



**MADE EASY**

Leading Institute for ESE, GATE & PSUs

## ESE 2025 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

### Mechanical Engineering

#### Test-2 : Strength of Materials + Machine Design + Engineering Mechanics

Name : .....

Roll No :

Test Centres	Student's Signature
Delhi <input checked="" type="checkbox"/> Bhopal <input type="checkbox"/> Jaipur <input type="checkbox"/> Pune <input type="checkbox"/> Kolkata <input type="checkbox"/> Hyderabad <input type="checkbox"/>	

#### Instructions for Candidates

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

#### FOR OFFICE USE

Question No.	Marks Obtained
Section-A	
Q.1	20
Q.2	—
Q.3	60
Q.4	—
Section-B	
Q.5	48
Q.6	22
Q.7	30
Q.8	—
<b>Total Marks Obtained</b>	<b>180</b>

Signature of Evaluator

Cross Checked by

*Cam Sun*

## IMPORTANT INSTRUCTIONS

**CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.**

### DONT'S

1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

### DO'S

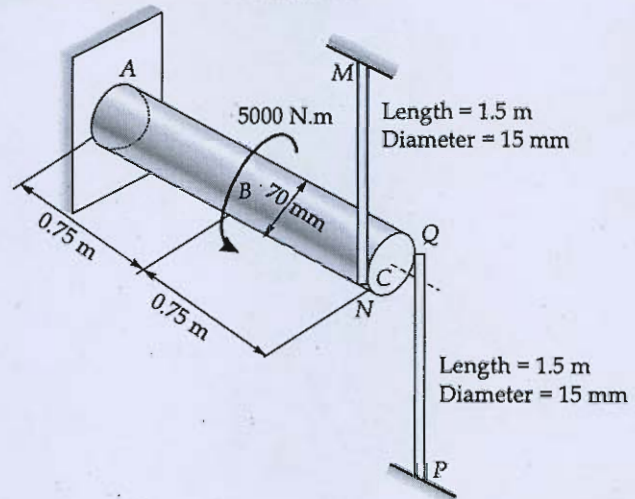
1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.



## Section A : Strength of Materials + Machine Design + Engineering Mechanics

- Q.1 (a) A steel shaft  $ABC$ , of constant circular cross-section and of diameter 70 mm, is clamped at the left end  $A$ , loaded by a twisting moment of 5000 N.m at its midpoint  $B$ , and elastically restrained against twisting at the right end  $C$  as shown in the figure.

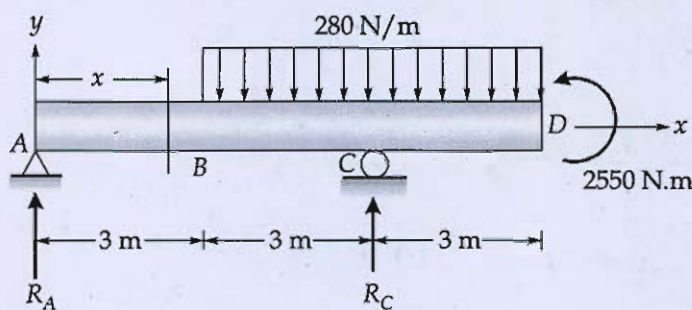
At end  $C$  the bar  $ABC$  is attached to vertical steel bars each of 15 mm diameter. The upper bar  $MN$  is attached to the end  $N$  of a horizontal diameter of the 70 mm bar  $ABC$  and the lower bar  $PQ$  is attached to the other end  $Q$  of this same horizontal diameter as shown in the figure. For all materials  $E = 200$  GPa and  $G = 80$  GPa. Determine the peak shearing stress in bar  $ABC$  as well as the tensile stress in the bar  $MN$ .



[12 marks]

2

- Q.1 (b) The beam AC is simply supported at A and C and subjected to the uniformly distributed load of  $280 \text{ N/m}$  and the couple of magnitude  $2550 \text{ Nm}$  as shown in the figure. Write the equations for shearing force and bending moment and make sketches of these equations.



[12 marks]

$$R_A + R_C = 280 \times 6 \rightarrow \textcircled{1}$$

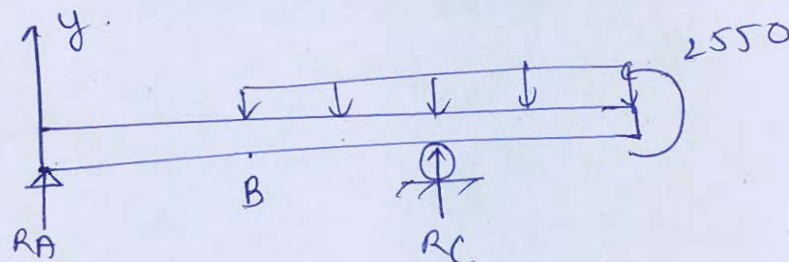
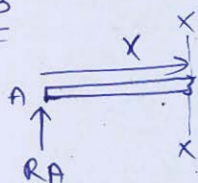
$$\sum M_A = 0 \Rightarrow (R_C \times 6) + 2550 - (280 \times 6 \times 6) = 0$$

$$R_C = 1255 \text{ N} \rightarrow \textcircled{11}$$

$$R_A = 425 \text{ N}$$

for writing SF, BM equations beam is divided into AB ( $0 < x < 3$ ), BC ( $3 < x < 6$ ) and CD ( $6 < x < 9$ )

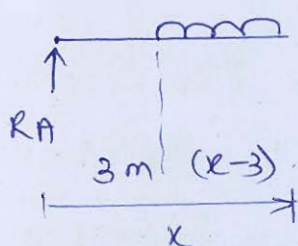


AB

$$SF = 425 \text{ N}$$

$$0 < x < 3$$

$$BM = 425x$$

BC

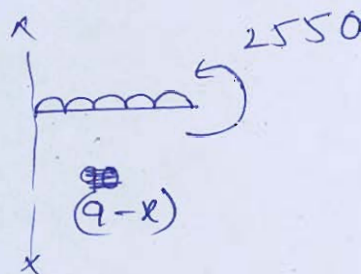
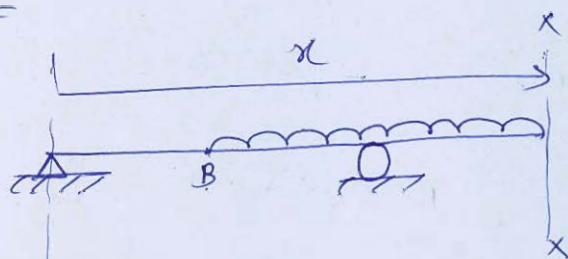
$$SF = 425 - 280(x-3)$$

$$BM = 425x - \frac{280(x-3)^2}{2}$$

$$SF = 425 - 280x + 840$$

$$SF = 1265 - 280x \quad \{3 < x < 6\}$$

$$BM = 425x - 140(x-3)^2 \quad \{3 < x < 6\}$$

CD

$$SF = 280(9-x)$$

$$SF = 2520 - 280x$$

$$BM = 2550 - \frac{280(9-x)^2}{2}$$

CD

$$SF = 2520 - 280x$$

$$BM = 2550 - 140(x-1)^2$$

Note

BM in N.m

SF in N

x in m.

- Q.1 (c) A plate clutch consists of one pair of contacting surface and transmits 30 kW power at 900 rpm. The ratio of outer diameter to inner diameter is 2. The coefficient of friction is 0.3 and the permissible intensity of pressure is  $1.5 \text{ N/mm}^2$ . Assuming uniform wear, calculate the inner and outer diameters.

[12 marks]

$$T = \frac{30 \times 60 \times 10^6}{2\pi \times 900} = 318.30988 \text{ N}\cdot\text{mm}$$

$$\frac{R_o}{R_i} = 2 ; \mu = 0.3 ; p_{per} = 1.5 \text{ N/mm}^2$$

$$T_f = n \mu \pi p_{per} R_i (R_o^2 - R_i^2) \quad n = 2$$

(one pair  
of contacting  
surfaces)

$$318.30988 = 2 \times 0.3 \times \pi \times 1.5 R_i^3 (2^2 - 1)$$

$$R_i^3 = 37256.3643$$

$$R_i = 33.4795 \text{ mm}$$

$$R_o = 66.9589 \text{ mm}$$

Ans

inner diameter = 66.958 mm

outer diameter = 133.917 mm



- Q.1 (d) A pair of spur gears with  $20^\circ$  full depth involute teeth consists of a 22 teeth pinion meshing with a 44 teeth gear. The module is 3 mm while the face width is 45 mm. The material for pinion as well as gear is steel with an ultimate tensile strength of  $600 \text{ N/mm}^2$ . The gears are heat treated to a surface hardness of 400 BHN. The pinion rotates at 1500 rpm and the service factor for the application is 1.75. Assume that velocity factor accounts for the dynamic load and the factor of safety is 2. Determine the rated power that the gears can transmit. Take Lewis form factor ( $Y$ ) = 0.33 for  $20^\circ$  full depth involute system and  $\sigma_b = 0.33 s_{ut}$ .

