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Detailed Solutions

ESE-2025
Mains Test Series

Electrical Engineering
Test No : 4

Section A : Electrical Machines + Analog Electronics + Control Systems

Q.1 (a) Solution:

(i) Speed of rotor field with respect to stator

$$= \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Speed of rotor field with respect to rotor

$$= \frac{120 f_2}{P} = \frac{120 \times 20}{4} = 600 \text{ rpm}$$

In a 3-phase induction motor, rotor speed (N_r) ± speed of rotor field w.r.t. rotor

= speed of stator field w.r.t. stator (N_s)

$$N_r \pm 600 = 1500 \text{ rpm}$$

For positive sign, rotor must be driven in the direction of stator field at a speed,

$$N_r = 1500 - 600 = 900 \text{ rpm}$$

For negative sign, rotor must be driven against the direction of stator field at a speed,

$$N_r = 1500 + 600 = 2100 \text{ rpm}$$

(ii) Rotor emf at any slip 's' is given by,

$$E_{2s} = \sqrt{2}\pi (s f_1) N_2 \phi k_{w2}$$

For, $N_r = 900$ rpm

Slip, $$s_1 = \frac{1500 - 900}{1500} = 0.4$$

$$E'_{2s} = \sqrt{2}\pi(0.4 f_1) N_2 \phi k_{w2}$$

For, $$N_r = 2100 \text{ rpm}$$

Slip, $$s_2 = \frac{1500 - 2100}{1500} = -0.4$$

$$E''_{2s} = \sqrt{2}\pi(-0.4 f_1) N_2 \phi k_{w2}$$

$$\therefore \quad \frac{E'_{2s}}{E''_{2s}} = \frac{\sqrt{2}\pi(+0.4 f_1) N_2 \phi k_{w2}}{\sqrt{2}\pi(-0.4 f_1) N_2 \phi k_{w2}} = -1$$

Q.1 (b) Solution:

$$\text{Input Velocity} = \frac{1}{2} \text{ revolution per second}$$

$$= \frac{1}{2} \times 2\pi = \pi = 3.14 \text{ rad/sec}$$

Therefore, the ramp input, R = 3.14 rad/sec

The steady-state error $$e_{ss} = 0.2° = \frac{0.2 \times \pi}{180} \text{rad}$$

For a ramp input of R units

$$e_{ss} = \frac{R}{K_V}$$

$$K_V = \frac{R}{e_{ss}} = \frac{3.14 \times 180}{0.2 \times 3.14} = 900$$

But $K_V = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} s \dfrac{10K}{s(1+0.1s)} = 10K$

$$\therefore \quad 10K = 900$$

$$K = 90$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{900}{s(1+0.1s)}}{1+\dfrac{900}{s(1+0.1s)}} = \frac{9000}{s^2 + 10s + 9000}$$

Comparing this with the standard form of the transfer function of a second-order system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \text{ we get } \omega_n^2 = 9000$$

Therefore, the natural frequency,

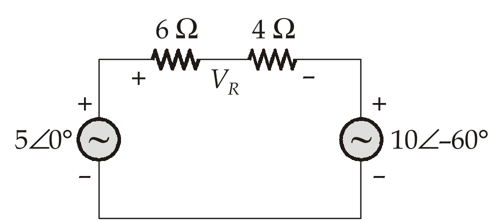
$$\omega_n = \sqrt{9000} = 94.87 \text{ rad/sec}$$

$$2\xi\omega_n = 10$$

Therefore, the damping ratio, $\xi = \dfrac{10}{2\omega_n} = \dfrac{10}{2\times 94.87} = 0.053$

Q.1 (c) Solution:

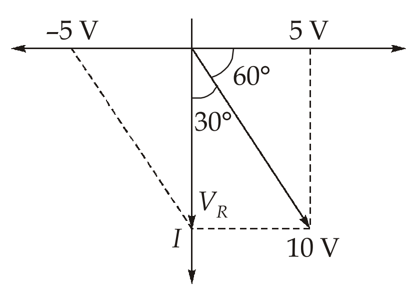
Secondary parameters 16 Ω and 20∠–60° when transferred to primary side become $16\times\dfrac{1}{4}$ = 4 Ω and $20\angle-60°\times\dfrac{1}{2}$ = 10∠–60° V respectively,



with 5 V as reference, the resultant voltage in the circuit is given by

$$V_R = 10\angle-60° - 5\angle 0°$$

$$= 10\left(0.5 - j\frac{\sqrt{3}}{2}\right) - 5 = -j5\sqrt{3} = 5\sqrt{3}\angle-90° \text{ V}$$



Magnitude of current I, in phase with V_R is given by

$$I = \frac{5\sqrt{3}}{6+4} = 0.5\sqrt{3} \text{ A}$$

Power dissipated in 6 Ω = $I^2R = (0.5\sqrt{3})^2\times 6$ = 4.5 W

Power dissipated in 4 Ω = $(0.5\sqrt{3})^2\times 4$ = 3.0 W

$$\text{Power delivered by 5 V source} = (5V)I\cos(5V, I)$$

$$= 5\times 0.5\sqrt{3}\times\cos 90^\circ = 0\text{ W}$$

$$\text{Power delivered by 20 V source} = 10\times 0.5\sqrt{3}\times\cos 30^\circ = 7.5\text{ W}$$

Q.1 (d) Solution:

$$\text{Total reluctance} = \frac{\text{Length of iron path}}{\mu_0\mu_r\times\text{Area}}+\frac{\text{Airgap length}}{\mu_0\times\text{Area}}$$

$$= \frac{120\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 25\times 10^{-4}}+\frac{0.5\times 10^{-2}}{4\pi\times 10^{-7}\times 25\times 10^{-4}}$$

$$= \frac{10^9}{4\pi\times 25}\left[\frac{120}{1500}+\frac{0.5}{1}\right] = 1.8462\times 10^6\text{ A/Wb}$$

$$\text{Flux, } \phi = \frac{NI}{Rl} = \frac{1000\times 2}{1.8462\times 10^6}\times 10^3 = 1.0833\text{ mWb}$$

$$\text{Field energy stored in iron} = \frac{1}{2}\phi^2\times\text{reluctance offered by iron path}$$

$$= \frac{1}{2}[1.0833\times 10^{-3}]^2\times\frac{120\times 10^{-2}}{4\pi\times 10^{-7}\times 1500\times 25\times 10^{-4}}$$

$$= 0.14922\text{ J}$$

$$\text{Field energy stored in air gap} = \frac{1}{2}\phi^2\times\text{Reluctance of airgap}$$

$$= \frac{1}{2}[1.0833\times 10^{-3}]^2\times\frac{0.5\times 10^{-2}}{4\pi\times 10^{-7}\times 25\times 10^{-4}} = 0.93387\text{ J}$$

$$\text{Energy density in iron} = \frac{\text{Energy stored in iron}}{\text{Volume of iron}}$$

$$= \frac{0.14942}{120\times 10^{-2}\times 25\times 10^{-4}} = 49.805\text{ J/m}^3$$

$$\text{Energy density in airgap} = \frac{0.93387}{0.5\times 10^{-2}\times 25\times 10^{-4}} = 74709.6\text{ J/m}^3$$

$$\frac{\text{Energy stored in airgap}}{\text{Energy stored in iron}} = \frac{0.93387}{0.14942} = 6.25$$

$$\frac{\text{Energy density in airgap}}{\text{Energy density in iron}} = \frac{74709.6}{49.807} = 1499.98\approx 1500$$

Q.1 (e) Solution:

$$I_{rated} = I_{base} = 1.00$$

$$V_{rated} = V_{base} = 1.00$$

Under short circuit, $I_{SC}Z_{e1} = V_{SC}$

$$I_{SC} = I_{rated}$$

$$1\ Z_{e1} = (0.03)\ (1)$$

$$Z_{e1} = 0.03$$

Short-circuit power factor,

$$\cos\theta_{SC} = 0.25,$$

$\therefore$ $\sin\theta_{SC} = 0.968$

In complex notation,

$$\bar{Z}_{e1} = 0.03(0.25 + j0.968)$$

$$= (0.0075 + j0.029)\text{ p.u.}$$

Similarly, $\bar{Z}_{e2} = 0.04(0.3 + j0.953)$

$$= 0.012 + j0.0381\text{ p.u.}$$

Common base kVA may be 200 kVA, 500 kVA or any other suitable base kVA

Choosing 500 kVA base arbitrarity, we get

$$\bar{Z}_{e1} = \frac{500}{200}(0.0075 + j0.029) = 0.075\angle 75.52°$$

$$\bar{Z}_{e2} = \frac{500}{500}(0.012 + j0.0381) = 0.04\angle 72.54°$$

$$\bar{Z}_{e1} + \bar{Z}_{e2} = 0.03 + j0.11 = 0.114\angle 74.74°$$

Total kVA, $S = \dfrac{560}{0.8} = 700\text{ kVA}$

$\therefore$ $\bar{S} = 700\angle\cos^{-1} 0.8 = 700\angle 36.9°\text{ kVA}$

We know that, in paralllel connection of transformers, power shared by transformer-1,

$$\bar{S}_1^* = \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}\right) \times \left(\vec{S}_2\right)^*$$

$$\bar{S}_1^* = \frac{\vec{Z}_{e2}}{\vec{Z}_{e1} + \vec{Z}_{e2}}\left(\vec{S}_L\right)^*$$

$$= (700\angle -36.9)\frac{0.04\angle 72.54°}{0.114\angle 74.74°} = 245.0\angle{-39.1°}\text{ kVA}$$

$$S_1 = (245)\,(\cos 39.1°) \text{ at power factor of } \cos 39.1° \text{ lag}$$

$$= 192 \text{ kW at } 0.776 \text{ p.f. lag}$$

$$\bar{S}_2^* = (700\angle -36.90)\frac{0.075\angle 75.52°}{0.114\angle 74.74°} = 460\angle{-36.1°}\text{ kVA}$$

$\therefore$ $$S_2 = 460 \cos 36.1° \text{ at power factor of } \cos 36.1° \text{ lag}$$

$$= 372 \text{ kW at } 0.808 \text{ p.f. lag}$$

Q.2 (a) Solution:

Since the plot begins with a constant magnitude of 20 dB.

