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Detailed Solutions

ESE-2025 Mains Test Series

Civil Engineering Test No : 3

Section A : Design of Concrete and Masonry Structure

Q.1 (a) Solution:

(i)

Ultimate Load Method:

1. It takes into account the elasto-plastic properties of the materials to decide upon the strength of a member. It is more close to actual load tests.
2. Sections are designed for ultimate loads for which suitable critical combination of various loads is anticipated.
3. Design procedure is based on actual experimental observations. Though suitable assumptions have to be made to simplify the computation. It stands quite close to actual tests.
4. Actual test strengths of two materials are made use of in the design of members.
5. It is more often an economical section.
6. It is desirable to use under-reinforced beam to exploit full strength of the materials and in practice, design is done so.

Elastic Theory Method:

1. It takes into account only the elastic properties of material and does not allow to exceed safe stresses on any fibre. It results in the under stressing of materials.
2. Only maximum probable load anticipated to be borne by the members are considered.
3. Design procedure is mathematically simple and is based on the laws of Mechanics.
4. Modular ratio (m) is used in the design of members.

5. It gives an uneconomical design in comparison with ultimate load method.
6. Mostly sections are either balanced or over-reintorced.

(ii)

Let effective depth of the beam $= d$

$\therefore$ Breadth of beam $b = \dfrac{d}{2}$

$$N = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}} = \frac{1}{1+\frac{1400}{18\times 50}} = 0.39$$

$$j = 1-\frac{N}{3} = 1-\frac{0.39}{3} = 0.87$$

$$Q = \frac{1}{2}\sigma_{cbc}JN = \frac{1}{2}\times 50\times 0.87\times 0.39 = 8.48$$

Equating the moment of resistance to the given bending moment.

$$Qbd^2 = 600{,}000$$

$$\Rightarrow \quad 8.48\times\frac{d}{2}\times d^2 = 600{,}000$$

$$\Rightarrow \quad d^3 = \frac{600{,}000\times 2}{8.48}$$

$$\therefore \quad d = 52.11 \simeq 54 \text{ cm (say)}$$

$$\therefore \quad b = \frac{d}{2} = 27 \text{ cm}$$

$$\text{Area of steel, } A_{st} = \frac{B.M.}{\sigma_{st}J\cdot d} = \frac{600{,}000}{1400\times 0.87\times 54} = 9.12 \text{ sq. cm}$$

Q.1 (b) Solution:

$$\text{Equation to the cable profile: } y = \frac{4e}{l^2}x(l-x) \qquad (e = 0.10 \text{ m})$$

$$y = \frac{4\times 0.1}{10\times 10}x(10-x)$$

$$\therefore \quad y = 0.004x\,(10-x)$$

$$\text{Slope, } \quad \frac{dy}{dx} = 0.004\,(10-2x)$$

Slope at ends A and B $= 0.004 \times 10 = 0.04$ rad

Slope at mid span $C = 0$

$\therefore$ Change of slope from A to $C = 0.04$ radian

Angle of deviation for the cable from A to $C = 0.04$ radian

i.e. $\alpha = 2\theta = 2 \times 0.04 = 0.08$ radian

Loss of stress from A to $B = (\mu\alpha + kx) \times$ Initial stress

$= (\mu\alpha + kx) \times f_o$, where f_o is initial stress

$\therefore$ Loss $= (0.35 \times 0.08 + 0.0015 \times 10) \times f_o$

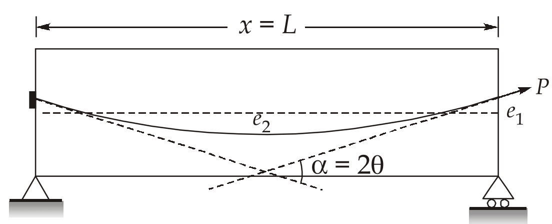
$= 0.043\, f_o$

Percentage loss of prestress in the cable

$$= \frac{0.043 f_o}{f_o} \times 100 = 4.3\%$$

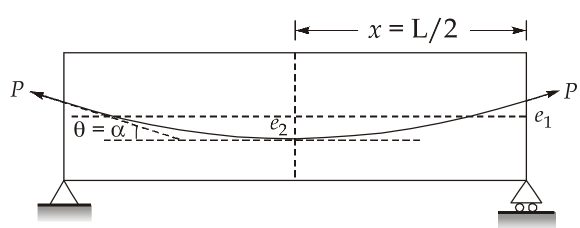
Note :

For one end jacking:



$$\alpha = 2\theta = \frac{8(e_1 + e_2)}{L}$$

For two end Jacking:



$$\alpha = \theta = \frac{4(e_1 + e_2)}{L}$$

Q.1 (c) Solution:

Anchorage length of bar at simply supported end of beam can be determined by,

$$\text{Development length, } L_d \leq \frac{1.30M_u}{V_u} + l_0$$

$$\text{Anchorage length, } l_0 \geq L_d - \frac{1.3M_u}{V_u}$$

where, $$L_d = \frac{0.87 \times f_y \times \phi}{4 \times \tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940 \text{ mm}$$

$$M_u = f_s A_{st} (d - 0.42x_u)$$

where $$x_u = \frac{f_{st} A_{st}}{0.36 f_{ck} b}$$

$f_{st} = 0.87 f_y$, if the section is balanced/under-reinforced.

$< 0.87 f_y$, if the section is over-reinforced.

Assume $x_u \leq x_{u,lim}$ (balanced/under-reinforced section)

$\therefore$ $$x_u = \frac{0.87 f_s A_{st}}{0.36 f_{ck} b}$$

where $A_{st} = 3 \times 20\phi = 942 \text{ mm}^2$

$\therefore$ $$x_u = \frac{0.87 \times 415 \times 942}{0.36 \times 20 \times 300} = 157.458 \text{ mm}$$

and $$x_{u,lim} = 0.48d = 0.48 \times 500 = 240 \text{ mm}$$

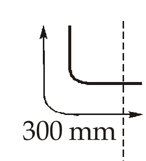
$\because$ $x_u < x_{u,\text{lim}}$, therefore section is under-reinforced and computed value of x_u is correct.

$\therefore$ $$M_u = 0.87 \times 415 \times 942 (500 - 0.42 \times 157.458) \text{ Nmm}$$

$$= 147.56 \text{ kNm}$$

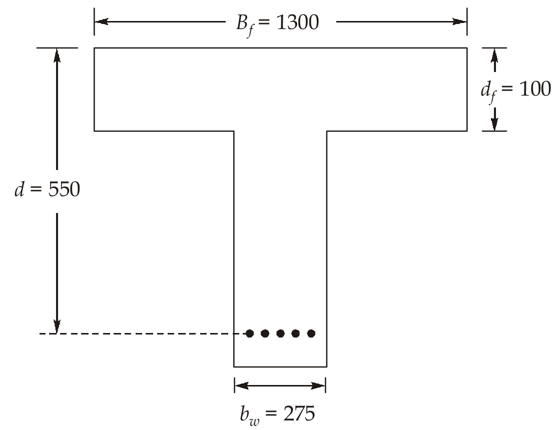
Hence, $$l_0 \geq 940 - \frac{1.3 \times 147.56 \times 10^6}{300 \times 10^3} \geq 300.573 \text{ mm}$$

$\therefore$ Therefore minimum anchorage length of 300 mm is required.



Q.1 (d)Solution:

Given effective flange width, B_f = 1300 mm



(All dimensions in mm)

Area of steel, $A_{st} = 5\times\frac{\pi}{4}\times 25^2 = 2454.3693 \text{ mm}^2$

Limiting depth of neutral axis, $(x_u)_{lim} = 0.48d$ (For Fe415)

$\Rightarrow$ $(x_u)_{lim} = 0.48 \times 550 = 264$ mm

Assuming actual depth of neutral axis $x_u < d_f$

$\therefore$ $C = T$

$\Rightarrow$ $0.36 f_{ck}.B_f.x_u = 0.87 f_y.A_{st}$

$\Rightarrow$ $0.36 \times 15 \times 1300 \times x_u = 0.87 \times 415 \times 5 \times \frac{\pi}{4} \times 25^2$

$\Rightarrow$ $x_u = 126.23 \text{ mm} > d_f$

Hence our assumption is wrong.

Assuming $x_u > d_f$

and $\frac{3}{7}x_u < d_f$

$\therefore$ $y_f = 0.15\, x_u + 0.65\, d_f = 0.15\, x_u + 0.65 \times 100$

$= (0.15\, x_u + 65)$ mm

Hence, $C_u = 0.36 f_{ck}.b_w.x_u + 0.45 f_{ck}(B_f - b_w).y_f = 0.87 f_y.A_{st}$

$\Rightarrow$ $0.36 \times 15 \times 275 \times x_u + 0.45 \times 15\,(1300 - 275)(0.15\, x_u + 65) = 0.87 \times 415 \times 2454.3693$

$\Rightarrow$ $x_u = 172.99 \text{ mm} > d_f$

$\therefore$ $\frac{3}{7}x_u = 74.14 \text{ mm} < d_f$

Hence our assumption is correct.

$\therefore$ Depth of neutral axis, $x_u = 172.99$ mm

$y_f = 0.15 \times 172.99 + 65 = 90.95$ mm

$\therefore$ Ultimate moment of resistance $= \left[0.36 f_{ck}.b_w.x_u(d-0.42\,x_u)+0.45\,f_{ck}(b_f-b_w).y_f.\left(d-\frac{y_f}{2}\right)\right]$

$\Rightarrow$ $(M.R)_u = [0.36 \times 15 \times 275 \times 172.99 \times (550 - 0.42 \times 172.99)]$

$$+\left[0.45\times15\times(1300-275)\times90.95\times\left(550-\frac{90.95}{2}\right)\right]$$

$\Rightarrow$ $(M.R)_u = 440102582.3$ N-mm

$= 440.1026$ kN-m $\simeq 440$ kNm

Q.1 (e)Solution:

(i)

Light weight or foam concrete is broadly classified into three categories:

1. **Light weight aggregate concrete:** Naturally available light weight aggregates are pumice, diatomite, scoria, volcanic cinders, rice husk and sawdust.
   Artificially light weight aggregates are cinder, foamed slag, sintered flyash, exfoliated vermiculite, bloated clay and expanded perlite. These light weight aggregates are used to form light weight concrete.
2. **Aerated concrete:** It is manufactured by raw materials that are calcarious and silicious in nature like cement, lime, pulverized sand, flash; by entrapping air in cells. It is also known as foam/gel/cellular concrete.
3. **No fines concrete:** Here, the fine aggregate fraction is omitted in concrete manufacturing. The aggregated/cement ratio ranges from 6 : 1 to 10 : 1. And the water-cement ratio ranges from 0.38 to 0.52.

(ii)

Underwater concreting is done with the use of tremie pipe. Concrete is poured with the help of funnel and bottom end is closed with polyethylene. Thus pipe is filled with concrete having slump 150 to 200 mm. Pipe is slightly lifted and jerk is given to cause tearing of polythene, resulting in the discharge of concrete at the place of deposition.

Q.2 (a)Solution:

Given, $b = 300$ mm, $d = 655$ mm, $d' = 45$ mm, $f_y = 415$ MPa and $f_{ck} = 20$ MPa

$A_{sc} = \frac{\pi(25)^2}{4}\times 2 = 491\times 2 = 982\text{ mm}^2$, $A_{st} = 491 \times 4 = 1964\text{ mm}^2$

- $\frac{x_{u,\max}}{d} = 0.479$ for Fe415

$\Rightarrow$ $x_{u,\max} = 0.479 \times 655 = 313.7$ mm

- Assuming (for first approximation) $f_{sc} = f_{st} = 0.87\,f_y$

$$C_{uc} = 0.362 \times 20 \times 300 \times x_u = (2172\, x_u) \text{ N}$$

$$C_{us} = (0.87 \times 415 - 0.447 \times 20) \times 982 = 345772 \text{ N}$$

$$T_u = 0.87 \times 415 \times 1964 = 709102 \text{ N}$$

- Considering force equilibrium : $C_{uc} + C_{us} = T_u$

$\Rightarrow$ $2172x_u + 345772 = 709102$

$\Rightarrow$ $x_u = 167.3 \text{ mm} < x_{u,\max} = 313.7 \text{ mm}$

$\therefore$ The assumption $f_{st} = 0.87 f_y$ is justified since section is under-reinforced and thus tension steel has yielded.

$$\text{Further } \varepsilon_{sc} = 0.0035\left(1 - \frac{45}{167.3}\right) = 0.00256$$

For Fe415, $$\varepsilon_y = \frac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.0038$$

As $\varepsilon_{sc} < \varepsilon_y$, the assumption $f_{sc} = 0.87 f_y$ is not justified, whereby the calculated value of C_{us} (and hence the value of $x_u = 167.3$ mm) is also not correct. The correct value has to be obtained iteratively using strain compatibility.

First cycle

- Assuming $\varepsilon_{sc} = 0.00256$

$$f_{sc} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)}(0.00256 - 0.00241)$$

$$= 346.7 \text{ MPa}$$

$\Rightarrow$ $C_{us} = (346.7 - 0.447 \times 20) \times 982 = 331680 \text{ N}$

$$C_{uc} + C_{us} = T_u$$

$\Rightarrow$ $x_u = \left(\frac{709102 - 331680}{2172}\right) = 173.8 \text{ mm}$

$\Rightarrow$ $\varepsilon_{sc} = 0.0035\left(1 - \frac{45}{173.8}\right)$

$$= 0.00259 \simeq 0.00256 \quad \text{(calculated earlier)}$$

Second cycle

- Assuming $\varepsilon_{sc} = 0.00259$

$$f_{sc} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)}(0.00259 - 0.00241)$$

$$= 347.4 \text{ MPa}$$

$\Rightarrow$ $C_{us} = (347.4 - 0.447 \times 20) \times 982 = 332368 \text{ N}$

$$\Rightarrow \quad x_u = \left(\frac{709102 - 332368}{2172}\right) = 173.4 \text{ mm} \quad \text{(Converged)}$$

- Taking $x_u = 173.4$ mm

$$M_{UR} = C_{uc}(d - 0.416x_u) + C_{us}(d - d')$$

$$= (2172 \times 173.4)(655 - 0.416 \times 173.4) + 332368(655 - 45)$$

$$= 422.3 \times 10^6 \text{ Nmm}$$

$$= 422 \text{ kNm}$$

Q.2 (b)Solution:

(i)

The assumptions made while analyzing the reinforced concrete beam using Limit State Method as per **IS 456:2000 Code** are as follows:

1. Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section. Thus, strain variation diagram is linear.
2. The maximum compressive strain in concrete at the outermost fiber (ε_{cu}) is taken as 0.0035, regardless of whether the beam is under-reinforced or over-reinforced, because collapse invariably occurs by the crushing of concrete.
3. IS 456: 2000 allows the use of any other possible shape of the stress-strain curve of concrete which results in substantial agreement with the results of the tests on reinforced concrete.
4. For design purposes, compressive strength of concrete may be assumed as **0.67** times the characteristic strength of concrete. The partial safety factor of $\gamma_c = 1.5$ shall be applied in addition to this.
5. The tensile strength of concrete is ignored i.e. not taken into account. **Cl. B-1.3(b) of IS 456: 2000** states that all tensile stresses are to be taken up by reinforcement and none by concrete, except as otherwise specifically permitted.
6. The stress in reinforcement is derived from representative stress-strain curve for the type of steel used.
7. For design purpose, the partial safety factor for steel is taken as $\gamma_s = 1.15$ i.e. design stress of steel = $\dfrac{f_y}{1.15} = 0.87 f_y$

