MADE EASY

Leading Institute for ESE, GATE & PSUs

Detailed Solutions

ESE-2025 Mains Test Series

Mechanical Engineering Test No : 4

Section A : Theory of Machines + Industrial and Maintenance Engineering

1. (a)

Given : $w = 1500$ N; $I_w = 25$ kgm^2; $a = 1.6$ m; $N_m = 3\,N_w$; $r_w = 0.42$ m; $I_m = 18$ kgm^2; $h = 1$ m; $R = 250$ m

Angular velocity of wheel, $\omega_w = \dfrac{2\times\pi\times N_w}{60} = \dfrac{\pi N_w}{30}$

So, $\omega_m = \dfrac{3\pi N_w}{30}$

and $V_w = \omega_w r_w = \dfrac{\pi N_w}{30}\times 0.42 = 0.04398\,N_w$ m/s

Angular velocity of precession,

$$\omega_p = \frac{V_w}{R} = \frac{\pi N_w}{30}\times\frac{0.42}{250}$$

$$= 1.7593\times10^{-4}\,N_w \text{ rad/s}$$

Gyroscopic couple due to four wheels

$$C_w = 4I_w\omega_w\omega_p$$

$$= 4\times25\times\left(\frac{\pi N_w}{30}\right)(1.7593\times10^{-4})\,N_w$$

$$= 18.4233\times10^{-4}\,N_w^2 \text{ Nm}$$

Note : In question it is given that each axle driven by motors means these are two motors.

Gyroscopic couple due to motor,

$$C_m = 2I_m\omega_m\omega_p$$

$$= 2\times 18\times\left(\frac{3\times\pi\times N_w}{30}\right)\times 1.7593\times 10^{-4}N_w$$

$$= 19.897 \times 10^{-4} N_w^2 \text{ Nm}$$

$$\text{Total gyroscopic couple, } C = C_w - C_m$$

$$= -1.474 \times 10^{-4} N_w^2 \text{ Nm}$$

Negative sign has been taken because the speed of motor is opposite to that of wheels. Vertical reaction at each of the outer or inner wheels due to gyroscopic effect.

$$\frac{P}{2} = \pm\frac{C}{2a}$$

$$= \pm\frac{(-1.474)\times 10^{-4}N_w^2}{2\times 1.6} = \pm 0.46\times 10^{-4}N_w^2 N$$

$$\text{Centrifugal couple, } C_c = \frac{WV_w^2 h}{gR} = \frac{1500(0.04398N_w)^2\times 1}{9.81\times 250}$$

$$= 1.183 \times 10^{-3} N_w^2 \text{ Nm}$$

Vertical reaction at each outer or inner wheel due to centrifugal effect.

$$\frac{Q}{2} = +\frac{C_c}{2a} = \frac{1.183\times 10^{-3}N_w^2}{2\times 1.6}$$

$$= 3.6968 \times 10^{-4} N_w^2 \text{ N}$$

For no wheels to leave the rails

$$W = 2(P+Q) = 4\left(\frac{P}{2}+\frac{Q}{2}\right)$$

$$1500 = 4\left(-0.46\times 10^{-4}N_w^2 + 3.6968\times 10^{-4}N_w^2\right)$$

$$375 = 3.2368 \times 10^{-4} N_w^2$$

$$N_w = 1076.36 \text{ rpm}$$

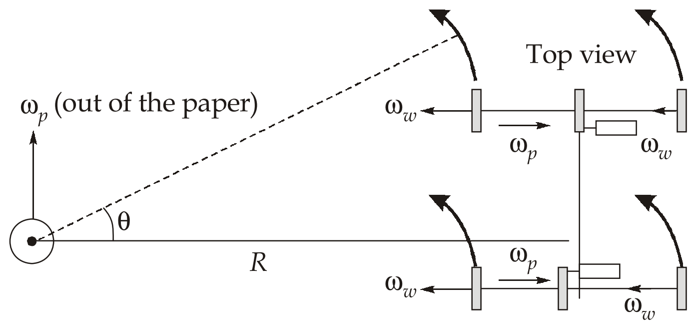
$$V_w = 0.04398\, N_w = 47.338 \text{ m/s}$$

$$V_w = 170.42 \text{ km/hr}$$

Alternatively,

Given : $W = 1500$ N; $I_w = 25$ kgm^2; $r_w = 0.42$ m; $\omega_m = 3\omega_w$;

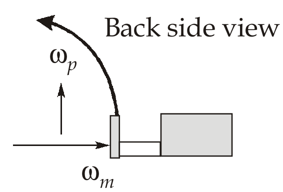
$I_m = 18$ kgm^2; $H = 1$ m; $R = 250$ m

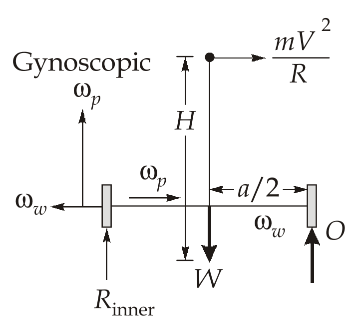


$$\frac{d\theta}{dt} = \omega_p = \frac{V}{R}$$

$$\omega_w = \frac{V}{r_w}$$

$$\omega_w = 3\omega_w = \frac{3V}{r_w}$$





For no wheels in air, the limiting condition is when the reaction on inner type becomes zero.

Taking moment around point (O).

$$W \times \frac{a}{2} = \frac{mv^2}{R} \times H + 4I_w\omega_w\omega_p - 2I_m\omega_m\omega_p$$

$$\frac{1500}{2} \times 1.6 = \frac{V^2}{250}\left[\frac{1500}{9.81} + \frac{4 \times 25 - 2 \times 3 \times 18}{0.42}\right]$$

$$V = 47.34 \text{ m/s} = 170.43 \text{ km/hr}$$

1. (b)

(a) **Plant management in maintenance work :** The main role of maintenance function is to provide safe and effective operation of the equipment to achieve the desired targets on time with economically usage of resources.

(b) **Production and maintenance objectives :** Minimizing the idle time of maintenance workers.

- Maximizing the efficient use of work tool, machines and equipment.
- Maintaining the operating equipment at a responsive level to the need to production in terms of delivery schedule and quality.

(c) **Establishment of work order and recording system :** The work order for the maintenance function indicates the nature of work to be performed and series of operations to be followed to execute a particular job. It is necessary to maintain the proper records and entries to monitor the maintenance functions.

(d) **Information based decision making :** The maintenance objectives are successfully achieved by the use of reliable information system. This system is used to meet the man power and spare parts requirements of the industry.

(e) **Adherence to planned maintenance strategy :** A sound maintenance management should be adhere to the planned maintenance strategy. This also includes use and maintenance schedules of the equipment and other material resources available.

(f) **Planning of maintenance functions :** All the maintenance functions are to be carefully executed by a way of proper planning to ensure the effectiveness of utilization of man power and materials.

(g) **Manpower for maintenance :** The manpower requirement of the maintenance system must be carefully evaluated based on time and motion study. The requirements should also satisfy the need arising in cases of overhauls component replacement, emergency and the un-scheduled repairs.

(h) **Work force control :** Determination of the exact work force required to meet the maintenance objectives of the system is difficult task due to the uncertainty of components. Hence the proper control and monitoring of work force are needs to be ensured.

(i) **Role spare parts :** A good maintenance management system requires appropriate tools, so system should have good quality tools and that too available in required quantities to ensure the proper functioning of the maintenance works.

(j) **Training of the maintenance work force :** Training helps the work force to learn about the modern techniques, recent trends in maintenance, knowledge of sophisticated instruments and to check out a strategy to meet the growing demand of the industry.

Objective of planned maintenance activity

- Analysis of repetitive equipment failures.
- Estimation of the maintenance costs and evaluation of alternatives.
- Forecasting of the spare parts.
- Assessing the needs for equipment replacement and the establish replacement programs, when due application of scheduling and project management principles to replacement programs.
- Assessing skills required for maintenance personnel.
- Reviewing personnel transfers to and from maintenance organization assessing and reporting safety hazards associated with maintenance of equipment.

1. (c)

Given : Present plant capacity, $n = 5000$; $S = 3 \times 10^7$; $\nu = ₹4500$; $F = 0.5 \times 10^7$

$$\text{Profit} = (s - \nu)n - F$$

$$s = \frac{S}{n} = 6000$$

$$P = (6000 - 4500) \times 5000 - 0.5 \times 10^7$$

$$P = ₹0.5 \times 10^7$$

$$\text{Break even point} = \frac{F}{s - v} = \frac{0.5 \times 10^7}{6000 - 4500} = \frac{10000}{3}$$

(i) $$\text{Full capacity, } n' = \frac{5000}{0.8} = 6250 \text{ units}$$

$$\text{Fixed cost, } F' = 0.5 \times 10^7 + 0.08 \times 10^7 = 0.58 \times 10^7$$

$$v' = 4500 - 800 = 3700 \text{ per unit}$$

$$\therefore \quad P = (6000 - 3700)6250 - 0.58 \times 10^7$$

$$P = -₹0.8575 \times 10^7$$

Thus proposal is more economical since profit increased.

(ii) $$\text{Selling price, } s'' = 6000 - 500 = 5500$$

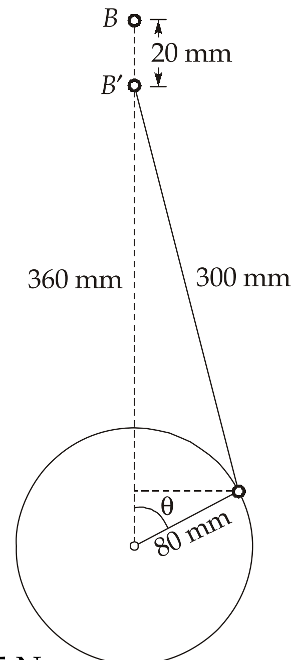
$$90\% \text{ of break even capacity, } n'' = \frac{0.9 \times 10000}{3} = 3000 \text{ units}$$

$$\therefore \quad \text{Profit} = [5500 - 4500] \times 3000 - 0.5 \times 10^7 = -0.2 \times 10^7$$

∴ This proposal is not better than the previous proposal.

