{ J}$$

(iii)

x-dir:-

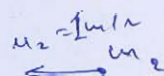
$$m_1 \cdot u_1 \cos 35^\circ - m_2 \cdot u_2 = (m_1 + m_2) v_x$$

$$60 \times 1 \times \cos 35^\circ - 130 \times 1 = 190 \times v_x$$

$$v_x = -0.4255 \text{ m/s}$$

$$v_y \rightarrow 0 \text{ (ground restriction)}$$

$$\therefore v_{\text{after jump}} = 0.4255 \text{ m/s (in -x dir, towards bridge)}$$



$$(K.E.)_{\text{tot}} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \frac{1}{2} (m_1 + m_2) v_n^2$$

$$(K.E.)_{\text{tot}} = 77.8 \text{ J}$$

- Q.6 (b) A welded connection, as shown in the figure is subjected to an eccentric force of 80 kN in the plane of the welds. Determine the size of the welds, if the permissible shear stress for the weld is  $410 \text{ N/mm}^2$ . Assume static condition.

$$(\tau)_{\text{per}} = 410 \text{ N/mm}^2$$

By symmetry, the centroid of the welds can be located at 'G', which is 50 mm from bottom & 25 mm from left.

$$\therefore e = 150 + 25 = 175 \text{ mm}$$

$$T = 80 \times 175 = 14000 \text{ kN-mm}$$

Due to 80 kN, a direct shear stress & torsion shear stress is applied on to weld.

$$\therefore \tau_{\text{dir}} = \frac{80 \times 10^3}{(50+50+100) \times h} \frac{\text{N}}{\text{mm}^2} = \frac{400}{h} \text{ N/mm}^2 \quad \text{--- (1)}$$

For torsional shear stress,

$$\tau_{\text{tor}} = \frac{T \cdot r}{J} \quad \rightarrow \text{This means } \tau_{\text{tor}} \text{ will be max. at } r_{\text{max}} \text{ i.e., A, B, C, D.}$$

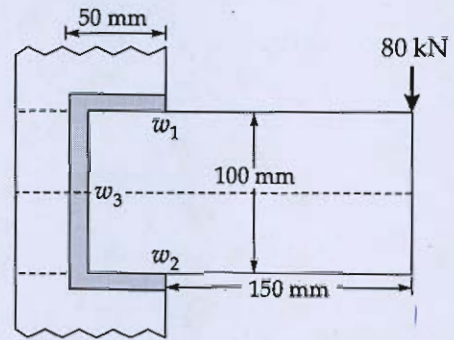
$$r = \sqrt{25^2 + 50^2} = 55.902 \text{ mm}$$

$$J = I_{xx} + I_{yy}$$

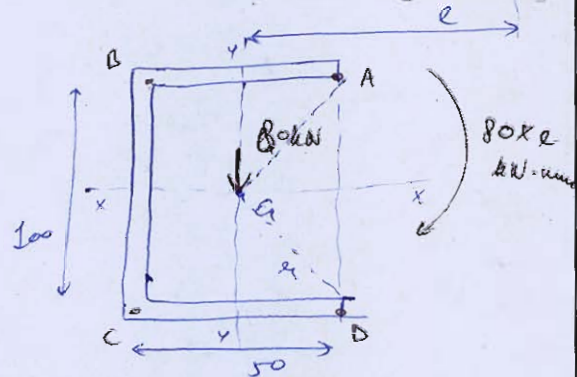
$$= \left[ \left( \frac{1}{12} \times 4 \times 100^3 + 4 \times 100 \times 25^2 \right) + 2 \left( \frac{1}{12} \times 50 \times h^3 + 50 \times h \times 50^2 \right) \right] + \left[ \left( \frac{1}{12} \times 100 \times h^3 + (100 \times h \times 25^2) \right) + 2 \left( \frac{1}{12} \times 4 \times 50^3 \right) \right]$$

$$J = 416.667 \times 10^3 \text{ mm}^4$$

$$(\tau_{\text{tor}})_{\text{max}} = \frac{14000 \times 10^3 \text{ N-mm} \times 55.902 \text{ mm}}{4 \times 416.667 \times 10^3 \text{ mm}^4} = \frac{1879.307}{h} \text{ N/mm}^2$$