8. The maximum strain (ε_{st}) in the tension reinforcement at the level of centroid of reinforcement steel at the ultimate limit state shall not be less than ε^*_{st} which is defined as:

$$\varepsilon^*_{st} = \frac{0.87\, f_y}{E_s} + 0.002$$

(ii)

Size of the column: 400 mm × 400 mm

$$l = 2.25 \text{ m}$$

Minimum eccentricity is greater than the following:

1. $$\frac{l}{500} + \frac{b}{30} = \frac{2250}{500} + \frac{400}{30} = 4.50 + 13.33 = 17.83 \text{ mm}$$

2. 20 mm

$$\therefore \quad e_{\min} = 20 \text{ mm}$$

$$0.05\, b = 0.05 \times 400 = 20 \text{ mm}$$

$\therefore$ $e_{\min}$ has not exceeded 0.05 b

$$\text{Gross area of the section, } A_g = 400 \times 400 = 160000 \text{ mm}^2$$

$$\text{Area of steel, } A_{sc} = 8 \times \frac{\pi}{4} \times 16^2 = 8 \times 201 = 1608 \text{ mm}^2$$

$$\therefore \quad \text{Area of concrete, } A_c = 160000 - 1608 = 158392 \text{ mm}^2$$

Since $e_{\min}$ has not exceeded 0.05 b, the ultimate load is given by,

$$P_u = 0.40\, f_{ck} A_c + 0.67\, f_y\, A_{sc}$$

$$\Rightarrow \quad P_u = 0.40 \times 20 \times 158392 + 0.67 \times 415 \times 1608$$

$$= 1267136 + 447104 = 1714240 \text{ N} = 1714.24 \text{ kN}$$

Q.2 (c) Solution:

- **Short column/slender column:**

Given; $l_x = l_y = 3000$ mm, $D_y = 450$ mm and $D_x = 600$ mm

$$\text{Slenderness ratio, } \begin{cases} \dfrac{l_{ex}}{D_x} = \dfrac{k_x l_x}{D_x} = k_x \times \dfrac{3000}{600} = 5k_x \\[2ex] \dfrac{l_{ey}}{D_y} = \dfrac{k_y l_y}{D_y} = k_y \times \dfrac{3000}{450} = 6.67k_y \end{cases}$$

As the column is braced against side sway in both directions, effective length ratios k_x and k_y are both less than unity and hence slenderness ratios are both less than 12.

Hence, the column may be designed as short column.

- **Minimum eccentricities:**

$$e_{x,\min} = \frac{l}{500} + \frac{D_x}{30} = \frac{3000}{500} + \frac{600}{30} = 26.0 \text{ mm } (> 20.0 \text{ mm})$$

$$e_{y,\min} = \frac{l}{500} + \frac{D_y}{30} = \frac{3000}{500} + \frac{450}{30} = 21.0 \text{ mm } (> 20.0 \text{ mm})$$

As $0.05D_x = 0.05 \times 600 = 30.0 \text{ mm} > e_{x,\min} = 26.0 \text{ mm}$

and $0.05D_y = 0.05 \times 450 = 22.5 \text{ mm} > e_{y,\min} = 21.0 \text{ mm}$

The formula for axially loaded column can be used as per IS 456 : 2000.

- **Factored load**

P_u = Service load × Partial load factor = 2000 × 1.5 = 3000 kN

- **Design of longitudinal reinforcement**

$$P_u = 0.4 f_{ck} A_g + \left(0.67 f_y - 0.4 f_{ck}\right) A_{sc}$$

$$\Rightarrow \quad 3000 \times 10^3 = 0.4 \times 20 \times (450 \times 600) + (0.67 \times 415 - 0.4 \times 20)A_{sc}$$

$$\Rightarrow \quad 3000 \times 10^3 = 2160 \times 10^3 + 270.05\, A_{sc}$$

$$\Rightarrow \quad A_{sc} = \frac{(3000 - 2160) \times 10^3}{270.05} \simeq 3111 \text{ mm}^2$$

In view of column dimensions (450 mm, 600 mm), it is necessary to place intermediate bars in addition to the four corner bars.

Provide 4-25ϕ bars at corners : $4 \times \frac{\pi}{4} \times 25^2 = 1964 \text{ mm}^2$

and 4-20ϕ at additional : $4 \times \frac{\pi}{4} \times 20^2 = 1256 \text{ mm}^2$

$$\therefore \quad A_{st} = 1964 + 1256 = 3220 \text{ mm}^2 > 3111 \text{ mm}^2$$

Percent reinforcement provided,

$$P = \frac{100 \times 3220}{(450 \times 600)}$$

$$= 1.193 > 0.8\% \text{ (minimum reinforcement) (OK)}$$

$$< 6\% \text{ (maximum reinforcement) (OK)}$$

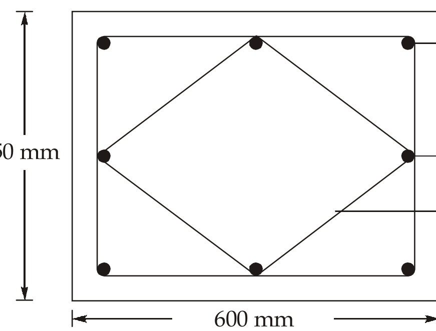
- **Lateral ties**

$$\text{Tie diameter, } \phi_t > \begin{cases} \dfrac{\phi_{main,long}}{4} = \dfrac{25}{4} = 6.25 \text{ mm} \\ 6 \text{ mm} \end{cases}$$

∴ Provide 8 mm diameter ties

$$\text{Tie spacing, } S_t < \begin{cases} 450 \text{ mm} \\ 16 \times 20 = 320 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

∴ Provide ties spacing = 300 mm



The detailing of reinforcement is shown above.

Q.3 (a) Solution:

∵ One end of short span is continuous and other discontinuous.

Assuming discontinuous end to be simply supported.

∴ Limiting span to depth ratio $= \dfrac{26+20}{2} = 23$

∴ Depth of slab required, $D = \dfrac{4000}{23} = 173.9 \text{ mm} \simeq 175 \text{ mm (say)}$

Let effective cover = 25 mm

∴ Effective depth of slab in short direction, $d_x = 175 - 25 = 150$ mm

∴ Effective depth of slab in long direction, $d_y = d_x$ – Bar diameter

$= 150 - 10 = 140$ mm

∴ **Effective spans:**

$$l_x = \text{clear span} + d_x = 4 + 0.15 = 4.15 \text{ m}$$

$$l_y = \text{clear span} + d_y = 5 + 0.14 = 5.14 \text{ m}$$

$$\therefore \quad r = \frac{l_y}{l_x} = \frac{5.14}{4.15} = 1.24 < 2$$

∴ Two way slab.

Loads

Self weight of slab $= 0.175 \times 1 \times 1 \times 25 = 4.375 \text{ kN/m}^2$

Finishes $= 1 \text{ kN/m}^2$

Live load $= 8 \text{ kN/m}^2$

$\therefore$ Total load = 13.375 kN/m^2

$\therefore$ Factored load, w_u = 1.5 × 13.375 = 20.0625 kN/m^2 = 20.06 kN/m^2

Moments

Positive mid-span moment in short direction $\left(M_{ux}^{+}\right)$:

$$M_{ux}^{+} = \alpha_x^{+} w_u l_x^2$$
$$= (0.047)\ 20.06\ (4.15)^2$$
$$= 16.24 \text{ kNm/m}$$

Positive mid-span moment in long direction $\left(M_{uy}^{+}\right)$:

$$M_{uy}^{+} = \alpha_y^{+} w_u l_x^2$$
$$= 0.035\ (20.06)\ (4.15)^2$$
$$= 12.09 \text{ kNm/m}$$

Negative support moment in short direction $\left(M_{ux}^{-}\right)$:

$$M_{ux}^{-} = \alpha_x^{-} w_u l_x^2$$
$$= 0.062\ (20.06)\ (4.15)^2$$
$$= 21.42 \text{ kNm/m}$$

Negative support moment in long direction $\left(M_{uy}^{-}\right)$:

$$M_{uy}^{-} = \alpha_y^{-} w_u l_x^2$$
$$= 0.047\ (20.06)\ (4.15)^2$$
$$= 16.24 \text{ kNm/m}$$

Depth of slab required :

Maximum moment, M = 21.42 kNm/m

For Fe415, $M_{u,\lim} = 0.138 f_{ck} b d^2$

$\therefore$ $M_{u,\lim} = M$

$\Rightarrow$ $0.138 f_{ck} b d^2 = 21.42 \times 10^6$

$\Rightarrow$ $0.138(20)(1000)d^2 = 21.42 \times 10^6$

$\Rightarrow$ d = 88.1 mm < 150 mm (OK)

Flexural reinforcement :

$$M_{ux}^{+} = 16.24 \text{ kNm/m}$$
$$d_x = 150 \text{ mm}$$

$\therefore \quad p^{+}_{tx,reqd.} = 0.23\% \quad \left[\text{From } \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left(1-\sqrt{1-\frac{4.598M_u}{f_{ck}bd^2}}\right)\right]$

$\therefore \quad A^{+}_{stx,reqd.} = \frac{0.23}{100} \times 1000 \times 150 = 345 \text{ mm}^2/\text{m}$

$\therefore \quad \text{Spacing of } 10\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{345} = 227.65 \text{ mm c/c} \simeq 220 \text{ mm c/c(say)}$

∴ Provide 10ϕ bars @ 220 mm c/c in short direction at mid-span.

$M^{+}_{uy} = 12.09 \text{ kNm/m}$

$d_y = 140 \text{ mm}$

$\therefore \quad p^{+}_{ty,reqd.} = 0.18\% \quad \left[\text{From } \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left(1-\sqrt{1-\frac{4.598M_u}{f_{ck}bd^2}}\right)\right]$

$\therefore \quad A^{+}_{sty,reqd.} = \frac{0.18}{100} \times 1000 \times 140 = 252 \text{ mm}^2/\text{m}$

$\therefore \quad \text{Spacing of } 10\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{252} = 311.7 \text{ mm} > 300 \text{ mm}$

∴ Provide 10ϕ bars @ 280 mm c/c (< 300 mm) in long direction at mid-span.

$M^{-}_{ux} = 21.42 \text{ kNm/m}$

$d_x = 150 \text{ mm}$

$\therefore \quad p^{-}_{tx,reqd.} = 0.28\% \quad \left[\text{From } \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left(1-\sqrt{1-\frac{4.598M_u}{f_{ck}bd^2}}\right)\right]$

$\therefore \quad A^{-}_{st,reqd.} = \frac{0.28}{100} \times 1000 \times 150 = 420 \text{ mm}^2/\text{m}$

$\therefore \quad \text{Spacing of } 10\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{420} = 187 \text{ mm c/c} \simeq 180 \text{ mm c/c (say)}$

∴ Provide 10ϕ bars 180 mm c/c in short direction at support.

$M^{-}_{uy} = 16.24 \text{ kNm/m}$

$d_y = 140 \text{ mm}$

$\therefore \quad p^{-}_{ty,reqd.} = 0.24\% \quad \left[\text{From } \frac{p_t}{100} = \frac{A_{st}}{bd} = \frac{f_{ck}}{2f_y}\left(1-\sqrt{1-\frac{4.598M_u}{f_{ck}bd^2}}\right)\right]$

$$\therefore \quad A^{-}_{sty,reqd.} = \frac{0.24}{100} \times 1000 \times 140 = 336 \text{ mm}^2/\text{m}$$

$$\therefore \quad \text{Spacing of } 10\phi \text{ bars} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{336} = 233.7 \text{ mm c/c} \simeq 220 \text{ mm c/c (say)}$$

∴ Provide 10ϕ bars @ 220 mm c/c in long direction at support.

Deflection check:

$$A^{+}_{stx,reqd} = 315 \text{ mm}^2/\text{m}$$

$$A^{+}_{stx,prov.} = \frac{1000 \times \frac{\pi}{4} \times 10^2}{220} = 357 \text{ mm}^2/\text{m}$$

$$\therefore \quad f_{st} = 0.58 f_y \cdot \frac{A^{+}_{stx,reqd}}{A^{+}_{stx,prov.}}$$

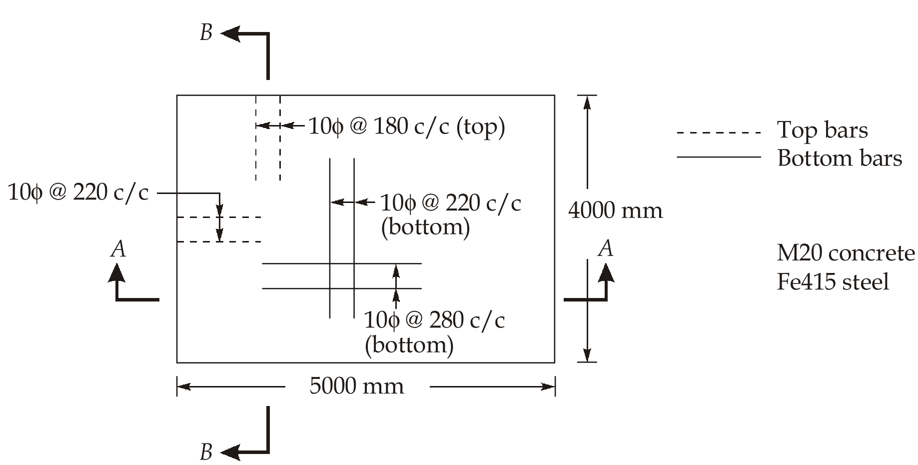
$$= 0.58(415)\frac{345}{357} = 232.6 \text{ N/mm}^2$$

$$\therefore \quad k_t = 1.6 \text{ (from figure 4 of IS 456:2000)}$$

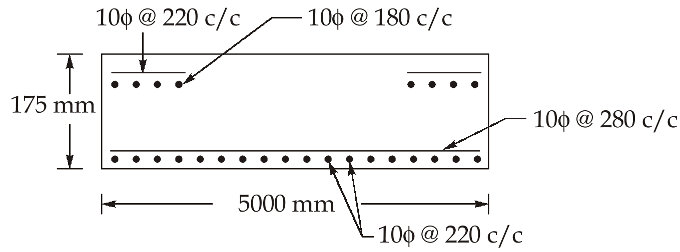
$$\therefore \quad \left(\frac{l}{d}\right)_{max} = \left(\frac{l}{d}\right)_{basic} \cdot k_t \cdot k_c \quad \text{where } k_c = 1 \qquad (\because p_c = 0)$$

$$= 23 \times 1.6 = 36.8$$

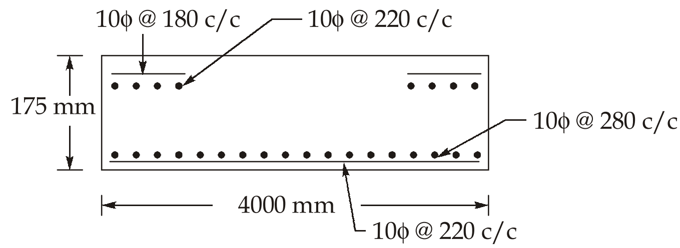
$$\left(\frac{l}{d}\right)_{prov} = \frac{4150}{150} = 27.7 < 36.8 \quad \text{(OK)}$$



Plan



Section A-A



Section B-B

Reinforcement details

Q.3 (b)Solution:

Size of foundation :

Column service load, $P = 1500$ kN

Let weight of the foundation, $P_f = 10\%$ of column load $(P) = 0.1 \times 1500$ kN $= 150$ kN

$\therefore$ Total load, $P_t = 1500 + 150$ kN $= 1650$ kN

$$\text{Area of footing required, } A = \frac{1650}{160} = 10.31 \text{ m}^2$$

Note: Do not use factored load of 1.5 × 1650 kN here, since safe bearing capacity of soil itself takes into account the factor of safety.