$\therefore$ $$20 \log K_{10} = 20 \Rightarrow \log K_{10} = 1 \Rightarrow K = 10$$

$\therefore$ $$\text{DC gain} = 10$$

–6 dB octave at $\omega = 1$ and–12 dB/octave at $\omega = 2$ indicate the existence of two simple poles at corner frequencies $\omega = 1$ and $\omega = 2$. Therefore, the open-loop transfer function is given by

$$G(s)H(s) = \frac{10}{(1+s)\left[1+\frac{s}{2}\right]}$$

or

$$G(j\omega)H(j\omega) = \frac{10}{(1+j\omega)\left(1+\frac{j\omega}{2}\right)}$$

$$= \frac{10}{\sqrt{1+\omega^2}\sqrt{1+\left(\frac{\omega}{2}\right)^2}}\angle\left(-\tan^{-1}\omega-\tan^{-1}\frac{\omega}{2}\right)$$

(i) To find the gain crossover frequency ω_g, we equate the magnitude part to 0 dB. Thus,

$$20\log\frac{10}{\sqrt{1+\omega_g^2}\sqrt{1+\left(\frac{\omega_g}{2}\right)^2}} = 0$$

$$\frac{10}{\sqrt{1+\omega_g^2}\sqrt{1+\left(\frac{\omega_g}{2}\right)^2}} = 10^0 = 1$$

$$\sqrt{1+\omega_g^2}\sqrt{1+\frac{\omega_g^2}{4}} = 10$$

$$(1+\omega_g^2)\left(1+\frac{\omega_g^2}{4}\right) = 100$$

$$4 + 5\omega_g^2 + \omega_g^4 = 400$$

$$\omega_g^2 = 17.56 \text{ and } -22.56$$

The latter value is discarded because the square of a frequency cannot be negative.

So, $\omega_g = \sqrt{17.56} = 4.19 \text{ rad/sec}$

(ii) To determine the gain margin, we need to know the phase crossover frequency ω_p. We do that by equating the phase factor to –180°. Thus,

$$-\tan^{-1}\omega_p - \tan^{-1}\frac{\omega_p}{2} = -180^\circ$$

$$\tan^{-1}\frac{\omega_p + \frac{\omega_p}{2}}{1-\omega_p\cdot\frac{\omega_p}{2}} = 180^\circ$$

$$\frac{\frac{3}{2}\omega_p}{1-\frac{\omega_p^2}{2}} = \tan 180^\circ = 0 \qquad \text{...(ii)}$$

The R.H.S. of equation (ii) becomes 0 if $\omega_p = 0$ or $\omega_p = \infty$. The former value, being impractical, is rejected. So, $\omega_p = \infty$ is accepted.

(iii)

$$|M| = \left.\frac{10}{\sqrt{1+\omega_p^2}\sqrt{1+\left(\frac{\omega_p}{2}\right)^2}}\right|_{\omega_p=\infty} = \frac{1}{\infty} = 0$$

Therefore, $\frac{1}{|M|} = \infty$

Thus, gain margin $GM = 20\log\infty = \infty$

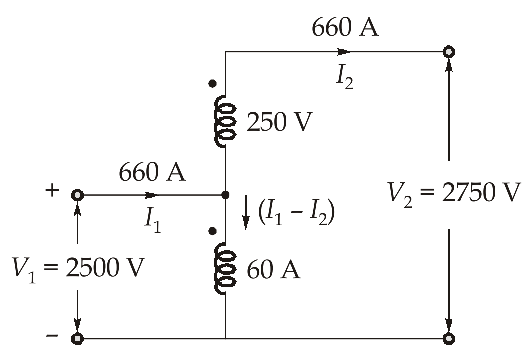
(iv) The phase margin is calculated as

$$\phi|_{\omega_g=4.19} = -\tan^{-1}4.19 - \tan^{-1}\frac{4.19}{2} = -141.06°$$

Therefore, Phase margin $= 180° - 141.06° = 38.9°$

Q.2 (b) Solution:

(i) To achive auto transformer, the two winding transformer can be connected as shown in figure,



For two winding transformer primary rated current,

$$I_1 = \frac{150000}{2500} = 60 \text{ A}$$

Secondary rated current, $I_2 = \frac{150000}{250} = 600 \text{ A}$

From figure, for auto transformer

Primary voltage = 2500 V

Primary current = 600 + 60 = 660 A

Secondary voltage = 2750 V

Secondary current = 600 A

(ii) kVA rating $= V_1I_1 = V_2I_2$

$= 2500 \times 660 = 2750 \times 600$

$= 1650$ kVA

(iii) Here transformation ratio, $k = \frac{2500}{2750} = 0.91$

Hence, percentage full load losses in auto transformer

$= (1 - k)$ full load losses in two winding

$= (1 - 0.91) \times 2.5 = 0.225\%$

Efficiency of auto transformer

$$= (100 - 0.225)\% = 99.775\%$$

(iv) Percentage impedance of auto-transformer

$$= (1 - k) \times Z_{PTW}$$

$$= (1 - 0.91) \times 4\% = 0.36\%$$

(v) Regulation of auto transformer

$$= (1 - k) \times \text{regulation of two winding transformer}$$

$$= (1 - 0.91) \times 3\% = 0.27\%$$

(vi) Short circuit current as auto transformer

$$= \frac{1}{(1-k)} \times \text{short circuit current of two windings transformer}$$

$$= \frac{1}{(1-0.91)} \times \frac{1}{\text{p.u. impedance of two winding transformer}}$$

$$= \frac{1}{0.09} \times \frac{1}{0.04} = 277.78 \text{ p.u.}$$

$$\text{Primary current} = 660 \times 277.78 = 183.33 \text{ kA}$$

$$\text{Secondary current} = 600 \times 277.78 = 166.67 \text{ kA}$$

Q.2 (c) Solution:

The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\dfrac{4}{s^2 + 1.0s}}{1 + \dfrac{4}{s^2 + 1.0s}} = \frac{4}{s^2 + 1.0s + 4}$$

Therefore, the characteristic equation is

$$s^2 + 1.0s + 4 = 0$$

Comparing it with

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = 4$$

$$\omega_n = 2 \text{ rad/s}$$

$$2\xi\omega_n = 1$$

$$\xi = \frac{1}{2\omega_n} = \frac{1}{2 \times 2} = 0.25$$

The damping ratio with derivative

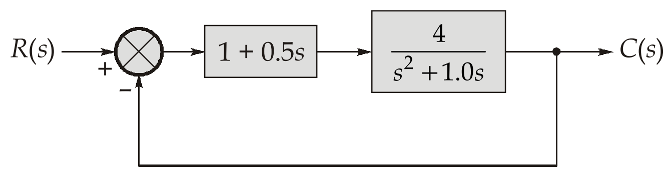
$$\xi' = \xi + \frac{\omega_n T_d}{2}$$

Since the damping ratio with derivative control is to be 0.75 and ω_n = 2, we have

$$0.75 = 0.25 + \frac{2 \times T_d}{2}$$

$$T_d = 0.5$$

Therefore, the block diagram with derivative control is as shown in figure below,



Without derivative control, the rise time

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi - \tan^{-1}\frac{\sqrt{1-0.25^2}}{0.25}}{2\times\sqrt{1-0.25^2}} = \frac{1.823}{1.936} = 0.942 \text{ sec}$$

The peak time,

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi}{2\sqrt{1-0.25^2}} = 1.62 \text{ sec}$$

The peak overshoot is given by

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi\times 0.25/\sqrt{1-0.25^2}} = 0.4443$$

The overall transfer function with derivative control is

$$\frac{C(s)}{R(s)} = \frac{\frac{4(1+0.5s)}{s(s+1)}}{1+\frac{4(1+0.5s)}{s(s+1)}} = \frac{4(1+0.5s)}{s^2+3s+4} = \frac{4+2s}{s^2+3s+4}$$

For a unit-step input, $r(t) = 1$

$\therefore$ $R(s) = \frac{1}{s}$

$$C(s) = \frac{4+2s}{s(s^2+3s+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+3s+4}$$

$$= \frac{1}{s} - \frac{(s+1)}{s^2+3s+4} = \frac{1}{s} - \frac{s+1}{(s+1.5)^2+1.75}$$

$$= \frac{1}{s} - \frac{(s+1)}{(s+1.5)^2+1.32^2} = \frac{1}{s} - \frac{(s+1.5)-0.5}{(s+1.5)^2+1.32^2}$$

$$= \frac{1}{s} - \frac{(s+1.5)}{(s+1.5)^2+1.32^2} + \frac{0.5}{1.32}\frac{1.32}{(s+1.5)^2+1.32^2}$$

$$c(t) = 1 - e^{-1.5t}\cos 1.32t + 0.378e^{-1.5t}\sin 1.32t$$

$$= 1 - e^{-1.5t}(\cos 1.32t - 0.378\sin 1.32t)$$

At $t = t_r$,

$$c(t) = 1$$

i.e. $c(t_r) = 1$

$\therefore$ $1 = 1 - e^{-1.5t_r}(\cos 1.32t_r - 0.378\sin 1.32t_r)$

$\therefore$ $\cos 1.32t_r = 0.378\sin 1.32t_r$

$$\tan 1.32t_r = \frac{1}{0.378} = 2.645$$

$\therefore$ $1.32t_r = \tan^{-1} 2.645 = 69.28° = 1.209 \text{ rad}$

$$t_r = \frac{1.209}{1.32} = 0.916 \text{ sec}$$

For peak time, differentiating the output equation with respect to t and equating to zero

$$\left.\frac{d}{dt}c(t)\right|_{t=t_p} = \left.\frac{d}{dt}[1 - e^{-1.5t}(\cos 1.32t - 0.378\sin 1.32t)]\right|_{t=t_p} = 0$$

i.e. $e^{-1.5t}[-\sin 1.32t(1.32) - 0.378\times 1.32\cos 1.32t] - e^{-1.5t}(-1.5)[\cos 1.32t - 0.378t]\Big|_{t=t_p} = 0$

i.e. $-1.32\sin 1.32t_p - 0.498\cos 1.32t_p + 0.567\sin 1.32t_p - 1.5\cos 1.32t_p = 0$

i.e. $-0.753\sin 1.32t_p - 1.998\cos 1.32t_p = 0$

$$\tan 1.32t_p = \frac{-1.998}{0.753} = -2.653$$

$$\therefore \qquad t_p = \frac{\tan^{-1}(-2.63)}{1.32} = \frac{\pi - \tan^{-1}2.63}{1.32} = \frac{\pi - 1.207}{1.32} = 1.46 \text{ sec}$$

Peak output occurs at $t = t_p$

$$c(t_p) = 1 - e^{-1.5t_p}(\cos 1.32t_p - 0.378 \sin 1.32 t_p)$$
$$= 1 - e^{-1.5 \times 1.46}(\cos 1.32 \times 1.46 - 0.378 \sin 1.32 \times 1.46)$$
$$= 1 - e^{-2.19}(\cos 1.927 - 0.378 \sin 1.927)$$
$$= 1 - 0.119(\cos 110.4° - 0.378 \sin 110.4°)$$
$$= 1 - 0.119(-0.348 - 0.354)$$
$$= 1.08$$

Therefore, the peak overshoot,

$$M_p = c(t_p) - c(\infty)$$
$$= 1.08 - 1 = 0.08$$
$$\%M_p = 8\%$$

Q.3 (a) Solution:

For first transformer,

$$\text{Primary voltage} = 11000 \text{ V}, \qquad f_1 = 50 \text{ Hz}$$
$$P_{c1} = 2400 \text{ W}, \qquad I_{e1} = 3.2 \text{ A}$$

For second transformer,

$$\text{Primary voltage} = 22000 \text{ V}, \qquad f_2 = 50 \text{ Hz}$$
$$I_{e2} = ?, \qquad P_{c2} = ?$$

For first transformer,

Core-loss component of I_{e1}

$$I_{c1} = \frac{P_{c1}}{\text{Primary voltage}} = \frac{2400}{11000} = 0.2182 \text{ A}$$

$$\text{Flux } (\phi_{m1}) = \frac{\text{Primary voltage}}{\sqrt{2}\pi f_1 N_1} = \frac{11000}{\sqrt{2}\pi f_1 N_1} \qquad \text{...(i)}$$

Magnetising component of I_{e1}

$$I_{\phi 1} = \sqrt{I_{e1}^2 - I_{c1}^2} = \sqrt{(3.2)^2 - (0.2182)^2} = 3.193 \text{ A}$$

As we know that core loss ∝ core volume

$$\frac{\text{Core loss of 2}^{\text{nd}} \text{ transformer}}{\text{Core loss of 1}^{\text{st}} \text{ transformer}} = \frac{\text{Core volume of 2}^{\text{nd}} \text{ transformer}}{\text{Core loss of 1}^{\text{st}} \text{ transformer}}$$

or $$\frac{P_{c2}}{P_{c1}} = \frac{(\sqrt{2})^3}{1}$$

∴ $$P_{c2} = 2\sqrt{2} \times P_{c1}$$

or $$P_{c2} = 2\sqrt{2} \times 2400 = 6788.23 \text{ W}$$

Now, core-loss component, $$I_{c2} = (\sqrt{2})^3 \times I_{c1} = 2\sqrt{2} \times 0.2182$$

$$= 0.6172 \text{ A}$$

Now, $$\phi_{m2} = \frac{V_2}{\sqrt{2}\pi f_2 N_2} = \frac{22000}{\sqrt{2}\pi f N_2} \quad [\because f_1 = f_2 = f \text{ and } N_1 = N_2]$$

$$= \frac{2 \times 11000}{\sqrt{2}\pi f N_1}$$

∴ $$\phi_{m2} = 2\ \phi_{m1} \quad \text{...(ii)}$$

Now, $$R_2 = \frac{R_1}{\sqrt{2}}$$

∴ $$R_1 = \sqrt{2} \times R_2 \quad \text{...(iii)}$$

We know that, $$R\phi = NI$$

∴ $$\varphi_{m1} = \frac{N_1 I_1}{R_1}$$

Multiplying both sides by 2,

$$2\ \varphi_{m1} = \frac{2N_1 I_{\phi 1}}{R_1}$$

or $$\varphi_{m2} = \frac{2N_1 I_{\phi 1}}{R_1}$$

or $$\frac{I_{\phi 2} \times N_1}{R_2} = \frac{2N_1 I_{\phi 1}}{R_1}$$

or $$I_{\phi 2} = \frac{2 \times R_2}{R_1} \times I_{\phi 2} = \frac{2 \times R_2}{\sqrt{2} \times R_2} I_{\phi 1}$$

$$= \sqrt{2} \times I_{\phi 1} = \sqrt{2} \times 3.193 = 4.515 \text{ A}$$

Now, no-load current of 2nd transformer,

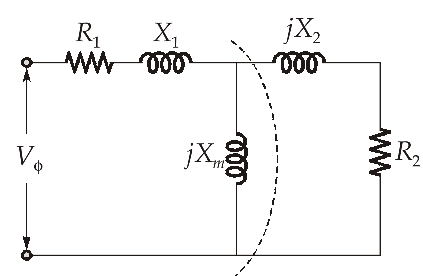
$$I_{e2} = \sqrt{(I_{\phi 2})^2 + (I_{c2})^2}$$

$$= \sqrt{(4.515)^2 + (0.6172)^2} = 4.557 \text{ A}$$

Thus, No-load current = 4.557 A,

Core loss = 6788.8 W

Q.3 (b) Solution:



Taking Thevenin's equivalent across the magnetizing branch

$$V_{\text{Th}} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} \times V_\phi$$

$$= \frac{266 \times 26.3}{\sqrt{(0.641)^2 + (1.106 + 26.3)^2}} = 251.5 \text{ V}$$

The Thevenin resistance is,

$$R_{\text{Th}} \simeq R_1\left(\frac{X_m}{X_1 + X_m}\right)^2 \simeq 0.641 \times \left(\frac{26.3}{1.106 + 26.3}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is,

$$X_{\text{Th}} \simeq X_1 = 1.106\ \Omega$$

(i) The slip at which maximum torque occurs is given as :

$$s_{\text{max}} = \frac{R_2}{\sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}}$$

$$= \frac{0.332}{\sqrt{(0.592)^2 + (1.106 + 0.464)^2}} = 0.198$$

This corresponding to a mechanical speed of

$$N_m = (1 - s_{\text{max}})N_s = (1 - 0.198) \times 1800 = 1444 \text{ rpm}$$

The torque at this speed is,

$$T_{max} = \frac{3V_{Th}^2}{2\omega_s\left[R_{Th} + \sqrt{R_{Th}^2 + (X_{Th} + X_2)^2}\right]}$$

$$T_{max} = \frac{3(255.2)^2}{2\times188.5\left[0.590 + \sqrt{(0.592)^2 + (1.106 + 0.464)^2}\right]}$$

$$T_{max} = 229 \text{ N-m}$$

(ii) The starting torque of this motor is found by setting $s = 1$,

$$T_{st} = \frac{3V_{Th}^2 \cdot R_2}{\omega_s[(R_{Th} + R_2)^2 + (X_{Th} + X_2)^2]}$$

$$= \frac{3\times(255.2)^2\times0.332}{188.5\times[(0.590 + 0.332)^2 + (1.106 + 0.464)^2]}$$

$$T_{st} = 104 \text{ N-m}$$

(iii) If the rotor resistance is doubled, then the slip at maximum torque doubles, too.

Therefore, $s_{max} = 0.396$

and the speed at maximum torque is

$$N_m = (1 - s_{max}).N_s = (1 - 0.396) \times 1800$$

$$N_m = 1087 \text{ rpm}$$

The maximum torque is still,

$$T_{max} = 229 \text{ N-m}$$

The starting torque now becomes

$$T_{st} = \frac{3\times(255.2)^2\times(0.664)}{(188.5)\times[(0.590 + 0.664)^2 + (1.106 + 0.464)^2]}$$

$$T_{st} = 170 \text{ N-m}$$

Q.3 (c) Solution:

Synchronous speed, $N_s = \frac{120f}{P} = \frac{120\times50}{6}$

$= 1000$ rpm or 104.72 rad/sec

$$\text{Full load slip} = \frac{1000 - 950}{1000} = 0.05$$

Full load rotor current, $$\vec{I}'_r = \frac{\frac{440}{\sqrt{3}}}{\sqrt{\left(0.5+\frac{0.4}{0.05}\right)^2 + (1.2+1.2)^2}} = 28.76\angle{-15.77°}\text{ A}$$

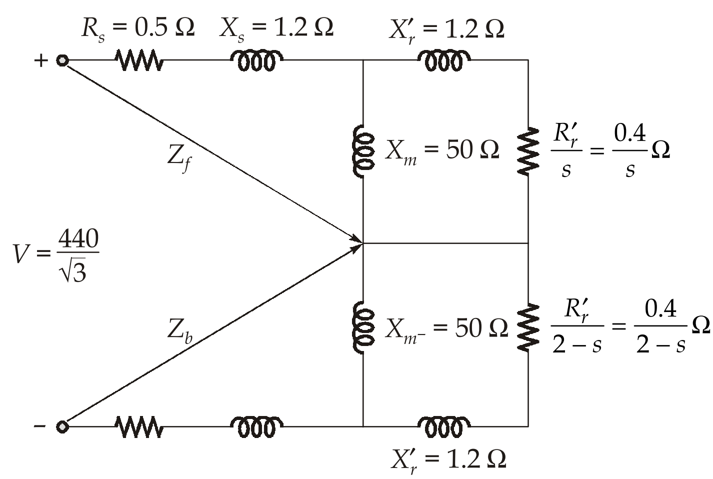
Stator current, $$\vec{I}_s = \vec{I}'_r + \vec{I}_m$$

$$= 28.76\angle-15.77° + \frac{440}{\sqrt{3}\times 50}\angle-90°$$

$$\vec{I}_s = 30.53\angle{-25°}\text{ A}$$

Now, one phase of motor is fall, then motor will operate in single phase operation.

Equivalent circuit diagram in single phasing,



Steady state slip, $$s = 0.05$$

$$Z_f = R_s + jX_s + \frac{jX_m\left(\frac{R'_r}{s} + jX'_r\right)}{\frac{R'_r}{s} + j(X_m + X'_r)}$$

$$= 0.5 + j1.2 + \frac{j50\left(\frac{0.4}{0.05} + j1.2\right)}{\frac{0.4}{0.05} + j51.2}$$

$$\vec{Z}_f = 7.947 + j3.536\ \Omega$$

Now, $$\vec{Z}_b = 0.5 + j1.2 + \frac{j50\left(\frac{0.4}{1.95} + j1.2\right)}{\frac{0.4}{1.95} + j51.2}$$

$$\vec{Z}_b = (0.696 + j2.37)\ \Omega$$

Equivalent impedance, $\vec{Z} = \vec{Z}_f + \vec{Z}_b$

$$\vec{Z} = 7.947 + j3.536 + 0.696 + j2.37 = (8.64 + j5.91)\ \Omega$$

Now, stator current, $|I| = \dfrac{V}{|Z|} = \dfrac{\frac{440}{\sqrt{3}}}{|Z|} = 24.26\text{ A}$

Now, positive sequence torque,

$$T_P = \frac{1}{\omega_{sm}} \cdot \frac{3I^2X_m^2 \cdot \frac{R_r'}{s}}{\left[\left(\frac{R_r'}{s}\right)^2 + (X_r' + X_m)^2\right]}$$

$$= \frac{3 \times (24.26)^2 \times 50^2 \times \frac{0.4}{0.05}}{104.72\left[\left(\frac{0.4}{0.05}\right)^2 + (51.2)^2\right]} = 125.457\text{ N-m}$$

Similarly, torque due to –ve sequence current,

$$T_N = \frac{-1}{\omega_{sm}} \cdot \frac{3I^2X_m^2 \cdot \frac{R_r'}{(2-s)}}{\left[\left(\frac{R_r'}{2s}\right)^2 + (X_r' + X_m)^2\right]} = \frac{3 \times (24.26)^2 \times 50^2 \times \frac{0.4}{1.95}}{104.72 \times \left[\left(\frac{0.4}{1.95}\right)^2 + (51.2)^2\right]}$$

$$T_N = -3.298\text{ N-m}$$

Net torque, $T = 125.57 - 3.298 = 122.27\text{ N-m}$

Also, for $s = 0.05$, $\omega_m = \omega_{sm}(1 - s) = 99.484\text{ rad/s}$

Hence, $T_L = 0.0123 \times (99.484)^2 = 121.734\text{ N-m}$

$\because T = T_L$, $s = 0.05$ is an approximate solution.