1. (d)

Given : $r = 0.08$ m; $l = 0.3$ m; $N = 2000$ rpm; $m = 2$ kg; $d = 0.09$ m



$$n = \frac{l}{r} = \frac{0.3}{0.08} = 3.75$$

$$\omega = \frac{2\pi \times 2000}{60} = 209.44 \text{ rad/s}$$

$$\cos\theta = \frac{360^2 + 80^2 - 300^2}{2 \times 360 \times 80}$$

$$\theta = 37°$$

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 37}{3.75}$$

$$\beta = 9.235°$$

Force due to gas pressure, F_p = Pressure × Area

$$F_p = \frac{\pi}{4} \times (0.09)^2 \times 1000 \times 10^3 = 6361.725 \text{ N}$$

Accelerating force, $F_b = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$= 2 \times 0.08 \times (209.44)^2\left(\cos 37 + \frac{\cos 2 \times 37}{3.75}\right)$$

$$= 6121.035 \text{ N}$$

(i) Force on the piston, $F = F_p + mg - F_b$

$$F = 6361.725 + (2 \times 9.81) - 6121.035$$

$$= 260.31 \text{ N}$$

(ii) Thrust in connecting rod,

$$F_c = \frac{F}{\cos\beta} = \frac{260.31}{\cos 9.235} = 263.73 \text{ N}$$

(iii) Thrust on the sides of cylinder walls,

$$F_n = F\tan\beta = 42.32 \text{ N}$$

(iv) The above values are zero at the speed when the force on the piston F is zero.

$$F = F_p - mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + mg$$

$$0 = 6361.725 - 2 \times 0.08 \times \omega^2 \left(\cos 37 + \frac{\cos(2 \times 37)}{3.75}\right) + 2 \times 9.81$$

$$0.1395 \times \omega^2 = (6361.7 + 2 \times 9.81)$$

$$\omega = 213.85$$

$$N = \frac{60 \times 213.85}{2\pi} = 2042.1 \text{ rpm}$$

1. (e)

Given : $\phi = 20°$; $VR = 9$, $C = 280$ mm; $P = 400$ kW; $P_r = 1$ kN/mm of width; $N_p = 2000$ rpm

$$VR = \frac{N_P}{N_G} = \frac{T_g}{T_p} = \frac{d_g}{d_p} = 9$$

$$C = \frac{d_p + d_g}{2} = d_p + d_g = 2 \times 280 = 560 \text{ mm}$$

$$9d_p = d_g$$

$$10d_p = 560$$

$$d_p = 56 \text{ mm}$$

$$d_g = 504 \text{ mm}$$

Minimum number of teeth on gear wheel,

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

$$= \frac{2 \times 1}{\sqrt{1 + \frac{1}{9}\left(\frac{1}{9} + 2\right)\sin^2 20°} - 1} = 146.769$$

(i) Number of teeth on pinion $= \dfrac{146.769}{9} = 16.3 \simeq 17(say)$

$\therefore$ Number of teeth on gear $= 17 \times 9 = 153$

Now, $m = \dfrac{d_p}{t} = \dfrac{56}{17} = 3$ mm

Exact $d_p = mT_p = 3 \times 17 = 51$ mm

and $d_g = mT_g = 3 \times 153 = 459$ mm

Exact centre distance, $C = \frac{d_p + d_g}{2} = \frac{51 + 459}{2} = 255 \text{ mm} << 280 \text{ mm}$

Now we take $T_p = 19$

then $d_p = 19 \times 3 = 57$

$d_g = 3 \times 171 = 513$

$$C = \frac{d_p + d_g}{2} = 285 \text{ mm}$$

which is more closer to 280 mm

So we will take $T_p = 19$ and $T_g = 171$

(ii) $P = \frac{2\pi NT}{60}$

or $400 \times 10^3 = \frac{2\pi \times 2000T}{60}$

$$T = 1909.86 \text{ Nm}$$

$$\text{Tangential force} = \frac{1909.86 \times 10^3}{\left(\frac{57}{2}\right)} = 67012.63 \text{ N}$$

$$\text{Normal pressure on the tooth} = \frac{F}{\cos\phi} = 71313.35 \text{ N}$$

$$\text{Width of pinion} = \frac{F_n}{\text{Limiting normal pressure}}$$

$$= \frac{71313.35}{1000} = 71.314 \text{ mm}$$

2. (a)

Order of machines is given, M1, M3, M2. In order to find optimal sequence,

$$(\text{Minimum Time})_{M1} \geq (\text{Maximum Time})_{M3}$$

or $(\text{Minimum Time})_{M2} \geq (\text{Maximum Time})_{M3}$

As second condition is satisfied. This problem can be replaced by an equivalent problem involving '*n*' jobs and two machines (fictitious) denoted by *G* and *H*.

$$G_i = M1 + M3$$

$$H_i = M3 + M2$$

Hence,

Spare part	1	2	3	4	5
G_i	14	7	13	13	9
H_i	16	15	20	17	16

Optimal sequence by Johnson's rule is

2	5	4	3	1

Job	M1			M3			M2		
	Time In	Machining Time	Time Out	Time In	Machining Time	Time Out	Time In	Machining Time	Time Out
2	0	5	5	5	2	7	7	13	20
5	5	5	10	10	4	14	20	12	32
4	10	6	16	16	7	23	32	10	42
3	16	4	20	23	9	32	42	11	53
1	20	8	28	32	6	38	53	10	63
		28			28			56	

Total elapsed time = 63 hours

Idle time on M1 = 63 – 28 = 35 hours

Idle time on M3 = 63 – 28 = 35 hours

Idle time on M2 = 63 – 56 = 7 hours

As P1 starts at 0^{th} hour and ends at 28^{th} hour (28 hours)

P2 starts at 7^{th} hour and ends at 63^{rd} hour (56 hours)

P3 starts at 5^{th} hour and ends at 38^{th} hour (33 hours)

Payment for P1 = 28×10 = Rs. 280 (all working hours)

Payment for P3 = $28 \times 10 + 5 \times 5$

= Rs. 305 (28 working hours and 5 waiting hours)

Payment for P2 = 56×10 = Rs. 560 (all working hours)

If optimal sequence is,

2	5	3	4	1

Job	M1			M3			M2		
	Time In	Machining Time	Time Out	Time In	Machining Time	Time Out	Time In	Machining Time	Time Out
2	0	5	5	5	2	7	7	13	20
5	5	5	10	10	4	14	20	12	32
3	10	4	14	14	9	23	32	11	43
4	14	6	20	23	7	30	43	10	53
1	20	8	28	30	6	36	53	10	63
		28			28			56	

Total elapsed time = 63 hours

Idle time on M1 = 63 – 28 = 35 hours

Idle time on M3 = 63 – 28 = 35 hours

Idle time on M2 = 63 – 56 = 7 hours

2. As P1 starts at 0^{th} hour and complete the task in 28^{th} hour. So, 28 hour total run time and no waiting time.

As P2 (machine M2) starts at 7^{th} hour and complete the task in 63^{th} hour. So 56 hours run time and no waiting time.

As P3 (machine M3) starts at 5^{th} hour and complete the task in 36^{th} hour. So total run time 28^{th} hour and waiting time 3 hour.

3. Payment for P1 = 28 × 10

= ₹ 280/- (All working hour)

Payment for P2 = 56 × 10

= ₹ 560/- (All working hour)

Payment for P3 = 28 × 10 + 3 × 5

= ₹ 295/- (28 working hour and 3 waiting hour)

So sequence

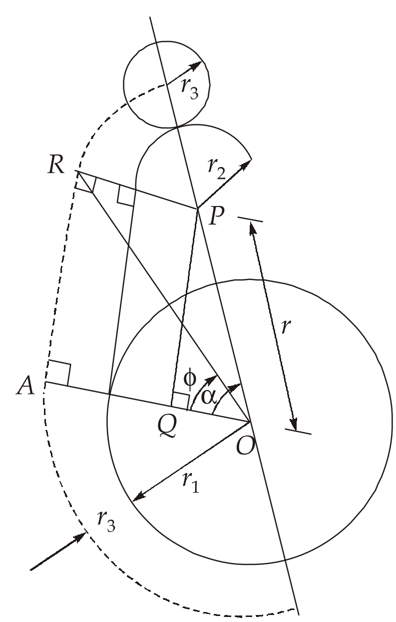
2	5	3	4	1

is more iconomical and optimal sequence.

2. (b)

Given : r_1 = 45 mm; r_3 = 25 mm; h = 25 mm; N = 360 rpm; α = 75°

(i) From the figure



$$OP + r_2 = r_1 + h = 45 + 25$$

$$OP = 70 - r_2$$

$$OQ + r_2 = r_1 = 45$$

$$OQ = 45 - r_2$$

From ΔOQP,
$$\cos\alpha = \frac{OQ}{OP}$$

$$\cos 75° = \frac{45 - r_2}{70 - r_2}$$

Nose radius, r_2 = 36.27 mm

Distance between cam and nose centre,

$$r = OP = 70 - 36.27$$

$$r = 33.73$$

$$\tan\phi = \frac{OP\sin\alpha}{OA} = \frac{33.73\sin 75°}{70}$$

$\Rightarrow$ $$P = 24.96°$$

(ii) Equation of displacement curve

1. When contact is with straight flank

$$x = (r_1 + r_3)\left(\frac{1}{\cos\theta} - 1\right)$$

$$x = (45 + 25)\left(\frac{1}{\cos\theta} - 1\right) = 70\left(\frac{1}{\cos\theta} - 1\right) \text{ mm}$$

2. When contact is with circular nose

$$l = r_2 + r_3$$

$$x = r(1 - \cos\theta_1) + (r_2 + r_3) - \left(l^2 - r^2\sin\theta_1\right)^{0.5}$$

$$= 33.73(1 - \cos\theta_1) + (36.27 + 25) - \sqrt{(61.27)^2 - (33.73)^2\sin^2\theta_1}$$

$$= 33.73(1 - \cos\theta_1) + 61.27 - \sqrt{3754 - (33.73)^2\sin^2\theta_1}$$

where θ_1 is measured from the apex position.

(iii) Acceleration of the follower

When in contat with straight flank

$$f = \omega^2(r_1 + r_3)\left[\frac{2 - \cos^2\theta}{\cos^3\theta}\right]$$

$$\omega = \frac{2 \times \pi \times 360}{60} = 37.7 \text{ rad/s}$$

when $\theta = 0$

$$f = (37.7)^2(0.045 + 0.025)\left[\frac{2-1}{1}\right]$$

$$f = 99.5 \text{ m/s}^2$$

When in contact with straight flank, $\theta = \phi$

$$f = \omega^2(r_1 + r_3) \times \left[\frac{2 - \cos^2\phi}{\cos^3\phi}\right]$$

$$= (37.7)^2 (0.07) \times \left[\frac{2 - \cos^2 24.96}{\cos^3 24.96}\right]$$

$$= 157.3 \text{ m/s}^2$$

When contact is an circular nose,

$$\theta_1 = \alpha - \phi$$

$$= 75 - 24.96$$

$$= 50.04°$$

$$f = \omega^2 r\left[\cos\theta_1 + \frac{\left(l^2 r\cos 2\theta_1 + r^3 \sin^4\theta_1\right)}{\left(l^2 - r^2\sin^2\theta_1\right)^{1.5}}\right]$$

$$f = (37.7)^2 (0.03373)\left[\cos(50.04) + \frac{(61.27)^2 \times (33.73)\cos(100.08) + (33.73)^3 \sin^4(50.04)}{\left[(61.27)^2 - (33.73)^2 \sin^2(50.04)\right]^{1.5}}\right]$$

$f = 28.3$ m/s^2

When at apex

$$\theta_1 = 0°$$

$$f = \omega^2 r\left[1 + \frac{r}{l}\right]$$

$$f = (37.7)^2 (0.03373)\left[1 + \frac{0.03373}{0.06127}\right]$$

$$f = 74.33 \text{ m/s}^2$$

2. (c)

Universal joint or Universal coupling/Hook's joint.

- This joint is basically used to connect shafts which are non parallel but coplanar (intersecting shaft).
- This joint gives variable velocities ratio i.e. driven shaft angular velocity is continuously changing with respect to the angle turned by driver shaft but mean velocity of driven shaft during one revolution is exactly same as the mean velocity of driving shaft during one revolution. $(\omega_1)_{avg} = (\omega_2)_{avg.}$
- The time taken by driver shaft to complete one revolution with constant angular velocity is exactly same as time taken by driven shaft to complete one revolution with variable angular velocity.

1-Driver

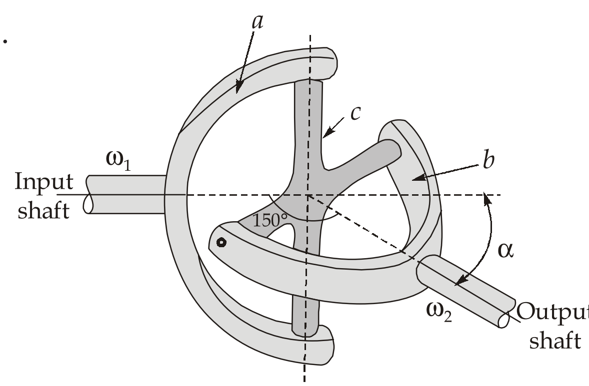
2-Driven

α - Angle between shafts.

ω_1 - Angular velocity of driver (constant).

ω_2 - Angular velocity of driven (variable).

θ - Angle turned by driver.