Given, $B = 2.5$ m

$$\therefore \quad L = \frac{A}{B} = \frac{10.31}{2.5} = 4.124 \text{ m} \simeq 4.2 \text{ m (say)}$$

$\therefore$ Area of footing provided, $A = 4.2 \times 2.5 \text{ m}^2 = 10.5 \text{ m}^2 > 10.31 \text{ m}^2$ (OK)

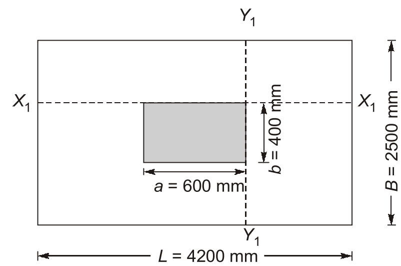
Net soil pressure

$$w_0 = \frac{P}{A} = \frac{1500}{10.5} = 142.86 \text{ kN/m}^2$$

$$< 160 \text{ kN/m}^2 \qquad \text{(OK)}$$

Factored soil pressure, $w_{u0} = 1.5 \times 142.86 = 214.29 \text{ kN/m}^2$

Depth of foundation from bending moment criterion:



The critical section for BM will be at the face of the column.

$$\text{Moment at section } X_1\text{-}X_1,\ M_{ux} = w_{u0}\left(\frac{B-b}{2}\right)\left(\frac{B-b}{4}\right)$$

$$= 214.29 \times \frac{(2.5-0.40)^2}{8} = 118.13\,\text{kNm}/\text{m}$$

$$\text{Moment at section } Y_1\text{-}Y_1,\quad M_{uy} = w_{u0}\left(\frac{L-a}{2}\right)\left(\frac{L-a}{4}\right)$$

$$= 214.29 \times \frac{(4.2-0.6)^2}{8} = 347.15\text{ kNm/m}$$

∴ Maximum moment, M_u = 347.15 kNm

For Fe 500, $M_{ulim} = 0.133 f_{ck} bd^2$

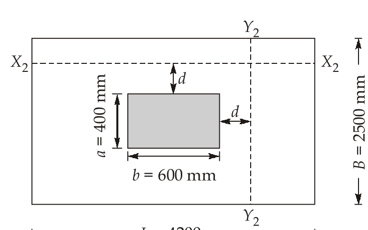
⇒ $347.15 \times 10^6 = 0.133\,(25)\,(1000)\,d^2$

∴ d = 323.12 mm

$$\text{Depth of foundation, } d = \sqrt{\frac{M_{uy}}{QB}} = \sqrt{\frac{347.15\times10^6}{3.34\times1000}} = 322.39\text{ mm}$$

Take d = 330 mm

Check for one way shear



Critical section for one way shear will be at a distance of 'd' from the face of the column.

Maximum shear force at section Y_2-Y_2

$$V_{uy} = w_{u0} \times 1 \times \left[\frac{L-b}{2} - d\right]$$

$$= 214.29\times 1\left[\frac{4.2-0.6}{2}-0.33\right] = 315 \text{ kN}$$

Nominal shear stress, $\tau_v = \dfrac{V_{uy}}{Bd} = \dfrac{315\times 10^3}{1000\times 330} = 0.955 \text{ N/mm}^2$

$k = 1.0$ for $d > 300$ mm

$\therefore$ $k\tau_c = 1 \times 0.29 \text{ N/mm}^2 < \tau_v$ $(= 0.955 \text{ N/mm}^2)$ (Not safe)

Revised depth of the footing is computed as below,

$$k\tau_c = 0.29 = \frac{V_{uy}}{Bd}$$

$$\Rightarrow \quad d = \frac{V_{uy}}{B\times 0.29} = \frac{315\times 10^3}{1000\times 0.29} = 1086.20 \text{ mm}$$

Average of 330 mm and 1086.20 mm = $\dfrac{330+1086.20}{2} = 708.10$ mm ≈ 700 mm (say)

Check for d = 700 mm

$$V_{uy} = 214.29\times\left[\frac{4.2-0.6}{2}-0.7\right] = 235.72 \text{ kN}$$

$$\tau_v = \text{c} \quad \frac{V_{uy}}{Bd} = \frac{235.72\times 10^3}{1000\times 700} = 0.34 \text{ N/mm}^2 > \tau_c\left(= 0.29 \text{ N/mm}^2\right)\text{(Not safe)}$$

Try d = 800 mm

$$\therefore \quad V_{uy} = 214.29\times\left[\frac{4.2-0.6}{2}-0.80\right] = 214.29 \text{ kN}$$

$$\therefore \quad \tau_v = \frac{V_{uy}}{Bd} = \frac{214.29\times 10^3}{1000\times 800} = 0.268 \text{ N/mm}^2 < 0.29 \text{ N/mm}^2$$

(OK)

$\therefore$ Adopt effective depth, d = 800 mm

Check for two-way shear (Punching shear) :

Net punching shear stress developed,

$$\tau_{vp\,(\text{developed})} = \frac{P_u - w_{u_0}(a+d)(b+d)}{2\left[(a+d)+(b+d)\right]d}$$

$$= \frac{(1.5\times 1500)-214.29(0.4+0.8)(0.6+0.8)}{2\left[(0.4+0.8)+(0.6+0.8)\right]\times 0.8}$$

$$= \frac{225-360}{4.16}$$

$$= 454.33 \text{ kN/m}^2 = 0.454 \text{ N/mm}^2$$

$$\tau_{v_p \text{ (permissible)}} = 0.25\sqrt{f_{ck}} \times k_\beta$$

$$\left[k_\beta = 0.5 + \frac{b}{a} = 0.5 + \frac{400}{600} = 1.167 \not> 1.0\right]$$

$$= 0.25\sqrt{25} \times 1.0$$

$$= 1.25 \text{ N/mm}^2 > 0.454 \text{ N/m}^2 \quad \text{(Hence safe)}$$

With 60 mm effective cover,

Total depth if footing = 800 + 60 = 860 mm

Q.3 (c)Solution:

(i)

When the bending moment required to be resisted is more than the moment of resistance of a balanced section of singly reinforced beam of given size, there are two alternatives:

(i) To use an **over-reinforced** section.

(ii) To use **doubly reinforced** section.

An over reinforced section is always uneconomical and also undesirable because of sudden failure probability. Also the increase in the moment of resistance is not in proportion to the increase in the area of tensile reinforcement. The reason behind this is that the concrete, having reached maximum allowable stress, cannot take more additional load without adding compression steel. The other alternative is to provide reinforcement in the compression side of the beam and thus to increase the moment of resistance of the beam beyond that of a balanced section.

Doubly reinforced sections are also useful in following situations:

(i) Where the members are subjected to probable reversal of external loads and thereby the bending moment in the section reverses, such as in case of wind and earthquake loads etc.

(ii) When the members are subjected to loading, eccentric to either side of the axis, such as in columns subjected to wind loads.

(iii) When the members are subjected to accidental lateral loads, shock or impact.

The steel reinforcement provided in the compression zone is subjected to compressive stress. However, concrete undergoes creep strains due to continued compressive stress, with the result that the strain in concrete goes on increasing with time. This increases compressive strain in steel in addition to creep strain in compressive steel. Thus the total compressive strain in compressive steel will be much greater than the strain in surrounding concrete due to flexure alone. Thus, compressive steel takes up all the additional compressive stresses beyond the permissible compressive stress for concrete making the section safe against failure in flexure.

(ii)

Approximate solution considering gross section:

$$\text{Area of a single wire, } A_w = \frac{\pi}{4}\times 7^2 = 38.48 \text{ mm}^2$$

$$\text{Total area of prestressing steel} = 9 \times 38.48 = 346.32 \text{ mm}^2$$

$$\text{Area of beam section, } A = 300 \times 250 = 75 \times 10^3 \text{ mm}^2$$

$$\text{Moment of inertia of beam section-}I = \frac{300\times 250^3}{12} = 3.91\times 10^8 \text{mm}^4$$

Distance of centroid of steel area from the soffit of beam,

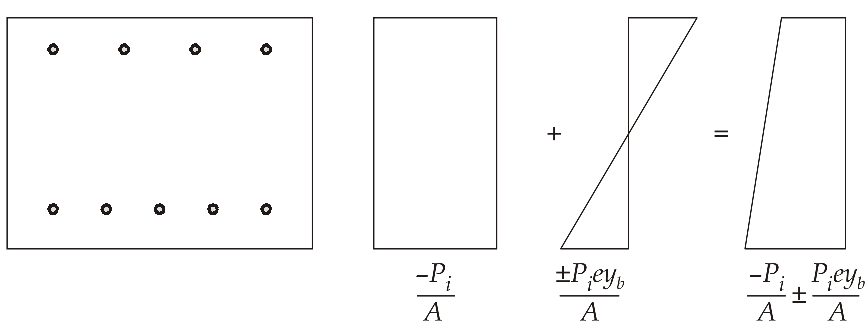
$$\bar{y} = \frac{4\times 38.48\times(250-40)+5\times 38.48\times 40}{9\times 38.4} = 115.56 \text{ mm}$$

$$\text{Prestressing force, } P_i = 0.8 \times 1570 \times 346.32 \text{ N} \simeq 435 \text{ kN}$$

Eccentricity of prestressing force,

$$e = \left(\frac{250}{2}\right) - 115.56 = 9.44 \text{ mm}$$

Stress diagrams due to P_i is shown below.



Since the wires are distributed above and below the CG, the losses are calculated for the top and bottom wires separately.

Stress at level of top wires ($y = y_c = 125 - 40 = 85$ mm)

$$(f_c)_t = -\frac{P_i}{A} + \frac{P_i e}{I}\cdot y_t$$

$$= -\frac{435\times 10^3}{75\times 10^3} + \frac{435\times 10^3\times 9.44}{3.91\times 10^8}(125-40)$$

$$= -5.8 + 0.9$$

$$= -4.9 \text{ N/mm}^2$$

$$\text{Stress at bottom, } (f_c)_b = -\frac{P_i}{A} - \frac{P_i e y_b}{I}$$

$$= -\frac{435 \times 10^3}{75 \times 10^3} - \frac{435 \times 10^3 \times 9.44}{3.91 \times 10^8}(125 - 40)$$

$$= -5.8 - 0.9 = -6.7 \text{ N/mm}^2$$

Loss of prestress in top wires = 6 × 4.9 × 4 × 38.48 = 4525.248 N = 4.525 kN

Loss of prestress in bottom wires = 6 × 6.7 × 5 × 38.48 = 7734.48N = 7.734 kN

$$\text{Total loss of prestress} = 4.525 + 7.734 = 12.259 \text{ kN} \simeq 12.3 \text{ kN}$$

$$\therefore \quad \text{Percentage loss} = \frac{12.3}{435} \times 100 = 2.83\%$$

Q.4 (a) Solution:

1. Given data : $h = 4.0 + 1.25 = 5.25$ m; $\mu = 0.5$, $\theta = 15°$, $\phi = 30°$, $\gamma_e = 16$ kN/m^3, $q_a = 160$ kN/m^2

- Earth pressure coefficients: $$C_a = \left[\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right]\cos\theta = 0.373$$

$$C_p = \frac{1 + \sin\phi}{1 - \sin\phi} = 3.0$$

2. **Preliminary proportions**

   - Thickness of footing base slab $\simeq 0.08h = 0.08 \times 5.25 = 0.42$ m

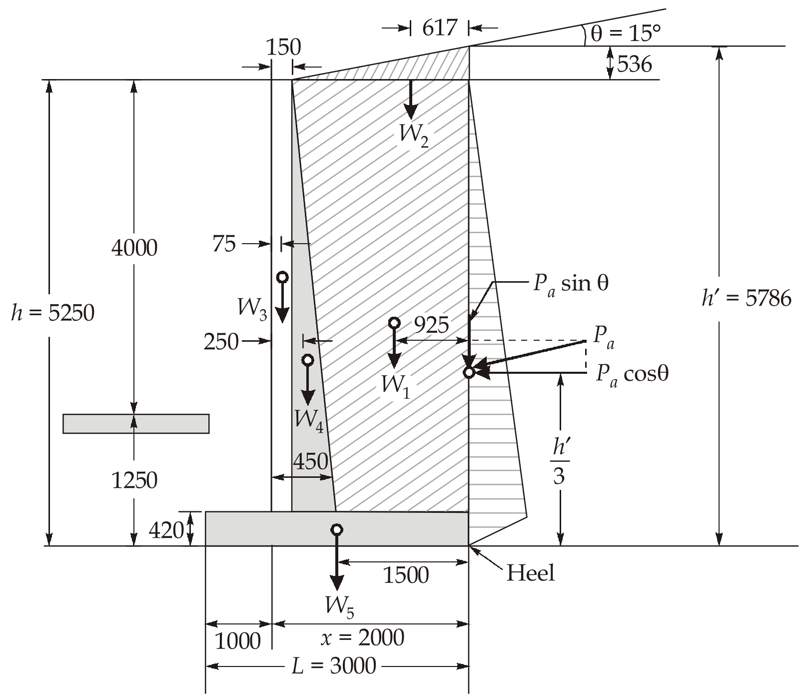
     Assume thickness of base slab as 420 mm.
   - Assume stem thickness of 450 mm at the base of stem, tapering to a value of 150 mm at the top of the wall.
   - For an economical proportioning of the length L of the base slab, it will be assumed that the vertical reaction R at the footing base is in line with the front face of the stem. For such a condition, (assuming the height above top of wall to be about 0.4 m), the length of the heel slab (inclusive of stem thickness).

$$X = \left(\sqrt{\frac{C_a}{3}}\right)h' = \sqrt{\frac{0.373}{3}}(5.25 + 0.4) \simeq 2.0 \text{ m}$$

Assuming a triangular base pressure distribution,

$$L = 1.5X = 3.0 \text{ m}$$

- The preliminary proportions are shown in figure below.