[20 marks]



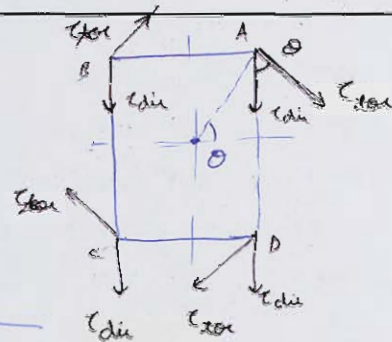
(2)



It can be seen, that A & D are critical.

$$\tan \theta = \frac{50}{25} = 2$$

$$\theta = 63.4349^\circ$$



$$\begin{aligned} \tau_{\text{res}} &= \sqrt{\tau_{\text{dir}}^2 + \tau_{\text{tor}}^2 + 2 \tau_{\text{dir}} \cdot \tau_{\text{tor}} \cos \theta} \\ &= \sqrt{\left(\frac{400}{h}\right)^2 + \left(\frac{1878.307}{h}\right)^2 + 2 \left(\frac{400}{h}\right) \left(\frac{1878.307}{h}\right) \cos 63.4349} \end{aligned}$$

$$\tau_{\text{res}} = \frac{2088.07}{h} \cdot \text{N/mm}^2$$

Now,  $\tau_{\text{res}} \leq \tau_{\text{per}}$

$$\frac{2088.07}{h} \leq 410$$

$$h \geq 5.0928 \text{ mm}$$

$$0.70 \times h \geq 5.0928$$

$$\therefore h \geq 7.2035 \text{ mm}$$

$$\therefore \boxed{h = 8 \text{ mm}}$$





- Q.6 (c) A cylindrical tank is 1.6 m diameter, 2.4 m long and 10 mm thick. Its ends are flat and are joined by nine tie bars, each 35 mm diameter equally spaced. If the tie bars are initially stressed to  $45 \text{ N/mm}^2$  and the tank is filled with water. Determine
- the increase in capacity when the pressure is raised to  $2 \text{ N/mm}^2$ .
  - the final stress in the tie bars.
- Taking  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\mu = 0.3$

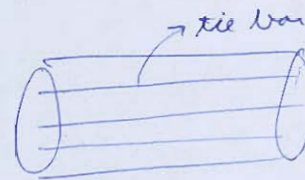
[20 marks]

(i) without tie bar,

$$\sigma_u = \frac{pD}{2t} = \frac{2 \times 1600}{2 \times 10} = 160 \text{ MPa}$$

Now with tie bar,

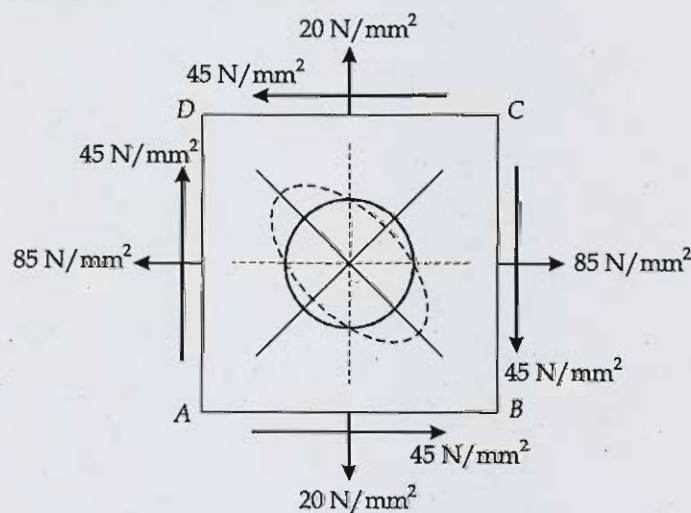
~~Pressure~~ ( $\sigma_L$ ) due to



Y  
B



- Q.7 (a) On a mild steel plate, a circle of diameter 60 mm is drawn before the plate is stressed as shown in the figure. Find the lengths of the major and minor axes of an ellipse formed as a result of the deformation of the circle marked.



Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\frac{1}{m} = \frac{1}{4} = \mu$

[20 marks]







Q.7 (b) Following data is given for a full hydrodynamic bearing used for electric motor.

Radial load = 1250 N; Journal speed = 1500 rpm; Journal diameter = 50 mm

Static load on the bearing = 400 N; Start up bearing pressure = 2 N/mm<sup>2</sup>

Permissible bearing pressure in application of elastic motor is 1 N/mm<sup>2</sup>

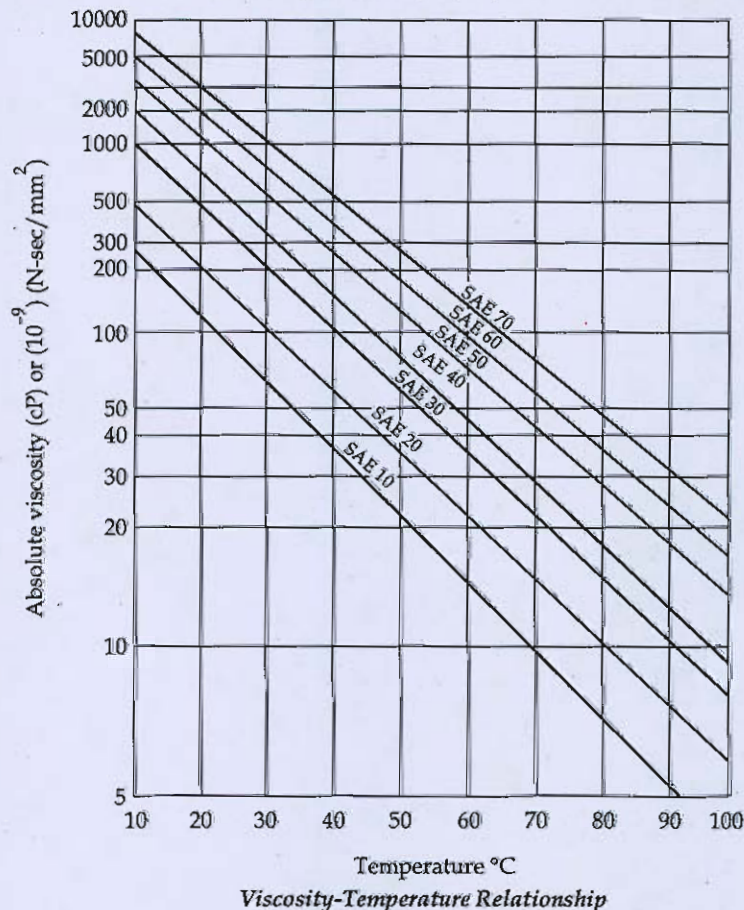
The value of surface roughness (CLA) of the journal and the bearing are 2 and 1 micron respectively. The minimum oil film thickness should be five times the sum of surface roughness of the journal and the bearings. Determine

- (i) length of the bearing                      (ii) radial clearance  
(iii) minimum oil film thickness              (iv) viscosity of lubricant  
(v) flow of lubricant

Select suitable oil for this application assuming the operating temperature as 65°C.

$\left(\frac{l}{d}\right)$	$\epsilon$	$\left(\frac{h_0}{c}\right)$	$S$	$\phi$	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{ren_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{max}}\right)$
$\left(\frac{1}{2}\right)$	0	1.0	$\infty$	88.5	$\infty$	$\pi$	0	—
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	0.206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	0.126
	1.0	0	0	0	0	—	1.0	0