ϕ - Angle turned by driven.

Fundamental equation,

$$\tan\theta = \cos\alpha \tan\phi$$

$$\tan\phi = \frac{\tan\theta}{\cos\alpha}$$

Differentiating both side,

$$\cos\alpha \cdot \sec^2\phi \frac{d\phi}{dt} = \sec^2\theta \frac{d\theta}{dt} \qquad \because \left(\frac{d\phi/dt}{d\theta/dt}\right) = \left(\frac{\omega_2}{\omega_1}\right)$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\alpha \cdot \sec^2\phi} = \frac{1}{\cos^2\theta \cos\alpha\left(1+\tan^2\phi\right)} \qquad \because \left(\tan\phi = \frac{\tan\theta}{\cos\alpha}\right)$$

$$= \frac{1}{\cos^2\theta \cos\alpha\left[1+\frac{\tan^2\theta}{\cos^2\alpha}\right]}$$

$$= \frac{1}{\cos^2\theta \times \cos\alpha\left[1+\frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}\right]}$$

$$= \frac{\cos\alpha}{\cos^2\theta\cos^2\alpha + \sin^2\theta} = \frac{\cos\alpha}{\cos^2\theta\left(1-\sin^2\alpha\right)+\sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}$$

$$\omega_2 = \frac{\omega_1\cos\alpha}{1-\cos^2\theta\sin^2\alpha} \qquad \text{... (i)}$$

As per given information, $\alpha = 180° - 150° = 30°$

ω_1 = Constant (uniform)

$$N_1 = 120 \text{ rpm}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 120}{60} = 12.566 \text{ rad/s}$$

$$T_{steady} = 135 \text{ Nm}$$

Flywheel mass, m = 45 kg

Radius of gyration, k = 0.15 m

Moment of inertia, $I_2 = mk^2$

$$I_2 = 45 \times 0.15^2$$

$$I_2 = 1.0125 \text{ kg.m}^2$$

For angular acceleration of driven shaft to be maximum,

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha} = \frac{2\times(\sin 30°)^2}{2-(\sin 30°)^2}$$

$$\cos 2\theta = 0.285714$$

$$2\theta = 73.398°,\ 286.602°$$

$$\theta = 36.699°, 143.301°$$

$$(\alpha_2) = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{\left(1-\cos^2\theta \sin^2\alpha\right)^2}$$

$$(\alpha_2)_{(\theta_1 = 36.699°)} = \frac{-(12.566)^2 \cos 30°(\sin 30°)^2 \sin(2\times 36.699°)}{\left[1-(\cos 36.699°)^2 \times (\sin 30°)^2\right]^2}$$

$$= \frac{-32.762}{0.7044} = -46.51 \text{ rad/s}^2$$

$$(\alpha_2)_{(\theta_2 = 143.301°)} = \frac{-(12.566)^2 \cos 30°(\sin 30°)^2 \sin(2\times 143.301°)}{\left(1-(\cos 143.301°)^2 (\sin 30°)^2\right)^2}$$

$$= \frac{+32.762}{0.7044} = 46.51 \text{ rad/s}^2$$

Inertia torque at driven at θ_1 = $I_2\alpha_2 = 1.0125 \times (-46.51)$

$= -47.098$ Nm

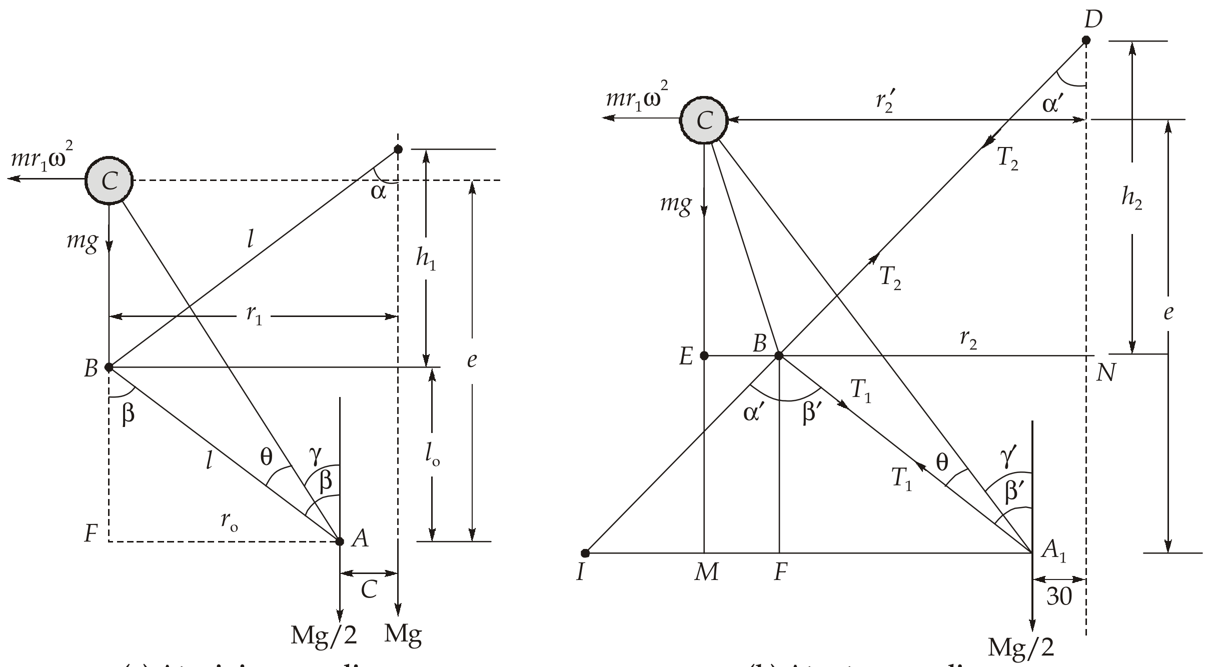
At θ_2, Inertia torque = $I_2\alpha_2 = 1.0125 \times (46.51)$

$= 47.092$ Nm

Maximum torque at driven,

$$(T_2)_{\max \text{ at } \theta = \theta_2} = (T_2)_{steady} + (T)_{inertia\ torque}$$
$$= 135 \text{ Nm} + 47.092 \text{ Nm}$$
$$= 182.092 \text{ Nm}$$

3. (a)



(a) At minimum radius **(b) At extreme radius**

Given : $m = 6$ kg; $M = 60$ kg; $l = 300$ mm; $c = 30$ mm; $BC = 100$ mm; $r_1 = 150$ mm; $r_2 = 200$ mm

$$AF = r_1 - c = 150 - 30 = 120 \text{ mm}$$

$$BF = l_o = \left(l^2 - r_o^2\right)^{1/2} = \left(300^2 - 120^2\right)^{1/2} = 274.96$$

$$h_1 = \left(l^2 - r_1^2\right)^{1/2} = \left(300^2 - 150^2\right)^{1/2} = 259.8 \text{ mm}$$

$$\tan\alpha = \frac{r_1}{h_1} = \frac{150}{259.8} = 0.577$$

$$\tan\beta = \frac{r_o}{l_o} = \frac{120}{274.96} = 0.436$$

$$k = \frac{\tan\beta}{\tan\alpha} = \frac{0.436}{0.577} = 0.755$$

$$CF = CB + BF = 100 + 274.96 = 374.96 \text{ mm}$$

$$\omega_1^2 = \left(\frac{BF}{CF}\right)\left(\frac{g}{h_1}\right)\left[1+(1+k)\frac{M}{2m}\right]$$

$$= \left(\frac{274.96}{374.96}\right)\left(\frac{9.81\times10^3}{259.8}\right)\left[1+(1+0.755)\frac{60}{2\times6}\right]$$

$$\omega_1^2 = 270.66$$

$$\omega_1 = 16.45 \text{ rad/sec}$$

$$N_1 = \frac{16.45\times60}{2\pi} = 157.10 \text{ rpm}$$

$$h_2 = \left(l^2 - r_2^2\right)^{1/2} = \left(300^2 - 200^2\right)^{1/2} = 223.6 \text{ mm}$$

$$\tan\alpha' = \frac{r_2}{h_2} = \frac{200}{223.6} = 0.894$$

$$\alpha' = 41.80°$$

$$r_o = r_2 - c = 200 - 30 = 170 \text{ mm}$$

$$\text{BF} = \left(l^2 - r_o^2\right)^{1/2} = \left(300^2 - 170^2\right) = 247.18 \text{ mm}$$

$$\tan\beta' = \frac{r_o}{BF} = \frac{170}{247.18} = 0.688$$

$$\beta' = 34.53°$$

$$k = \frac{\tan\beta'}{\tan\alpha'} = 0.77$$

$$\omega_2^2 = \left(\frac{BF}{CF}\right)\left(\frac{g}{h_2}\right)\left(1+(1+k)\frac{M}{2m}\right)$$

$$= \frac{247.18}{347.18}\times\frac{9.81\times10^3}{223.6}\left(1+(1.77)\frac{60}{12}\right)$$

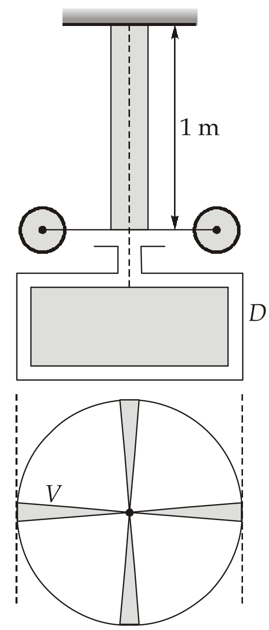
$$\omega_2^2 = 307.67$$

$$\omega_2 = 17.54$$

$$N_2 = 167.50 \text{ rpm}$$

3. (b)

As per given information,



$$I = 25 \text{ kg.m}^2, G = 85 \text{ GPa}, \dot{\theta} = 1 \text{ rad/s}$$

$$d = 2.54 \text{ cm} = 0.0254 \text{ m}$$

$$L = 1 \text{ m}$$

$$X_3 = \frac{1}{20} X_0$$

$$\frac{X_0}{X_3} = 20$$

$$\frac{X_0}{X_1} = \frac{X_1}{X_2} = \frac{X_2}{X_3}$$

$$\left(\frac{X_0}{X_1}\right)^3 = 20$$

$$\frac{X_0}{X_1} = (20)^{1/3}$$

$$\ln\left(\frac{X_0}{X_1}\right) = \ln(20)^{1/3}$$

$$\frac{2\pi\xi}{\sqrt{1-\xi^2}} = \ln(20)^{1/3} = 0.99857$$

$$2\pi\xi = 0.99857\sqrt{(1-\xi^2)}$$

$$39.59156\xi^2 = 1 - \xi^2$$

$$\xi = 0.15695$$

$$\omega_n = \sqrt{\frac{q}{I}} = \sqrt{\frac{GJ}{LI}} = \sqrt{\frac{85\times10^9\times\pi\times0.0254^4}{32\times1\times25}}$$

$$= 11.787 \text{ rad/s}$$

$$2\xi\omega_n = \frac{C}{I}$$

$$C = 2\xi\omega_n I = 2 \times 0.15695 \times 11.787 \times 25$$

$$C = 92.4985 \left(\frac{\text{Nm}}{\text{rad/s}}\right)$$

Magnitude of damping torque = $C.\dot{\theta} = 92.4985 \times 1 = 92.4985$ Nm

$$\text{Damped frequency} = \omega_n\sqrt{1-\xi^2} = 11.787\sqrt{1-0.15695^2}$$

$$= 11.641 \text{ rad/s}$$

$$\text{Ratio } \frac{\omega_d}{\omega_n} = \frac{11.641}{11.787} \times 100\% = 98.76\%$$

3. (c)

Wear debris analysis is the analysis of two things.