[12 marks]

 $20^\circ$  FD involute

$z_1 = 22$

$z_2 = 44$

$m = 3 \text{ mm}$

$b = 45 \text{ mm}$

$s_{ut} = 600 \text{ N/mm}^2$

$\text{BHN} = 400$

$N_1 = 1500 \text{ rpm}$

$C_s = 1.75$

$N = 2$

(factor of safety)

$C_v = \frac{3}{3+V}$  {velocity factor}

$Y = 0.33$

$V = \frac{\pi D_1 N}{60} = \frac{\pi \times 3 \times 22 \times 1500}{60 \times 1000} = 5.1836 \text{ m/s}$

$C_v = 0.3666$

Acc: to Lewis Beam strength Equation,

$F_b = \left\{ \frac{\sigma Y}{N} \right\} w \cdot g \cdot b \cdot m$

$= \left[ \frac{0.33 \times 600}{2} \right] \times 0.33 \times 45 \times 3$

$F_b = 4410.45 \text{ N}$

considering velocity factor,

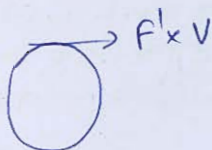
$F = F_b \cdot C_v$

$F = 1616.87 \text{ N}$

considering service factor,

$$F' = \frac{F}{C_g} = \frac{F}{1.75}$$

$$F' = 923.9262 \text{ N}$$



$$\text{Power (Rated)} = 923.9262 \times V$$

$$V = 5.1836 \text{ m/s} \quad \{\text{pitch line velocity}\}$$

$$\text{Power} = 4789.2642 \text{ W}$$

$$\boxed{\text{Power} = 4.79 \text{ kW}} \quad \underline{\text{Ans}}$$

Wear strength

$$F_w = D_1 B Q K$$

$$D_1 = 3 \times 22 = 66 \text{ mm}$$

$$B = 45 \text{ mm}$$

$$Q = \frac{2G}{G+1} = \frac{4}{3}$$

$$K = 0.16 \left( \frac{\text{BHN}}{100} \right)^2 = 2.56$$

$$F_w = 10137.6 \text{ N}$$

$$F_w > F_b$$

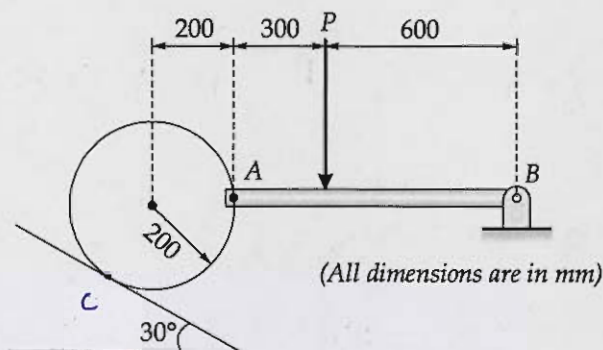
So we calculate  
rated power  
based on  $F_b$ .

$$\text{Rated power} = 4.79 \text{ kW}$$

12

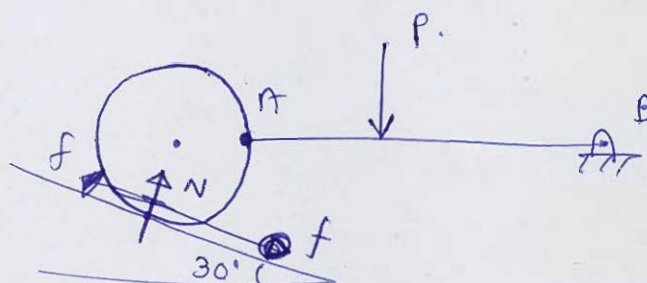


- Q.1 (e) A 40 kg disc rests on an inclined surface for which  $\mu_s = 0.3$  as shown in the figure. Determine the maximum vertical force  $P$  that may be applied to link AB without causing the disc to slip at C.

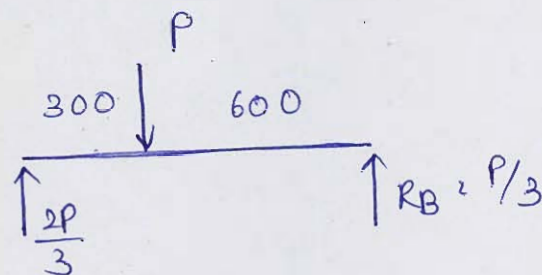
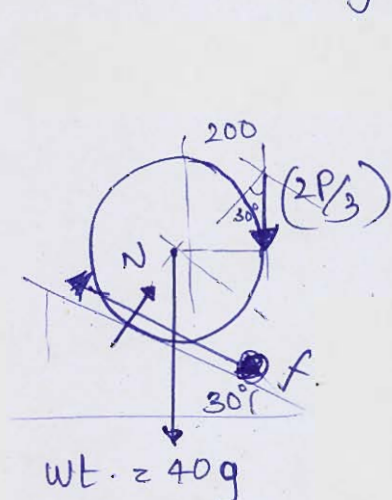


[12 marks]

Assume pt C is @ verge of slipping.



$f, N$  are acting on disc by inclined surface.



Egm of Disc:-

$$N - \left(\frac{2P}{3}\right) \cos 30^\circ - 40g \cos 30^\circ = 0$$

$$N = \left[ \frac{2P}{3} \times \frac{\sqrt{3}}{2} + 40g \cos 30^\circ \right]$$

$$-f + \left(\frac{2P}{3}\right) \sin 30^\circ + 40g \sin 30^\circ = 0$$

$$f = \frac{2P}{3} \times \frac{1}{2} + 40g \times \frac{1}{2} = \mu N$$

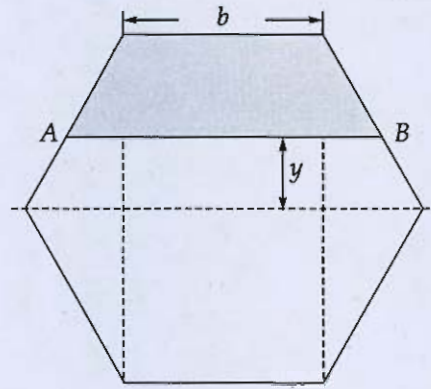


$$\frac{P}{3} + 20g = \mu \left[ \frac{P}{\sqrt{3}} + 20\sqrt{3}g \right]$$

$$\frac{P}{3} + 20g = \frac{0.3P}{\sqrt{3}} + 0.3 \times 20 \times \sqrt{3}g$$

$$(20g - 0.3 \times 20 \times \sqrt{3}g) = \left( \frac{0.3}{\sqrt{3}} - \frac{1}{3} \right) P$$

- Q.2 (a) A bar of hexagonal cross-section of side length  $b$  mm is used as a cantilever with one of its diagonal being horizontal. Derive an expression for the shear stress  $\tau$  at the fibre  $AB$  in terms of  $b$  and  $y$ . Determine the shear stress when  $y = 10$  mm,  $b = 30$  mm and shear force applied is 6 kN. Also plot the shear stress distribution plot across the depth of the hexagonal section.



[20 marks]

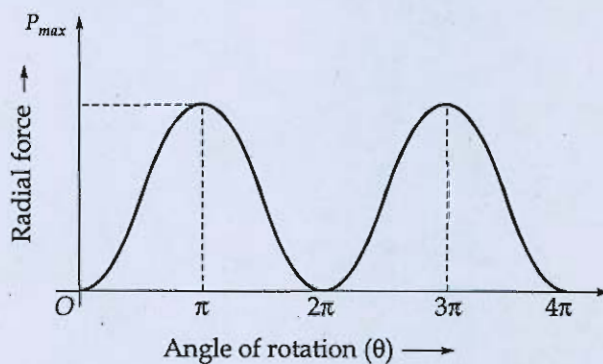
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70

- Q.2 (b) A ball bearing is subjected to a radial force which varies in sinusoidal way as shown in the figure. The direction of force remains fixed. The amplitude of the force is 2000 N and the speed of rotation is 750 rpm. Determine the dynamic load capacity of the bearing for the expected life of 9000 hr.



[20 marks]

4

✓





- Q.2 (c) If the density of a hemisphere varies as the distance from the bounding plane, show that the distance of the centre of gravity from that plane is  $\frac{8}{15}$ <sup>th</sup> of its radius.

[20 marks]



y

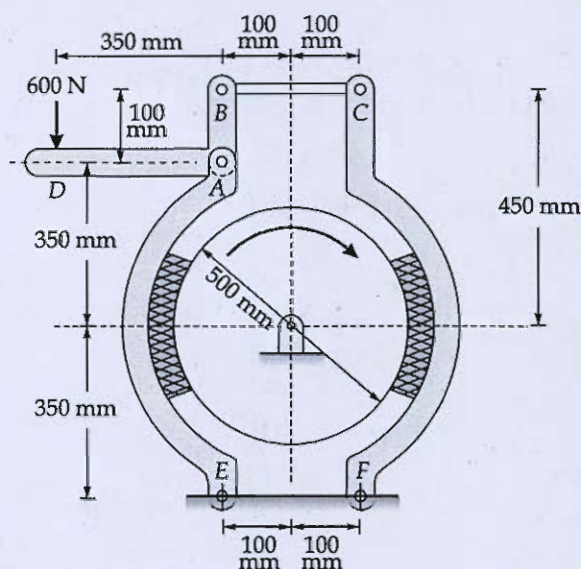


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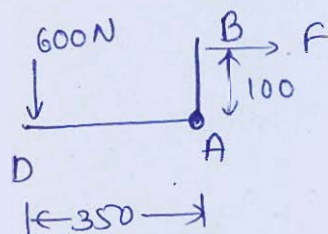
**Q.3 (a)** A double block brake is as shown in the figure. The brake drum rotates in clockwise direction and the actuating force is 600 N. The coefficient of friction between the blocks and the drum is 0.3 Calculate.