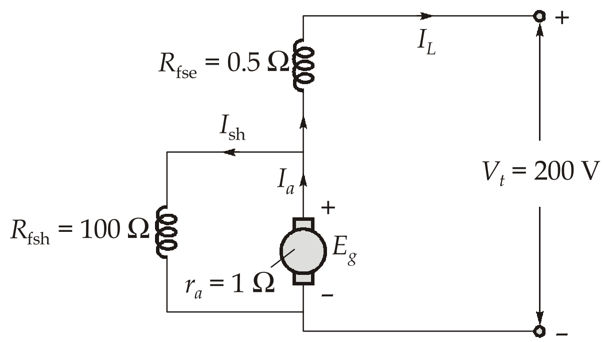
Now, motor current = 24.26 A and motor speed = 950 rpm.

As the motor current is less than full load, the motor will run safely.

Q.4 (a) Solution:

Calculation for short shunt:

The equivalent circuit for short shunt can be drawn as



$$\text{Load current} = \frac{\text{Load power}}{\text{Terminal voltage}} = \frac{4 \times 1000}{200} = 20 \text{ A}$$

(i) Voltage drop across series field resistance

$$= I_{se} \times R_{fse} = 0.5 \times 20 = 10 \text{ V}$$

Voltage drop across shunt field resistance

$$= V_t + V_{se} = 200 + 10 = 210 \text{ V}$$

Shunt field current, $I_{sh} = \dfrac{V_f}{R_{fsh}} = \dfrac{210}{100} = 2.1$ A

Series field current = Load current = 20 A

Armature current, I_a = 20 + 2.1 = 22.1 A

Total voltage drop across brushes

= 1 × 4 V = 4 V

[∵ In lap winding, no. of brushes = no. of poles]

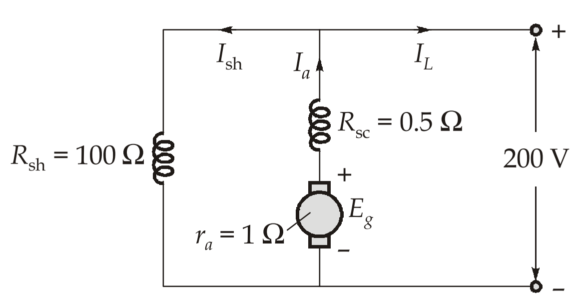
For generator, $E_g = V_b + I_a r_a + I_{se} \times R_{fse} + V_t$

= 4 + 22.1 × 1 + 10 + 200 = 236.1 V

Thus, generated emf = 236.1 V

Calculation for long shunt:

The equivalent circuit can be drawn as below



Shunt field current, $I_{sh} = \frac{200}{100} = 2\text{ A}$

Armature current = Series field current = 2 + 20 = 22 A

For generated emf, $E_g = V_t + V_b + I_a(R_{se} + r_a)$

$= 200 + 4 + 22\,(1 + 0.5) = 237\text{ V}$

Thus, generated emf = 237 V

(ii) To calculate flux per pole

Applying the equation, $E_g = \frac{\phi N Z P}{60\,A}$

For lap winding, $A = P = 4$

$$\phi = \frac{E_g \times 60\,A}{Z \times P \times N} = \frac{236.1 \times 60 \times 4}{200 \times 4 \times 750} = 94.44\text{ mWb}$$

Thus, flux per pole = 94.44 mWb

Again, to calculate flux per pole,

Applying the equation, $E_g = \frac{\phi NZP}{60\,A}$

For lap winding, $A = P = 4$

$$\Rightarrow \quad \phi = \frac{E_g \times 60\,A}{Z \times P \times N}$$

$$= \frac{237 \times 60 \times 4}{750 \times 4 \times 200} = 94.80\text{ mWb}$$

Thus, flux per pole = 94.80 mWb

Q.4 (b) Solution:

The response of the system in s-domain is given by

$$X(s) = [sI - A]^{-1}\, x(0^+) + [sI - A]^{-1}\, BU(s)$$

$$X(s) = \phi(s)\, x(0^+) + \phi(s) BU(\text{s})$$

where $\phi(s) = [sI - A]^{-1}$ is called the resolvent matrix.

$$X(s) = \phi(s)[x(0) + BU(s)]$$

$$= \phi(s)\left[\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix}\frac{1}{s}\right] = \phi(s)\begin{bmatrix}1/s\\1-1/s\end{bmatrix}$$

Given,
$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, [sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}$$

$\therefore$
$$\phi(s) = [sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^T}{\Delta}$$

$\therefore$
$$X(s) = \frac{\begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}}{(s+1)^2}\begin{bmatrix} \frac{1}{s} \\ 1-\frac{1}{s} \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 1+\frac{2}{s}+1-\frac{1}{s} \\ -\frac{1}{s}+s-1 \end{bmatrix}}{(s+1)^2} = \frac{\begin{bmatrix} 2+\frac{1}{s} \\ -1+s-\frac{1}{s} \end{bmatrix}}{(s+1)^2}$$

$$= \begin{bmatrix} \frac{2s+1}{s(s+1)^2} \\ \frac{s^2-s-1}{s(s+1)^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{s}-\frac{1}{s+1}+\frac{1}{(s+1)^2} \\ \frac{-1}{s}+\frac{2}{s+1}\ \frac{1}{(s+1)^2} \end{bmatrix}$$

Taking the inverse Laplace transform, the time response is

$$x(t) = \begin{bmatrix} 1-e^{-t}+te^{-t} \\ -1+2e^{-t}-te^{-t} \end{bmatrix}$$

The output response is given by

$$Y(s) = [0 \quad 1]\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$
$$= X_2(s) = -\frac{1}{s}+\frac{2}{s+1}-\frac{1}{(s+1)^2}$$

Taking the inverse Laplace transform, the output response is

$$y(t) = x_2(t) = -1 + 2e^{-t} - te^{-t}$$
$$x_2 = -1 + 2e^{-t} - te^{-t}$$

Q.4 (c) Solution:

(i)

$$r_a = 1\ \Omega, \quad X_s = 15\ \Omega,$$

$$\bar{Z} = 1 + j15 = 15.033\angle 86.19°\ \Omega$$

$$\text{Load power} = 8\text{ kW},$$

$$\text{p.f.} = 0.8 \text{ lagging}$$

Load current, $$I_a = \frac{\text{Load power}}{\sqrt{3} \times V_L \times \text{p.f.}} = \frac{8 \times 1000}{\sqrt{3} \times 380 \times 0.8} = 15.193\text{ A}$$

Now, $$I_a r_a \text{ drop} = 15.193 \times 1 = 15.193\text{ V}$$

$$I_a X_s \text{ drop} = 15.193 \times 15 = 227.895\text{ V}$$

Let us take terminal voltage as reference phasor

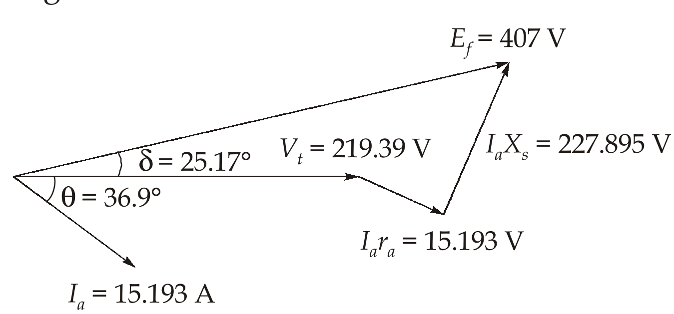
$$\bar{E}_f = \bar{V}_t + \bar{I}_a \bar{Z}_s$$

$$= \frac{380\angle 0°}{\sqrt{3}} + 15.193\angle -36.9° \times 15.033\angle 86.19°$$

$$= 219.39 + 228.396\angle 49.29°$$

$$= 407\angle 25.17° \text{ volt/phase} \quad ...(i)$$

Now, the phasor diagram can be drawn as



(ii) Base impedance, $$Z_{\text{base}} = \frac{(V_t)^2}{\text{Rating}} = \frac{(380)^2}{10 \times 1000} = 14.44\ \Omega$$

$$\text{p.u. resistance} = \frac{r_a}{Z_{\text{base}}} = \frac{1}{14.44} = 0.0639 \text{ p.u.}$$

$$\text{p.u. synchronous reactance} = \frac{x_s}{Z_{\text{base}}} = \frac{15}{14.44} = 1.039 \text{ p.u.}$$

(iii) From equation (i),

$$E_f = 407 \text{ volts}, \quad V_t = 219.39 \text{ volts}$$

Now, $$\text{voltage regulation} = \frac{E_f - V_t}{V_t} = \frac{407V_t - 219.39}{219.39} = 0.8551 = 85.51\%$$

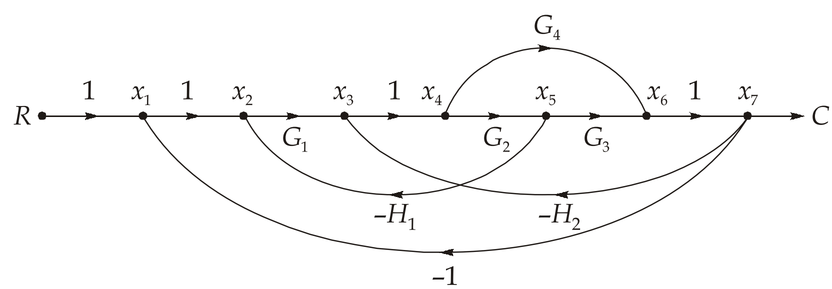
(iv) When the load is suddenly removed, terminal voltage is equal to excitation voltage

We have, $V_t = E_f = 407$ volts $(\because I_a = 0)$

Section B : Electrical Machines + Analog Electronics + Control Systems

Q.5 (a) Solution:

Signal flow graph:



The forward paths and the gains associated with them are given as follows:

Forward path : $R - x_1 - x_2 - x_3 - x_4 - x_6 - x_7 - C$

$$M_1 = (1)(1)(G_1)(1)(G_2)(G_3)(1)$$
$$= G_1G_2G_3 \qquad \Delta_1 = 1$$

Forward path : $R - x_1 - x_2 - x_3 - x_4 - x_6 - x_7 - C$

$$M_2 = (1)(1)(G_1)(1)(G_4)(1)$$
$$= G_1G_4 \qquad \Delta_2 = 1$$

The loops and the gains associated with them are given as follows:

Loop: $x_3 - x_4 - x_5 - x_6 - x_7 - x_3$

$$L_1 = (1)(G_2)(G_3)(1)(-H_2) = -G_2G_3H_2$$

Loop: $x_3 - x_4 - x_6 - x_7 - x_3$

$$L_2 = (1)(G_4)(1)(-H_2) = -G_4H_2$$

Loop: $x_2 - x_3 - x_4 - x_5 - x_2$

$$L_3 = (G_1)(1)(G_2)(-H_2) = -G_1G_2H_1$$

Loop: $x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_1$

$$L_4 = (1)(G_2)(1)(G_3)(G_3)(1)(-1) = -G_1G_2G_3$$

Loop: $x_1 - x_2 - x_3 - x_4 - x_6 - x_7 - x_1$

$$L_5 = (1)(G_1)(1)(G_4)(1)(-1) = -G_1G_4$$

The determinant of the signal flow graph is

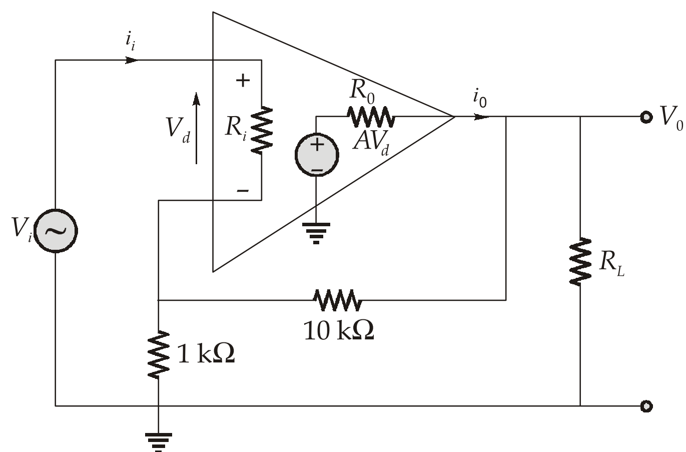
$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$$

$$= 1 - (-G_2G_3H_2 - G_4H_2 - G_1G_2H_1 - G_1G_2G_3 - G_1G_4)$$
$$= 1 + G_2G_3H_2 + G_4H_2 + G_1G_2H_1 + G_1G_2G_3 + G_1G_4$$

Applying Mason's gain formula, the transfer function is

$$\frac{C}{R} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta}$$
$$= \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_4H_2 + G_1G_2H_1 + G_1G_2G_3 + G_1G_4}$$

Q.5 (b) Solution:



$$V_d = V_i - V_0\left(\frac{1}{10+1}\right) \quad \text{...(i)}$$

Let us assume that, $R_L = \infty$

$$\Rightarrow \quad V_0 = AV_d\left(\frac{10+1}{10+1+R_0(\text{k}\Omega)}\right)$$

$$\Rightarrow \quad V_0 = AV_d\left(\frac{11}{11+0.075}\right)$$

$$\Rightarrow \quad V_0 = AV_d\left(\frac{11}{11.075}\right)$$

$$\Rightarrow \quad V_d = \frac{V_0}{A}\left(\frac{11.075}{11}\right) \quad \text{...(ii)}$$

From equation (i) and (ii),

$$\Rightarrow \quad \frac{V_0}{A}\left(\frac{11.075}{11}\right) = V_i - V_0\left(\frac{1}{11}\right)$$

$$\Rightarrow \quad \frac{V_0}{11}\left[\frac{11.075}{A} + 1\right] = V_i$$

$$V_i = 0.091\ V_0$$

$$V_0 = 11\ V_i \quad \text{...(iii)}$$

$$\frac{V_0}{V_i} = A_F = 11$$

Given that, output voltage swing $= \pm 13$ V

$\Rightarrow$ $$V_0 = 13 \sin \omega t$$

$\Rightarrow$ $$V_i = \frac{V_0}{11} = 1.182 \sin\omega t$$

Now, from (ii), $$V_d = 0.000654 \sin \omega t$$

So, $$i_i = \frac{V_d}{R_i} = \frac{V_d}{2\times 10^6} = 3.2726 \times 10^{-10} \sin\omega t \text{ A}$$

So, $$R_{iF} = \frac{V_i}{i_i} = 3611.738 \text{ M}\Omega$$

Now, $$R_{0F} = (R_0) \,||\, (10 + 1) = (0.075) \,||\, (11)$$

$$= \frac{0.075\times 11}{0.075 + 11} = 0.0745 \text{ k}\Omega = 74.5\ \Omega$$

Q.5 (c) Solution:

The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(1+sT)}}{1+\dfrac{K}{s(1+sT)}} = \frac{\dfrac{K}{T}}{s^2+\dfrac{s}{T}+\dfrac{K}{T}}$$

Comparing the characteristic equation $s^2+\frac{s}{T}+\frac{K}{T} = 0$, with the standard form of the characteristic equation $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ of a second-order system,

$$2\xi\omega_n = \frac{1}{T}$$

and $$\omega_n^2 = \frac{K}{T}$$

i.e. $$\omega_n = \sqrt{\frac{K}{T}}$$

$\therefore$ $$2\xi\sqrt{\frac{K}{T}} = \frac{1}{T}$$

or $$\xi = \frac{1}{2\sqrt{KT}}$$

(i) When $\xi = \xi_1 = 0.2$, let $K = K_1$

When $\xi = \xi_2 = 0.8$, let $K = K_2$

$$\frac{\xi_1}{\xi_2} = \frac{0.2}{0.8} = \frac{1}{4}$$

$$= \frac{1}{2\sqrt{K_1 T}} \times 2\sqrt{K_2 T} = \sqrt{\frac{K_2}{K_1}}$$

i.e., $$\frac{K_2}{K_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 = \frac{1}{16}$$

or $$K_2 = \frac{1}{16} K_1$$

Hence the gain K_1 , at which $\xi = 0.2$ should be multiplied by $\frac{1}{16}$ to increase the damping ratio from 0.2 to 0.8.

(ii) When $\xi = \xi_1 = 0.9$, let $T = T_1$

When $\xi = \xi_2 = 0.3$, let $T = T_2$

$$\frac{\xi_1}{\xi_2} = \frac{0.9}{0.3} = 3 = \frac{1}{2\sqrt{KT_1}} \times 2\sqrt{KT_2} = \sqrt{\frac{T_2}{T_1}}$$

$\therefore$ $$\frac{T_2}{T_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 = 9$$

or $$T_2 = 9T_1$$

Hence the original constant T_1 should be multilied by 9 to reduce the damping ratio from 0.9 to 0.3.

Q.5 (d) Solution:

Given: From the characteristic equation

$\therefore$ $$G(s)H(s) = \frac{K}{(1+s)(1.5+s)(2+s)}$$

Put $$s = -1 + j\omega$$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(1.5-1+j\omega)(2-1+j\omega)}$$

$$GH(-1 + j\omega) = \frac{K}{(j\omega)(0.5 + j\omega)(1 + j\omega)}$$

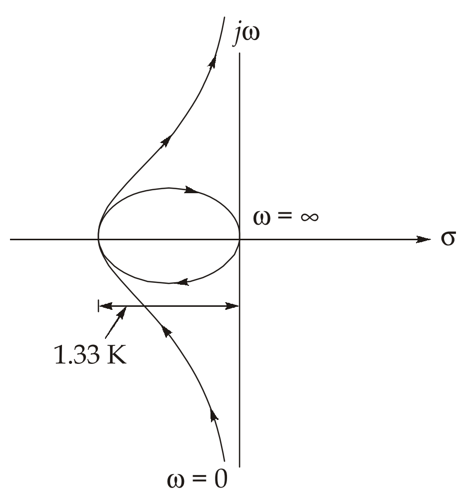
$$M = |GH(-1 + j\omega)| = \frac{K}{\omega\left(\sqrt{1+\omega^2}\right)\left(\sqrt{0.25+\omega^2}\right)}$$

$$M = \frac{2K}{\omega\left(\sqrt{1+4\omega^2}\right)\left(\sqrt{1+\omega^2}\right)}$$

Phase angle $\angle GH(-1 + j\omega) = \phi = -90 - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$

ω	M	ϕ
0	∞	–90°
∞	0	–270°
0.5	$2.53K$	–161.56°
0.1	$19.5K$	–107°
1	$0.63K$	–198°
10	$9.94\times10^{-4}K$	–261°
0.707	$1.33K$	–180°

Nyquist plot for the system is given as



Now, the frequency at which phase is –180° is given by

$$-180° = -90° - \tan^{-1}\omega - \tan^{-1}(2\omega)$$

$$\frac{\omega+2\omega}{1-2\omega^2} = \frac{1}{0}$$

$$\omega = \frac{1}{\sqrt{2}}\text{rad/sec}$$

At $\omega = \frac{1}{\sqrt{2}}$ rad/sec, the value of M is given as,

$$M = \frac{2K}{\frac{1}{\sqrt{2}} \times \sqrt{1+4\times\frac{1}{2}} \times \sqrt{1+\frac{1}{2}}} = \frac{4}{3}\text{K}$$

From the Nyqusit criterion, $Z = P - N$

Here, Z = No. of closed-loop poles inside the right half of S = –1 plane.

P = No. of open-loop poles inside the right half of S = –1 plane.

N = No. of encirclement around S = –1.

As $P = 0,$

∴ For system to be stable, N should be zero.

For N to be zero, $1.33K < 1$

$$K < \frac{1}{1.33}$$

$$K < 0.75$$

Hence, K = 0.75 is the largest value of K.