1. **Wear particles (contaminants) get deposited in the oil**: Concentration of wear particle, nature of the wear particles and the rate at which it is deposited in the oil gives us some clue about machine condition.
2. **Oil undergo change in its physical and chemical properties**: The properties of virgin oil will get change when machine is running.
   - Unlike in vibration monitoring and noise monitoring, in wear debris analysis, you have to collect the oil in which contaminant is there and given this sample to the lab for analysis. In wear debris analysis In-situ analysis is not possible.
   - Quality and quantity of wear metals allows to set alarm level.
   - Knowledge of metallurgical composition is helpful in localizing source of wear metal production.

Wear debris characteristics:

1. Quantity (it tells about the severity of machine component).
2. Size of the particle.
3. Morphology (shape/structure).
4. Composition (It tells about the source i.e. parent material from where wear takes place). If lets of silica/sand grain is found in contaminants, it means that let of foreign dirt has gone into the machine.

Wear Mechanism:

1. Abrasive wear
2. Adhesive wear
3. Diffusion wear
4. Oxidation/corrosion

Wear mode:

1. Running in (during infant mortality zone of bathtub curve).
2. Steady wear (during the useful life of machine).
3. Wear out (towards the end of the life of the machine).
4. Pitting (it is the surface failure of a material as a result of stresses that exceed the endurance limit of the material).
5. Scuffing (due to insufficient lubrication between the mating part).

Wear Debris analysis method:

1. Ferrography:

i. Analytical ferrography

ii. Direct reading ferrography

2. Spectrophotometric technique:

i. Atomic absorption spectroscopy (AAS)

ii. Atomic emission spectroscopy (AES)

iii. X-ray fluorescence (XRF)

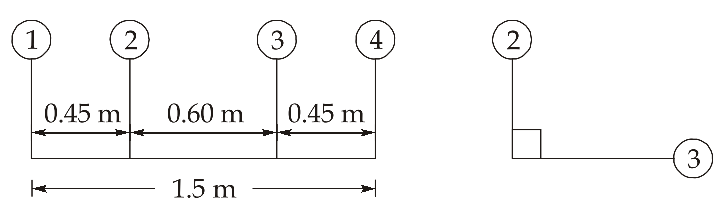
iv. Inductively coupled plasma (ICP)

v. Direct current plasma (DCP)

vi. Energy dispersive x-ray analysis(EDX)

4. (a)

$$\text{Mass to be balanced} = 350 + \left(\frac{1}{2} \times 400\right) = 550 \text{ kg}$$



(i)

Plane	M (kg)	r (m)	Mr	θ	$Mr\cos\theta$	$Mr\sin\theta$	l	Mrl	$Mrl\cos\theta$	$Mrl\sin\theta$
M_2	550	0.4	220	0	220	0	0.45	99	99	0
M_3	550	0.4	220	90°	0	220	1.05	231	0	231
M_1	M_1	0.6	$0.6M_1$	θ_1	$0.6M_1 \times \cos\theta_1$	$0.6M_1 \times \sin\theta_1$	0	0	0	0
M_4	M_4	0.6	$0.6M_4$	θ_4	$0.6M_4 \times \cos\theta_4$	$0.6M_4 \times \sin\theta_4$	1.5	$0.9M_4$	$0.9M_4 \times \cos\theta_4$	$0.9M_4 \times \sin\theta_4$

For table,

$$\sum M_i r_i \cos\theta_i = 220$$

$$\sum M_i r_i \sin\theta_i = 220$$

$$\sum M_i r_i l_i \cos\theta_i = 99$$

$$\sum M_i r_i l_i \sin\theta_i = 231$$

$$\left[\left(\sum M_i r_i l_i \cos\theta_i\right)^2 + \left(\sum M_i r_i l_i \sin\theta_i\right)^2\right]^{1/2} = M_4 r_4 l_4$$

$$\sqrt{(99)^2 + (231)^2} = 0.9 M_4$$

$$M_4 = 279.245 \text{ kg}$$

$$\tan\theta_4 = \frac{-\sum M_i r_i l_i \sin\theta_i}{-\sum M_i r_i l_i \cos\theta_i} = \frac{-231}{-99} = \frac{7}{3} = 2.33$$

$$\theta_4 = 180° + 66.8 = 246.8° \text{ (CCW)}$$

Since numerator and denominator are both negative.

$\therefore$ θ_4 lies in the third quadrant.

$$M_1 r_1 = \left[\left(\sum M_i r_i \cos\theta_i + \sum M_4 r_4 \cos\theta_4\right)^2 + \left(\sum M_i r_i \sin\theta_i + \sum M_4 r_4 \sin\theta_4\right)^2\right]^{1/2}$$

$$0.6 M_1 = \left[(220 + 0.6 \times 279.245 \cos 246.8)^2 + (220 + 0.6 \times 279.245 \sin 246.8)^2\right]^{1/2}$$

$$M_1 = 279.24 \text{ kg}$$

$$\tan\theta_1 = \frac{-\left(\sum M_i r_i \cos\theta_i + \sum M_4 r_4 \sin\theta_4\right)}{-\left(\sum M_i r_i \cos\theta_i + \sum M_4 r_4 \cos\theta_4\right)}$$

$$\tan\theta_1 = \frac{-66}{-153.99} = 0.428$$

$$\theta_1 = 180 + 23.199 = 203.199°$$

Since the numerator and denominator are both negative, therefore θ_1 lies in the third quadrant.

$$v = 54 \times \frac{5}{18} = 15 \text{ m/s}$$

$$w = \frac{v}{r} = 16.667 \text{ rad/s}$$

(ii) Swaying couple $= \left[\frac{1-C}{\sqrt{2}}\right] \times Rr\omega^2 l$

$$= \left[\frac{1-\frac{1}{2}}{\sqrt{2}}\right] \times 350 \times 0.4 \times (16.667)^2 \times 0.6 = 8249.9 \text{ Nm}$$

(iii) Variation in tractive effort $= \pm\sqrt{2}(1-C)R\omega^2 r$

$$= \pm\sqrt{2}\left(1-\frac{1}{2}\right) \times 350 \times (16.667)^2 (0.4)$$

$$= \pm 27499.70 \text{ N}$$

(iv) Balance mass for the reciprocating parts only

$$R_1 = 279.245 \times \frac{1}{2} \times \frac{400}{550} = 101.54 \text{ kg}$$

Hammer below $= R_1 b\omega^2 = 101.54 \times 0.6 \times (16.667)^2$

$$= 16924.616 \text{ N}$$

Dead weight = 40 kN

Maximum pressure on rails = 40000 + 16924.616

= 56924.616 N

Minimum pressure on rails = 40000 – 16924.616

= 23075.38 N

(v) Let w_1 be the speed then,

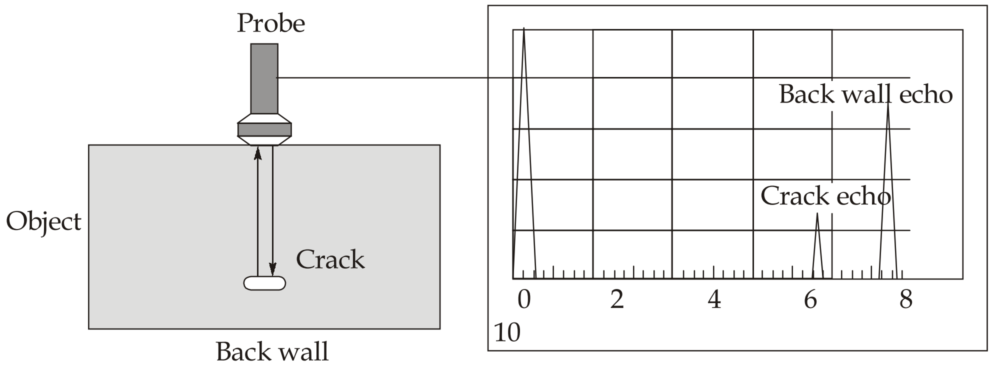
$$101.54 \times 0.6 \times \omega_1^2 = 40000$$

$$\omega_1 = 25.6 \text{ rad/s}$$

$$v = 25.6 \times 0.9 \times \frac{3600}{1000} = 83.019 \text{ km/h}$$

4. **(b)**

(i) **Ultrasonics :** The ultrasonic principle is based on the fact that solid materials are good conductors of sound waves. Sound waves are not only reflected at the interfaces but also by internal flaws such as cracks. Because the interaction effect of sound waves with the material is stronger for high frequencies, ultrasound wave frequencies used are in the range 0.5-25 MHz.



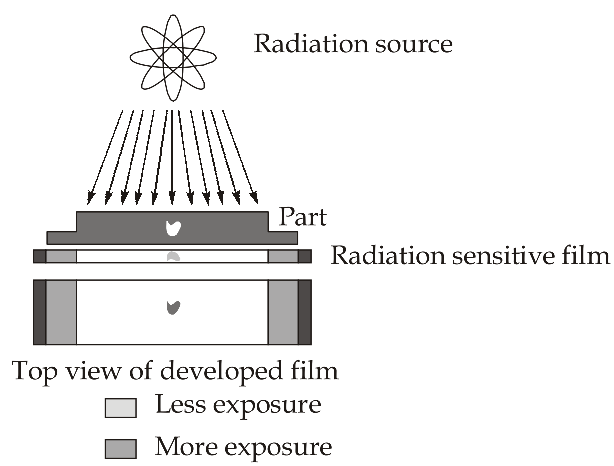
The essential element is the probe. When the pulse produced by the sensor is reflected by a surface or flaw in the scanned material, the same element generates an electrical signal that can be registered on a display screen as shown in figure. The probe is coupled to the surface of the scanned object with a liquid or coupling paste, so that the sound waves from the probe are able to be transmitted into the object. The object is scanned by moving the probe evenly to and fro across the surface. Any signals caused by reflections from internal discontinuities can be observed on an instrument display. The three signals shown in the display in figure correspond to the first surface of the object, the crack, and the back surface or wall, respectively. Radiography and ultrasonic testing are probably the most frequently used methods of testing different objects for internal flaws. NDT ultrasonic testing is used widely to test welded gas and oil pipelines, railway track, and other shaped engineered object pieces.

(ii) **Acoustic emissions :** AE refers to the generation of transient elastic waves during the rapid release of energy from localized sources within a material. The dislocation movement accompanying plastic deformation of material and the initiation and extension of cracks under stress are examples of sources of AE. AE piezoelectric sensors are used for monitoring defects in real time while the phenomenon is taking place. Anomalies that can be detected include corrosion, fatigue cracks, fiber breakage, and cracking in concrete or reinforced concrete structures.

(iii) **Eddy current testing :** When the eddy currents in the tested object are distorted by the presence of flaws or material variations such as cracks, the impedance in the coil is changed. This change is measured and displayed in a manner that indicates the type of flaw or material condition. This method is used to detect small surface flaws in an item and for measuring the thickness of a non-conductive coating of an item.

(iv) Liquid color penetrant testing : This is mainly a surface crack detection technique. It may be applied to all non-porous materials, but only cracks open to the surface can be detected. The surface to be inspected is coated with a film of a special liquid or penetrant. This is drawn into any surface-breaking cracks by capillary action. Then the surface is cleaned with a suitable remover. After this, a layer of developer (chalk powder or similar absorbent material) suspended in a suitable solvent is applied. As this layer dries, it draws the liquid out of the crack and spreads it over a larger area of the surface, making it more visible. The liquid used as the penetrant is frequently colored to contrast with the chalk developer. It is also possible to use a fluorescent penetrant that can be inspected under ultra-violet light. Liquid penetrant inspection is an extension of visual inspection and is used for detecting surface-breaking flaws, such as cracks, laps, and folds on any non-absorbent material's surface.

(v) Radiography : RT is an NDT method of inspecting materials for hidden flaws by using the ability of short-wavelength electromagnetic radiation (X-rays or gamma rays) to penetrate various materials.