Forces on wall (with preliminary proportions)

3. Stability against overturning

- Force due to active pressure:

$$P_a = \frac{C_a \gamma_e (h')^2}{2}$$

where, $h' = h + X \tan\theta = 5250 + 1850 \tan 15° \simeq 5746$ mm

$$P_a = \frac{(0.373)(16)(5.746)^2}{2}$$

$$= 98.52 \text{ kN} \quad \text{(per m length of wall)}$$

$\therefore$ $P_a \cos\theta = 98.52 \cos 15° = 95.16$ kN

$P_a \sin\theta = 98.52 \sin 15° = 25.5$ kN

- Overturning moment $M_0 = \dfrac{(P_a \cos\theta)h'}{3}$

$= (95.16)(5.746/3) = 182.26$ kNm

- Line of action of resultant of vertical forces as shown in figure above with respect to the heel can be located by applying statics, considering 1 m length of the wall:

Force (kN)		Distance from heel (m)	Moment (m)
$W_1 = (16)(1.85)(5.25 - 0.42)$	$= 143.0$	0.925	132.3
$W_2 = (16)(1.85)(0.5 \times 0.536)$	$= 7.9$	0.617	4.9
$W_3 = (25)(0.15)(5.25 - 0.42)$	$= 18.1$	1.925	34.8
$W_4 = (25 - 16)(4.83)(0.5 \times 0.30)$	$= 6.5$	1.750	11.4
$W_5 = (25)(3.0)(0.42)$	$= 31.5$	1.500	47.2
$P_a \sin\theta$	$= 25.5$	0.000	0.0
	W = 232.5 kN		**M_W = 230.6 kNm**

Distance of resultant vertical force from heel

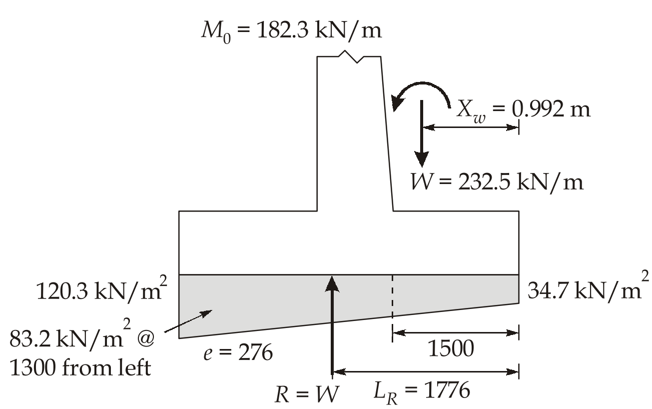
$$x_W = \frac{M_W}{W} = \frac{230.6}{232.5} = 0.992 \text{ m}$$

- Stabilising moment (about toe)

$$M_r = W(L - X_W) = 232.5 \times (3.0 - 0.992)$$
$$= 466.86 \text{ kNm (per m length of wall)}$$

$$\therefore \quad (FS)_{\text{overtuning}} = \frac{0.9M_r}{M_0} = \frac{0.9 \times 466.86}{182.26} = 2.31 > 1.40 \quad \text{(OK)}$$

4. Soil pressures at footing base



Calculation of soil pressures

- Resultant vertical reaction $R = W$ = 232.5 kN (per m length of wall)

- Distance of R from heel : $L_R = \dfrac{(M_w + M_0)}{R}$

$$= \frac{(230.6 + 182.26)}{232.5} = 1.776 \text{ m}$$

- $$\text{Eccentricity, } e = L_R - \frac{L}{2} = 1.776 - \frac{3.0}{2} = 0.276 \text{ m} < \frac{L}{6} = 0.5 \text{ m}$$
- Hence, the resultant lies within the middle third of the base.

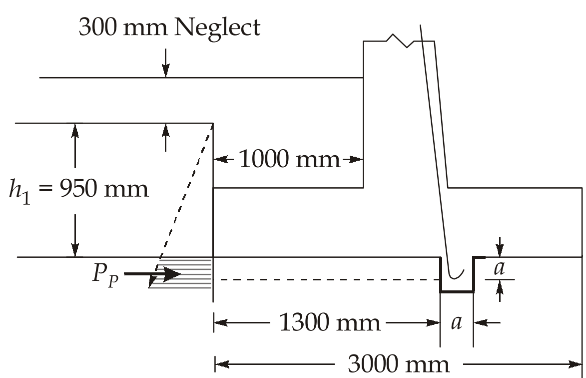
$$\frac{6e}{L} = \frac{6 \times 0.276}{3.0} = 0.552$$

$$\therefore \quad q_{max} = \frac{R}{L}\left(1 + \frac{6e}{L}\right)$$

$$= \frac{232.5}{3.0}(1 + 0.552) = 120.28 \text{ kN/m}^2 < q_a \; (= 160 \text{ kN/m}^2)$$

and $$q_{min} = \frac{232.5}{3.0}(1 - 0.552) = 34.72 \text{ kN/m}^2 > 0 \qquad \text{(OK)}$$

5. Stability against sliding



Design of shear key

- Sliding force = $P_a \cos\theta$ = 95.16 kN
- Resisting force (ignoring passive pressure on the toe side) $F = \mu R$

$$= 0.5 \times 232.5 = 116.25 \simeq 116.3 \text{ kN}$$

$$\therefore \quad (FS)_{sliding} = \frac{0.9F}{P_a \cos\theta} = \frac{0.9 \times 116.3}{95.16} = 1.1 < 1.40 \text{ (Not safe)}$$

Hence, a shear key may be provided to mobilise the balance force through passive resistance.

- Assume a shear key of size $a \times a$, at 1300 mm from toe

$$F = \frac{C_p \gamma_e \left[(h_1 + a)^2 - h_1^2\right]}{2} + \mu_s R_{toe} + \mu R_{heel}$$

$$= \frac{3 \times 16 \times \left[(0.95 + a)^2 - 0.95^2\right]}{2} + \tan 30 \times \frac{120.28 + 83.2}{2} \times 1.3 + 0.5 \times \frac{(83.2 + 34.74)}{2} \times 1.7$$

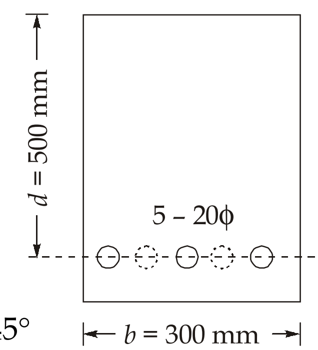
$$= 24a^2 + 45.6a + 126.48$$

$$(\text{FS})_{\text{sliding}} = \frac{0.9\left(24a^2 + 45.6a + 126.48\right)}{95.16} > 1.4$$

$\Rightarrow$ $a > 0.36$ m.

Hence provide a shear key of size 400 mm × 400 mm.

Q.4 (b) Solution:



Given: Material used, M20, Fe 415

Ultimate shear force, $V_u = 300$ kN

Shear resistance of bent up bar

$$V_{uc} = 0.87 f_y \cdot A_{sv} \cdot \sin\alpha$$

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4}(20)^2 \times \sin 45°$$

$$= 160410.2885 \text{ N} = 160.41 \text{ kN}$$

Design of shear reinforcement:

$$\text{Nominal shear stress, } (\tau_v) = \frac{V_u}{bd} = \frac{300 \times 10^3}{300 \times 500} = 2 \text{ N/mm}^2$$

$$< \tau_{c,\max} = 0.625\sqrt{f_{ck}} = 0.625\sqrt{20} = 2.8 \text{ N/mm}^2 \text{ (O.K)}$$

Design shear strength of concrete (τ_c)

Percentage of tensile reinforcement at section where bars are bent-up:

$$P_t(\%) = \frac{A_{st}}{bd} \times 100 = \frac{3 \times \frac{\pi}{4} \times (20)^2}{300 \times 500} \times 100 = 0.63\%$$

From table given in question, using linear interpolation

$$\tau_c = 0.48 + \frac{0.56 - 0.48}{0.75 - 0.50}(0.63 - 0.50)$$

$$= 0.5216 \text{ N/mm}^2$$

$\therefore$ $\tau_v > \tau_c$

Hence, additional shear reinforcement consisting of vertical stirrups shall be required to resist shear force.

Assuming 2-legged 8 mm ϕ stirrups

$\therefore$ Design shear force, $V_{us} = V_u - V_c - V_{uc}$

$$= 300 - (\tau_c \cdot bd) - 160.41$$

$$= 300 - \frac{0.521 \times 300 \times 500}{1000} - 160.41$$

$$= 61.44 \text{ kN but } \nless 0.5\,(V_u - V_c) = 0.5\left(300 - \frac{0.521 \times 300 \times 500}{1000}\right) = 110.925 \text{ kN}$$

$\therefore$ Design shear force, $V_{us} = 110.925$ kN

Spacing of stirrups:

$$V_{us} = \frac{0.87\, f_y A_{sv} d}{S_v}$$

$$\Rightarrow \quad S_v = \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} \times (8)^2 \times 500}{110.925 \times 10^3} = 163.60 \text{ mm}$$

Spacing from minimum shear reinforcement criterion.

$$\frac{A_{sv}}{b \cdot S_v} \geq \frac{0.4}{0.87\, f_y}$$

$$\Rightarrow \quad S_v \leq \frac{0.87\, f_y A_{sv}}{0.4 b}$$

$$= \frac{0.87 \times 415 \times 2 \times \frac{\pi}{4} (8)^2}{0.4 \times 300} = 302.47 \text{ mm}$$

Spacing $S_v \ngtr$

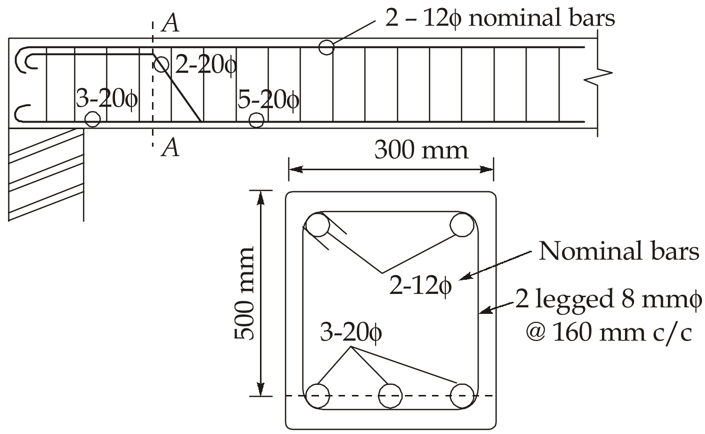
(i) 0.75d = 0.75 × 500 = 375 mm

(ii) 300 mm

(iii) 163.60 mm (spacing of stirrups computed above)

(iv) 302.47 mm (from minimum shear reinforcement consideration)

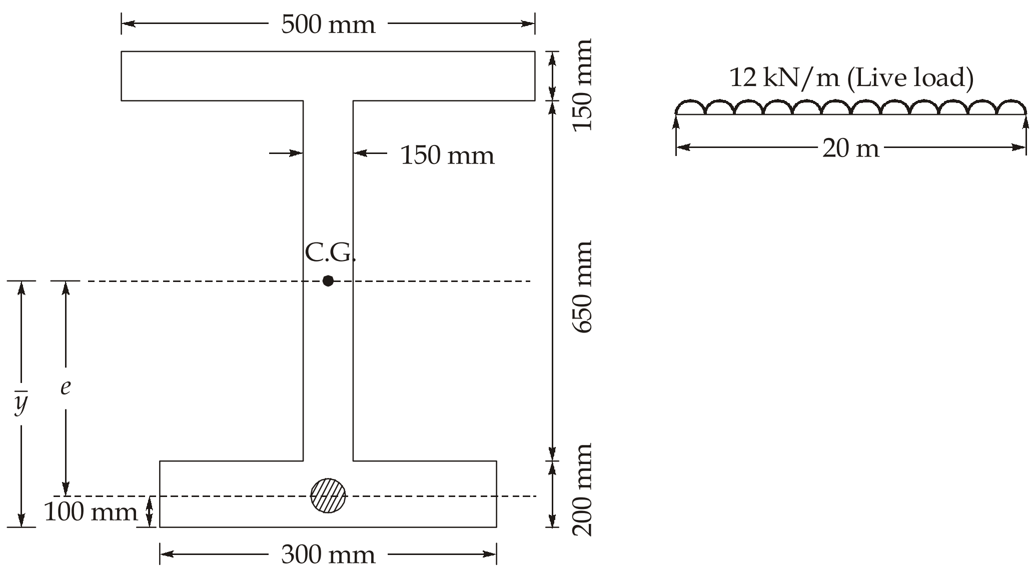
$\therefore$ Provide additional shear reinforcement of 8 mm ϕ 2-legged vertical stirrups at spacing of 160 mm c/c.



Section (A-A)

Q.4 (c) Solution:

Given: Beam cross-section



Let $\bar{y}$ be the distance of C.G. from bottom of the flange,

$$\therefore \qquad \bar{y} = \frac{300 \times 200 \times 100 + 150 \times 650 \times 525 + 500 \times 150 \times 925}{300 \times 200 + 150 \times 650 + 500 \times 150}$$

$$= 544.4 \text{ mm}$$

Moment of inertia of beam about C.G of the cross-section

$$I_{C.G.} = \Sigma I_{self} + \Sigma A_i \left(\bar{y}_i - \bar{y}\right)^2$$

$$= \frac{300\times(200)^3}{12} + 300\times200\times(100-544.4)^2 + \frac{150\times(650)^3}{12} + 150\times650\times(525-544.4)^2$$

$$+\frac{500\times(150)^3}{12} + 500\times150\times(925-544.4)^2$$

$= 265.24 \times 10^8$ mm^4

Area of cross-section of beam

$= 500 \times 150 + 150 \times 650 + 300 \times 200$

$= 232500$ mm^2

$$z_{top} = \frac{I_{C.G.}}{y_t} = \frac{265.24\times10^8}{(1000-544.4)} = 5.822\times10^7 \text{ mm}^3$$

$$z_{bot} = \frac{I_{C.G.}}{y_b} = \frac{265.24\times10^8}{544.4} = 4.872\times10^7 \text{ mm}^3$$

Now, self weight of beam $= \gamma_c A = 24 \times (232500 \times 10^{-6}) = 5.58$ kN/m

Prestressing force, $P = (1385.44\times1100)\times\frac{1}{1000}\text{kN} = 1523.984\text{ kN}$

At mid span:

Moment due to self weight, $M_d = \frac{w_d \cdot L^2}{8} = \frac{5.58\times(20)^2}{8} = 279\text{ kNm}$

Moment due to live load, $M_l = \frac{w_l \cdot L^2}{8} = \frac{12\times(20)^2}{8} = 600\text{ kNm}$

Eccentricity due to prestressing force

$e = 544.4 - 100 = 444.4$ mm

Now, stresses at top and bottom are computed as below:

Case : 1 At transfer stage:

Only prestressing force and DL are considered.

At top: $f_{top} = \frac{P}{A} - \frac{Pe}{z_{top}} + \frac{M_d}{z_{top}}$

$$= \frac{1523.984\times10^3}{232500} - \frac{1523.984\times10^3}{5.822\times10^7}\times444.4 + \frac{279\times10^6}{5.822\times10^7}$$

$= 6.555 - 11.6327 + 4.792 = -0.286$ N/mm^2

At bottom: $$f_{bot} = \frac{P}{A}+\frac{Pe}{z_b}-\frac{M_d}{z_b}$$

$$= \frac{1523.984\times10^3}{232500}+\frac{1523.984\times10^3\times444.4}{4.872\times10^7}-\frac{279\times10^6}{4.872\times10^7}$$

$$= 6.555 + 13.901 - 5.7266 = 14.73 \text{ N/mm}^2$$

Case : 2 At final stage.