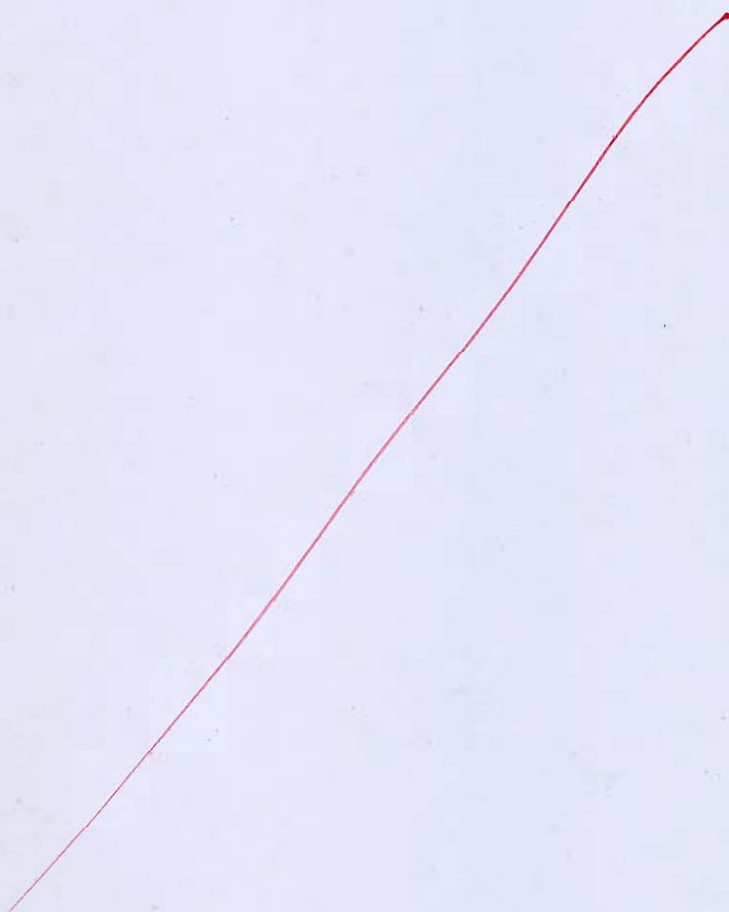
Table : Dimensionless performance parameters for full journal bearing with side flow



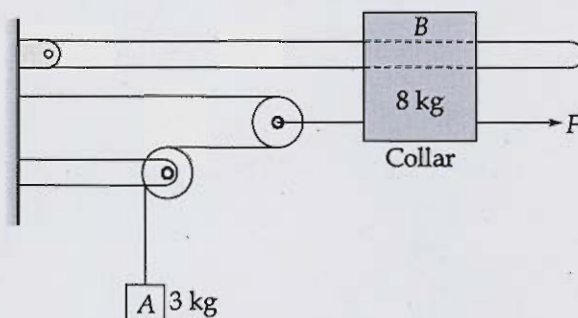
[20 marks]