- (i) The torque absorbing capacity of the brake.
- (ii) The dimensions of the blocks, if the intensity of pressure between the blocks and brake drum is  $1.2 \text{ N/mm}^2$ .

Assume that the blocks are identical and the length of each block is twice its width.

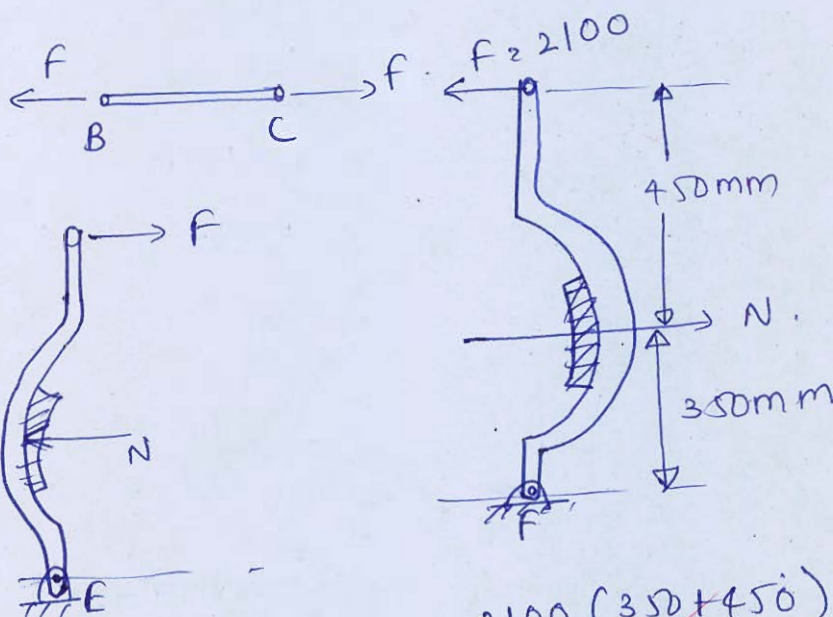


[20 marks]



$$600 \times 350 = F \times 100$$

$$F = 2100 \text{ N}$$



FBD of  
individual  
members

$$2100 (350 + 450) = 350 \times N$$

$$N = 4800 \text{ N}$$

Normal Rxn at blocks due to actuating force of 600 N.

$$f = \mu N = 0.3 (4800) \text{ N}$$

$$f = 1440 \text{ N}$$

$$R = 250 \text{ mm}$$

Torque absorbing capacity of  
each block =  $fR$

$$= 360 \text{ N}\cdot\text{m}$$

$$\text{Torque absorbing capacity of Brake} = (360 \times 2) \text{ N}\cdot\text{m} \\ = 720 \text{ N}\cdot\text{m}.$$

$$N = 4800 \text{ N}.$$

$$\frac{N}{L \times W} = 1.2 \text{ N/mm}^2 = \frac{4800}{L \times W}$$

$$L \times W = 4000 \text{ mm}^2.$$

$$L = 2W.$$

$$2W^2 = 4000$$

$$W = 44.72 \text{ mm}$$

$$L = 89.44 \text{ mm}$$

20



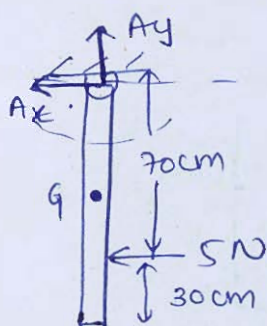
- Q.3 (b) A wooden metre stick AB of 500 grams mass and length 1 m hangs vertically as shown in figure. If a horizontal force of 5 N is applied at a point that is 30 cm from the bottom end B, determine (a) the angular acceleration of the stick, (ii) the components of reaction at the hinge at A. In addition, determine the point of application of the horizontal force at which the horizontal component of the reaction at A is zero.

[20 marks]



$$m = 0.5 \text{ kg} \quad L = 1 \text{ m} \quad P = 5 \text{ N}$$

$$I_A = \frac{mL^2}{3} = \frac{0.5 \times 1^2}{3} = 0.1667 \text{ kg m}^2 \quad \left\{ \begin{array}{l} \text{mass} \\ \text{MOI} \end{array} \right\}$$



$$\alpha = \frac{\tau}{I_A} = \frac{(5 \times 0.7) \text{ N.m}}{I_A}$$

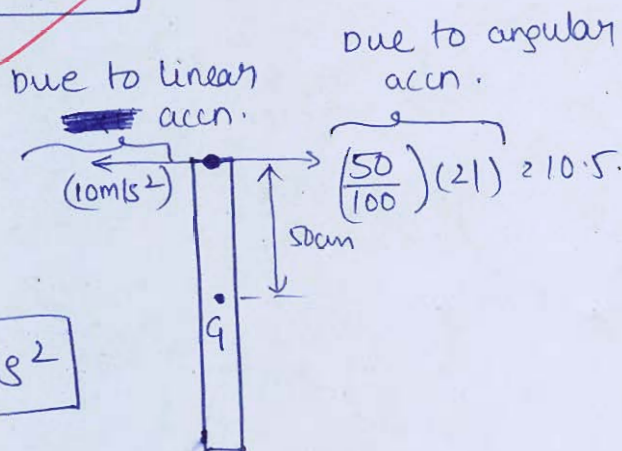
$$\alpha = 21 \text{ rad/s}^2 \quad \rightarrow (a)$$

Newton's 2nd law

$$m(a_{CG}) = F_{ex}$$

$$0.5 a_{CG} = 5 \text{ N}$$

$$a_{CG} = 10 \text{ m/s}^2$$



FBD of stick

$$A_y = 0.5 \times 9.81 = 4.905 \text{ N} \quad (\uparrow)$$

$$A_x = 0.5 \times (10.5 - 10) \quad (\leftarrow)$$

$$A_x = 0.25 \text{ N} \quad (\leftarrow)$$

$$A_y = 4.905 \text{ N} \quad (\uparrow)$$

Reactions at 'A'

Point of application of Hg force at which horizontal reaction is zero is known as center of percussion.

$h$  = distance of center of percussion from hinge

$$h = \bar{x} + \frac{\bar{x}^2}{K}$$

~~$$\frac{mL^2}{12} = mK^2$$~~

~~$$K^2 = \frac{L^2}{12}$$~~

~~$$h = \frac{L}{\sqrt{3}} + \frac{L^2}{(\frac{L}{\sqrt{3}})}$$~~

~~$$h = \frac{L}{\sqrt{3}} + \sqrt{3}L$$~~

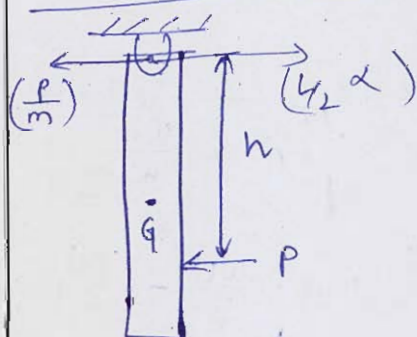
~~$$h = \frac{L}{2} + \frac{L^2}{K}(\sqrt{3})$$~~

~~$$h = \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{1+\sqrt{3}}{\sqrt{3}}$$~~

~~$$h = \frac{4}{\sqrt{3}} \text{ m}$$~~

$$mK^2 = \frac{mL^2}{12} \Rightarrow K^2 = \frac{L^2}{12}$$

$$h = \bar{x} + \frac{\bar{x}^2}{K} = \frac{L}{2} + \frac{\frac{L^2}{4}}{\frac{L}{\sqrt{3}}} = \frac{L}{2} + \frac{\sqrt{3}L}{4}$$



$$a_{CG} = \frac{P}{m}$$

$$\alpha = \frac{Ph}{\left(\frac{mL^2}{3}\right)}$$

$$\frac{P}{m} = \frac{L}{2} \alpha = \frac{L}{2} \times \frac{Ph}{\left(\frac{mL^2}{3}\right)}$$

$$\frac{P}{m} = \frac{K}{2} \frac{Ph(3)}{mL^2}$$

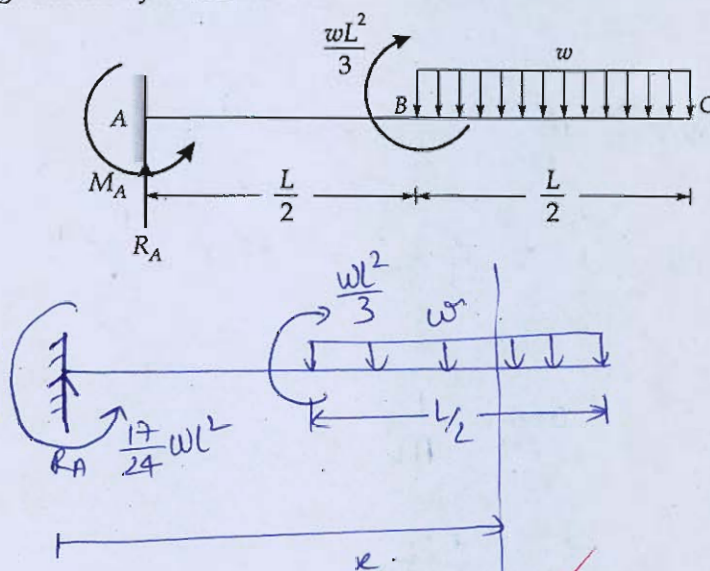
$$\frac{2L}{3} = h$$

$$h = \frac{2}{3} m$$

20



- Q.3 (c) The cantilever beam ABC as shown below is subjected to a uniform load  $w$  per unit length distributed over its right half, together with a concentrated couple  $\frac{wL^2}{3}$  applied at B. Using Macaulay's method determine and the maximum deflection of the beam.