At this stage, prestressing force with losses + DL + LL are considered,

At top: $$f_{top} = \frac{kP}{A}-\frac{kPe}{z_t}+\frac{M_d}{z_t}+\frac{M_l}{z_t}$$

$$k = \left(1-\frac{P_l\%}{100}\right)=\left(1-\frac{15}{100}\right)=0.85$$

$$f_{top} = \frac{0.85\times1523.984\times10^3}{232500}-\frac{0.85\times1523.984\times10^3}{5.822\times10^7}\times444.4+\frac{279\times10^6}{5.822\times10^7}+\frac{600\times10^6}{5.822\times10^7}$$

$$= 5.572 - 9.888 + 4.792 + 10.306$$

$$= 10.78 \text{ N/mm}^2$$

At bottom:

$$f_{bot} = \frac{kP}{A}+\frac{kPe}{z_b}-\frac{M_d}{z_b}-\frac{M_l}{z_b}$$

$$= \frac{0.85\times1523.984\times10^3}{232500}+\frac{0.85\times1523.984\times10^3}{4.872\times10^7}\times444.4-\frac{279\times10^6}{4.872\times10^7}-\frac{600\times10^6}{4.872\times10^7}$$

$$= 5.572 + 11.816 - 5.7266 - 12.315$$

$$= -0.65 \text{ N/mm}^2$$

Section B : Design of Steel Structures

Q.5 (a) Solution:

For Fe410 steel, f_u = 410 MPa, f_y = 250 MPa

For 4.6 grade bolts, f_{ub} = 400 MPa, $f_y = 0.6 \times 400 = 240$ MPa

A_{nb} = Stress area of bolts

$$= 0.78\times\frac{\pi}{4}\times d^2 = 0.78\times\frac{\pi}{4}\times20^2 = 245\text{ mm}^2$$

Diameter of bolt hole, d_0 = 20 + 2 = 22 mm

γ_{mb} = Partial factor of safety of bolt material = 1.25

∵ Bolts are in single shear and strength of bolt in single shear,

$$V_{sb} = A_{nb} \times \frac{f_{ub}}{\sqrt{3}\gamma_{mb}} \times 10^{-3} = 245 \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3} = 45.26\,\text{kN}$$

Strength of bolt in bearing, $V_{pd} = 2.5 k_b dt \frac{f_u}{\gamma_{mb}}$ $\quad (f_u = \text{minimum of } f_u \text{ and } f_{ub})$

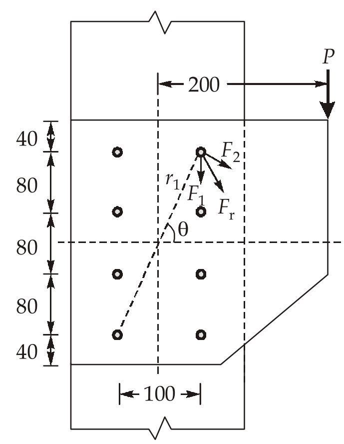
Taking, $k_b = 0.606$ (given)

$$V_{pd} = 2.5 \times 0.606 \times 20 \times 9.1 \times \frac{400}{1.25} \times 10^{-3}$$

$$= 88.23 \text{ kN}$$

So, Bolt value = Minimum of V_{sb} and V_{pb} = 45.26 kN

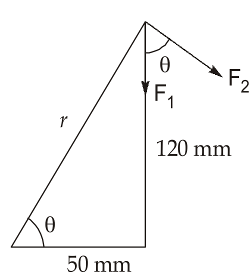
Critical bolt is top-right bolt,



(All dimensions are in mm)

Direct force in bolt, $F_1 = \frac{P'}{n} = \frac{P'}{8}$

Force in bolt due to torque, $F_2 = \frac{P'er}{\Sigma r^2}$



$$r = \sqrt{50^2 + 120^2} = 130\,\text{mm}$$

$$\Sigma r^2 = 4 \times [(50^2 + 40^2) + (50^2 + 120^2)]$$

$$= 84000 \text{ mm}^2$$

$$\cos\theta = \frac{50}{130} = 0.3846$$

$$\Rightarrow \quad F_2 = \frac{P' \times 200 \times 130}{84000} = \frac{P'}{3.23}$$

$$\text{Resultant force on bolt, } F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= P'\sqrt{\frac{1}{8^2} + \frac{1}{3.23^2} + 2 \times \frac{1}{8} \times \frac{1}{3.23} \times 0.3846}$$

$$= 0.376\, P'$$

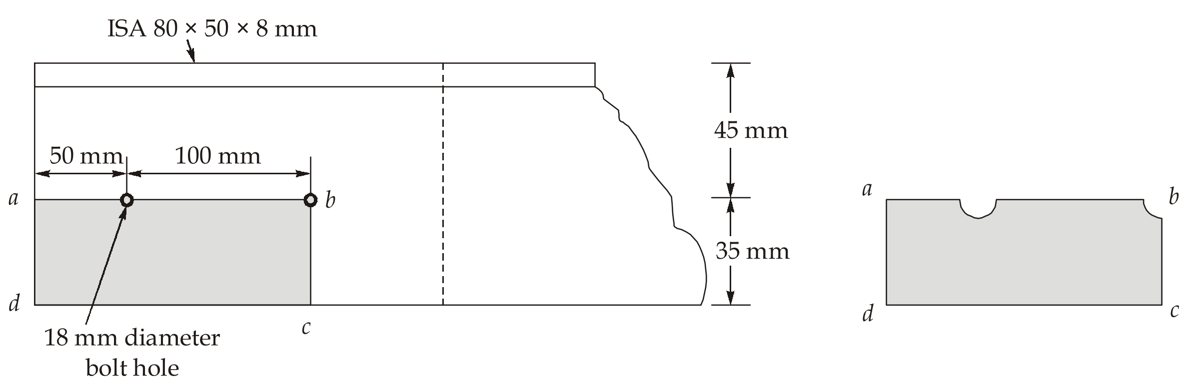
Now, Resultant force $\leq$ Bolt value

$\therefore$ $0.376\, P' = 45.26$ kN

$\therefore$ Factored Load, $P' = 120.37$ kN

$$\text{Service load, } P = \frac{P'}{1.5} = \frac{120.37}{1.5} = 80.25 \text{ kN} \simeq 80 \text{ kN}$$

Q.5 (b)Solution:



For Fe410 grade steel : f_u = 410 MPa, f_y = 250 MPa

Partial safety factors for materials : $\gamma_{m0} = 1.10$; $\gamma_{m1} = 1.25$

The shaded area will shear out

$$A_{vg} = (100 + 50) \times 8 = 1200 \text{ mm}^2$$

$$A_{vn} = \left(100 + 50 - \left(2 - \frac{1}{2}\right) \times 18\right) \times 8 = 984 \text{ mm}^2$$

$$A_{tg} = 35 \times 8 = 280 \text{ mm}^2$$

$$A_{tn} = \left(35 - \frac{1}{2} \times 18\right) \times 8 = 208 \text{ mm}^2$$

The block shear strength will be minimum of T_{db1} and T_{db2} as calculated below:

$$T_{db1} = \frac{A_{vg} f_y}{\sqrt{3}\gamma_{m0}} + \frac{0.9 A_{tn} f_u}{\gamma_{m1}}$$

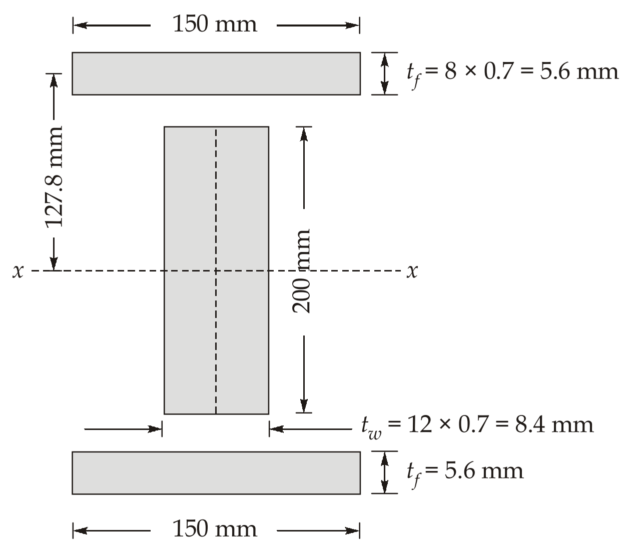
$$= \left[\frac{1200 \times 250}{\sqrt{3} \times 1.1} + \frac{0.9 \times 208 \times 410}{1.25}\right] \times 10^{-3} \text{ kN} = 218.86 \text{ kN}$$

$$T_{db2} = \frac{0.9 A_{vn} f_u}{\sqrt{3}\gamma_{m1}} + \frac{A_{tg} f_y}{\gamma_{m0}}$$

$$= \left(\frac{0.9 \times 984 \times 410}{\sqrt{3} \times 1.25} + \frac{280 \times 250}{1.1}\right) \times 10^{-3} \text{ kN} = 231.34 \text{ kN}$$

Hence, the block shear strength of the tension member is 218.86 kN

Q.5 (c) Solution:



$$I_{xx} = \frac{150 \times 5.6^3}{12} \times 2 + 150 \times 5.6 \times 2 \times 127.8^2 + \frac{200^3 \times 8.4}{12}$$

$$= 33.04 \times 10^6 \text{ mm}^4$$

Direct shear stress, $f_1 = \frac{W}{A} = \frac{W \times 10^3}{150 \times 5.6 \times 2 + 8.4 \times 200}$

$$= 0.297 \text{ W N/mm}^2$$

Due to moment

$$f_2 = \frac{M}{z} = \frac{W \times 10^3 \times 200 \times 130.6}{33.04 \times 10^6} = 0.79 \text{ W}$$

$$\because \quad \sqrt{f_2^2 + 3f_1^2} \leq \frac{f_u}{\sqrt{3} \times \gamma_m}$$

$$\Rightarrow \quad W\sqrt{0.79^2 + 3 \times 0.297^2} \leq \frac{410}{\sqrt{3} \times 1.25}$$

$$W \leq 200.876 \text{ kN}$$

$$\text{Service load, } W_s = \frac{200.876}{1.5} = 133.92 \text{ kN}$$

Q.5 (d)Solution:

(i)

Limit states are the states beyond which the structure no longer satisfies the specified performance requirement. **As per IS 800: 2007, the limit states are generally grouped under.**

1. **Limit state of strength:** Limit state of strength are associated with failure of structure under the worst combination of loading including appropriate partial factor of safety. The limit state of strength include
   (a) Loss of stability/equilibrium of structure (including the effect of sway) or any of parts including supports and foundation.
   (b) Strength limit (general yielding, formation of mechanism, rupture of structure or any of its parts of components)
   (c) Fatigue and brittle failure.

2. **Limit state of serviceability:** These are limit states beyond which specified service criteria are no longer met. These include
   (a) Deformation and deflections
   (b) Vibrations in the structure or any of its component causing discomfort to people or damages to the structure
   (c) Ponding of structures
   (d) Corrosion and durability
   (e) Repairable damage due to fatigue

(ii)

For Fe410 grade steel: f_u = 410 MPa

For shop welding: partial safety factor for material, γ_{mw} = 1.25

Size of weld, S = 6 mm

Effective throat thickness, KS = 0.7 × 6 = 4.2 mm

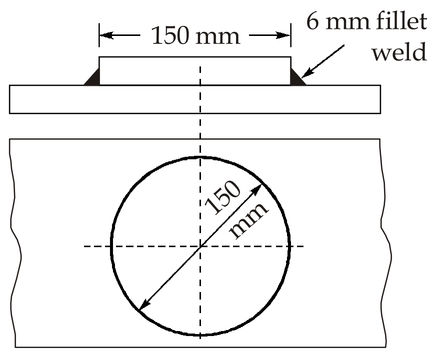
$$\text{Strength of weld per mm length} = 1 \times t_t \times \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

$$= 1 \times 4.2 \times \frac{410}{\sqrt{3} \times 1.25} = 795.36 \text{ N/mm}$$

Total length of the weld provided, $\pi d = \pi \times 150 = 471.24$ mm

$$\text{Greatest twisting moment} = 795.36 \times 471.24 \times \frac{150}{2}$$

$$= 28110408.5 \text{ Nmm} = 28.11 \text{ kNm}$$



Q.5 (e) Solution:

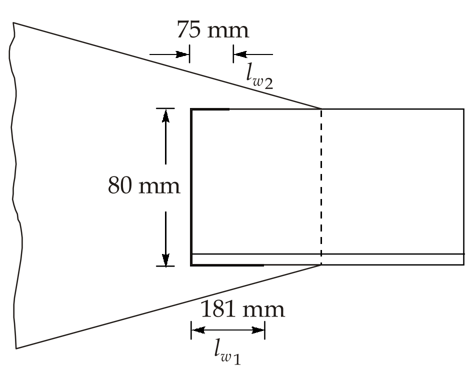
For steel for grade Fe410: f_u = 410 MPa

For site weld: partial safety factor for material $\gamma_{mw} = 1.5$

Total weld length = $l_{w1} + l_{w2} + 80$ mm

Strength of the weld required = design strength of the tie member

= 222.27 kN



$$\text{Design strength of the weld, } T_{dw} = l_w t_t \frac{f_u}{\sqrt{3}\gamma_{mw}}$$

Design strength of 6 mm ($t_t = 0.7 \times 6 = 4.2$ mm) weld per mm length

$$= 1 \times 4.2 \times \frac{410}{\sqrt{3} \times 1.5} = 662.79\,\text{kN}$$

Equating the strength of weld to the load,

$$(l_{w1} + l_{w2} + 80) \times 662.79 = 222.27 \times 10^3$$

$$\Rightarrow \quad l_{w1} + l_{w2} = 255.355 \text{ mm} \simeq 256 \text{ mm}$$

Assume $C_{yy} = 27.3$ mm

Now, taking moments about top edge of angle section,

$$662.79 \times 80 \times \frac{80}{2} + 662.79 \times l_{w1} \times 80 = 222.27 \times 10^3 \times (80 - 27.3)$$

$$l_{w1} = 180.91 \simeq 181 \text{ mm}$$

$$\therefore \quad l_{w2} = 256 - 181 = 75 \text{ mm}$$

Q.6 (a) Solution:

(i)

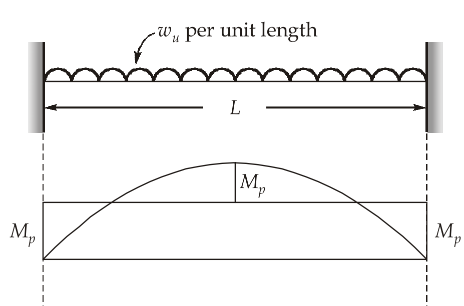
Lower bound theorem:

If any bending moment distribution can be found which satisfies

(i) Equilibrium condition and

(ii) Yield condition (i.e. bending moment no where exceeds M_p)

That system is safe and statically sufficient and corresponding load is less than or equal to the true collapse load.