Q.5 (e) Solution:

Given, $G(s) = \frac{1}{s(1+s)(1+2s)}$

The sinusoidal transfer function is

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+j2\omega)}$$

Rationalizing, $G(j\omega) = \frac{1(1-j\omega)(1-j2\omega)}{j\omega(1+j\omega)(1-j\omega)(1+j2\omega)(1-j2\omega)}$

$$= \frac{(1-2\omega^2)-j3\omega}{j\omega(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{-3}{(1+\omega^2)(1+4\omega^2)} - j\frac{1-2\omega^2}{\omega(1+\omega^2)(1+4\omega^2)}$$

When $\omega = 0$, $G(j0) = -3 - j\infty$

When $\omega = \infty$, $G(j\infty) = -0 + j0$

The frequency at which the polar plot crosses the real axis is given by the solution of

$$\frac{1-2\omega^2}{\omega(1+\omega^2)(1+4\omega^2)} = 0$$

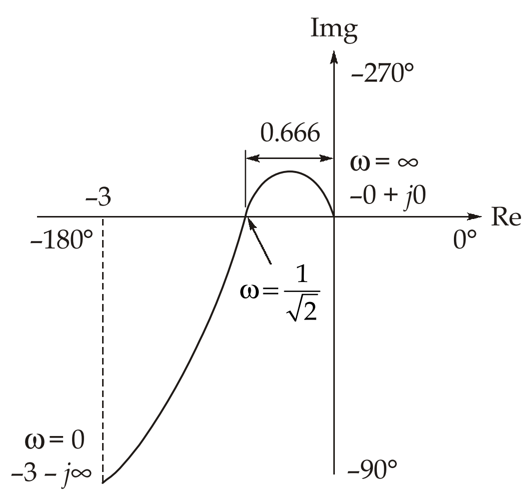
i.e. $$1 - 2\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{2}}$$

The value of $|G(j\omega)|$ at this frequency is

$$\left|\frac{-3}{(1+\omega^2)(1+4\omega^2)}\right|_{\omega=\frac{1}{\sqrt{2}}} = \left|\frac{-3}{\left(1+\frac{1}{2}\right)\left(1+4\times\frac{1}{2}\right)}\right| = \frac{2}{3} = 0.66$$

Based on the above information, an approximate polar plot is drawn as shown in figure,



Q.6 (a) Solution:

(i) Using the open-loop poles and zeros, we represent the open-loop system whose root locus is given as

$$G(s)H(s) = \frac{k(s-3)(s-5)}{(s+1)(s+2)} = \frac{k(s^2-8s+15)}{(s^2+3s+2)}$$

But for all points along the root locus,

$$kG(s)H(s) = -1, \text{ and along the real axis, } s = \sigma$$

Hence, $$\frac{k(\sigma^2-8\sigma+15)}{(\sigma^2+3\sigma+2)} = -1$$

$$k = \frac{-(\sigma^2+3\sigma+2)}{(\sigma^2-8\sigma+15)}$$

for break points, $$\frac{dk}{d\sigma} = 0$$

$$\frac{dk}{d\sigma} = \frac{-\left[(2\sigma+3)(\sigma^2-8\sigma+15)-(\sigma^2+3\sigma+2)(2\sigma-8)\right]}{(\sigma^2-8\sigma+15)^2} = 0$$

$$\frac{dk}{d\sigma} = \frac{11\sigma^2-26\sigma-61}{(\sigma^2-8\sigma+15)^2} = 0$$

$$11\sigma^2 - 26\sigma - 61 = 0$$

$$\sigma = \frac{26 \pm \sqrt{26^2 + 4\times 61\times 11}}{2\times 11}$$

$\therefore$ $\sigma = -1.45;\ 3.82$

$\therefore$ The breakaway point is,

$$\sigma_p = -1.45$$

The break-in point, $\sigma_z = 3.82$

(ii) Given system involves one integrator and two delay integrators. The output of each integrator or delayed integrator can be a state variable. Let us define the output of the plant as x_1, the output of the controller as x_2 and output of the sensor as x_3. Then we obtain,

$$\frac{X_1(s)}{X_2(s)} = \frac{10}{s+5}$$

$$\frac{X_2(s)}{U(s)-X_3(s)} = \frac{1}{s}$$

$$\frac{X_3(s)}{X_1(s)} = \frac{1}{s+1}$$

$$Y(s) = X_1(s)$$

which can be rewritten as

$$sX_1(s) = -5X_1(s) + 10X_2(s) \quad \text{...(i)}$$

$$sX_2(s) = -X_3(s) + U(s) \quad \text{...(ii)}$$

$$sX_3(s) = X_1(s) - X_3(s) \quad \text{...(iii)}$$

$$Y(s) = X_1(s) \quad \text{...(iv)}$$

by taking the inverse Laplace transform of the above equations (i), (ii), (iii), (iv)

$$\dot{x}_1 = -5x_1 + 10x_2$$

$$\dot{x}_2 = -x_3 + u$$

$$\dot{x}_3 = x_1 - x_3$$

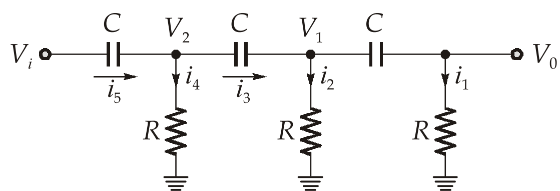
$$y = x_1$$

Thus, the state space model of the system in the standard form is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & 10 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Q.6 (b) Solution:



$$i_1 = \frac{V_0}{R} \quad \text{...(i)}$$

$$V_1 = V_0 + i_1 X_C$$

$$= V_0 + V_0 \frac{X_C}{R}$$

$$= V_0 \left(\frac{X_C + R}{R} \right) \quad \text{...(ii)}$$

$$i_2 = \frac{V_1}{R} = V_0 \left(\frac{X_C + R}{R^2} \right) \quad \text{...(iii)}$$

$$i_3 = i_1 + i_2 = \frac{V_0}{R} + V_0 \left(\frac{X_C + R}{R^2} \right)$$

$$\Rightarrow \quad i_3 = V_0 \left(\frac{X_C + 2R}{R^2} \right) \quad \text{...(iv)}$$

$$V_2 = V_1 + X_C i_3$$

$$= V_0 \left(\frac{X_C + R}{R} \right) + V_0 \left(\frac{X_C^2 + 2X_C R}{R^2} \right)$$

$$= V_0 \left[\frac{X_C R + R^2 + X_C^2 + 2X_C R}{R^2} \right]$$

$$\Rightarrow \quad V_2 = V_0 \left[\frac{X_C^2 + R^2 + 3X_C R}{R^2} \right] \quad \text{...(v)}$$

$$i_4 = \frac{V_2}{R} = V_0\left[\frac{X_C{}^2 + R^2 + 3X_C R}{R^3}\right] \quad \text{...(vi)}$$

$$i_5 = i_4 + i_3 = V_0\left[\frac{X_C{}^2 + R^2 + 3X_C R}{R^3}\right] + V_0\left[\frac{X_C + 2R}{R^2}\right]$$

$$= V_0\left[\frac{X_C{}^2 + R^2 + 3X_C R + X_C R + 2R^2}{R^3}\right]$$

$$= V_0\left[\frac{X_C{}^2 + 3R^2 + 4X_C R}{R^3}\right]$$

Now,
$$V_i = V_2 + X_C\, i_5$$

$$V_i = V_0\left[\frac{X_C{}^2 + R^2 + 3X_C R}{R^2}\right] + V_0\left[\frac{X_C{}^3 + 3R^2 X_C + 4X_C{}^2 R}{R^3}\right]$$

$$V_i = V_0\left[\frac{X_C^2 R + R^3 + 3X_C R^2 + X_C^3 + 3R^2 X_C + 4X_C^2 R}{R^3}\right]$$

$$= V_0\left[\frac{5X_C^2 R + R^3 + X_C^3 + 6X_C R^2}{R^3}\right]$$

$$\frac{V_0}{V_i} = \frac{R^3}{5X_C^2 R + R^3 + X_C^3 + 6X_C R^2}$$

$$\frac{V_0}{V_i} = \frac{R^3}{5R\left[\frac{1}{-\omega^2 C^2}\right] + R^3 + \left[\frac{1}{-j\omega^3 C^3}\right] + \frac{6R^2}{j\omega C}} \qquad \left(\because X_C = \frac{1}{j\omega C}\right)$$

$$\frac{V_0}{V_i} = \frac{R^3}{-\frac{5R}{\omega^2 C^2} + R^3 + \frac{j}{\omega^3 C^3} - j\frac{6R^2}{\omega C}}$$

For satisfying Barkhausen's criteria, the imaginary part must be zero.

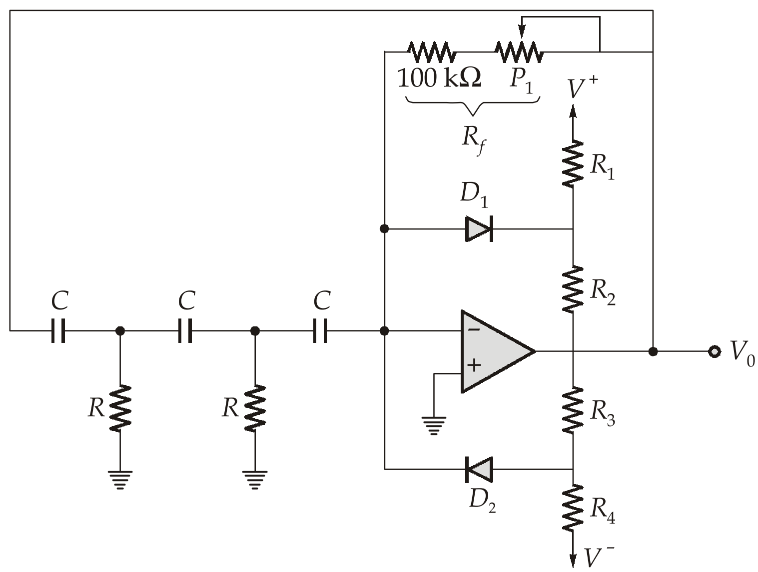
$$\frac{1}{\omega^3 C^3} - \frac{6R^2}{\omega C} = 0$$

$$\Rightarrow \quad \frac{1}{\omega^2 C^2} = 6R^2$$

$$\Rightarrow \quad \omega = \frac{1}{RC\sqrt{6}} \text{ rad/sec}$$

This is the frequency of oscillation.

Actual oscillator circuit:



Q.6 (c) (i) Solution:

The Routh table is formulated as follows:

s^4	1	6	8
s^3	2	8	0
s^2	$\frac{2\times6-1\times8}{2}=2$	$\frac{2\times8-1\times0}{2}=8$	
s^1	$\frac{2\times8-2\times8}{2}=0$		
s^0			

All the elements in the s^1 row are zero. That means there are symmetrically located roots of the characteristic equation with respect to the origin of the s-plane. So the system can be unstable or marginal stable.