[20 marks]

$$R_A = \frac{wL}{2}$$

$$M_A - \frac{wL^2}{3} - \frac{wL}{2} \left( \frac{3L}{4} \right) = 0$$

$$M_A = \frac{wL^2}{3} + \frac{3wL^2}{8}$$

$$M_A = \frac{17wL^2}{24}$$

Reactions

$$M(x) = R_A x - \frac{17wL^2}{24} x^0 + \frac{wL^2}{3} (x - \frac{L}{2})^0 + \left( -\frac{w(x - \frac{L}{2})^2}{2} \right)$$



$$m(x) = \frac{wL}{2} x - \frac{17}{24} wL^2 x^0 + \frac{wL^2}{3} \left(x - \frac{L}{2}\right)^0 - \frac{w(x - L/2)^L}{2}$$

Acc. to macaulays method;

$$EI \frac{d^2 y}{dx^2} = m(x)$$

$$EI \frac{dy}{dx} = \frac{wL}{2} \left(\frac{x^2}{2}\right) - \frac{17}{24} wL^2 (x) + \frac{wL^2}{3} \left(x - \frac{L}{2}\right) - \frac{w(x - L/2)^3}{6} + C_1$$

$$EI \cdot y = \frac{wL}{4} \frac{x^3}{3} - \frac{17}{24} wL^2 \left(\frac{x^2}{2}\right) + \frac{wL^2}{3} \frac{(x - L/2)^2}{2} - \frac{w(x - L/2)^4}{24} + C_1 x + C_2$$

①  $x = 0 \quad \frac{dy}{dx} = 0$

②  $x = 0 \quad y = 0$

$$0 = 0 - 0 - 0 + 0 + C_1$$

$$0 = C_2$$

$$C_1 = 0$$

$$C_2 = 0$$

$$\left\{ y = \frac{1}{EI} \left[ \frac{wL}{12} x^3 - \frac{17}{48} wL^2 x^2 + \frac{wL^2}{6} \left(x - \frac{L}{2}\right)^2 - \frac{w}{24} (x - L/2)^4 \right] \right.$$

→ Elastic Eqn.

more deflection at free end

ie  $x = L$ .

$$Y_{max} = \frac{1}{EI} \left[ \frac{\omega L}{12} (L)^3 - \frac{17}{24 \times 2} \omega L^2 (L^2) + \frac{\omega L^2}{6} \left(\frac{L}{2}\right)^2 - \frac{\omega}{24} \left(\frac{L^4}{16}\right) \right]$$

$$Y_{max} = \frac{\omega L^4}{EI} \left[ \frac{1}{12} - \frac{17}{48} + \frac{1}{24} - \frac{1}{24 \times 16} \right]$$

$$= \frac{-89}{384} \frac{\omega L^4}{EI}$$

$$Y_{max} = \frac{89}{384} \frac{\omega L^4}{EI} \quad (\downarrow) \quad @ \text{ free end}$$

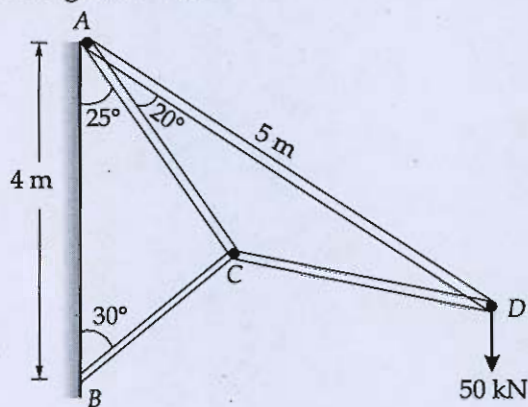
$$\left(\frac{dy}{dx}\right) = \frac{1}{EI} \left[ \frac{\omega L}{4} (L)^2 - \frac{17}{24} \omega L^3 + \frac{\omega L^2}{3} \left(\frac{L}{2}\right) - \frac{\omega}{6} \left(\frac{L^3}{8}\right) \right]$$

$$\left(\frac{dy}{dx}\right)_{max} = \frac{\omega L^3}{EI} \left[ \frac{1}{4} - \frac{17}{24} + \frac{1}{6} - \frac{1}{48} \right]$$

$$\frac{dy}{dx} = \frac{-5\omega L^3}{16EI} \rightarrow @ \text{ free end}$$

20

- Q.4 (a) Find the force its nature in member  $AD$  and  $BC$  for given cantilever truss loaded by  $50\text{ kN}$  as shown in the figure below.



[20 marks]

✓





- Q.4 (b) A long thin bar of length  $L$  and rigidity  $EI$  is pinned at end  $A$ , and at  $B$  rotation is resisted by a restoring moment of magnitude  $\lambda$  per radian of rotation at that end. Derive the equation for the axial buckling load  $P$ . Neither  $A$  nor  $B$  can displace in the  $y$ -direction, but  $A$  is free to approach  $B$ .

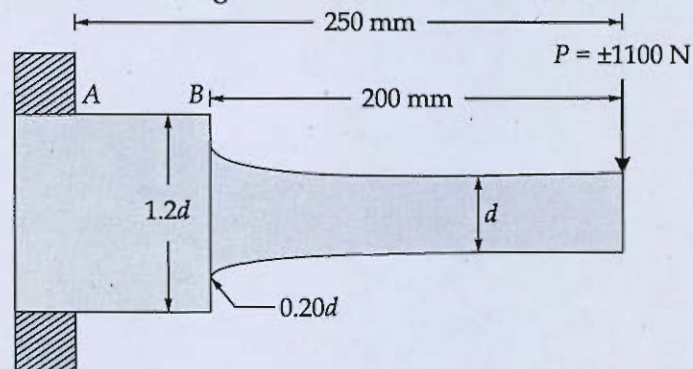
[20 marks]







- Q.4 (c) A cantilever beam made of cold draw steel having surface finish factor ( $k_a$ ) 0.78 and  $S_{ut} = 540 \text{ N/mm}^2$  is subjected to a completely reversed load of 1100 N as shown in the figure. The notch sensitivity factor  $q$  at the fillet can be taken as 0.85 and the expected reliability is 90%. Determining the diameter  $d$  of the beam for a life cycle of 11000 cycles.



Take, reliability factor,  $k_c = 0.897$  for 90% reliability and size factor  $k_b = 0.85$ .

[Use Stress Concentration Factor Chart attached at the end]

[20 marks]

4



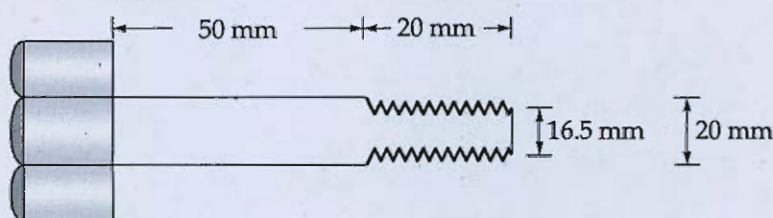


Y

### Section B : Strength of Materials + Machine Design + Engineering Mechanics

- Q.5 (a)** Calculate the strain energy of the bolt as shown in the figure under a tensile load of 20 kN. Show that the strain energy is increased for the same maximum stress, by turning down the shank of the bolt to the root diameter of the thread.

Take  $E = 2 \times 10^5 \text{ N/mm}^2$



Assuming that the load is distributed evenly over the core of screwed portion.

[12 marks]

$$\sigma_{\text{core}} = \frac{20 \times 10^3}{\frac{\pi}{4} (16.5)^2}$$

$$\sigma_{\text{shank}} = \frac{20 \times 10^3}{\frac{\pi}{4} (20)^2}$$

$$\sigma_{\text{core}} = 98.53 \text{ N/mm}^2$$

$$\sigma_{\text{shank}} = 63.6619 \frac{\text{N}}{\text{mm}^2}$$

$$U_1 = \frac{\sigma_{\text{core}}^2}{2E} \left( \frac{\pi}{4} d_c^2 L_c \right) + \frac{\sigma_{\text{shank}}^2}{2E} \left( \frac{\pi}{4} d^2 L_s \right)$$

↓

initial  
strain energy

$$U_1 = \frac{93.53^2}{2 \times 2 \times 10^5} \left[ \frac{\pi}{4} \times 16.5^2 \times 20 \right] + \frac{63.6619^2}{2 \times 2 \times 10^5} \left[ \frac{\pi}{4} \times 20^2 \times 50 \right]$$

$$U_1 = 252.68 \text{ N}\cdot\text{mm} \longrightarrow \textcircled{1}$$

let  $U_2$  be the S.E after turning down the shank to root dia.

$$U_2 = \frac{93.53^2}{2 \times 2 \times 10^5} \left[ \frac{\pi}{4} \times 16.5^2 \times 70 \right]$$

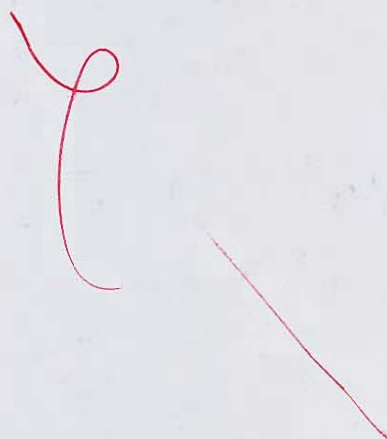
$$U_2 = 327.3389 \text{ N}\cdot\text{mm} \longrightarrow \textcircled{2}$$

$U_2 > U_1 \Rightarrow$  S.E increased after turning.