Alternatively, the theorem can be stated in a simple manner as follows:

"A load computed on the basis of an assumed equilibrium moment diagram in which moments $M \le M_p$, is less than or at best equal to the true ultimate load."

$$\frac{w_u L^2}{8} = M_p + M_p$$

$$\Rightarrow \quad 2M_p = \frac{w_u L^2}{8}$$

$$\Rightarrow \qquad w_u = \frac{16\,M_p}{L^2}$$

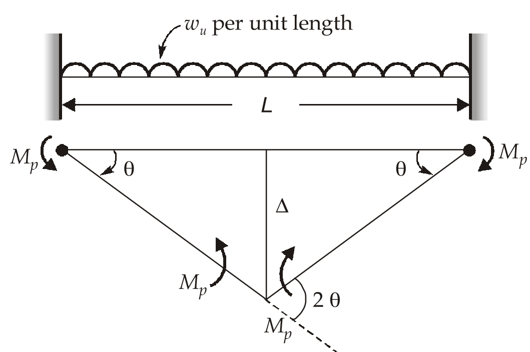
Upper bound theorem :

If a collapse mechanism can be found such that the associated moments satisfy

(i) Equilibrium condition and

(ii) Mechanism condition

The mechanism is kinematically sufficient and the corresponding load is greater than or equal to the true collapse load.



Alternatively the theorem can be stated as:

"A load computed on the basis of an assumed mechanism will be always greater than or at best equal to the true ultimate load."

Applying principle of virtual work, we get

$$w_u \times \frac{1}{2} \times L \times \Delta = M_p\theta + M_p(\theta + \theta) + M_p\theta \qquad \text{where, } \left[\Delta = \frac{L}{2} \times \theta\right]$$

$$\Rightarrow \qquad \frac{w_u L}{2} \times \frac{L}{2} \times \theta = 4\,M_p\theta$$

$$\Rightarrow \qquad w_u = \frac{16 M_p}{L^2}$$

(ii)

Gross diameter of rivets = 18 + 1.5 = 19.5 mm

For 150 mm leg, checking for critical section along section ABC

Deduction in width for hole = 1 × 19.5 = 19.5 mm

Along section ABDE, deduction in width for hole

$$= 2 \times 19.5 - \frac{(40)^2}{4 \times 60} = 32.33 \text{ mm}$$

∴ Maximum deduction for hole in 150 mm leg = 32.33 mm

∴ Net area of connected leg,

$$A_1 = \left(150 - 32.33 - \frac{t}{2}\right)t$$

Area of outstanding leg,

$$A_2 = \left(75 - \frac{t}{2}\right)t$$

Where, t = thickness of the angle = 12 mm (given)

∴ $$A_1 = \left(150 - 32.33 - \frac{12}{2}\right)12 = 1340.04 \text{ mm}^2$$

$$A_2 = \left(75 - \frac{12}{2}\right)12 = 828 \text{ mm}^2$$

∴ $$A_{net} = A_1 + kA_2$$

$$k = \frac{3A_1}{3A_1 + A_2} = \frac{3 \times 1340.04}{3 \times 1340.04 + 828} = 0.829$$

∴ $$A_{net} = 1340.04 + 0.829 \times 828 = 2026.452 \text{ mm}^2$$

∴ Allowable stress = 150 MPa (given)

∴ Allowable load

$$= 150 \times A_{net} = 150 \times 2026.452 \text{ N}$$
$$= 303.97 \text{ kN} \simeq 304 \text{ kN}$$

Q.6 (b) Solution:

Given: Fe 410 grade steel,

$$f_u = 410 \text{ N/mm}^2$$
$$f_y = 250 \text{ N/mm}^2$$
$$f_y = f_{yp} = f_{yw} = 250 \text{ N/mm}^2$$
$$\mu = 0.3$$
$$E = 2 \times 10^5 \text{ N/mm}^2$$
$$\gamma_{mw} = 1.25$$
$$\varepsilon = \varepsilon_w = \varepsilon_f = \sqrt{\frac{250}{250}} = 1.0$$

Design forces:

Total superimposed load = 80 kN/m

Factored superimposed load = 1.5 × 80 = 120 kN/m

Self weight of plate girder, $\frac{WL}{400} = \frac{(80 \times 25) \times 25}{400} = 125$ kN.

Self weight of girder per metre length = 125/25 = 5 kN/m

Factored self weight = 1.5 × 5 = 7.5 kN/m

Total uniform factored load,

$$w_u = 120 + 7.5 = 127.5 \text{ kN/m}$$

Factored maximum bending moment

$$M = \frac{w_u l^2}{8} = \frac{127.5 \times (25)^2}{8} = 9960.9375 \text{ kNm}$$

Factored maximum shear force,

$$V = \frac{w_u . L}{2} = \frac{127.5 \times 25}{2} = 1593.75 \text{ kN}$$

Design of web:

Optimum depth of plate girder,

$$d = \left(\frac{MK}{f_y}\right)^{1/3}$$

When, intermediate stiffeners are not to be provided,

$$\frac{d}{t_w} \leq 200\,\varepsilon = 200$$

Let $K = \dfrac{d}{t_w} = 180$

$$\therefore \quad d = \left(\frac{9960.9375 \times 10^6 \times 180}{250}\right)^{1/3} = 1928.5 \text{ mm}$$

$\therefore$ Thickness of Web, $t_w = \left(\dfrac{M_z}{f_y K^2}\right)^{1/3} = \left(\dfrac{9960.9375 \times 10^6}{250 \times (180)^2}\right)^{1/3}$

$= 10.71 \simeq 12 \text{ mm}$ (say)

(Thickness provided is more since intermediate transverse stiffeners are not to be provided).

Provide web of size 1950 mm × 12 mm

Design of flanges:

Let the bending moment will be resisted by the flanges and shear by the web.

Required area of flange, $A_f = \dfrac{M}{f_y . d} . \gamma_{mo} = \dfrac{9960.9375 \times 10^6}{250 \times 1950} \times 1.1 = 22475.96 \text{ mm}^2$

Assume width of flange equal to 0.3 times the depth of girder.

$$b_f = 0.3 \times 1950 = 585 \text{ mm} \simeq 600 \text{ mm (say)}$$

$\therefore$ Thickness of flanges plate,

$$t_f = \frac{22475.96}{600} = 37.46 \text{ mm} \simeq 40 \text{ mm (say)}$$

Provide flange plates each of size 600 mm × 40 mm.

Classification of flanges:

For the flange to be classifiable as plastic

$$\frac{b}{t_f} \leq 8.4\varepsilon$$

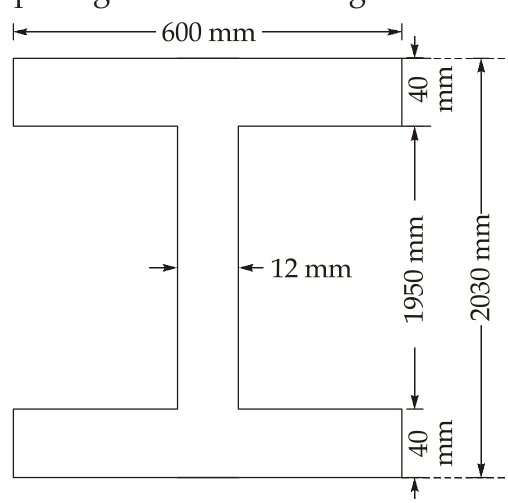
The out stand of flange, $b = \frac{b_f - t_w}{2} = \frac{600 - 12}{2} = 294 \text{ mm}$

$$\frac{b}{t_f} = \frac{294}{40} = 7.35 < 8.4$$

$\therefore$ Flanges are plastic ($\beta_b = 1.0$)

Check for bending strength:

The trial section of the plate girder shown in figure below.



Plastic modulus of the section,

$$Z_{Pz} = 2 \times b_f t_f \cdot \left(\frac{D - t_f}{2}\right)$$

$$= 2 \times 600 \times 40 \left[\frac{2030}{2} - \frac{40}{2}\right] = 47.76 \times 10^6 \text{ mm}^3$$

Bending moment capacity, $M_d = \beta_b \cdot Z_{pz} \cdot \frac{f_y}{\gamma_{Mo}}$

$$= 1.0 \times 47.76 \times 10^6 \times \frac{250}{1.1} \times 10^{-6} \text{ kNm}$$

$$= 10854.5 \text{ kNm} > 9960.9375 \text{ kNm} \quad \text{(OK)}$$

Also, $$\frac{d}{t_w} = \frac{1950}{12} = 162.5 \quad (< 200)$$

Shear capacity of web:

Elastic critical shear stress

$$\tau_{cr}, e = \frac{K_v.\pi^2 E}{12(1-\mu^2)\left[\frac{d}{t_w}\right]^2}$$

Since, intermediate stiffeners are not provided and only transverse stiffeners are provided at each end,

$$K_v = 5.35$$

$$\tau_{cr}, e = \frac{5.35 \times \pi^2 \times 2 \times 10^5}{12(1-0.3^2)(162.5)^2} = 36.62 \text{ N/mm}^2$$

Non-dimensional web slenderness ratio for shear buckling stress

$$\lambda_w = \sqrt{\frac{f_{yw}}{\sqrt{3}\,\tau_{cr,e}}} = \sqrt{\frac{250}{\sqrt{3} \times 36.62}} = 1.985$$

$\because$ $\lambda_w > 1.2$

$\therefore$ Shear stress corresponding to web buckling

$$\tau_b = \frac{f_{yw}}{\sqrt{3} \times \lambda_w^2} = \frac{250}{\sqrt{3} \times (1.985)^2} = 36.63 \text{ N/mm}^2$$

Shear force corresponding to web buckling

$$V_{cr} = A_v . \tau_b$$

$$= (1950 \times 12) \times 36.63 \text{ N} = 857.142 \times 10^3 \text{ N}$$

$$= 857.142 \text{ kN} < 1593.75 \text{ kN.}$$

Which is not safe.

Let us revise the section

Let the thickness of the web be 16 mm.

$$\frac{d}{t_w} = \frac{1950}{16} = 121.875$$

$$\tau_{cr,e} = \frac{5.35 \times \pi^2 \times 2 \times 10^5}{12(1-0.3^2) \times 121.875^2} = 65.11 \text{ N/mm}^2$$

$$\lambda_w = \sqrt{\frac{250}{\sqrt{3} \times 65.11}} = 1.489 > 1.2$$

$$\tau_b = \frac{250}{\sqrt{3} \times (1.489)^2} = 65.10 \text{ N/mm}^2$$

$$V_{cr} = A_v \cdot \tau_b = 1950 \times 16 \times 65.10 = 2031.12 \times 10^3 \text{ N}$$

$$= 2031.12 \text{ kN} > 1593.75 \text{ kN} \quad \text{(OK)}$$

Connection of flange to web:

The welding is done on both sides of the web for its connection to the flange. Since there are two weld lengths, longitudinal load transmitted to each weld length per mm

$$q_W = \frac{1}{2} \cdot \left[\frac{Va\bar{y}}{I}\right]$$

$$I = \frac{600 \times (2030)^3}{12} - \frac{(600 - 16) \times (1950)^3}{12}$$

$$I = 5.74141 \times 10^{10} \text{ mm}^4$$

$$q_W = \frac{1}{2}\left[\frac{1593.75 \times 10^3 \times (600 \times 40) \times 995}{5.74141 \times 10^{10}}\right] = 331.4 \text{ N/mm}$$

Providing 6 mm weld size,

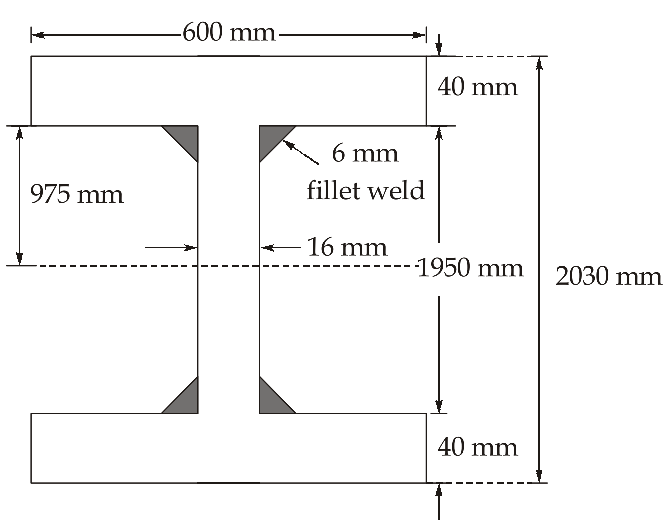
Strength of the weld per mm length,

$$= \frac{f_y}{\sqrt{3}} \cdot \frac{1}{\gamma_{mw}} \cdot [0.7S] = \frac{250}{\sqrt{3} \times 1.25} \times 0.7 \times 6$$

$$= 485 \text{ N/mm} > 331.4 \text{ N/mm}$$

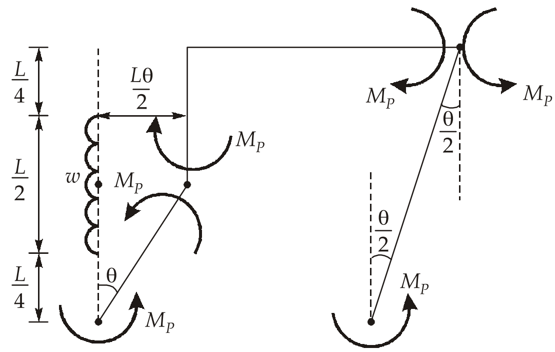
Which is all right.

Designed cross-section



Q.6 (c) Solution:

(i)

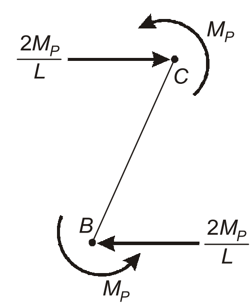


External work done = Intensity of loading × Area swept

$$= w_u\left[\frac{L}{4}\times\frac{L\theta}{2}\right]+w_u\left[\frac{1}{2}\times\frac{L}{4}\right]\left(\frac{L\theta}{2}+\frac{L\theta}{4}\right)$$

$$= \frac{7w_uL^2\theta}{32}$$

Internal work done = $M_P\theta+M_P\theta+\frac{M_P\theta}{2}+\frac{M_P\theta}{2}=3M_P\theta$



$\therefore$ $$\frac{7w_uL^2\theta}{32} = 3M_P\theta$$

$\therefore$ $$W_u = \frac{1}{2}w_uL = \frac{48M_P}{L}$$

Considering member BC

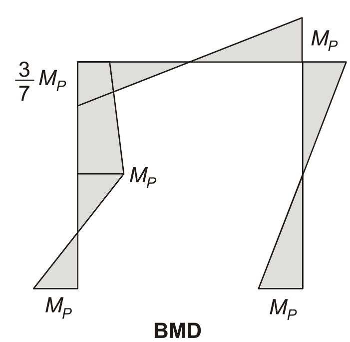
$$H_B \times L = M_P + M_P$$

$$H_B = \frac{2M_P}{L}$$

$$H_A = \frac{48M_P}{7L}-\frac{2M_P}{L}=\frac{34M_P}{L}$$

$$\text{Moment at } D = \frac{34M_P}{7L}\times L-\frac{48}{7}\frac{M_P}{L}\times\frac{L}{2}-M_P$$

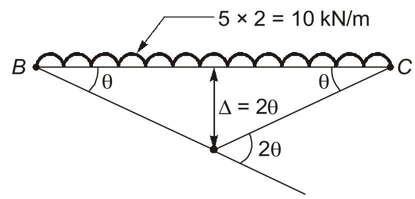
$$= \frac{3M_P}{L}$$



(ii)

As service loads are given, collapse loads are obtained by multiplying service loads with load factor.