The three basic elements of RT are: (i) a radiation source, (ii) the test piece being evaluated, and (iii) a recording medium. These elements are combined to produce a radiograph, as depicted in figure. The intensity of the radiation that penetrates and passes through the material is either captured by a radiation-sensitive film (Film Radiography) or by a planar array of radiation-sensitive sensors (Real-time Radiography).

The underlying principle can be described as follows. The radiation source emits energy that travels in straight lines and penetrates the tested object, producing an image on the recording medium opposite to the X-ray source.

This image is used to evaluate the condition of the object being examined. The image is produced on film or electronically for real-time systems.

RT offers a number of advantages over other NDT methods, including the ability to detect surface and internal discontinuities, the production of a permanent test record, and good portability, especially for gamma-ray sources. However, one of its major disadvantages is the health risk associated with the radiation.

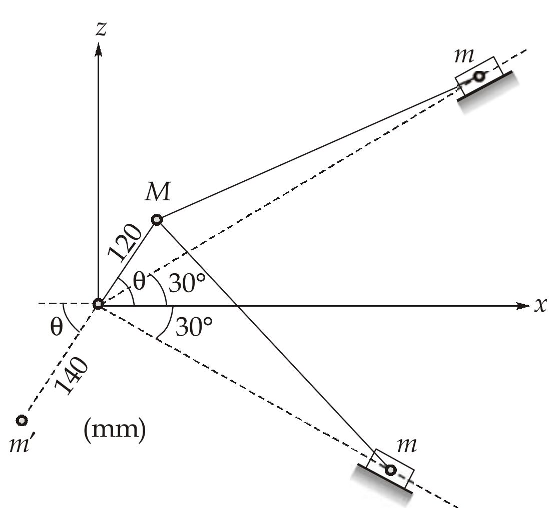
Radiographic testing can be used for detecting internal defects in thick and complex shapes in metallic and non-metallic materials, structures, and assemblies. Applications include detection of surface and subsurface features in welded pipes, corrosion mapping, detection of blockages inside sealed equipment, and so on.

4. (c)

Given : $m = 1.1$ kg; $l = 540$ mm; $r = 120$ mm; $M = 2.2$ kg; $m' = 2.3$ kg; $r' = 140$ mm; N = 900 rpm

$$\omega = \frac{2\times\pi\times N}{60} = \frac{2\times\pi\times 900}{60} = 94.25$$

$$n = \frac{l}{r} = \frac{540}{120} = 4.5$$



Total primary force along x-axis = $2mr\omega^2\cos^2\alpha\cos\theta$ (where $\alpha = 30°$)

Centrifugal force due to rotating mass along x-axis = $Mr\omega^2\cos\theta$

Centrifugal force due to balancing mass along x-axis = $-m'r'\omega^2\cos\theta$

Total unbalanced force along x-axis

$$= 2mr\omega^2\cos^2\alpha\cos\theta + Mr\omega^2\cos\theta - m'r'\omega^2\cos\theta$$

$$= \omega^2\cos\theta\left[2mr\cos^2\alpha + Mr - m'r'\right]$$

$$= (94.25)^2\cos\theta\left[2(1.1)(0.12)\cos^2 30° + 2.2(0.12) - 2.3(0.14)\right]$$

$$= 1243.63\cos\theta \text{ N}$$

In z-direction,

Total primary force along z-direction = $2mr\omega^2\sin^2\alpha\sin\theta$

Centrifugal force due to rotating mass along z-axis = $Mr\omega^2\sin\theta$

Centrifugal force due to balancing mass along z-axis = $-m'r'\omega^2\sin\theta$

Total unbalanced force along z-axis

$$= 2mr\omega^2\sin^2\alpha\sin\theta + Mr\omega^2\sin\theta - m'r'\omega^2\sin\theta$$

$$= \omega^2\sin\theta\left[2mr\sin^2\alpha + Mr - m'r'\right]$$

$$= (94.25)^2\sin\theta\left[2(1.1)(0.12)\sin^2 30° + 2.2(0.12) - 2.3(0.14)\right]$$

$$= 71.0645\sin\theta \text{ N}$$

$$\text{Resultant primary force} = \sqrt{(1243.63)^2\cos^2\theta + (71.0645)\sin^2\theta}$$

$$= \sqrt{1546615.58\cos^2\theta + 5050.163\sin^2\theta}$$

The resultant primary force is maximum when $\theta = 0°$ and minimum when $\theta = 90°$.

Maximum primary force = 1243.63 N

Minimum primary force = 71.0645 N

Secondary force

The rotating masses do not affect the secondary forces as they are only due to second harmonics of the piston acceleration.

$$\text{Resultant secondary force} = \frac{2mr\omega^2}{n}\sqrt{(\cos\alpha\cos 2\theta\cos 2\alpha)^2 + (\sin\alpha\sin 2\theta\sin 2\alpha)^2}$$

$$= \frac{2\times 1.1(0.12)(94.25)^2}{4.5}\sqrt{(\cos 30°\cos 2\theta\cos 60°)^2 + (\sin 30°\sin 2\theta\sin 60°)^2}$$

$$= 521.14\sqrt{(0.433\cos 2\theta)^2 + (0.433\sin 2\theta)^2}$$

Maximum primary force (when $\theta = 0°$) = 521.14 × 0.433 = 225.66 N

Maximum primary force (when $\theta = 90°$) = 521.14 × 0.433 = 225.66 N

Section B : Theory of Machines + Industrial and Maintenance Engineering

5. (a)

Work element	Cycle → 1	2	3	4	5	Average	Rating	Normal time
1	0.5	3.3	5.7	8.2	10.85	5.71	1.1	6.28
2	0.7	3.45	5.95	8.55	11.10	5.95	1.2	7.14
3	1.45	4.05	6.5	9.25	11.75	6.6	1.2	7.92
4	2.75	5.25	7.6	10.35	13.05	7.8	0.9	7.02

Normal time = Average observed time × Rating factor

(i) Normal time of the activity = [6.28 + 7.14 + 7.92 + 7.02] = 28.36

Standard time = Normal time × (1 + Allowance)

= 28.36 × (1 + 0.20) = 34.032

(ii) Standard deviation, $\sigma_P = \sqrt{\dfrac{P(1-P)}{N}}$

N = Number of observations

Given, $\sigma_P = 0.0742 = \sqrt{\dfrac{P(1-P)}{N}}$

Also, $PS = Z.\sigma_P$

where, Z = 1.96 for 95% confidence interval.

s = Accuracy required

$\Rightarrow \quad P \times 0.2 = 1.96 \times 0.0742$

$\Rightarrow \quad P = 0.727$

$\therefore \quad 0.0742 = \sqrt{\dfrac{0.727 \times (1-0.727)}{N}}$

N = 36 observations

5. (b)

Operation	Revolution of			
	Arm A	Gear 1	Gear 2	Gear 3
		100	120	80
1. Arm A fixed, $+x$ revolution or gear 1, CCW		$+x$	$\dfrac{+x \times 100}{120}$	$\dfrac{+x \times 100}{80}$
2. Add y	y	$y + x$	$y + \dfrac{x10}{12}$	$y + \dfrac{x10}{8}$

Now, arm $A = 1$

$y = 1$

Gear 1 is fixed, $y + x = 0$

$x = -1$

Gear 2, $$y + \frac{10x}{12} = 1 - \frac{10}{12} = \frac{1}{6}$$

Gear 3, $$y + \frac{10x}{8} = 1 - \frac{10}{8} = \frac{-1}{4}$$

5. (c)

Given : $N_m = 60$ rpm; $C_s = 1.5\%$; $k = 1.5$ m

The T-θ diagram is shown in figure

Moment of inertia of motor armature and gear wheel $= \dfrac{5 \times 10^3 \times 1^2}{9.81} = 509.68 \text{ kgm}^2$

$$T_m = \frac{1}{\pi}\int_0^{\pi} 8\sin\theta d\theta = \frac{8}{\pi}\left|-\cos\theta\right|_0^{\pi}$$

$$= \frac{8}{\pi} \times 2 = 5.093 \text{ kNm}$$

At points A and B, $8 \sin\theta = 5.093$

$\sin\theta = 0.6366$

$\theta = 39.54 \text{ and } 140.46°$

$$E_f = \int_{39.54}^{140.46} (8\sin\theta - 5.093)d\theta$$

$$= \left|-8\cos\theta - 5.093\theta\right|_{39.54}^{140.46}$$

$$= 8(-\cos 140.46 + \cos 39.54) - 5.093(140.46 - 39.54) \times \frac{\pi}{180}$$

$= 3.368$ kNm

Effective inertia for $c_s = 0.015$

$$I\omega^2 c_s = 3.368 \text{ kNm}$$

$$I = \frac{3.368 \times 1000}{\left(\frac{2\pi \times 60}{60}\right)^2 \times 0.015} = 5687.5 \text{ kgm}^2$$

$$I = I_{fly} + I_{motion}$$

$$I_{fly} = 5687.5 - 509.68$$

$$m = \frac{I_{fly}}{R^2} = 2301.25 \text{ kg}$$

5. (d)

(i) Here, we have:

D = 3000 units; C_0 = ₹15 per order; C = ₹35 per unit

$$C_1 = ₹3 \text{ per unit per year.}$$

$$\therefore \quad Q^\circ = \sqrt{\frac{2DC_0}{C_1}} = \sqrt{\frac{2 \times 3000 \times 15}{3}} = 173.2 \text{ Units}$$

n° = Optimal number of orders

$$= \frac{D}{Q^\circ} = \frac{3000}{173.2} = 17.32$$

(ii) We are given: D = 3000 units

C_0 = ₹240 per production run

C_1 = ₹3 per unit per year

$$\therefore \quad Q^\circ = \sqrt{\frac{2DC_0}{C_1}} \times \sqrt{\frac{k}{k-r}}$$

$$= \sqrt{\frac{2 \times 3000 \times 240}{3}} \times \sqrt{\frac{4500}{4500 - 3000}} = 1200 \text{ units}$$

t° = Average duration of the production run

$$= \frac{1200}{4500} = 0.2667 \text{ year/run}$$

(iii) When item is purchased from outside:

$$\text{Total cost} = D \times C + \sqrt{2DC_0C_1}$$
$$= 3000 \times 35 + \sqrt{2 \times 3000 \times 15 \times 3}$$
$$= 75000 + 547.72$$
$$= ₹105519.6 \text{ or } ₹105520 \text{ (approx)}$$

When item is produced internally:

$$\text{Cost per unit, } C = 80\% \text{ of } ₹35 = ₹28$$

and C_1 ₹250 per production run,

$$\therefore \quad \text{Total cost} = D \times C + \frac{D}{Q^\circ} \times C_0 + \frac{1}{2} Q_1^\circ \times \frac{k-r}{k} \times C_1$$
$$= (3000 \times 28) + \frac{3000}{1200} \times 240 + \frac{1200}{2} \times \left(\frac{4500-3000}{4500}\right) \times 3$$
$$= ₹85200$$

From the above calculation, we observe that the company should manufacture the product internally.