12

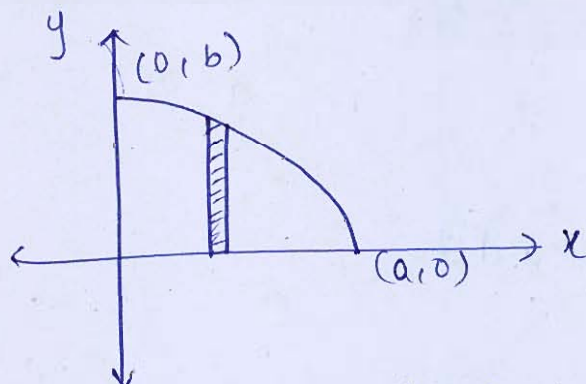
- Q.5 (b) A golf ball is launched with an initial velocity of  $75 \text{ m/s}$  at an angle of  $15^\circ$  with horizontal. Determine the radius of curvature of the trajectory and the time rate of change of the speed of the ball  
(a) just after launch, and (b) at apex  
Neglect aerodynamic drag.

[12 marks]





Q.5 (c) Determine the centroid of quadrant of an ellipse, whose equation is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .



[12 marks]

$$\bar{x} = \frac{\int_0^a yx dx}{A}$$

area of a quadrant of ellipse  $A = \left(\frac{\pi ab}{4}\right)$

$$\bar{x} = \frac{\int_0^a b \sqrt{1 - \frac{x^2}{a^2}} \cdot x \cdot dx}{\left(\frac{\pi ab}{4}\right)}$$

$$y^2 = b^2 \left(1 - \left(\frac{x^2}{a^2}\right)\right)$$

$$y = b \sqrt{1 - \frac{x^2}{a^2}}$$

$$\bar{x} = \frac{\int_0^a \sqrt{1 - \frac{x^2}{a^2}} \cdot x \cdot dx}{\frac{\pi ab}{4}}$$

$$\text{let } \frac{x}{a} = u$$

$$dx = a du$$

$$\bar{x} = \frac{4}{\pi} \int_0^1 \sqrt{1 - u^2} \cdot a du$$

$$= \frac{4a}{\pi} \int_0^1 \sqrt{1 - u^2} \cdot u du$$

$$\bar{x} = \frac{4a}{\pi} \int_0^1 \sqrt{1-u^2} u \, du$$

$$\boxed{\bar{x} = \frac{4a}{\pi \cdot 3}}$$

$$\bar{y} = \frac{\int_0^a y \, dx \cdot \frac{y}{2}}{\left(\frac{\pi ab}{4}\right)} = \frac{\frac{1}{2} \int_0^a y^2 \, dx}{\left(\frac{\pi ab}{4}\right)}$$

$$\bar{y} = \frac{\int_0^a b^2 \left(1 - \frac{x^2}{a^2}\right) dx}{\left(\frac{\pi ab}{2}\right)}$$

$$\bar{y} = \frac{2b^4}{\pi ab} \int_0^a \left(1 - \left(\frac{x}{a}\right)^2\right) dx$$

$$\text{let } \frac{x}{a} = u \quad dx = a \, du$$

$$\bar{y} = \frac{2b}{\pi a} \int_0^1 (1-u^2) a \, du$$

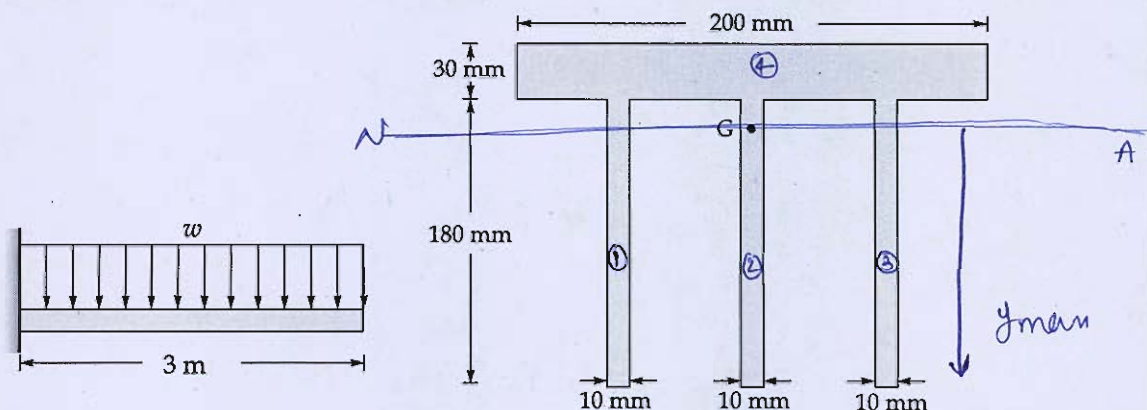
$$\boxed{\bar{y} = \frac{4b}{3\pi}}$$

$$\boxed{\text{Centroid} = \left( \frac{4a}{3\pi}, \frac{4b}{3\pi} \right)}$$

(Ans)

(12)

- Q.5 (d) The extruded beam as shown below is made of aluminium having an allowable working stress in either tension or compression of 90 MPa. The beam is a cantilever, subjected to a uniform vertical load. Determine the allowable intensity of uniform loading.



$$M_{max} = \frac{w \cdot 3 \times 3}{2} = \frac{9w}{2} \times 10^6$$

[12 marks]

$$\sigma_{per} = 90 \text{ MPa}$$

$$A_1 = A_2 = A_3 = 10 \times 180$$

$$A_4 = 30 \times 200$$

$$\bar{y}_1 = \bar{y}_2 = \bar{y}_3 = 90$$

$$\bar{y}_4 = 195$$

$$\bar{y}_{NA} = 145.263 \text{ mm}$$

$$I_1 = \frac{10 \times 180^3}{12} + 10 \times 180 \times (145.263 - 90)^2$$

$$I_1 = 10 \cdot 3572 \times 10^6 \text{ mm}^4 = I_2 = I_3$$

$$I_4 = \frac{200 \times 30^3}{12} + 200 \times 30 \times (195 - 145.263)^2$$

$$I_4 = 15.2926 \times 10^6 \text{ mm}^4$$

$$I_{NA} = 46.3426 \times 10^6 \text{ mm}^4$$

$$y_{max} = 145.263$$

$$Z_{NA} = 319.025 \times 10^3 \text{ mm}^3$$



$$(\sigma_b)_{max} = \frac{M}{Z_{NA}}$$

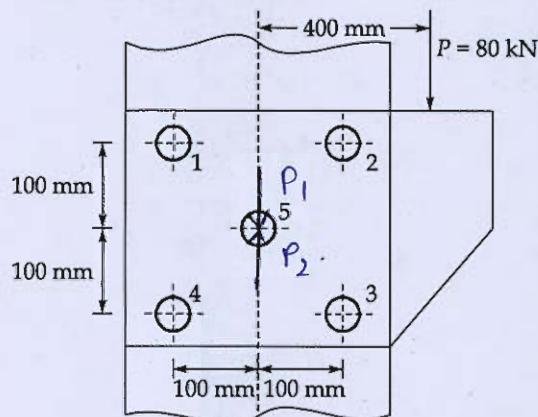
$$= \frac{4.5 \times 10^6 \text{ Nmm}}{319.0256 \times 10^3} \leq 90$$

$$\omega = 6.3805 \text{ N/mm}^2$$

→ Allowable intensity

(12)

- Q.5 (e) A bracket is attached to a steel channel by means five identical rivets as shown in figure. Determine the diameter of rivets, if the permissible shear stress is  $100 \text{ N/mm}^2$ .