(1) Beam mechanism

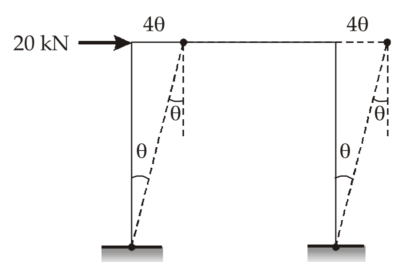


External work done = Internal energy stored

$$10 \times \frac{1}{2} \times (4 \times 2\theta) = 0.5 M_P \times \theta + 0.5 M_P \times 2\theta + 0.5 M_P \times \theta$$

$\Rightarrow$ $$M_p = 20 \text{ kN-m} \quad \text{...(i)}$$

(2) Sway mechanism

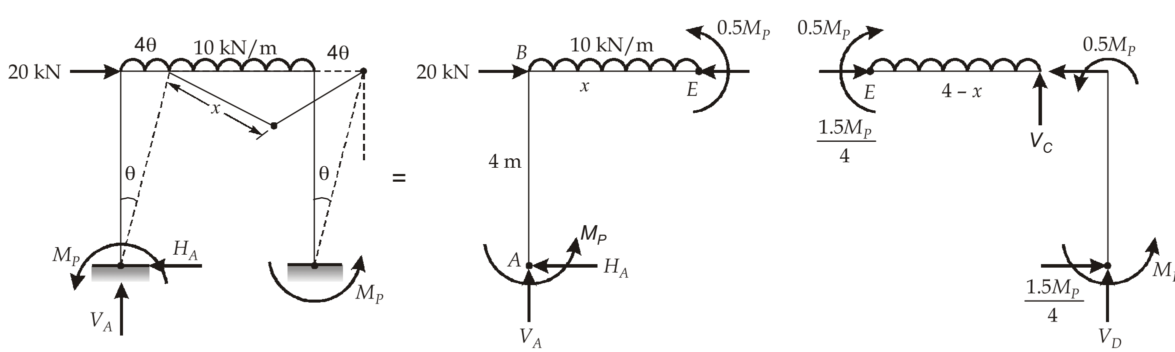


$$20 \times 4\theta = M_P \times \theta + 0.5 M_P \times \theta + 0.5 M_P \times \theta + M_P \times \theta$$

$\Rightarrow$ $$80\theta = 3M_p\theta$$

$\Rightarrow$ $$M_p = 26.67 \text{ kN-m} \quad \text{...(ii)}$$

(3) Combined mechanism



Let plastic hinge forms at a distance 'x' from B.

FBD of ABE

$$\Sigma M_A = 0$$

$$= \frac{1.5M_P \times 4}{4} - 0.5M_P + 10x\frac{x}{2} + 20 \times 4 - M_P = 0$$

$$= 3M_P = 5x^2 + 80 \quad \text{...(i)}$$

From FBD of EC

$$\Sigma M_C = 0$$

$$= 0.5M_P + 0.5M_P - 10(4-x)\times\frac{4-x}{2} = 0$$

$$M_p = 5\,(4 - x)^2 \qquad \text{...(ii)}$$

Put (ii) in equation (i)

$$3 \times 5\,(4 - x)^2 = 5x^2 + 80$$

$\therefore$ $x = 1.528$ m from B

$\therefore$

$$M_p = 5(4-x)^2$$

$$= 30.6 \text{ kN-m} \qquad \text{...(iii)}$$

Minimum M_p required to prevent any failure = 30.6 kN-m.

Q.7 (a) Solution:

The relevant properties of ISMC 400@484.6 N/m are,

A = 6293 mm², b_f = 100 mm, I_{xx} = 15082.8 × 10⁴ mm⁴,

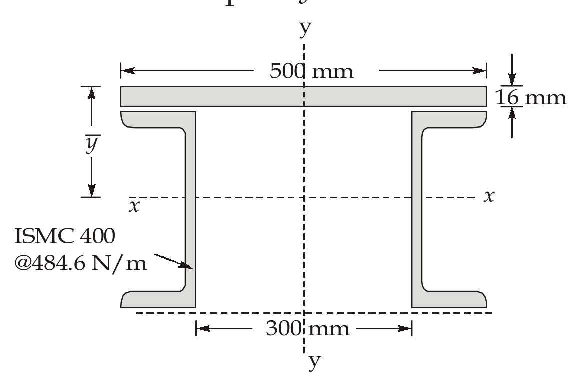
I_{yy} = 504.8 × 10⁴ mm⁴, C_{yy} = 24.2 mm

Area of channel section = 2 × 6293 = 12586 mm²

Area of plate section = 500 × 16 = 8000 mm²

Total area provided = 12586 + 8000 = 20586 mm²

Let the distance of neutral axis from top be $\bar{y}$



$$\bar{y} = \frac{2\times 6293\times\left(\frac{400}{2}+16\right)+8000\times\frac{16}{2}}{20586} = 135.168 \text{ mm}$$

$$I_{xx} = 2\left[15082.8\times 10^4 + 6293\times(216-135.168)^2 + \frac{500\times 16^3}{12} + 500\times 16\times(135.168-8)^2\right]$$

$$= 64297.91 \times 10^4 \text{ mm}^4$$

$$I_{yy} = 2\times\left[\left(504.8\times10^4\right)+6293\times(150+24.2)^2\right]+16\times\frac{500^3}{12}$$

$$= 55869.29 \times 10^4 \text{ mm}^4$$

As I_{yy} is less than I_{xx}, the minimum radius of gyration will be r_{yy}

Minimum radius of gyration, $r_{min} = \sqrt{\dfrac{55869.29\times10^4}{164.74}} = 164.74$ mm

$$\frac{l}{r_{min}} = \frac{5.5\times10^3}{164.74} = 33.39$$

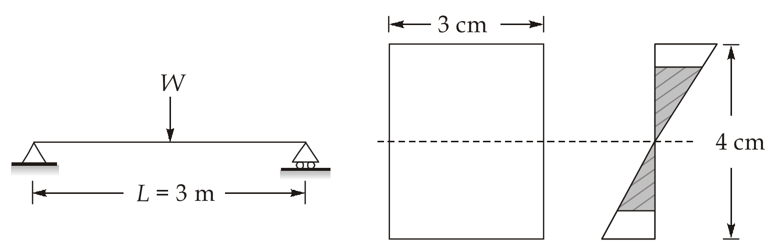
The allowable compressive stress for $\dfrac{l}{r} = 33.39$ and $f_y = 250 \text{ N/mm}^2$

From interpolation, $\sigma_{ac} = 142.966 \text{ N/mm}^2$ (from interpolation)

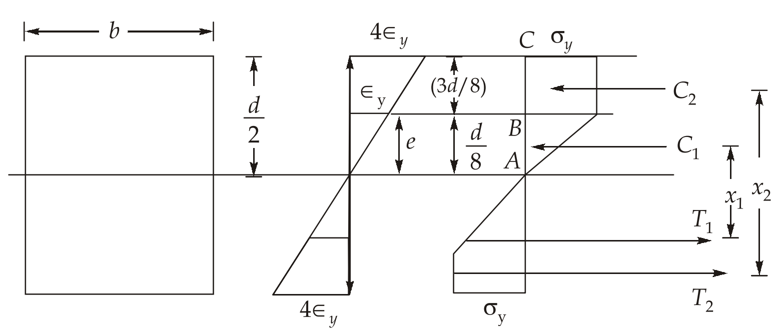
Allowable safe load $= 142.966 \times 20586$

$= 2943098.1 \text{ N} = 2943 \text{ kN (say)}$

Q.7 (b) Solution:



From the strain diagram



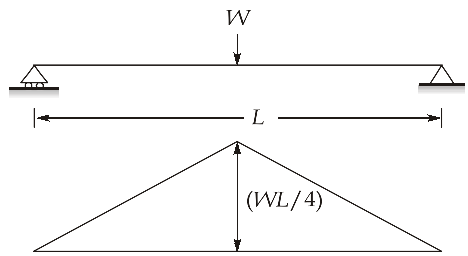
Strain diagram

From strain diagram

$$\frac{e}{\varepsilon_y} = \frac{d/2}{2\times\epsilon_y} \quad \text{...(i)}$$

$$\Rightarrow \quad e = \frac{d}{8}$$

$$\therefore \quad BC = \left(\frac{d}{2} - \frac{d}{8}\right) = \left(\frac{3d}{8}\right)$$



Moment of resistance of the beam section

$$M.R. = C_1x_1 + C_2x_2$$

$$\therefore \quad M.R. = \left[\frac{1}{2}\sigma_y\left(\frac{d}{8}\right)b \times \frac{2}{3}\left(\frac{d}{8}\right) \times 2 + \sigma_y\left(\frac{3d}{8}\right)b \times 2\left\{\frac{d}{8} + \frac{3d}{16}\right\}\right]$$

$$\therefore \quad M.R. = \frac{\sigma_y bd^2}{96} + \frac{15\sigma_y bd^2}{64} = \frac{47\sigma_y bd^2}{192} \quad \text{...(ii)}$$

$$\text{Maximum bending moment, } (BM)_{max} = \left(\frac{WL}{4}\right) \quad \text{...(iii)}$$

From (ii) and (iii) we have

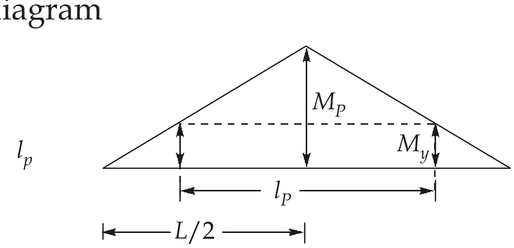
$$\left(\frac{WL}{4}\right) = \frac{47\sigma_y bd^2}{192}$$

$$\therefore \quad W = \left(\frac{47}{48}\right)\left(\frac{\sigma_y bd^2}{L}\right)$$

$$\Rightarrow \quad W = \frac{47}{48} \times \left(\frac{250 \times 30 \times (40)^2}{3000}\right)$$

$$W = 3916.67 \text{ N} \quad \text{...(iv)}$$

From bending moment diagram



(Collapse beam)

$$\Rightarrow \quad \left(\frac{M_P}{M_y}\right) = \frac{L/2}{\left(\frac{L-l_p}{2}\right)}$$

For rectangular section, S.F. $= \dfrac{3}{2}$

$$\left(\frac{L}{2}-\frac{l_p}{2}\right) = \frac{L}{2}\times\left(\frac{1}{S.F.}\right)$$

$$\left(\frac{l_p}{2}\right) = \frac{L}{2}\left(1-\frac{1}{S.F.}\right)$$

$$l_p = L\left(L-\frac{1}{S.F.}\right) = L\left(1-\frac{2}{3}\right) = \frac{L}{3} = \frac{3\,\text{m}}{3} = 1\,\text{m}$$

Q.7 (c) Solution:

(i)

For Fe410 grade of steel, f_u = 410 MPa, f_y = 250 MPa

For bolts of grade 4.6, f_{ub} = 400 MPa, f_y = 0.6 × 400 = 240 MPa

For 16 mm diameter bolt, $A_{nb} = 0.78\times\frac{\pi}{4}\times(16)^2 = 156.83\,\text{mm}^2 \simeq 157\,\text{mm}^2$

Diameter of bolt hole, d_0 = 16 + 2 = 18 mm

1. Since bolts are in two way shear, strength of bolt in shear per pitch length,

$$V_{sb} = 2\times A_{nb}\times\frac{f_{ub}}{\sqrt{3}\gamma_{mb}} = 2\times157\times\frac{400}{\sqrt{3}\times1.25}\times10^{-3}\,\text{kN} = 58\,\text{kN}$$

Strength of bolt in bearing per pitch length,

$$V_{pb} = 2.5k_b dt\frac{f_u}{\gamma_{mb}} \qquad (f_u \text{ will be lesser of } f_u \text{ and } f_{ub})$$

$$e = 1.5\,d_0 = 1.5\times18 = 27\text{ mm} \simeq 30\text{ mm}$$

Here, k_b is least of $\dfrac{e}{3d_0} = \dfrac{30}{3\times18} = 0.56$

$$\frac{p}{3d_0}-0.25 = \frac{45}{3\times18}-0.25 = 0.58;\ \frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975;\ \text{and } 1.0$$

Hence, $k_b = 0.56$

$$V_{pb} = 2.5\times0.56\times16\times8\times\frac{400}{1.25}\times10^{-3}\,\text{kN} = 56.34\text{ kN}$$

Net strength of plate per pitch length

$$T_{nd} = 0.9\frac{f_u}{\gamma_{m1}}A_n$$

$$= 0.9\frac{f_u}{\gamma_{m1}}(p-d_0)t$$

$$= 0.9\times\frac{410}{1.25}\times(45-18)\times 8\times 10^{-3} = 63.76\,\text{kN}$$

Strength of the solid plate per pitch length

$$= 0.9\frac{f_u}{\gamma_{m1}}pt = 0.9\times\frac{410}{1.25}\times 45\times 8\times 10^{-3} = 106.27\,\text{kN}$$

Hence, the strength of the joint per pitch length will be the least of the strength per pitch length in shear, bearing for bolts and net strength of plate.

Hence, strength of the joint per pitch length = 56.34 kN

2. Efficiency of the joint $= \dfrac{56.34}{106.27}\times 100 = 53.02\%$

(ii)

For Fe410 grade steel, $f_u = 410\ \text{N/mm}^2$, $f_y = 250\ \text{N/mm}^2$

Gross shear area, $A_{vg} = (2 \times 150) \times 10 = 3000\ \text{mm}^2$

Net shear area, $A_{vn} = (2 \times 150) \times 10 = 3000\ \text{mm}^2$

Gross tensile area, $A_{tg} = 300 \times 10 = 3000\ \text{mm}^2$

Net tensile area, $A_{tn} = 300 \times 10 = 3000\ \text{mm}^2$

$\gamma_{m0} = 1.1$ and $\gamma_{m1} = 1.25$

Strength due to shear yield and tension fracture,

$$T_{d_{b_1}} = \frac{A_{vg}f_y}{\sqrt{3}\gamma_{m_0}} + \frac{0.9\,A_{tn}f_u}{\gamma_{m_1}}$$

$$= \frac{3000\times 250}{\sqrt{3}\times 1.1} + \frac{0.9\times 3000\times 410}{1.25} = 1279.25\,\text{kN}$$

Strength due to tension yield and shear fracture,

$$T_{d_{b_2}} = \frac{0.9\,A_{vn}f_u}{\sqrt{3}\gamma_{m_1}} + \frac{A_{tg}f_y}{\gamma_{m_0}}$$

$$= \frac{0.9\times 3000\times 410}{\sqrt{3}\times 1.25} + \frac{3000\times 250}{1.1} = 1193.12\,\text{kN}$$

So, block shear strength is minimum of the above two i.e. T_{db1} and T_{db2} = 1193.12 kN

Q.8 (a) Solution:

For steel of grade Fe410, f_u = 410 MPa, f_y = 250 MPa

For bolts of grade 4.6: f_{ub} = 400 MPa

Partial safety factor for material:

$$\gamma_{m_0} = 1.10$$

$$\gamma_{m_b} = 1.25$$

Assume design axial compressive stress = 168 MPa,

$$\text{Cross-sectional area required} = \frac{700 \times 10^3}{168} = 4166.7 \text{ mm}^2$$

Let us try 4, ISA 90 × 90 × 6 mm

The relevant properties are,

A = 1047 mm^2, $C_{xx} = C_{yy}$ = 24.2 mm, $r_z = r_y$ = 27.7 mm, $I_z = I_y$ = 80.1 × 10^4 mm^4

Area provided = 4 × 1047 = 4188 mm^2 > 4166.7 mm^2 (OK)

Spacing of angles

For design compressive stress of 168 MPa, f_y = 250 MPa and buckling curve c, the effective slenderness ratio is 60.

