Leading Institute for ESE, GATE & PSUs

Detailed Solutions

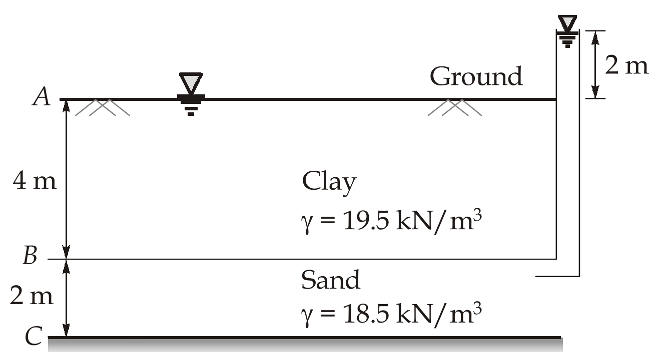
ESE-2025 Mains Test Series

Civil Engineering Test No : 2

Section A : Geo-technical Engineering and Foundation Engineering

Q.1 (a) Solution:

(i)



Impermeable

At point A:

$$\sigma_A = 0$$

$$u_A = 0$$

$\therefore$

$$\bar{\sigma}_A = 0$$

At point B: (in clay layer)

$$(\sigma_B)_{Clay} = \gamma H = 19.5 \times 4 = 78 \text{ kN/m}^2$$

$$(u_B)_{Clay} = \gamma_w H = 10 \times 4 = 40 \text{ kN/m}^2$$

$\therefore$

$$(\bar{\sigma}_B)_{Clay} = \sigma_B - u_B = 78 - 40 = 38 \text{ kN/m}^2$$

At point B: (in sand layer)

$$\sigma_B = \gamma H = 19.5 \times 4 = 78.0 \text{ kN/m}^2$$

$$u_B = \gamma_w H = 10 \times (4 + 2) = 60 \text{ kN/m}^2$$

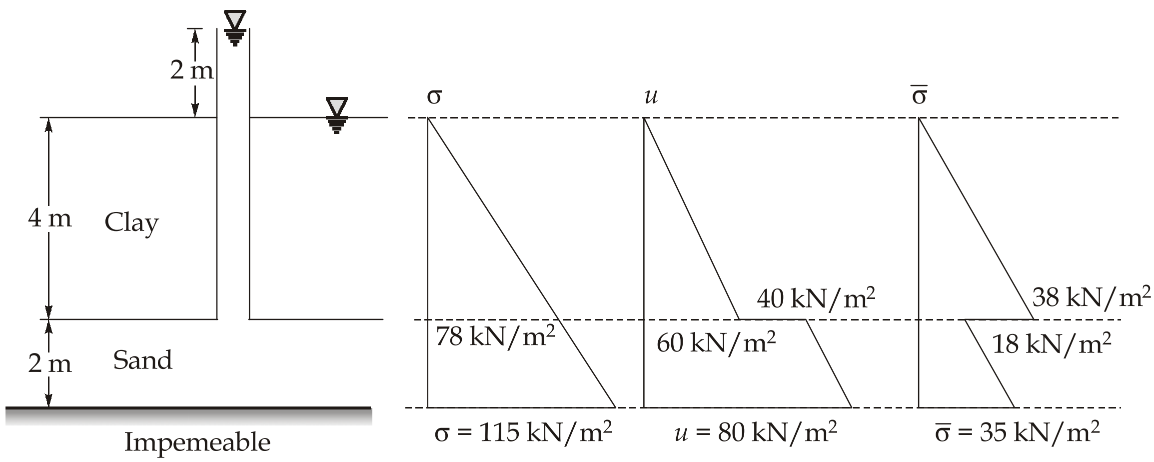
$\therefore \quad \bar{\sigma}_B = \sigma_B - u_B = 78 - 60 = 18 \text{ kN/m}^2$

At point C:

$$\sigma_C = 19.5 \times 4 + 18.5 \times 2 = 115.0 \text{ kN/m}^2$$

$$u_C = \gamma_w H = 10 \times (4 + 2 + 2) = 80 \text{ kN/m}^2$$

$\therefore \quad \bar{\sigma}_C = \sigma_C - u_C = 115 - 80 = 35 \text{ kN/m}^2$



(ii) When artesian pressure is reduced by 1m

At point C:

$$\sigma_C = 4 \times 19.5 + 2 \times 18.5 = 115 \text{ kN/m}^2$$

$$u_C = 7 \times 10 = 70 \text{ kN/m}^2$$

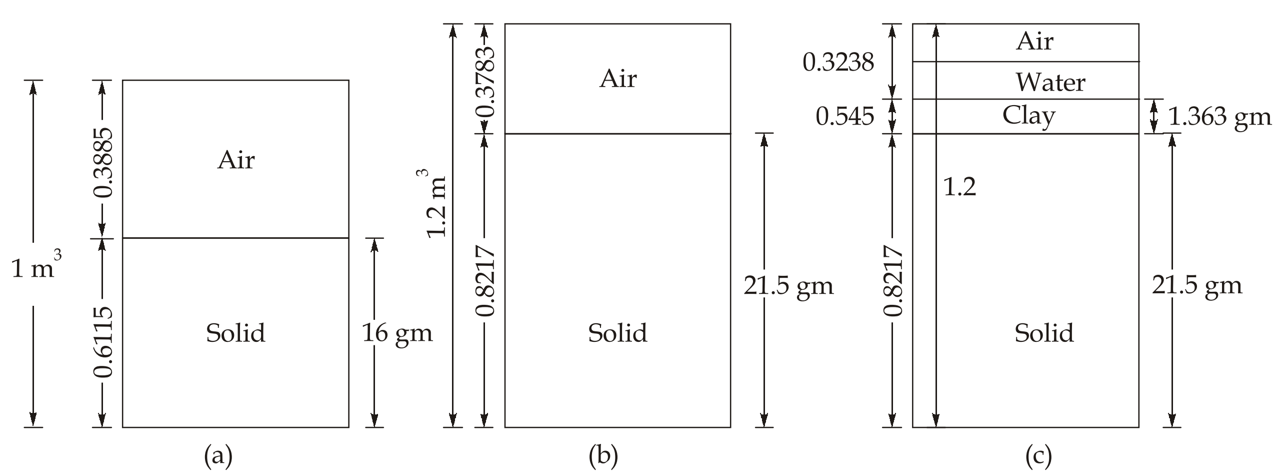
$$\bar{\sigma} = \sigma - u = 115 - 70 = 45 \text{ kN/m}^2$$