[12 marks]

$$\tau_{\text{per}} = 100 \text{ N/mm}^2$$

1. Apply  $P_1, P_2$  at C.G. of group of rivets.

2. Effect of  $P_1$ .

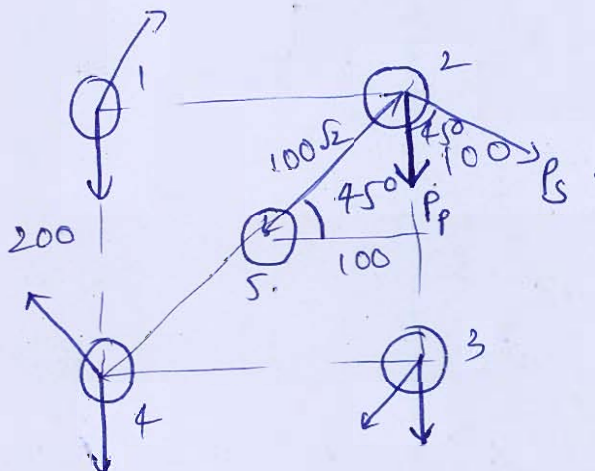
$$\tau_s = \frac{(P_1/5)}{\left(\frac{\pi}{4} d^2\right)} = \frac{80,000/5}{\frac{\pi}{4} d^2}$$

$$\tau_p = \frac{20371.83}{d^2}$$

3. Effect of  $P_2, P$

$$e = 400 \text{ mm} ; T = 80 \times 10^3 \times 400 \text{ mm}$$

$$T = 32 \times 10^6 \text{ N}\cdot\text{mm}$$



$$K = \frac{T}{r_1^2 + r_2^2 + r_3^2 + r_4^2 + r_5^2}$$

$$r_1 = r_2 = r_3 = r_4 = 100\sqrt{2} \text{ mm}$$

$$r_5 = 0$$

$$K = 400 \text{ N/mm}$$

$$P_1 = P_2 = P_3 = P_4 = K \sigma_p = 56568.54 \text{ N}$$

$$\tau_s = \frac{56568.54}{\frac{\pi}{4} d^2} = \frac{72075.3053}{d^2}$$

Rivets 2, 3 are critical.

$$\tau_{Res} = \sqrt{\tau_p^2 + \tau_s^2 + 2\tau_p \tau_s \cos 45^\circ} \leq 100$$

$$\frac{87622.56}{d^2} \leq 100$$

$$d \geq 29.6 \text{ mm}$$

12

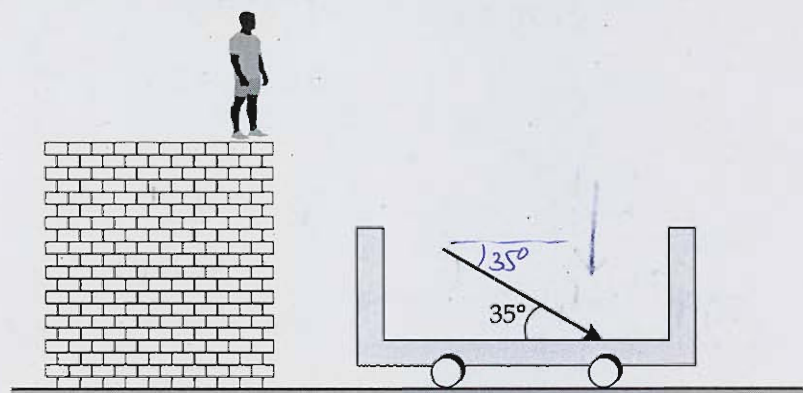
so

$$d = 30 \text{ mm}$$

Q.6 (a) A man of 60 kg mass standing on a bridge jumps on to a cart below him such that he lands with a velocity of 5 m/s at an angle of  $35^\circ$  to the horizontal direction. If the cart is free to move, determine its velocity after he has jumped in for the following cases : the cart is initially

- (i) at rest
- (ii) moving with a velocity of 1 m/s away from the bridge.
- (iii) moving with a velocity of 1 m/s towards the bridge.

Take the mass of the cart as 130 kg. Also determine the loss in kinetic energy in each case.



[20 marks]



① man

$$m_1 = 60 \text{ kg}$$

$$(\vec{v}_i)_1 = 5 [\cos 35^\circ \hat{i} - \sin 35^\circ \hat{j}]$$

② cart

$$m_2 = 130 \text{ kg}$$

$$(\vec{v}_i)_2$$

After landing on the cart, (man + cart) together move with same velocity  $= v_f$

Case I

$(\vec{v}_i)_2 = 0$  (initial velocity of cart)

No force  $\Rightarrow$  in horizontal direction. Linear momentum conservation

$$m_1(\vec{v}_i)_1 + m_2(\vec{v}_i)_2 = (m_1 + m_2) \vec{v}_f \quad \text{2n Hg.}$$

$$(60 \times 5 \cos 35^\circ) + 130(0) = (190)(v_f)$$

$$v_f = 1.2934 \text{ m/s} \quad \text{(away from bridge)}$$

$$\text{loss of KE} = KE_i - KE_f$$

$$= \left( \frac{1}{2} \times 60 \times 5^2 \right) - \frac{1}{2} \times 190 \times 1.2934^2$$

$$(KE)_{\text{loss}} = 591.07 \text{ J}$$

(ii) Case II

$$(\vec{v}_i)_2 = +1 \text{ m/s} \quad \left\{ \begin{array}{l} \text{cart moving away} \\ \text{from bridge} \end{array} \right\}$$

$$(60 \times 5 \cos 35^\circ) + (130 \times 1) = 190 v_f$$

$$v_f = 1.9776 \text{ m/s}$$

(away from bridge)



$$(KE)_{\text{loss}} = \frac{1}{2}(60)(5^2) + \frac{1}{2}(130)(1^2) - \frac{1}{2}(190)(1.9776)^2$$

$$= 482.57 \text{ J}$$

(iii) Case III

$$(V_i)_2 = -1 \text{ m/s} \quad (\text{towards the bridge})$$

$$m_1(V_i)_1 + m_2(V_i)_2 = (m_1 + m_2)V_f$$

$$(60 \times 5 \cos 35) + (130(-1)) = 190 V_f$$

$$V_f = 0.6092 \text{ m/s}$$

away from bridge.

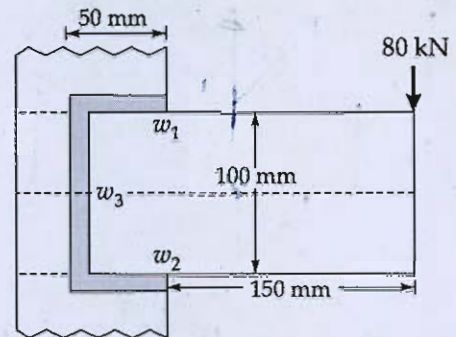
$$\text{Loss of KE} = KE_1 - KE_2$$

$$= \frac{1}{2} \times 60 \times 5^2 + \frac{1}{2} \times 130 (1)^2 - \frac{1}{2} (190) (0.6092)^2$$

$$\text{Loss of KE} = \underline{\underline{783.45 \text{ J}}}$$

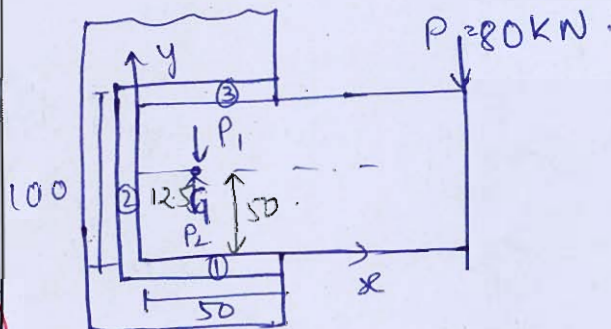
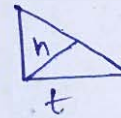
14

- Q.6 (b) A welded connection, as shown in the figure is subjected to an eccentric force of 80 kN in the plane of the welds. Determine the size of the welds, if the permissible shear stress for the weld is  $410 \text{ N/mm}^2$ . Assume static condition.



[20 marks]

let  $h$  be throat of weld



$$A_1 = 50h \quad x_1 = 25$$

$$A_2 = 100h \quad x_2 = 0$$

$$A_3 = 50h \quad x_3 = 25$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{(50h \times 25) + 0 + (50h \times 25)}{200h} = 12.5$$

$$\bar{y} = 50 \text{ mm (symmetry)}$$

1. Due to  $P_1 \Rightarrow$  direct shear stress in weld.

$$\tau_d = \frac{80 \times 10^3}{200 \times h} = \frac{80 \times 10^3}{200 \times 0.707t}$$

$$\tau_d = \frac{565.77}{t}$$

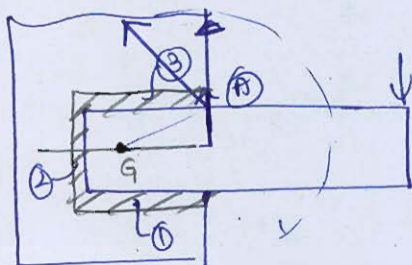
2. Due to  $P_2$  &  $P$

$$T = P \times e$$

$$e = (37.5 + 150) \text{ mm}$$

$$T = 15 \times 10^6 \text{ N}\cdot\text{mm}$$





$$J_G = J_{G1} + J_{G2} + J_{G3}$$

$$J_{G1} = J_1 + A_1 (50^2)$$

$$J_1 = \frac{50 h^3}{12} + \frac{h (50^3)}{12} \quad A_1 = 50h$$

$$J_{G1} = h \left( \frac{50^3}{12} + 50^3 \right) = \frac{13}{12} \times 50^3 h$$

$$J_{G1} = \frac{13}{12} \times 50^3 \times 0.707 t = J_{G3} \quad (\text{due to symmetry})$$

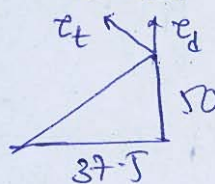
$$J_{G1} = 95.739 \times 10^3 t = J_{G3}$$

$$J_{G2} = \frac{h(100)^3}{12} = 83.33 \times 10^3 (0.707 t)$$

$$J_{G2} = 58.916 \times 10^3 t$$

$$J_G = 250.394 \times 10^3 t$$

polar moment of  
inertia of weld  
about G. (centroid)