5. (e)

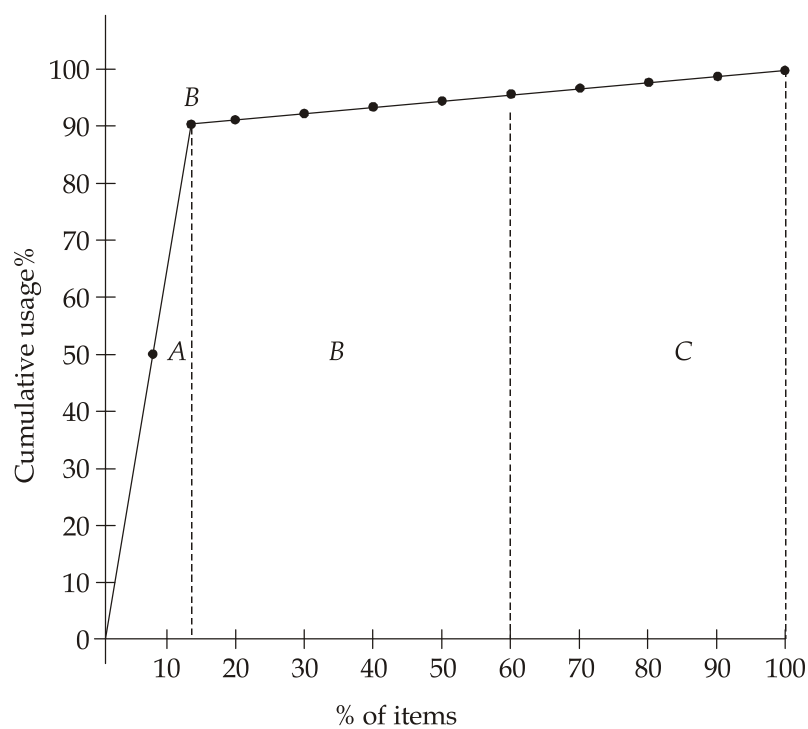
The first step is to compute the annual usage value of each item and to rank then in the descending order of their annual usage values.

Item no.	Quantity consumed in a year	Cost per unit (₹)	Annual usage value	Ranking
1	3	45	135	13
2	200	6	1200	4
3	40	1200	48000	1
4	30	25	750	9
5	5	20	100	14
6	17	2100	35700	2
7	25	50	1250	3
8	6	40	240	11
9	100	7	700	10
10	100	8	800	8
11	250	4	1000	5
12	120	8	960	7
13	140	7	980	6
14	9	10	90	15
15	20	10	200	12

The next step is to accumulate the total number of items and their usage values and then to convert the accumulate values into the percentages of the grand totals.

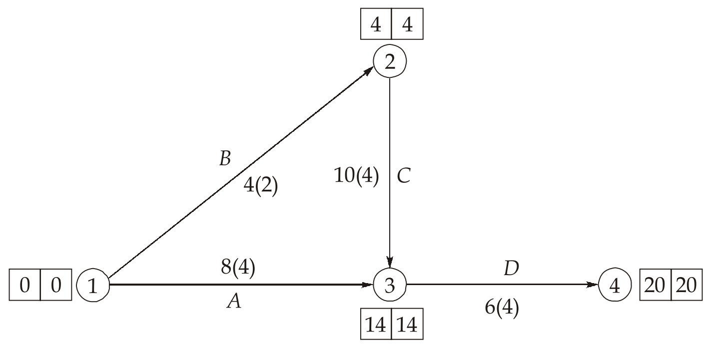
This is done in the table

Item no.	Annual usage	Cumulative Annual usage	Cumulative usage (%)	Percentage of items	Category
3	48000	48000	52.11	6.67	A
6	35700	83700	90.87	13.33	
7	1250	84950	92.23	20.00	B
2	1200	56150	93.53	26.67	
11	1000	87150	94.62	33.33	
13	980	88130	95.68	40.00	
12	960	89090	96.73	46.67	
10	800	89890	97.59	53.33	
4	750	90640	98.41	60.00	
9	700	91340	99.16	66.67	C
8	240	91580	99.43	73.33	
15	200	91780	99.64	80.00	
1	135	91915	99.79	86.67	
5	100	92015	99.90	93.33	
14	90	92105	100.0	100.00	



6. (a)

The CPM network for the given activities is shown below:



Normal duration of the project = 20 days

Total direct cost of the project = 6000 + 2000 + 4000 + 4000 = 16000

Total indirect cost of the project = 1000 × 20 = 20000

Total project cost = Total direct cost + Total indirect cost

= 16000 + 20000 = 36000

Activity	Duration in days		Direct cost in ₹		Cost slope = $(C_c - C_n)/(t_n - t_c)$
	Normal	Crash	Normal	Crash	
A	8	4	6000	12000	1500
B	4	2	2000	14000	6000
C	10	4	4000	8000	666.67
D	6	4	4000	8000	2000

First stage of crashing:

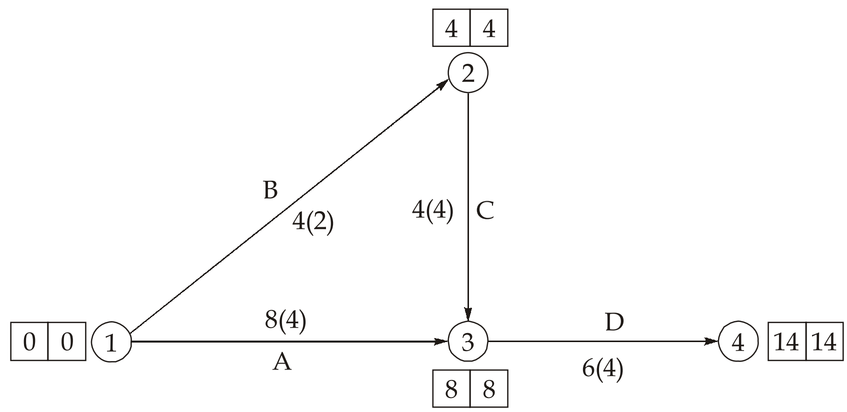
As it is evident from the network diagram above that the critical path is $B - C - D$. Among the critical activities minimum cost slope is of activity C. Hence, it should be crashed first. The activity C can be crashed by 6 days.

Increase in direct cost = 666.67 × 6 = 4000

Decrease in indirect cost = 1000 × 6 = 6000

Net change = 4000 – 6000 = – 2000 (decrease)

Total cost of project after first stage of crashing = 36000 – 2000 = 34000



Total project duration after first stage of crashing = 14 days

Second Stage of crashing:

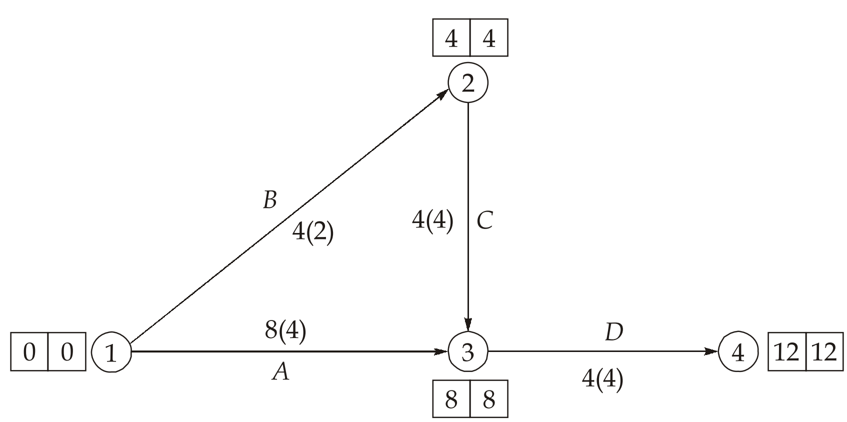
After first stage of crashing the number of critical paths are increased i.e. either $B - C - D$ or $A - D$ may be the critical path. In the second stage possible alternatives for crashing are activity D which can be crashed by 2 days and activity A and B combined which can be crashed by 2 days each. But the cost slope A and B combined is (1500 + 6000 = 7500), which is more than cost slope of activity D. Hence activity D will be crashed by 2 days.

$$\text{Increase in direct cost} = 2000 \times 2 = 4000$$

$$\text{Decrease in indirect cost} = 1000 \times 2 = 2000$$

$$\text{Net change} = 4000 - 2000 \text{ (increase)}$$

Total cost of project after second stage of crashing = 34000 + 2000 = 36000



Total project duration after second stage of crashing = 12 days

Third stage of crashing:

After second stage of crashing, the number of critical path will remain same i.e. $B - C - D$ and $A - D$.

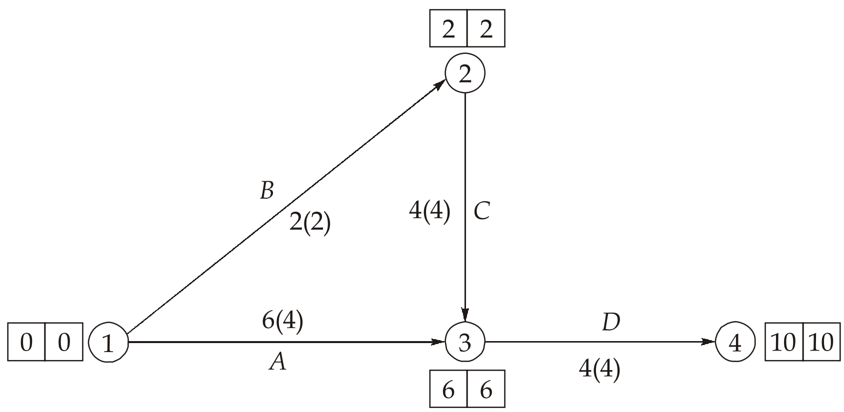
In third stage only possible alternative is to crash activity B and A simultaneously by 2 days.

Increase in direct cost = (1500 + 6000) × 2 = 15000

Decrease in indirect cost = 1000 × 2 = 2000

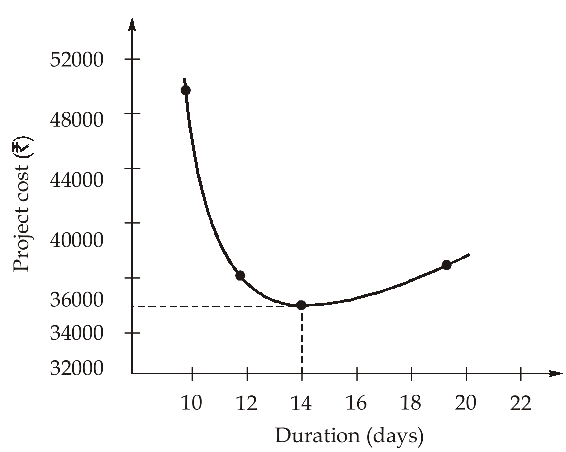
Net change = 15000 – 2000 = 13000 (increase)

Total cost of project after third stage of crashing = 36000 + 13000 = 49000



Total project duration after third stage of crashing = 10 days

Now, A can further be crashed by 2 days but it cannot happen because activity B and C are parallel to A and they have been already crashed to their maximum limits.



Optimum cost of the project = ₹ 34000

Optimum duration of the project completion = 14 days

6. **(b)**

(a) Let x_1, x_2 and x_3 represent the number of Deluxe, Standard and Economy tour packages.