For laced column, the effective slenderness ratio = 1.05 × 60 = 63

$$\therefore \quad l = 0.65 \times 12 \times 10^3 = 7800 \text{ mm}$$

$$\therefore \quad r = \frac{7800}{63} = 123.8 \text{ mm}$$

$$\text{Moment of inertia of section, } I = Ar^2$$

$$= 4188 \times 123.8^2$$

$$= 6418.7 \times 10^4 \text{ mm}^4$$

Moment of inertia required = Moment of inertia provided

$$\Rightarrow \quad 6418.7 \times 10^4 = 4 \times 80.1 \times 10^4 + 4188\,\bar{y}^2$$

$$\Rightarrow \quad \bar{y} = 120.67 \text{ mm}$$

$$\therefore \quad \text{Spacing of angles, } S = 2 \times (120.67 + 24.2)$$

$$= 289.74 \simeq 290 \text{ mm}$$

$$\text{Design compressive strength, } P_d = f_{cd} A_e = 168 \times 4188 \text{ N}$$

$$= 703.5 \text{ kN} > 700 \text{ kN}$$

Which is safe.

Connecting system

Let us provide a double lacing system with the lacing flats inclined at 45°. Bolts are provided at the centre of the leg of the angle.

$$\text{Spacing of lacing bars, } a_1 = (290 - 45 - 45)\cot 45^\circ$$
$$= 200 \text{ mm}$$

$$\frac{a_1}{r_y} = \frac{200}{27.7} = 7.22 < 50$$

Also it should be < 0.7 × 63 (= 44.1), which it is.

$$V_t = \frac{2.5}{100} \times 700 \times 10^3$$
$$= 17.5 \times 10^3 \text{ N}$$

$$\text{Transverse shear in each plane} = \frac{V_t}{N} = \frac{17.5 \times 10^3}{2} = 8.75 \times 10^3 \text{N}$$

As double lacing is provided,

$$\text{Compressive force in lacing bar} = \frac{1}{2} \times \frac{V_t}{N} \operatorname{cosec}\theta$$
$$= \frac{1}{2} \times 8.75 \times 10^3 \times \operatorname{cosec} 45^\circ$$
$$= 6.19 \times 10^3 \text{ N}$$

Section of lacing flat

Let us provide 20 mm diameter bolts of grade 4.6

For 20 mm diameter bolts

$$\text{Thickness of lacing flat} = 3 \times 20 = 60 \text{ mm}$$

$$\text{Thickness of lacing flat} = \frac{1}{60} \times (290 - 45 - 45)\operatorname{cosec} 45^\circ$$
$$= 4.714 \text{ mm} \simeq 8 \text{ mm (say)}$$

Provide 60 ISF 8 mm flat section.

$$\text{Minimum radius of gyration, } r = \frac{t}{\sqrt{12}} = \frac{8}{\sqrt{12}} = 2.31 \text{ mm}$$

$$\text{Slenderness ratio, } \frac{l_1}{r} = \frac{0.7 \times (290 - 45 - 45) \times \operatorname{cosec} 45^\circ}{2.31}$$
$$= 85.70 < 145$$

Which is safe.

For $\frac{l_1}{r} = 85.70, f_y = 250 \text{ MPa},$ and buckling curve c, the design compressive stress,

$$f_{cd} = 127.45 \text{ N/mm}^2$$

Design compressive strength, $P_d = f_{cd} A_e = 127.45 \times 60 \times 8 \times 10^{-3}$ kN

$= 61.176 \text{ kN} > 6.19 \text{ kN}$

Which is safe.

Tensile strength of lacing flat is minimum of, (i) and (ii) as below.

Diameter of bolt hole, $d_h = 20 + 2 = 22$ mm

(i) $$0.9 \times (B - d_h) t \frac{f_u}{\gamma_{m_1}} = 0.9 \times (60 - 22) \times 8 \times \frac{410}{1.25} \times 10^{-3} = 89.74 \text{ kN (Fracture)}$$

(ii) $$\frac{A_g f_u}{\gamma_{m0}} = (60 \times 8) \times \frac{250}{1.1} \times 10^{-3} = 109.90 \text{ kN (Gross section yielding)}$$

Hence, tensile strength of lacing flat is 89.74 kN > 6.19 kN. (OK)

Connection design

Strength of 20 mm diameter bolt in double shear

$$= 2 \times A_{nb} \times \frac{f_{ub}}{\sqrt{3} \times \gamma_{mb}}$$

$$= 2 \times 245 \times \frac{400}{\sqrt{3} \times 1.25} \times 10^{-3} \text{kN} = 90.52 \text{ kN}$$

Strength of 20 mm diameter bolt in bearing (on 6 mm plate)

$$= \frac{2.5 k_b \, dt \, f_u}{\gamma_{mb}} \qquad \text{(Let } k_b = 1.0\text{)}$$

$$= 2.5 \times 1.0 \times 20 \times 6 \times \frac{410}{1.25} \times 10^{-3} \text{ kN} = 98.4 \text{ kN}$$

Hence, the strength of bolt $= 90.52$ kN

$\therefore$ Number of bolts required $= \frac{2 \times 6.19 \times \cot 45^\circ}{90.52} = 0.136 \simeq 1$

Provide one bolt of 20 mm diameter of grade 4.6 at the ends of flat.

Tie plate

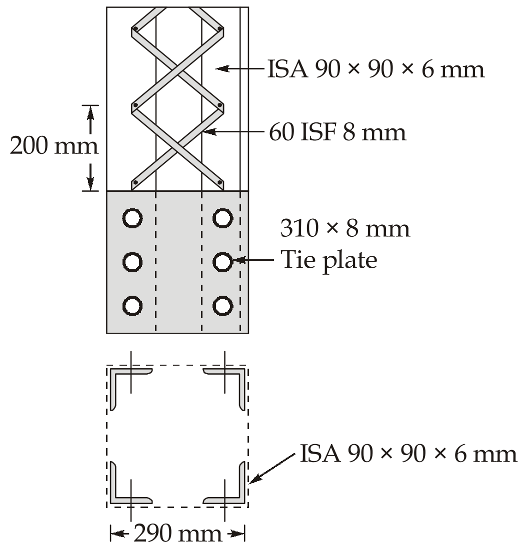
Tie plates are provided at each end of the built-up column.

Effective depth of tie plate $= 290 - 2 \times 24.2 = 241.6 \text{ mm} > 2 \times 90$ mm

Overall depth of the tie plate = 241.6 + 2 × 1.5 × 22 = 307.6 mm

$\simeq$ 310 mm (say)

Length of the tie plate = 290 mm

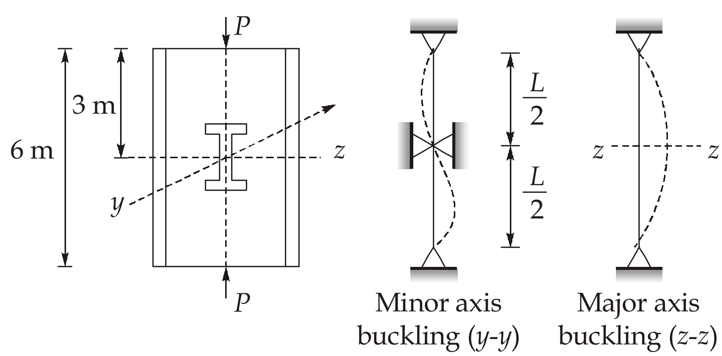


$$\text{Thickness of the tie plate} = \frac{1}{50}\times(290-45-45)$$

$$= 4.00 \text{ mm} \simeq 6 \text{ mm}$$

Provide 290 × 310 × 6 mm tie plate and connect it with bolts as shown in above figure.

Q.8 (b) Solution:



1. Buckling about y-y axis.

   $\because$ The column is effectively restrained at mid height by a bracing member in y-y direction as shown in figure.

   $\therefore$ Effective length $L_y = K_y \cdot \frac{L}{2} = 0.8\times\frac{6}{2} = 2.4$ for buckling in y-y axis

   Now, for ISMB 350,

   The buckling curve to be used along y-y will be curve b.

   $\therefore$ $\phi = 0.34$

Non dimensional slenderness ratio,

$$\lambda_y = \sqrt{\frac{f_y \times \left(\frac{L_y}{r_y}\right)^2}{\pi^2 E}} = \sqrt{\frac{250 \times \left(\frac{2.4 \times 10^3}{28.4}\right)^2}{\pi^2 \times 2 \times 10^5}} = 0.951$$

$$\therefore \quad \phi_y = 0.5\left[1 + \alpha(\lambda_y - 0.2) + \lambda_y^2\right]$$

$$= 0.5[1 + 0.34\,(0.951 - 0.2) + (0.951)^2] = 1.0799$$

Design compressive stress, $f_{cd} = \dfrac{f_y/\gamma_{m0}}{\left[\phi_y + \left(\phi_y^2 - \lambda_y^2\right)^{0.5}\right]} = \dfrac{250/1.1}{\left[1.0799 + \left\{1.0799^2 - 0.951^2\right\}^{0.5}\right]}$

$$= 142.8 \text{ N/mm}^2$$

Area of cross-section $= 6671 \text{ mm}^2$

$\therefore$ Design compressive strength, $P_d = A_c \cdot f_{cd}$

$$= 142.8 \times 6671 \text{ N}$$

$$= 952.62 \text{ kN}$$

2. Buckling about z-z axis

$$l_{eff} = 1 \times 6 = 6 \text{ m}$$

Buckling curve *a* will be used

$$\therefore \quad \alpha = 0.21$$

$$\therefore \quad \lambda_z = \sqrt{\frac{f_y \left(\frac{L_z}{r_z}\right)^2}{\pi^2 E}} = \sqrt{\frac{250 \times \left(\frac{6000}{142.9}\right)^2}{\pi^2 \times 2 \times 10^5}} = 0.473$$

$$\phi_z = 0.5\left[1 + \alpha(\lambda_z - 0.2) + \lambda_z^2\right]$$

$$= 0.5[1 + 0.21\,(0.473 - 0.2) + 0.473^2] = 0.641$$

$$f_{cd} = \frac{f_y/\gamma_{m0}}{\phi_z + \left(\phi_z^2 - \lambda_z^2\right)^{0.5}} = \frac{250/1.1}{0.641 + (0.641^2 - 0.473^2)^{0.5}}$$

$$= 211.69 \text{ N/mm}^2$$

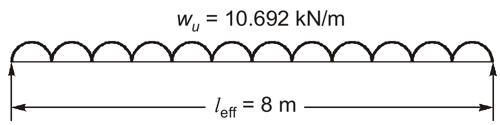
$$P_d = A \times f_{cd}$$

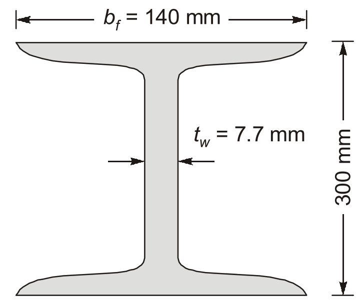
$$= 6671 \times 211.69 \text{ N} = 1412.2 \text{ kN}$$

Hence, the design compressive strength is the least of compressive strength about *yy* and *zz* axis i.e., 952.62 kN.

So, the service load will be 952.62/1.5 = 635.08 kN.

Q.8 (c) Solution:





- **Check for bending moment**

Factored live load (UDL) = 10 kN/m

Dead load = 0.461 kN/m

Factored dead load = 1.5 × 0.461 = 0.6915 ≃ 0.692 kN/m

Total factored load = 10 + 0.692 = 10.692 kN/m

Factored bending moment

$$M_u = \frac{w_u l^2}{8} = \frac{10.692 \times 8^2}{8} = 85.53 \text{ kNm}$$

Plastic section modulus required,

$$Z_{P, \text{ required}} = \frac{M_u}{\left(\frac{f_y}{\gamma_{mo}}\right)} = \frac{85.53 \times 10^6}{\left(\frac{250}{1.10}\right)} = 376332 \text{ mm}^3$$

$$I_{xx, \text{ available}} = 8985.7 \times 10^4 \text{ mm}^4$$

$$\therefore \quad Z_{e, \text{ available}} = \frac{I_{xx}}{y_{max}} = \frac{8985.7 \times 10^4}{150} = 599046.7 \text{ mm}^3$$

Assume shape factor for the I-section = 1.12

∴ Plastic section modulus available

$$Z_{P, \text{ available}} = Z_{e, \text{ available}} \times SF$$

$$= 599046.70 \times 1.12 \text{ mm}^3 = 670932.30 \text{ mm}^3$$

$$\therefore \quad Z_{P, \text{ available}} > Z_{P, \text{ required}} \qquad \text{(OK)}$$

Design bending moment

$$M_d = \beta_b Z_p \left(\frac{f_y}{\gamma_{mo}}\right) \le 1.2 Z_e \left(\frac{f_y}{\gamma_{mo}}\right)$$

(β_b = 1.0 for plastic section)

$$= 1.0 \times 670932.30 \times \left(\frac{250}{1.10}\right) \le 1.20 \times 599046.70 \times \frac{250}{1.10}$$

$$= 152.48 \times 10^6 \text{ Nmm} \le 163.38 \times 10^6 \text{ Nmm}$$

$$= 152.48 \text{ kNm} \le 163.386 \text{ kNm} \qquad \text{(OK)}$$

$\because \quad M_u < M_d$

Section is safe in bending.

- **Check for shear force**

Maximum shear force, $V_u = \frac{w_u l}{2} = \frac{10.692 \times 8}{2} = 42.768 \text{ kN}$

Design shear force, $V_d = \left(\frac{f_y}{\sqrt{3}\,\gamma_{mo}}\right)(h \times t_w)$

$$= \left(\frac{250}{\sqrt{3} \times 1.10}\right)(300 \times 7.70)\text{N} = 303.11 \text{ kN}$$

$$0.60\ V_d = 0.60 \times 303.11 = 181.86 \text{ kN}$$

$\because \quad V_u < 0.60\ V_d$

It is case of low shear.

- **Check for deflection**

Maximum deflection of the beam due to UDL,

$$\delta_1 = \frac{5}{384}\frac{wl^4}{EI} \qquad (w = \text{service load})$$

$$= \frac{5}{384} \times \frac{\left(\frac{10.692}{1.5}\right) \times 8^4 \times (10^3)^4}{2.10 \times 10^5 \times 8985.70 \times 10^4} \text{ mm} = 19.71 \text{ mm}$$

As per IS 800 : 2007, the maximum deflection for beam is

$$\delta_{max} = \frac{l}{300} = \frac{8000}{300} = 26.67 \text{ mm}$$

$\therefore \quad \delta_1 = 20.15 \text{ mm} \le \delta_{max} = 26.67 \text{ mm}$

$\therefore$ It is safe in deflection.

Thus ISMB 300 is suitable to be used as a beam section for the given loading and span.

■■■■