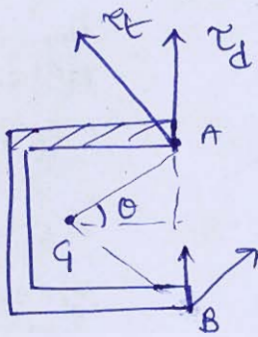
$$\sigma_{\max} = \sqrt{37.5^2 + 50^2}$$

$$= 62.5 \text{ mm}$$

$$z_p = \frac{J_G}{\sigma_{\max}} = 4006.304 t$$

$$\tau_t = \frac{T}{z_p} = \frac{15 \times 10^6}{4006.304 t} = \frac{3744.1}{t}$$

$$\tau_t = \frac{3744.1}{t}$$



A, B pts are critical.

$$\tan \theta = \frac{50}{37.5}$$

$$\theta = 53.13^\circ$$

$$\tau_{Res} = \sqrt{\tau_t^2 + \tau_d^2 + 2\tau_t\tau_d \cos \theta}$$

$$\tau_{Res} = \frac{4108.846}{t}$$

$$\tau_{Res} \leq \{\tau_{per} = 410\}$$

$$\frac{4108.846}{t} \leq 410$$

$$10.0216 \text{ mm} < t$$

$$t = 11 \text{ mm (or)} \quad t > 10.02 \text{ mm}$$

$$\boxed{\text{size of weld} = 11 \text{ mm}}$$

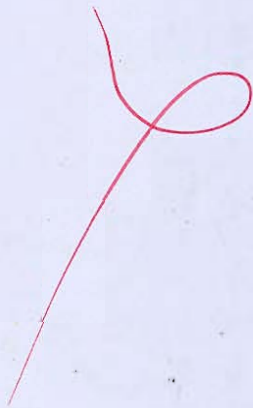


- Q.6 (c) A cylindrical tank is 1.6 m diameter, 2.4 m long and 10 mm thick. Its ends are flat and are joined by nine tie bars, each 35 mm diameter equally spaced. If the tie bars are initially stressed to  $45 \text{ N/mm}^2$  and the tank is filled with water. Determine
- (i) the increase in capacity when the pressure is raised to  $2 \text{ N/mm}^2$ .
  - (ii) the final stress in the tie bars.

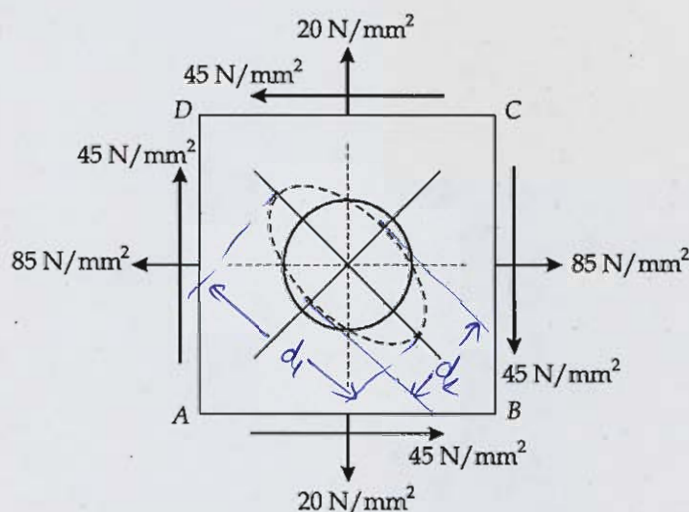
Taking  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\mu = 0.3$

[20 marks]





- Q.7 (a) On a mild steel plate, a circle of diameter 60 mm is drawn before the plate is stressed as shown in the figure. Find the lengths of the major and minor axes of an ellipse formed as a result of the deformation of the circle marked.



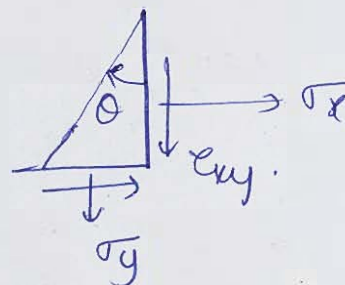
Take  $E = 2 \times 10^5 \text{ N/mm}^2$  and  $\frac{1}{m} = \frac{1}{4} = \mu$

[20 marks]

$$\sigma_x = 85 \text{ N/mm}^2$$

$$\sigma_y = 20 \text{ N/mm}^2$$

$$\tau_{xy} = 45 \text{ N/mm}^2$$





$$\sigma_{1,2} = \frac{85+20}{2} \pm \sqrt{\left(\frac{85-20}{2}\right)^2 + 45^2}$$

$$\left. \begin{array}{l} \sigma_1 = 108.009 \text{ mpa} \\ \sigma_2 = -3.009 \text{ mpa} \end{array} \right\} \text{ prnl stresses.}$$

dirn of prnl stresses,

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2 \times 45}{85 - 20}$$

$$2\theta = 54.16^\circ$$

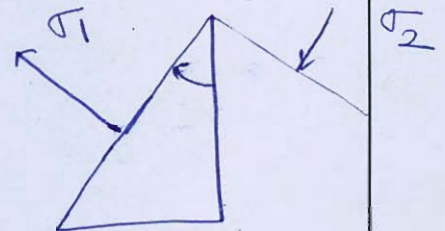
$$\theta_1 = 27.08^\circ$$

$$\theta_2 = 117.08^\circ$$

$\epsilon_1$  = major prnl strain

$$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\mu \sigma_2}{E}$$

$$= \frac{108.009 - (0.25 \times (-3.009))}{2 \times 10^5}$$



$$\boxed{\epsilon_1 = 5.438 \times 10^{-4}} = \frac{\Delta d_1}{60} \Rightarrow \boxed{\Delta d_1 = 0.03263 \text{ mm}}$$

$\epsilon_2$  = minor prnl strain

$$\epsilon_2 = \frac{(-3.009) - 0.25(108.009)}{2 \times 10^5}$$

$$\boxed{\epsilon_2 = -1.50056 \times 10^{-4}} = \frac{\Delta d_2}{60}$$

$$\boxed{\Delta d_2 = -9.003375 \times 10^{-3} \text{ mm}}$$

$$d_1 = \text{major axis} = d + \Delta d_1$$

$$d_1 = (60 + 0.03263) \text{ mm}$$

$$d_1 = 60.03263 \text{ mm} \rightarrow \text{major axis}$$

Ans.

$$d_2 = \text{minor axis} = d + \Delta d_2$$

$$= 60 - (9.003375 \times 10^{-3})$$

$$d_2 = 59.99099 \text{ mm} \rightarrow \text{minor axis}$$

Ans.

20



Q.7 (b) Following data is given for a full hydrodynamic bearing used for electric motor.

Radial load = 1250 N; Journal speed = 1500 rpm; Journal diameter = 50 mm

Static load on the bearing = 400 N; Start up bearing pressure = 2 N/mm<sup>2</sup>

Permissible bearing pressure in application of elastic motor is 1 N/mm<sup>2</sup>

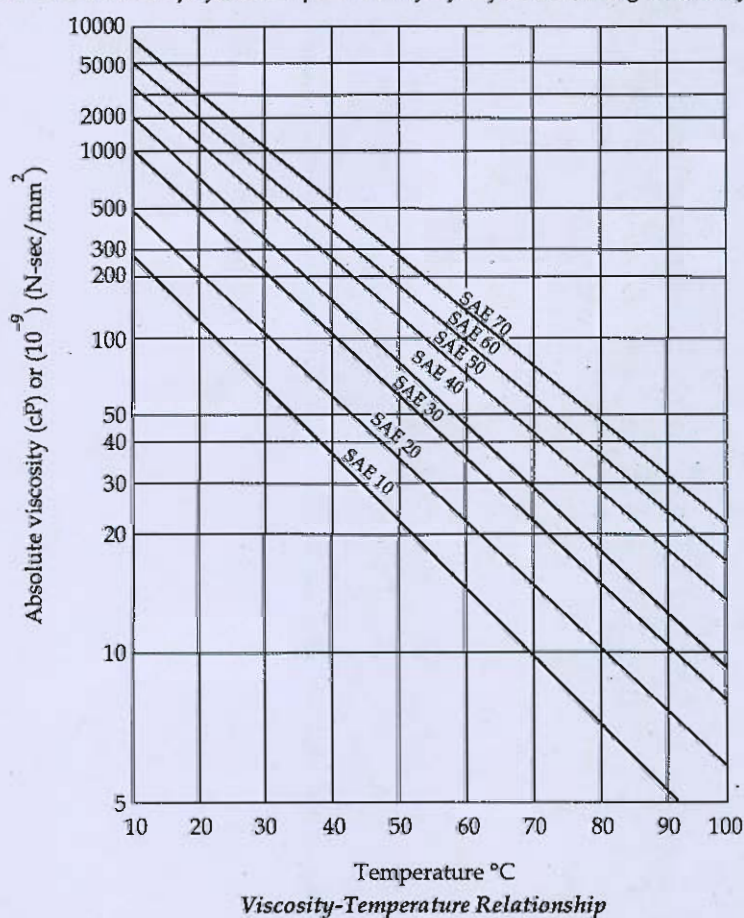
The value of surface roughness (CLA) of the journal and the bearing are 2 and 1 micron respectively. The minimum oil film thickness should be five times the sum of surface roughness of the journal and the bearings. Determine

- (i) length of the bearing                      (ii) radial clearance  
(iii) minimum oil film thickness              (iv) viscosity of lubricant  
(v) flow of lubricant

Select suitable oil for this application assuming the operating temperature as 65°C.