Then the problem can be expressed as

$$\text{maximize } Z = \text{Rs. }[x_1(12000 - 4000 - 5500) + x_2(8000 - 2600 - 2750) + x_3(6750 - 1950 - 2350) - 215000]$$

$$= \text{Rs.}[2500x_1 + 2650x_2 + 2450x_3 + 215000]$$

Subject to the following constraints:

(i) $\dfrac{x_1}{x_1 + x_2 + x_3} \geq 0.10 \text{ or } 9x_1 - x_2 - x_3 \geq 0$

(ii) $\dfrac{x_2}{x_1 + x_2 + x_3} \geq 0.35 \text{ or } 7x_1 - 13x_2 + 7x_3 \leq 0, \text{ or } -7x_1 + 13x_2 - 7x_3 \geq 0$

$\dfrac{x_2}{x_1 + x_2 + x_3} \leq 0.70 \text{ or } -7x_1 + 3x_2 - 7x_3 \leq 0$

(iii) $\dfrac{x_3}{x_1 + x_2 + x_3} \geq 0.30 \text{ or } -3x_1 - 3x_2 + 7x_3 \geq 0$

(iv) $x_1 \leq 60$

(v) $x_1 + x_2 \geq 120$

(vi) $x_1 + x_2 + x_3 = 200$

(b) $x_3 = 200 - x_1 - x_2$

∴ Objective function is to

$$\text{maximize } Z = \text{Rs.}[2500x_1 + 2650x_2 + 2450(200 - x_1 - x_2) - 215000]$$

$$= \text{Rs.}[50x_1 + 200x_2 + 275000]$$

The constraints can be expressed as

(i) $9x_1 - x_2 - (200 - x_1 - x_2) \geq 0 \text{ or } x_1 \geq 20$

(ii) $-7x_1 + 13x_2 - 7(200 - x_1 - x_2) \geq 0 \text{ or } x_2 \geq 70 \text{ and}$

$-7x_1 + 3x_2 - 7(200 - x_1 - x_2) \leq 0 \text{ or } x_2 \leq 140$

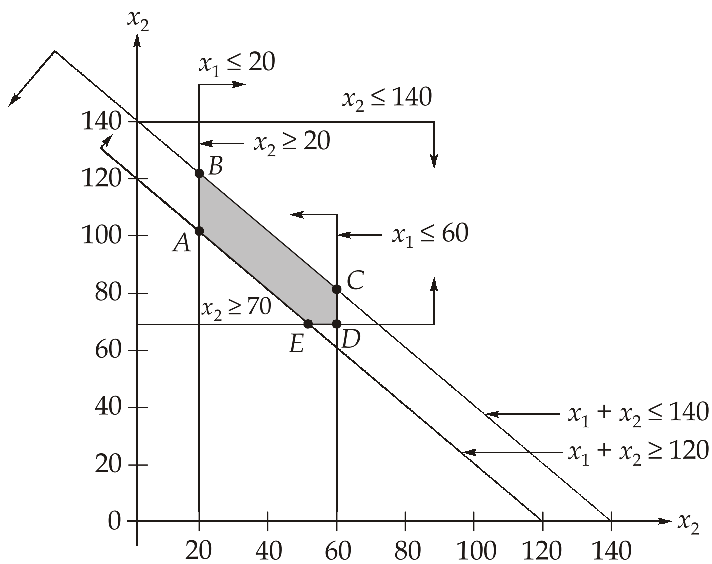
(iii) $-3x_1 - 3x_2 + 7(200 - x_1 - x_2) \geq 0 \text{ or } x_1 + x_2 \leq 140$

(iv) $x_1 \leq 60$

(v) $x_1 + x_2 \geq 120$

(c) The region of feasible solutions satisfying all the above constraints and meeting the non-negativity restrictions is shown shaded in figure. The co-ordinates of the vertices of the closed polygon ABCDE are

A(20, 100), B(20, 120), C(60, 80), D(60, 70) and E(50, 70)



Values of the objective function at these vertices are

$$Z_A = \text{Rs.}[50(20) + 200(100) + 275000] = \text{Rs.}296000$$

$$Z_B = \text{Rs.}[50(20) + 200(120) + 275000] = \text{Rs.}300000$$

$$Z_C = \text{Rs.}[50(60) + 200(80) + 275000] = \text{Rs.}294000$$

$$Z_D = \text{Rs.}[50(60) + 200(70) + 275000] = \text{Rs.}292000$$

$$Z_E = \text{Rs.}[50(50) + 200(70) + 275000] = \text{Rs.}291500$$

∴ B(20, 120) is the optimal point which means the number of deluxe, standard and economy packages should be 20, 120 and 60 respectively to earn maximum profit of Rs.300000.

6. (c)

Given data: $m = 1$ kg, $x = 100$ mm = 0.1 m, $y = 50$ mm = 0.05 m,

$r = 80$ mm = 0.08 m, $r_1 = 75$ mm = 0.075 m, $r_2 = 112.5$ mm = 0.1125 m,

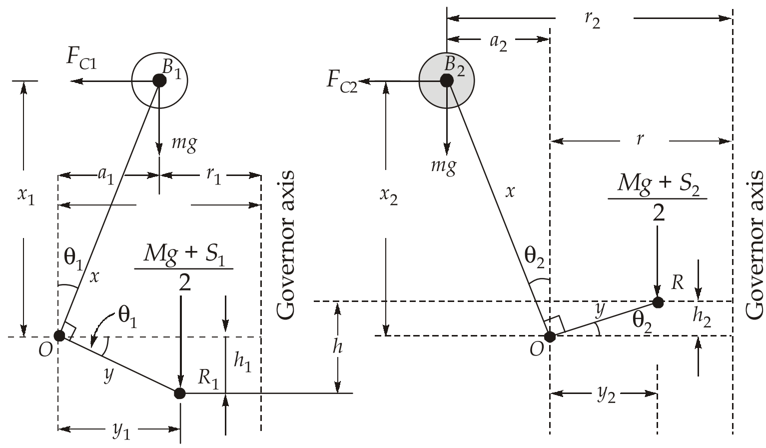
$N_1 = 360$ rpm or $\omega_1 = \dfrac{2\pi \times 360}{60} = 37.7$ rad/s

Since the maximum equilibrium speed is 5% greater than the minimum equilibrium speed (ω_1), therefore maximum equilibrium speed,

$$\omega_2 = 1.05 \times 37.7 = 39.6 \text{ rad/s}$$

We know that centrifugal force at the minimum equilibrium speed,

$$F_{C1} = m(\omega_1)^2 r_1 = 1(37.7)^2 \times 0.075 = 106.6 \text{ N}$$



(a) Minimum position (b) Maximum position

and centrifugal force at the maximum equilibrium speed,

$$F_{C2} = m(\omega_2)^2 r_2 = 1(39.6)^2 \times 0.1125 = 176.4 \text{ N}$$

Initial compression of the spring,

Let, S_1 = Spring force corresponding to ω_1 and

S_2 = Spring force corresponding to ω_2.

Since the obliquity of arms is neglected, therefore for minimum equilibrium position,

$$Mg + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 106.6 \times \frac{0.1}{0.05} = 426.4 \text{ N}$$

$\therefore$ $S_1 = 426.4$ N $\quad (\because M = 0)$

and for maximum equilibrium position,

$$Mg + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 176.4 \times \frac{0.1}{0.05} = 705.6 \text{ N}$$

$\therefore$ $S_2 = 705.6$ N $\quad (\because M = 0)$

We know that lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x} = (0.1125 - 0.075)\frac{0.05}{0.1} = 0.01875 \text{ m}$$

and stiffness of the spring,

$$s = \frac{S_2 - S_1}{h} = \frac{705.6 - 426.4}{0.01875} = 14890 \text{ N/m}$$

$$= 14.89 \text{ N/mm}$$

$\therefore$ Initial compression of the spring $= \dfrac{S_1}{s} = \dfrac{426.4}{14.89} = 28.6$ mm Ans.

Equilibrium speed corresponding to radius of rotation, r = 100 mm = 0.1 m

Let N = Equilibrium speed in rpm

Since the obliquity of the arms is neglected, therefore the centrifugal force at any instant.

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$= 106.6 + (176.4 - 106.6)\left(\frac{0.1 - 0.075}{0.1125 - 0.075}\right) = 153 \text{ N}$$

We know that centrifugal force (F_C),

$$153 = m\omega^2 r = 1\left(\frac{2\pi N}{60}\right)^2 0.1 = 0.0011N^2$$

$$N^2 = \frac{153}{0.0011} = 139090 \text{ or } N = 373 \text{ rpm}$$

Q.7 (a)

Step I: Converting the problem of maximization into equivalent minimization problem by subtracting each element of the matrix from the highest element of the matrix.

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

(Here, Highest element = 14)

Step II: Develop opportunity cost matrix

(a) Subtract the smallest element of each row from every element of the corresponding row.

(b) Subtract the smallest element of each column from every element of the corresponding column .

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	3	7
5	3	4	0	7

→

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

Step III: Make allocations in the opportunity cost matrix (i.e. put square on the zeros and cross all zeros (if any) of the corresponding row/column).

3	1	2	[0]	7
[0]	2	~~0~~	3	~~0~~
7	2	9	~~0~~	7
4	[0]	10	3	6
1	3	4	~~0~~	6

Since total number of allocations is less than the size of the matrix. So, current solution is not optimal. We have to perform optimally.

Step IV: Draw the minimum number of lines to cover all the zeros. Select the smallest element that do not have line through them. Subtract it from all the elements that do not have line through them. Add it to elements at the intersection of two lines and leave the remaining element of the matrix unchanged. Then again make allocations.

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

→

2	~~0~~	1	[0]	6
~~0~~	2	[0]	4	~~0~~
6	1	8	~~0~~	6
4	[0]	10	4	6
[0]	2	3	~~0~~	5

Since total number of allocations is less than the size of the matrix, so current solution is not optimal. Again we have to perform optimally.

Step V: Repeat step IV

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	4	6
0	2	3	0	5

→

2	~~0~~	[0]	~~0~~	5
1	3	~~0~~	5	[0]
6	1	7	[0]	5
4	[0]	9	4	5
[0]	2	2	~~0~~	4

Since total number of allocations is equal to the size of the matrix. So, current solution is optimal.

Optimal Assignment:

Machine	Job
1	3
2	5
3	4
4	2
5	1

Maximum profit = 10 + 5 + 14 + 14 + 7 = Rs. 50

7. (b) (i)

$$\lambda = \frac{1}{9} \text{ per minute}, \quad \mu = \frac{1}{3} \text{ per minute}$$

1. Probability that a person will have to wait:

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{9}}{\frac{1}{3}} = 0.33$$

2. Average queue length $= \dfrac{\lambda^2}{\mu(\mu-\lambda)} = \dfrac{\left(\frac{1}{9}\right)^2}{\left(\frac{1}{3}\right)\left(\frac{1}{3}-\frac{1}{9}\right)} = \dfrac{1}{6}$ person

3. If arrival is greater than the waiting time, the length of queue will go on increasing and that justifies the provision of a second milk booth.

$$E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)}$$

Here $E(W_q) = 4$ minutes, $\mu = \dfrac{1}{3}$ per min.

We have to find new λ'

$$4 = \frac{\lambda'}{\frac{1}{3}\left(\frac{1}{3}-\lambda'\right)}$$

or,
$$\frac{\lambda'}{\frac{1}{3}-\lambda'} = \frac{4}{3}$$

or
$$\lambda' = \frac{4}{21} \text{ arrival/minute} = 0.19/\text{minute}$$

∴ Increase in the flow of arrivals

$$= \frac{4}{21} - \frac{1}{9} = \frac{5}{63} \text{ per minute}$$

4. Probability (waiting time ≥ 10 min)

$$= \rho\, e^{-(\mu-\lambda)t}$$

$$= \frac{1}{3} e^{-\left(\frac{1}{3}-\frac{1}{9}\right)10} = \frac{0.1083}{3} = 0.036$$

5. Probability [time in the system ≥ 10]

$$= e^{-(\mu - \lambda)t}$$

$$= e^{-\left(\frac{1}{3}-\frac{1}{9}\right)10} = 0.1087$$

7. (b) (ii)

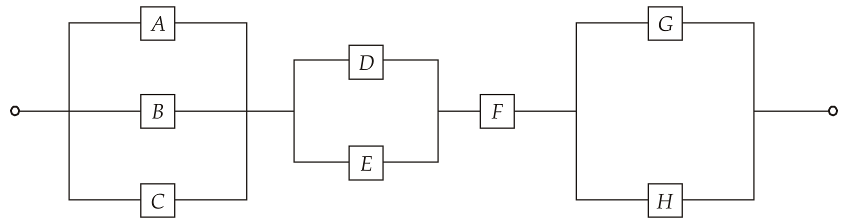
Reliability is the probability that an item or system will not fail in a given time period, and Availability is the probability that a component or system will be ready to use when it is needed. Availability takes into account maintainability, repair time, spares availability, lead time and other logistics considerations.