To determine the location of the roots form the auxiliary equation $A(s)$ by using the coefficients of the row just above the row of zeros, i.e.,

$$A(s) = 2s^2 + 8 = 0$$

Take the first derivative of the auxiliary equation, i.e,

$$\frac{dA(s)}{ds} = 4s + 0 = 0$$

Replace the row of zeros with the coefficients of the first derivative of the auxiliary equation and complete the formation of the Routh table,

$$\begin{array}{c|ccc} s^4 & 1 & 6 & 8 \\ s^3 & 2 & 8 & \\ s^2 & 2 & 8 & \\ s^1 & 4 & 0 & \\ s^0 & 8 & & \end{array}$$

There are no sign changes in the elements of the first column of the Routh array and hence there are no roots of the characteristic equation in the right-half of the s-plane. There must be roots on the imaginary axis of the s-plane which can be determined by solving the auxiliary equation,

$$2s^2 + 8 = 0$$

$$s = \pm j2$$

This shows that there is a pair of roots at $s = \pm j2$, and so the system oscillates and the frequency of sustained oscillations is $\omega = 2$ rad/sec

To determine the other two roots, factorize the characteristic equation,

$$s^4 + 2s^3 + 6s^2 + 8s + 8 = (s^2 + 4)(s^2 + 2s + 2) = 0$$

$$s^2 + 2s + 2 = (s + 1 + j1)(s + 1 - j1)$$

The other two roots are a pair of complex conjugate roots in the left-half of the s-plane.

Q.6 (c) (ii) Solution:

For the given control system:

$$G(s)H(s) = \frac{K}{(s^2 + 4s + 4)(s + 3)} = \frac{K}{s^3 + 7s^2 + 16s + 12}$$

$$G(j\omega)H(j\omega) = \frac{K}{-j\omega^3 - 7\omega^2 + j16\omega + 12}$$

$$= \frac{K}{j(16\omega - \omega^3) + (12 - 7\omega^2)}$$

Multiplying by conjugate,

For $$G(j\omega)\,H(j\omega) = \frac{K(12 - 7\omega^2) - j(16\omega - \omega^3)}{(12 - 7\omega^2)^2 + (16\omega - \omega^3)^2} \quad ...(i)$$

For imaginary part to be zero,

$$(16\omega - \omega^3) = 0$$

$$\omega(16 - \omega^2) = 0$$

$$\omega = 4 \text{ rad/sec}$$

At, ω = 4 rad/sec, phase cross-over frequency, (ω_{pc}) occurs.

Now, $$G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}=4\text{ rad/sec}} = \frac{K}{12-7(4)^2} = \frac{-K}{100}$$

$$\therefore \quad \text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} = \frac{1}{\frac{K}{100}} = \frac{100}{K} \quad \text{...(ii)}$$

Required value of gain margin,

$$GM \geq 4$$

Comparing with equation (ii), we get

$$\frac{100}{K} \geq 4$$

$$K \leq 25$$

For position error constant :

$$K_p > 2$$

We know, $$K_p = \lim_{s\to 0} G(s)H(s)$$

$$K_p = \lim_{s\to 0} \frac{K}{(s+2)^2(s+3)} = \frac{K}{12}$$

$$\therefore \quad \frac{K}{12} > 2;\ K > 24$$

∴ Allowable range of K :

$$24 < K \leq 25$$

Q.7 (a) Solution:

(i) With error rate control

$$G(s) = \frac{10(1+sk_e)}{s(s+2)}$$

Therefore, the closed-loop transfer function is

$$\frac{\theta_C(s)}{\theta_R(s)} = \frac{\frac{10(1+sk_e)}{s(s+2)}}{1+\frac{10(1+sk_e)}{s(s+2)}} = \frac{10+10sk_e}{s^2+s(2+10k_e)+10}$$

Comparing the characteristic equation $s^2 + s(2 + 10k_e) + 10 = 0$ with the standard form of the characteristic equation of a second-order system.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0, \text{ we get}$$

$$\omega_n^2 = 10, \quad \omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$

$$2\xi\omega_n = 2 + 10k_e$$

$$\therefore \quad k_e = \frac{2\xi\omega_n - 2}{10} = \frac{2\times0.6\times3.16-2}{10} = 0.18$$

The settling time, $$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.6\times3.16} = 2.11 \text{ sec}$$

The peak overshoot, $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-\pi\times0.6}{\sqrt{1-0.6^2}}} = 0.0949$$

The peak %overshoot

$$M_p \times 100\% = 0.0949 \times 100\% = 9.49\%$$

Therefore, the steady-state error,

$$e_{ss} = \frac{R}{k_v} = \frac{1}{k_v} = \frac{1}{\lim\limits_{s\to0} sG(s)} = \frac{1}{\lim\limits_{s\to0} s\frac{10(1+sk_e)}{s(s+2)}}$$

$$= \frac{1}{5} = 0.2 \text{ rad}$$

(ii) Without error rate

When $k_e = 0$

$$G(s) = \frac{10}{s(s+2)}$$

Therefore, the closed-loop transfer function is

$$\frac{\theta_C(s)}{\theta_R(s)} = \frac{10}{s(s+2)}$$

Therefore, the closed-loop transfer function is

$$\frac{\theta_C(s)}{\theta_R(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s^2+2s+10}$$

Comparing this transfer function with the standard form of the transfer function of a second-order system $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, we get

$$\omega_n^2 = 10$$

$$\therefore \quad \omega_n = \sqrt{10} = 3.16 \text{ rad/sec}$$

$$2\xi\omega_n = 2$$

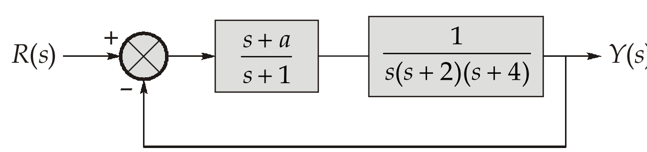
$$\therefore \quad \xi = \frac{2}{2\omega_n} = \frac{2}{2\times 3.16} = 0.32$$

The settling time, $$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.32\times 3.16} = 4 \text{ sec}$$

The peak overshoot, $$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = e^{\frac{-\pi\times 0.32}{\sqrt{1-0.32^2}}} = 0.351$$

The steady-state error, $$e_{ss}(t) = \frac{1}{k_v} = \frac{1}{\lim\limits_{s\to 0} sG(s)} = \frac{1}{\lim\limits_{s\to 0} s\frac{10}{s(s+2)}} = \frac{1}{5} = 0.2 \text{ rad}$$

Q.7 (b) Solution:



The characteristic equation of unity negative feedback system:

$$1 + G(s)H(s) = 0$$

$$s(s+1)(s+2)(s+4) + (s+a) = 0$$

$$s^4 + 6s^3 + 8s^2 + s^3 + 6s^2 + 8s + s + a = 0$$

$$s^4 + 7s^3 + 14s^2 + 9s + a = 0$$

Forming routh array:

$$\begin{array}{c|ccc} s^4 & 1 & 14 & a \\ s^3 & 7 & 9 & \\ s^2 & \frac{89}{7} & a & \\ s^1 & \frac{114.43-7a}{12.71} & & \\ s^0 & a & & \end{array}$$

From s^1 : $$\frac{114.43-7a}{12.71} > 0$$

$$a < 16.35$$

and $a > 0$

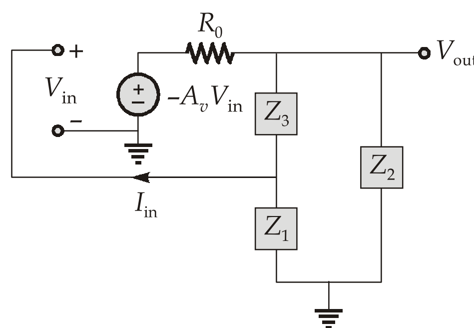
So the range of value of $0 < a < 16.35$,

At critical damping compensator will be $\dfrac{s+16.35}{s+1}$

So the compensator is lag compensator.

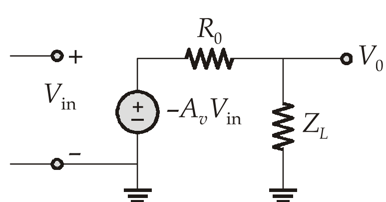
Q.7 (c) Solution:

Small signal model of the amplifier:



$\because$ $I_{in} = 0$

The above circuit can be reduced as,

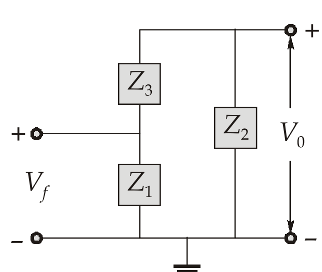


Thus, the overall gain of the amplifier,

$$A = \frac{V_0}{V_{in}} = \frac{-A_v Z_L}{Z_L + R_0}$$

$$Z_L = \frac{(Z_1 + Z_3)Z_2}{(Z_1 + Z_2 + Z_3)}$$

For the feedback circuit,



The feedback gain, $\beta = \frac{V_f}{V_0} = \frac{Z_1}{Z_1 + Z_3}$

$\because$ The phase shift of the feedback circuit is negative,

$$A\beta = \frac{-A_v Z_1 Z_L}{(R_0 + Z_L)(Z_1 + Z_3)}$$

$$= \frac{-A_v Z_1 \left[\frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}\right]}{\left[R_0 + \frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}\right](Z_1 + Z_3)}$$

$$= \frac{-A_v Z_1 Z_2}{R_0[(Z_1 + Z_2 + Z_3)] + Z_2(Z_1 + Z_3)}$$

Now, $Z_1 = jX_1$, $Z_2 = jX_2$ and $Z_3 = jX_3$

$$A\beta = \frac{A_v(X_1 X_2)}{jR_0(X_1 + X_2 + X_3) - X_2(X_1 + X_3)}$$

To produce sustained oscillations the phase shift of the loop gain $A\beta$ should be 0°.

Thus,
$$R_0(X_1 + X_2 + X_3) = 0$$
$$X_1 + X_2 + X_3 = 0$$
$$(X_1 + X_3) = -X_2$$
$$A\beta = \frac{-A_v X_1}{X_1 + X_3} = \frac{A_v X_1}{X_2}$$

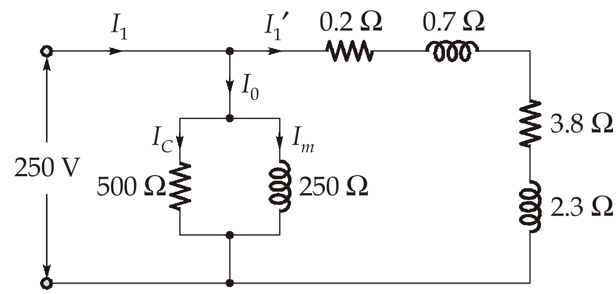
Hence, X_1 and X_2 should be of the same type of reactance.