$\left(\frac{l}{d}\right)$	$\varepsilon$	$\left(\frac{h_0}{c}\right)$	$S$	$\phi$	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{ren_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{max}}\right)$
$\left(\frac{1}{2}\right)$	0	1.0	$\infty$	88.5	$\infty$	$\pi$	0	—
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	0.206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	0.126
	1.0	0	0	0	0	—	1.0	0

Table : Dimensionless performance parameters for full journal bearing with side flow



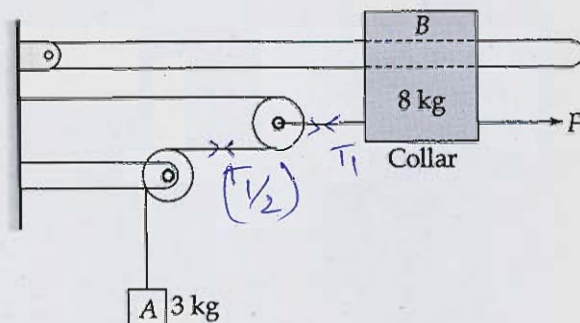
[20 marks]



4



- Q.7 (c) (i) A lift is operated by four ropes each having 30 wires of 1.6 mm diameter. The cage weighs 1.5 kN and the weight of the rope is 4.6 N/m. Determine the maximum load carried by the lift if each wire is of 40 m length and the lift operates
- without any drop
  - with a drop of 100 mm during operation.
- [Take  $E_{\text{rope}} = 70 \text{ GPa}$  and allowable stress = 120 MPa]
- (ii) System shown in the figure is initially at rest. Neglecting friction determine the force  $F$  required if velocity of collar  $B$  becomes 8 m/s in 3 seconds after the start.

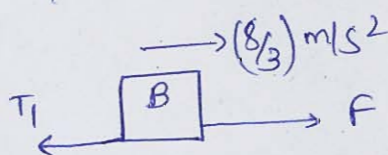


[10 + 10 marks]

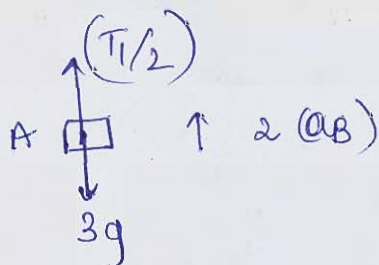
(ii)

$$a_B = \left( \frac{8-0}{3} \right) = \frac{8}{3}$$

~~$$F = m_B a_B = 8 \times \frac{8}{3} = \frac{64}{3} \text{ N}$$~~



$$F - T_1 = \left( 8 \times \frac{8}{3} \right) \text{ N} \quad \text{--- (1)}$$



$$\frac{T_1}{2} - (3 \times 9.81) = 3 \times \left( 2 \times \frac{8}{3} \right)$$

$$\frac{T_1}{2} = (3 \times 9.81) + 16$$

$$T_1 = 90.86 \text{ N}$$

$$F = T_1 + \left( \frac{8 \times 8}{3} \right)$$

$$F = 90.86 + \frac{64}{3}$$

$$F = 112.193 \text{ N}$$

→ force F reqd.

10






Q.8 (a) (i) Discuss the five important parameters involved in the selection and design of journal bearings. Explain in detail how each parameter effects the performance and reliability of the bearing.

(ii) The torque developed by an engine is given by following equation:

$$T = 15000 + 2000 \sin 2\theta - 1500 \cos 2\theta$$

where  $T$  is the torque in N-m and  $\theta$  is the crank angle from inner dead centre position. The resisting torque of the machine is constant throughout the work cycle. The coefficient of speed fluctuations is 0.02. The engine speed is 200 rpm. A circular solid steel disc, 60 mm thick, is used as flywheel. The mass density of steel is  $7800 \text{ kg/m}^3$ . Calculate the radius of the flywheel disk.

[10 + 10 marks]



4



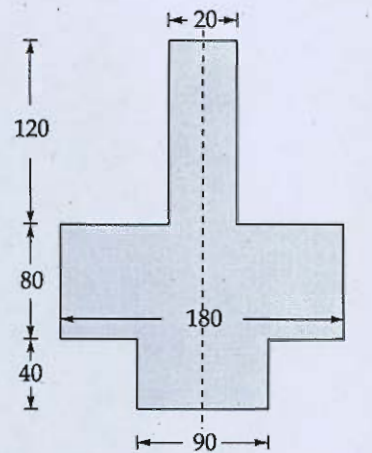


Q.8 (b) The cross-section of a conveyor beam is shown in the figure. The beam is subjected to a bending moment in the plane  $y-y$ . Determine the maximum permissible bending moment.

- (i) for the bottom flange to be in tension.
- (ii) for the bottom flange to be in compression.

The safe bending stress in tension and compression are  $40 \text{ N/mm}^2$  and  $140 \text{ N/mm}^2$  respectively.

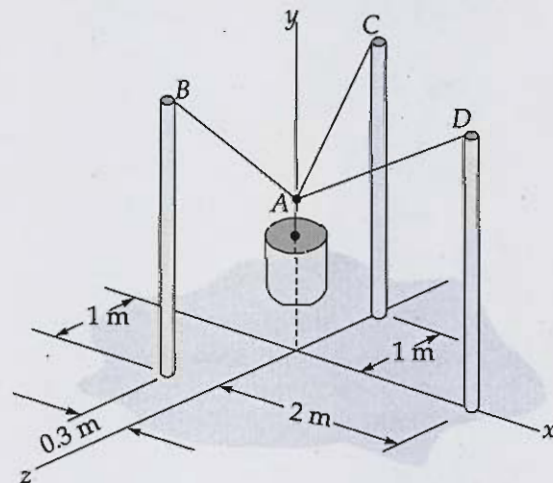
[20 marks]







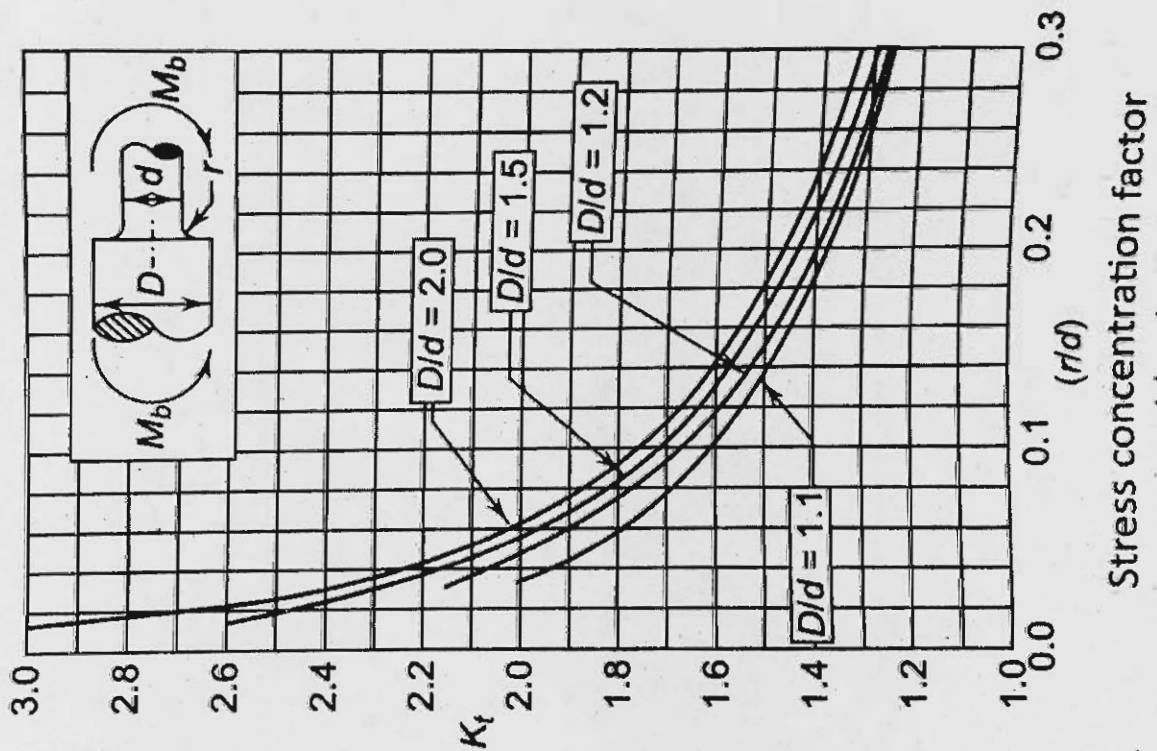
- Q.8 (c) The 20 kg mass is suspended by cables attached to three vertical 2 m posts. Point A is at (0, 1.2, 0) m. Determine the tensions in cables AB, AC and AD.



[20 marks]



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## Space for Rough Work

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## Space for Rough Work

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**Space for Rough Work**

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## Space for Rough Work

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