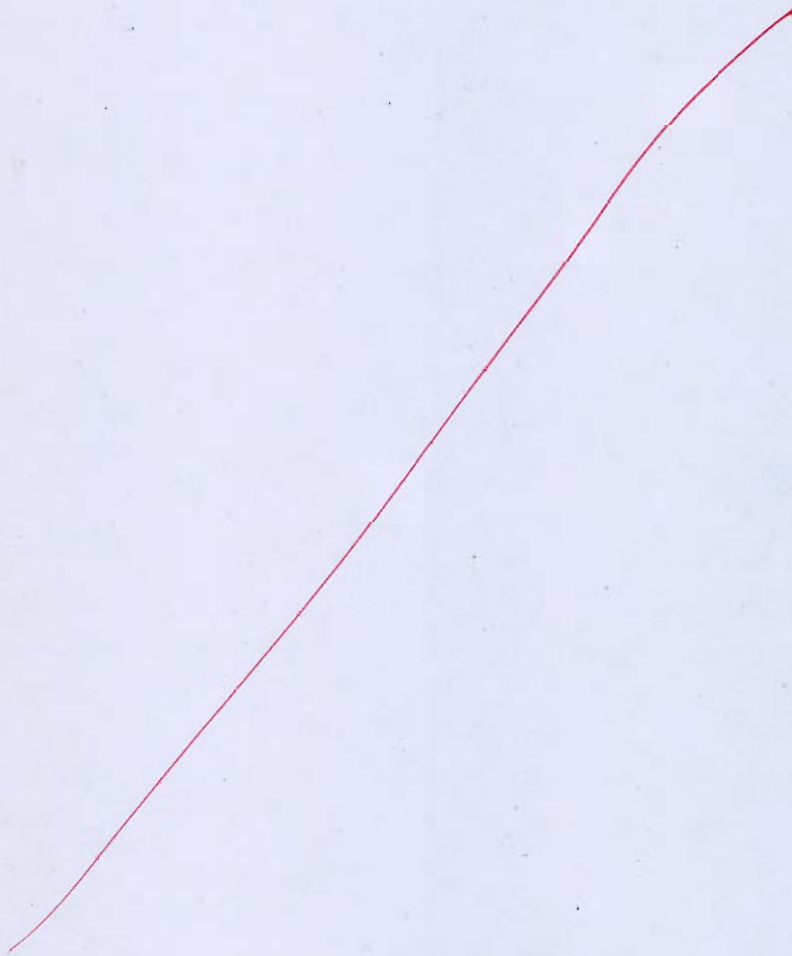




- Q.7 (c) (i) A lift is operated by four ropes each having 30 wires of 1.6 mm diameter. The cage weighs 1.5 kN and the weight of the rope is 4.6 N/m. Determine the maximum load carried by the lift if each wire is of 40 m length and the lift operates
1. without any drop
  2. with a drop of 100 mm during operation.
- [Take  $E_{\text{rope}} = 70 \text{ GPa}$  and allowable stress = 120 MPa]
- (ii) System shown in the figure is initially at rest. Neglecting friction determine the force  $F$  required if velocity of collar  $B$  becomes 8 m/s in 3 seconds after the start.



[10 + 10 marks]







- Q.8 (a) (i) Discuss the five important parameters involved in the selection and design of journal bearings. Explain in detail how each parameter effects the performance and reliability of the bearing.

- (ii) The torque developed by an engine is given by following equation:

$$T = 15000 + 2000 \sin 2\theta - 1500 \cos 2\theta$$

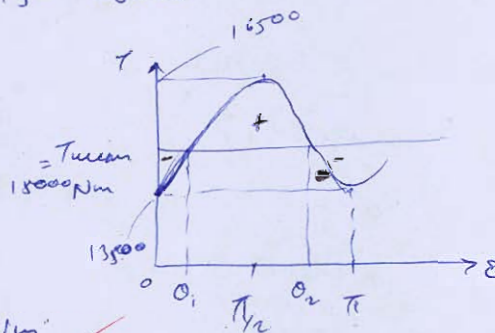
where  $T$  is the torque in N-m and  $\theta$  is the crank angle from inner dead centre position. The resisting torque of the machine is constant throughout the work cycle. The coefficient of speed fluctuations is 0.02. The engine speed is 200 rpm. A circular solid steel disc, 60 mm thick, is used as flywheel. The mass density of steel is  $7800 \text{ kg/m}^3$ . Calculate the radius of the flywheel disk.

[10 + 10 marks]

$$(i) T = 15000 + 2000 \sin 2\theta - 1500 \cos 2\theta$$

$$\begin{aligned} \text{Period} &= \pi \\ \text{Work} &= \int_0^\pi T \cdot d\theta \\ &= 15000\pi \text{ J} \end{aligned}$$

$$T_{\text{mean}} = \frac{\text{Work}}{\text{Period}} = 15000 \text{ Nm}$$



$$\text{at } \theta_1 \Rightarrow T - T_{\text{mean}} = 0$$

$$\Rightarrow 2000 \sin 2\theta = \frac{1500}{2000}$$

$$\theta_1 = 10.4349^\circ, \theta_2 = 100.435^\circ$$

It can be seen that,  $(\Delta E)_{\text{max}}$  is in  $\theta_1$  to  $\theta_2$ .

$$\therefore (\Delta E)_{\text{mean}} = \int_{\theta_1}^{\theta_2} (T - T_{\text{mean}}) \cdot d\theta$$

$$= 2500 \text{ J} = I \omega^2 \cdot C_s$$

$$2500 = \left( \frac{m R^2}{2} \times \left( \frac{200 \times 2\pi}{60} \right)^2 \times 0.02 \right)$$

$$\rightarrow 4 \times \pi R^2 \times \frac{60}{1000}$$

$$\Rightarrow \frac{2500 \times 2}{\left( \frac{200 \times 2\pi}{60} \right)^2 \times 0.02} = 7800 \times \pi R^4 \times \frac{60}{1000}$$

$$R = 0.70905 \text{ m} \Rightarrow R = 709.05 \text{ mm}$$

10

- (i) ① ~~Type of load~~ → ~~whether~~
- ② Type of speed → ~~if~~ whether rpm to work on is high or low  
if high → hydrodynamic works.  
if low → hydrostatic required to provide lift at static
- ③ Type of lubrication →  
Thin film, Thick film, hydrostatic.
- ④ working condition → ~~bearing use~~
- ⑤ Bearing modulus  $(K_0)$  → based on Bearing characteristic number (BCN), specifications & amount of wear & and stability can be checked.
- ⑥ Pump can be used or not as per space & power requirement.