Q.8 (a) Solution:

The load impedance referred to low-tension side is

$$Z_L' = (38 + j230)\left(\frac{N_1}{N_2}\right)^2 = (3.8 + j2.3)\Omega$$

Transformer leakage impedance $(0.2 + j0.7)\Omega$ and this load impedance $3.8 + j2.3\Omega$ are in series as shown,



Therefore, total series impedance is $4 + j3 = 5\angle 36.9°$

(i) Taking V_1 as the reference phasor,

$$\bar{V}_1 = 250\angle 0°$$

$$\because \quad \bar{I}_1' = \frac{250\angle 0°}{5\angle 36.9°} = 50\angle -36.9°$$

$$= 50(\cos 36.9° - j\sin 36.9°) = (40 - j30)\text{ A}$$

or $$I_1' = 50\text{ A and } I_2 = I_1'\frac{N_1}{N_2} = 50 \times \frac{1}{10} = 5\text{ A}$$

$\therefore$ Secondary terminal voltage $= I_2 Z_L$

$$= 5\left[380^2 + 230^2\right]^{1/2} = 2220\text{ V}$$

(ii) The core loss current, $$\bar{I}_C = \frac{\bar{V}_1}{R_C} = \frac{250\angle 0°}{500\angle 0°} = 0.5 + j0$$

The magnetizing current, $$\bar{I}_m = \frac{\bar{V}_1}{jX_m} = \frac{250\angle 0°}{250\angle 90°} = 1\angle -90° = 0 - j1$$

$\therefore$ Exciting current, $$\bar{I}_0 = \bar{I}_c + \bar{I}_m = (0.5 - j1)\text{A}$$

Hence total primary current,

$$\bar{I}_1 = \bar{I}_1' + \bar{I}_0$$

$$= (40 - j30) + (0.5 - j1)$$

$$= 40.5 - j31$$

$$= 51\angle -37.4°$$

$\therefore$ Primary current, $I_1 = 51$ A

and primary power factor $= \cos 37.4° = 0.794$ lagging

Load power factor $$= \cos\theta_2 = \frac{380}{\left[380^2 + 230^2\right]^{1/2}} = 0.855$$

$$\text{Power output} = V_2 I_2 \cos\theta_2 = 2220 \times 5 \times 0.855 = 9500 \text{ W}$$

Also, $$\text{power output} = |I_1'|^2 R_L$$

$$= (50)^2 \times 3.8 = 9500 \text{ W}$$

Core loss, $$P_C = \frac{V_1^2}{R_C} = \frac{(250)^2}{500} = 125 \text{ W}$$

$$P_C = I_C^2 R_C = (0.5)^2 (500) = 125 \text{ W}$$

Ohmic loss, $$P_{0h} = (I_1')r_{e1} = (50)^2 \times 0.2 = 500 \text{ W}$$

$$\text{Power input} = V_1 I_1 \cos\phi_1$$

$$= 250 \times 51 \times 0.794 = 10125 \text{ W}$$

$\therefore$ $$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

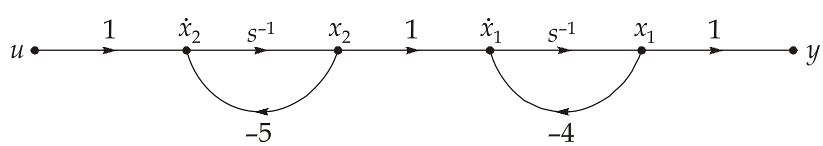
$$= 1 - \frac{\text{Losses}}{\text{Input}} = 1 - \frac{500 + 125}{10125} = 0.9383 \text{ p.u.} = 93.83\%$$

Q.8 (b) Solution:

(i) Cascade composition :

$$\frac{Y(s)}{U(s)} = \frac{s^{-1}}{1-(-5s^{-1})} \cdot \frac{s^{-1}}{1-(-4s^{-1})}$$

The signal flow graph for above transfer function



The state equations are

$$\dot{x}_1 = -4x_1 + x_2$$

$$\dot{x}_2 = -5x_2 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$A = \begin{bmatrix} -4 & 1 \\ 0 & -5 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Test for controllability, $$AB = \begin{bmatrix} -4 & 1 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$$

Controllability test matrix,

$$Q_C = [B \;\vdots\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & -5 \end{bmatrix}$$

$$|Q_C| \neq 0 = -1 \neq 0$$

So, matrix A is controllable. Hence, the state variable feedback can be applied.

Let, $K = [K_1 \quad K_2]$

Then $$[A - BK] = \begin{bmatrix} -4 & 1 \\ 0 & -5 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}[K_1 \quad K_2]$$

$$[A - BK] = \begin{bmatrix} -4 & 1 \\ -K_1 & -(5+K_2) \end{bmatrix}$$

$$|[sI - (A - BK)]| = \left|\begin{bmatrix} s+4 & -1 \\ K_1 & s+K_2+5 \end{bmatrix}\right|$$

$$s^2 + (9 + K_2)s + (20 + 4K_2 + K_1) = 0 \quad \text{...(i)}$$

Desired characteristic equation

$$(s + 1 + j2)(s + 1 - j2) = 0$$

$$s^2 + 2s + 5 = 0 \quad \text{...(ii)}$$

Comparing equation (i) and (ii)

$$9 + K_2 = 2$$

$$K_2 = -7$$

and $$20 + 4K_2 + K_1 = 5$$

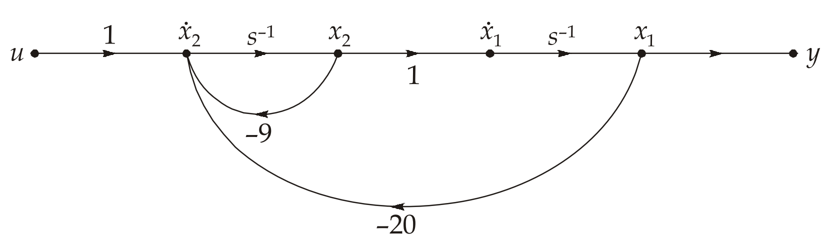
$$20 - 4 \times 7 + K_1 = 5$$

$$K_1 = 13$$

So, K-matrix, $$K = [13 \quad -7]$$

(ii) Direct Composition :

$$\frac{Y(s)}{U(s)} = \frac{1}{(s^2+9s+20)} = \frac{s^{-2}}{1-(-9s^{-1}-20s^{-2})}$$



Let
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -20x_1 - 9x_2 + u$$
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -20 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$
$$A = \begin{bmatrix}0 & 1\\ -20 & -9\end{bmatrix};\ B = \begin{bmatrix}0\\ 1\end{bmatrix}$$
$$A' = (A - BK) = \begin{bmatrix}0 & 1\\ -20 & -9\end{bmatrix} - \begin{bmatrix}0\\ 1\end{bmatrix}[K_1 \quad K_2]$$
$$= \begin{bmatrix}0 & 1\\ -(20+K_1) & -(9+K_2)\end{bmatrix}$$
$$[sI - A'] = \begin{bmatrix}s & -1\\ (20+K_1) & s+9+K_2\end{bmatrix}$$
$$\det[sI - A'] = s^2 + (9 + K_2)s + 20 + K_1 = 0 \qquad ...(iii)$$

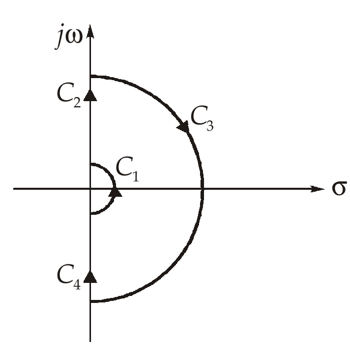
But required characteristic equation
$$s^2 + 2s + 5 = 0 \qquad ...(iv)$$

Comparing equation (iii) and (iv),
$$9 + K_2 = 2 \Rightarrow K_2 = -7$$
$$20 + K_1 = 5 \Rightarrow K_1 = -15$$

So, feedback matrix, $K = [-15 \quad -7]$

Q.8 (c) Solution:

Consider the contour in s-plane



Let $K = 1$,
$$G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega+3)(j\omega+5)}$$

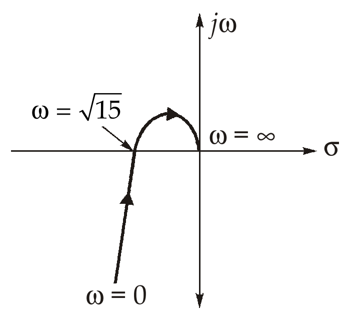
$$= \frac{1}{j\omega(-\omega^2 + 8j\omega + 15)} = \frac{1}{-8\omega^2 + j(15\omega - \omega^3)}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\omega\sqrt{9+\omega^2}\sqrt{25+\omega^2}}$$

$$\angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\left(\frac{\omega}{3}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

For C_2 : $s = j\omega$

ω	$\lvert GH \rvert$	$\angle GH$
$\omega = 0$	∞	$-90°$
$\omega = \infty$	0	$-270°$



At real axis crossing, $15\omega - \omega^3 = 0$

$$\omega = \sqrt{15} = 3.87 \text{ rad/sec}$$

For C_4 : $s = -j\omega$ inverse polar plot

For C_1 : $s = \lim\limits_{R\to 0} Re^{j\theta}$; $\theta \Rightarrow \left(\frac{-\pi}{2} \to \frac{\pi}{2}\right)$

$$GH = \lim_{R\to 0} \frac{1}{Re^{j\theta}\left(Re^{j\theta}+3\right)\left(Re^{j\theta}+5\right)} = \infty e^{-j\theta}$$

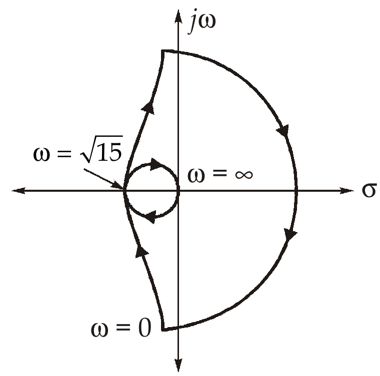
As θ varies from $\to \left(\frac{\pi}{2} \to \frac{-\pi}{2}\right)$

For C_3 : $s = \lim\limits_{R\to\infty} Re^{j\theta}$; $\theta \Rightarrow \left(\frac{\pi}{2} \to \frac{-\pi}{2}\right)$

$$GH = \lim_{R\to\infty} \frac{1}{Re^{j\theta}(Re^{-i\theta}+3)(Re^{i\theta}+5)} = 0\, e^{-j3\theta}$$

As θ varies from $\to \left(\frac{-3\pi}{2} \to \frac{3\pi}{2}\right)$

Nyquist plot,



At $\omega = \sqrt{15}$, $|GH| = \frac{1}{15 \times 8} = \frac{1}{120} = 0.0083$

Using Nyquist Criterion, $N = P - Z$

P = No. of poles of $G(s)H(s)$ in R.H.P.

$N = 0 - Z$

$N = -Z$

If $\frac{K}{120} < 1 \Rightarrow N = 0$

$K < 120$

$Z = 0$

∴ System is stable for $K < 120$

At marginal stability, frequency of oscillation

$$\omega = \sqrt{15} = 3.87 \text{ rad/sec}$$

and gain $K_{mar} = 120$

❍❍❍❍