High reliability contributes to high availability, but it is possible to achieve a high availability even with an unreliable product by minimizing repair time and ensuring that spares are always available when they are needed. This is usually done with a simple swap. When something fails, just swap it out with one on the shelf, and the system can resume operation. The failed component can be then repaired while the system continues to operate.

Definition of Reliability: The conditional probability, at a given confidence level, that the equipment will perform its intended functions satisfactorily or without failure, i.e., within specified performance limits, at a given age, for a specified length of time, function period, or mission time, when used in the manner and for the purpose intended while operating under the specified application and operation environments with their associated stress levels.

Definition of Availability: Availability is a measure of the degree to which an item is in an operable state and can be committed at the start of a mission when the mission is called for at an unknown (random) point of time. Availability as measured by the user is a function of how often failures occur and corrective maintenance is required, how often preventative maintenance is performed, how quickly indicated failures can be isolated and repaired, how quickly preventive maintenance tasks can be performed, and how long logistics support delays contribute to down time.

The reliability of the given system:



Reliability of three parallel units A, B and C is:

$$R_1 = [1 - (1 - R_A)(1 - R_B)(1 - R_C)]$$

Reliability of two parallel units D and E is:

$$R_2 = [1 - (1 - R_D)(1 - R_E)]$$

Reliability of two parallel units G and H is

$$R_3 = [1 - (1 - R_G)\,(1 - R_H)]$$

Therefore system reliability is the series system containing R_1, R_2, R_F and R_3, which is

$$R_s = R_1 \times R_2 \times R_F \times R_3$$

$$R_s = [1 - (1 - R_A)(1 - R_B)(1 - R_C)][1 - (1 - R_D)(1 - R_E)]R_F[1 - (1 - R_G)(1 - R_H)]$$

$$R_s = [1 - 0.65 \times 0.65 \times 0.65][1 - 0.65 \times 0.65] \times 0.35 \times [1 - 0.65 \times 0.65]$$

$$R_s = 0.725375 \times 0.5775 \times 0.35 \times 0.5775$$

$$R_s = 0.08467 \text{ or } 8.467\,\%$$ **Ans.**

7. (c)

Draw the configuration diagram as per the dimensions of a given mechanism on suitable scale as shown.

Here, velocity of input link

$$V_{AO} = \omega_{AO} \times OA$$

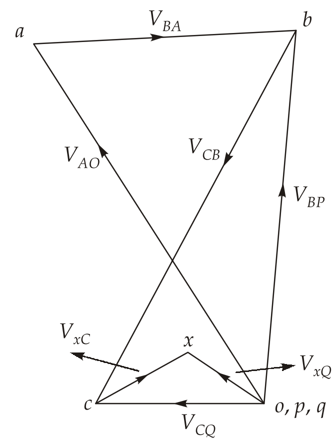
$$= \frac{2\pi N}{60} \times OA$$

$$= \frac{2\pi \times 60}{60} \times 7.5 = 47.1 \text{ cm/s}$$

Drawing velocity diagram :

Link	Direction	Magnitude	
OA	Perpendicular to link OA	$\overline{OA} = 47.1$ cm/s	
V_{BA}	Perpendiulcar to link BA, from point 'a'	Unknown	We will get intersecting point as 'b'
V_{BP}	Perpendicular to link PB, from pole 'O'	Unknown	
V_{CB}	Perpendicular to link BC, from point 'b'	Unknown	Intersection will give point as 'c'
V_{CQ}	Perpendicular to link QC, from pole 'O'	Unknown	

Now to get point 'X' draw $V_{XC} = \overrightarrow{cx}$ (perpendicular to XC). From 'C' point and $V_{XQ} = \overrightarrow{qx}$ perpendicular to QX from point 'q'. Intersection of these two lines will locate the point 'x'.



Velocity Diagram

From velocity diagram, $V_x = \vec{qx} = 13 \text{ cm/s}$

$$P_{o/p} = V_x \times F_x = \left(\frac{13}{100} \text{ m/s}\right) \times (20{,}000)\text{N} = 2600 \text{ Nm/s} = 2.6 \text{ kW}$$

$$P_{i/p} = \frac{P_{o/p}}{\eta_{\text{mech}}} = \frac{2.6}{0.8} = 3.25 \text{ kW} = T \times \omega$$

So, torque at crank, $T = \dfrac{3.25 \times 10^3}{2\pi} = 517.25 \text{ N/m}$

8. (a)

Given : $m = 5$ kg; $e = 2$ mm; $l = 60$ cm; $E = 2 \times 10^{11}$ N/m^2; $d = 20$ mm; $c = 60$ N-sec/m

(i) The static deflection, $\delta = \dfrac{WL^3}{48EI}$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.02)^4 = 7.854 \times 10^{-9}$$

$$\delta = \frac{(5 \times 9.81) \times (0.6)^3}{48 \times 2 \times 10^{11} \times 7.854 \times 10^{-9}} = 1.405 \times 10^{-4} \text{ m}$$

Critical speed, $\omega_c = \sqrt{\dfrac{g}{\delta}} = \sqrt{\dfrac{9.81}{1.405 \times 10^{-4}}} = 264.22 \text{ rad/s}$

Angular speed of shaft, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 360}{60} = 37.699 \text{ rad/s}$

Frequency ratio, $\beta = \dfrac{\omega}{\omega_c} = \dfrac{37.699}{264.22} = 0.1427$

$$\text{Damping factor, } \xi = \frac{c}{2\sqrt{sm}} = \frac{c}{2m\sqrt{\omega_c^2}}$$

$$= \frac{60}{2 \times 5 \times 264.22} = 0.0227$$

$$\frac{x}{e} = \frac{\beta^2}{\sqrt{\left(1-\beta^2\right)^2 + (2\varepsilon\beta)^2}}$$

$$\frac{x}{2} = \frac{(0.1427)^2}{\sqrt{\left[1-(0.1427)^2\right]^2 + (2 \times 0.1427 \times 0.0227)^2}}$$

$$x = 0.0416 \text{ mm}$$

For the dynamic load on the bearing

$$F_d = \sqrt{(\text{Spring force})^2 + (\text{Damping force})^2}$$

$$= \sqrt{(k_t x)^2 + (cwx)^2} = x\sqrt{(k_t)^2 + (cw)^2}$$

$$w_c = \sqrt{\frac{k_t}{m}}$$

$$\Rightarrow \quad k_t = w_c^2 \cdot m = 264.22^2 \times 5$$

$$k_t = 349.067 \text{ kN/m}$$

$$F_d = 0.0416 \times 10^{-3}\sqrt{(349.067)^2 + (60)^2 \times (37.70)^2} = 14.52$$

The dead load on the shaft, $w = mg = 5 \times 9.81 = 49.05$ N

Total maximum load on the shaft under the above condition

$$= 14.52 + 49.05 = 63.57 \text{ N}$$

By stress relation,

$$\frac{\sigma}{y} = \frac{M}{I}$$

$$\sigma = \frac{M}{I} \cdot y = \frac{F \cdot ld}{4\left(\frac{\pi}{64}d^4\right) \times 2} = \frac{8F \cdot l}{\pi d^3}$$

$$I = \frac{\pi}{64}d^4$$

$$y = \frac{d}{2}$$

$$\sigma_{max} = \frac{8F \cdot l}{\pi d^3} = \frac{8F \times 0.6}{\pi \times \left(20 \times 10^{-3}\right)^3}$$

$$= 12.141 \times 10^6 \text{ N/m}^2 = 12.141 \text{ MPa}$$

(ii) The power required to drive the shaft

$$P = \frac{2\pi NT}{60}$$

Damping torque, T = Damping force × x

$$= cwxx = cwx^2 = 60 \times (37.7) \times (0.0416 \times 10^{-3})^2$$

$$= 3.915 \times 10^{-6} \text{ Nm}$$

$$P = \frac{2\pi \times 360 \times 3.915 \times 10^{-6}}{60}$$

$$P = 1.47 \times 10^{-4} \text{ Watt}$$

8. (b)

Let us first draw the table for period and demand.

Period (x)	Demand (y)	x^2	xy
1	95	1	95
2	98	4	196
3	102	9	306
4	106	16	424
5	110	25	550
6	120	36	720
$\Sigma x = 21$	$\Sigma y = 631$	$\Sigma x^2 = 91$	$\Sigma xy = 2291$

The linear equation will be

$$y = a + bx$$

$$a = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{n\Sigma x^2 - (\Sigma x)^2} = \frac{631(91) - 21(2291)}{6(91) - (21)^2}$$

$$a = \frac{266}{3}$$

$$b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{6(2291) - 21 \times 631}{6(91) - (21)^2}$$

$$b = \frac{33}{7}$$

$$y = \frac{266}{3} + \frac{33}{7}x$$

Forecast of the 12th period, $y = 145.24$

Coefficient of determination, $r^2 = \dfrac{\left[n\sum xy - \sum x \sum y\right]^2}{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}$

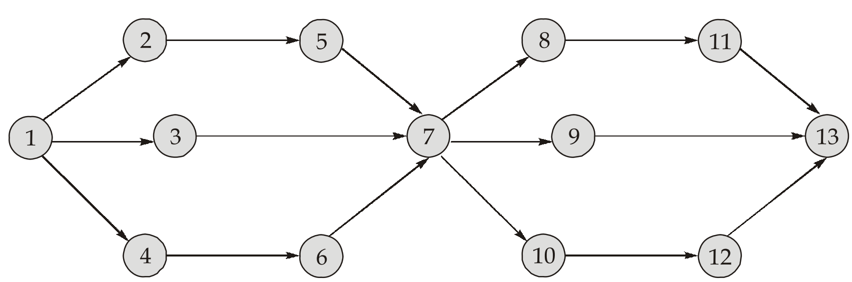
$$r^2 = \frac{[6(2291) - 21 \times 631]^2}{\left[6 \times 91 - (21)^2\right]\left[6 \times 66769 - (631)^2\right]}$$

$$r^2 = 0.95$$

Period (x)	Demand (y)	f	$(y-f)^2$
1	95	93.38	2.624
2	98	98.095	0.009
3	102	102.809	0.65
4	106	107.524	2.323
5	110	112.24	5.0176
6	120	116.95	9.3025
$n = 6$			$\sum(y-f)^2 = 19.98$

$$\text{Mean square error} = \frac{\sum_{i=1}^{n}(y-f)^2}{n} = \frac{19.98}{6} = 3.33$$

8. (c)



(i) Total work content,

$$\text{Total time} = 9 + 4 + 4 + 4 + 6 + 8 + 6 + 4 + 3 + 6 + 8 + 6 + 11$$
$$= 79 \text{ minutes}$$

Minimum number of workstation,

$$N_{min} = \frac{79}{17} = 5 \text{ workstations}$$

Sort tasks in descending order of duration,

Task	Time	Predecessors
13	11	9, 11, 12
1	9	–
6	8	4
11	8	8
5	6	2
7	6	3, 5, 6
10	6	7
12	6	10
2	4	1
4	4	1
3	4	1
8	4	7
9	3	7

Assigning work elements to workstations,

Workstation	Task assigned	Time	Total time
1	1	9	17
	2	4	
	4	4	
2	5	6	14
	6	8	
3	3	4	10
	7	6	
4	10	6	10
	8	4	
5	11	8	17
	12	6	
	9	3	
6	13	11	11

1. Number of workstations = 6

2. Line efficiency, $\eta = \dfrac{\text{Total work content}}{\text{Number of workstation} \times \text{Cycle time}}$

$$= \frac{79}{6 \times 17} = \frac{79}{102} = 77.45\%$$

3. Balance delay = 100 – 77.45 = 22.55%

4. Smoothness index $= \sqrt{\sum_{i=1}^{n} (\text{maximum station time – station line})^2}$

$$= \sqrt{(17-17)^2 + (17-14)^2 + (17-10)^2 + (17-10)^2 + (17-17)^2 + (17-11)^2}$$

$$= \sqrt{143} = 11.95$$

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