5





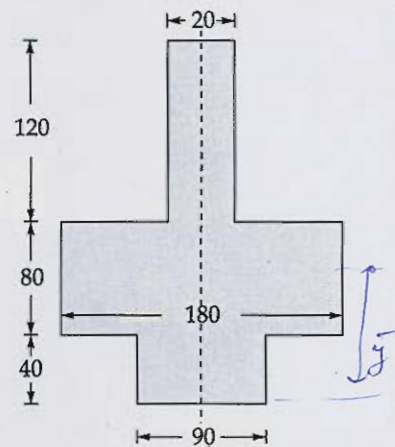
Q.8 (b) The cross-section of a conveyor beam is shown in the figure. The beam is subjected to a bending moment in the plane  $y-y$ . Determine the maximum permissible bending moment.

(i) for the bottom flange to be in tension.

(ii) for the bottom flange to be in compression.

The safe bending stress in tension and compression are  $40 \text{ N/mm}^2$  and  $140 \text{ N/mm}^2$  respectively.

[20 marks]



(All dimensions are in mm)

$$\bar{y} = \frac{(20 \times 120 \times 180) + (80 \times 180 \times 80) + (40 \times 90 \times 20)}{(20 \times 120) + (80 \times 180) + (40 \times 90)}$$

$$\bar{y} = 69.417 \text{ mm}$$

$$I = \frac{1}{12} 20 \times$$

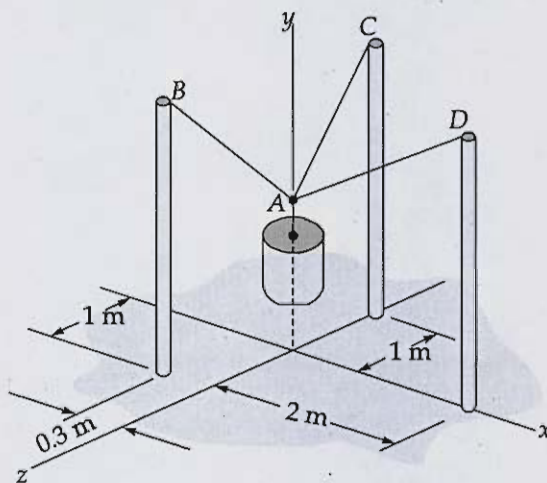
$$I_{y-y} = \frac{1}{12} \times 120 \times 20^3 + \frac{1}{12} \times 80 \times 180^3 + \frac{1}{12} \times 40 \times 90^3$$

$$= 41.38 \times 10^6 \text{ mm}^4$$

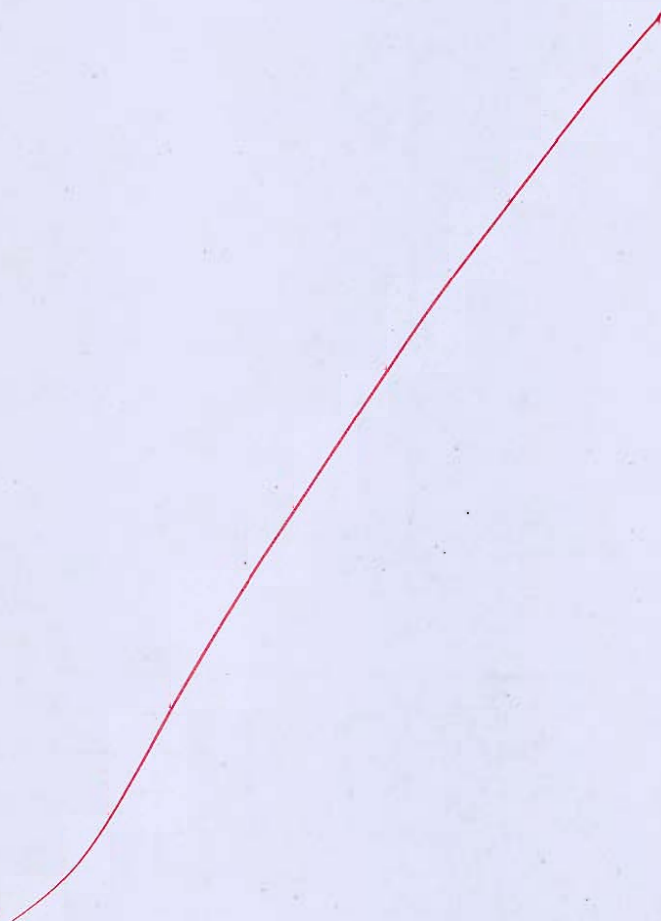


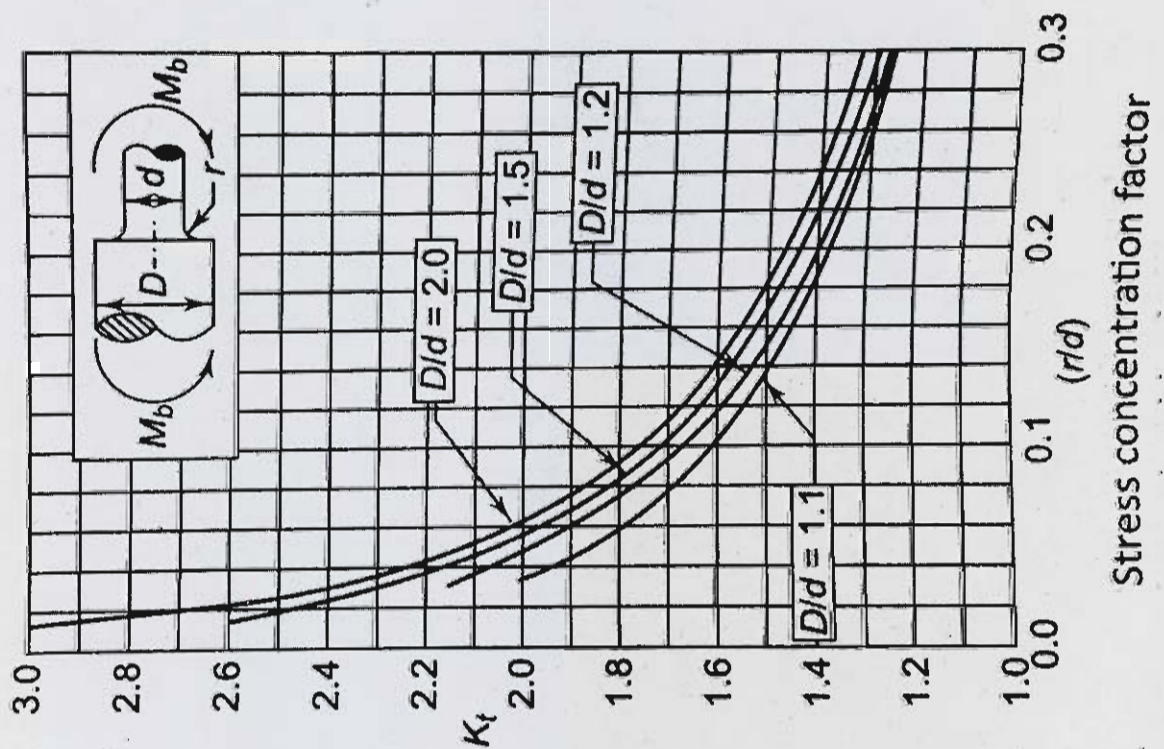


- Q.8 (c) The 20 kg mass is suspended by cables attached to three vertical 2 m posts. Point A is at (0, 1.2, 0) m. Determine the tensions in cables AB, AC and AD.



[20 marks]





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## Space for Rough Work

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## Space for Rough Work

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## Space for Rough Work

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## Space for Rough Work

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2