Increase in effective stress $= 45.0 - 35.0 = 10 \text{ kN/m}^2$

Q.1 (b) Solution:

(i)

Figure below shows phase diagram corresponding to (a) the in-situ condition (b) condition after addition of sand and silt (c) condition after addition of day.



In figure (a)
$$V_S = \frac{W_S}{G_S\gamma_w} = \frac{16}{2.67\times 9.8} = 0.6115\ m^3$$
$$V_V = 1 - 0.6115 = 0.3885\ m^3$$

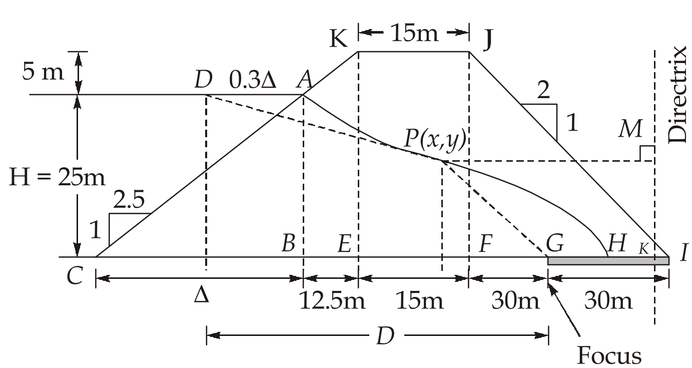
Porosity,
$$n = \frac{V_V}{V} = \frac{0.3885}{1} = 0.3885 \simeq 38.85\%$$

In figure (b) ,

$$\text{Volume of solids, } V_S = \frac{16+5.5}{2.67\times 9.8} = \frac{21.5}{2.67\times 9.8} = 0.8217 m^3$$
$$\therefore \quad V_V = 1.2 - 0.8217 = 0.3783\ \text{m}^3$$
$$\text{Porosity, } n = \frac{V_v}{V} = \frac{0.3783}{1.2} = 0.3153 \text{ or } 31.53\%$$
$$\text{Reduction in porosity} = 38.85 - 31.53 = 7.32\%$$

In figure (c)
$$W_{S(\text{clay})} = \frac{W_{\text{clay}}}{1+w_{\text{clay}}} = \frac{1.5}{1+0.1}$$
$$\Rightarrow \quad W_{S\ (\text{clay})} = \frac{1.5}{1.1} = 1.364\ \text{kN}$$
$$V_{S\,(\text{clay})} = \frac{W_S}{G_S\gamma_W} = \frac{1.364}{2.67\times 9.8} = 0.05213\ \text{m}^3$$
$$V_V = 1.2 - 0.8217 - 0.05213 = 0.32617\ \text{m}^3$$
$$\text{Porosity, } n = \frac{0.32617}{1.2} = 0.2718 \text{ or } 27.18\%$$
$$\text{Further reduction in porosity} = 31.53 - 27.18 = 4.35\%$$

Q.1 (c) Solution:



In ΔABC,
$$\frac{25}{\Delta} = \frac{1}{2.5}$$

$\Rightarrow \quad \Delta = 62.5$ m

In ΔJFI, $\quad \dfrac{JF}{FI} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{30}{FI} = \dfrac{1}{2}$

$\Rightarrow \quad FI = 60$ m

$\therefore \quad FG = 60 - 30 = 30$ m

Now, $\quad BE = 5 \times 2.5 = 12.5$ m

$$D = 0.3\,\Delta + BE + EF + FG = 0.3 \times 62.5 + 12.5 + 15 + 30$$

$$D = 76.25 \text{ m}$$

Equation of Parabola, $\quad PG = PM$

$$\sqrt{x^2 + y^2} = x + GK$$

$\because$ Parabola passing through the point (D, H) ≅ (76.25m, 25m)

$\therefore \quad \sqrt{76.25^2 + 25^2} = 76.25 + GK$

$\Rightarrow \quad GK = 3.994$ m (Focal distance)

(**Note:** In case of earthen dam the distance between the focus and direction is called as focal distance)

Seepage discharge per unit length is given by

$$q = kS$$

where: $\quad S = \sqrt{D^2 + H^2} - D$

$\Rightarrow \quad S = \sqrt{76.25^2 + 25^2} - 76.25$

$\Rightarrow \quad S = 3.994$ m

$\therefore \quad q = 40 \times 3.994$ m^3/day

$\Rightarrow \quad$ q = 159.76 m^3/day

Q.1 (d) Solution:

For free aquifer we have $q = \dfrac{\pi k\left(h^2 - h_1^2\right)}{\ln\left(\dfrac{r_2}{r_1}\right)}$

Where,

r_1 = 10 cm = 0.1 m (radius of well)

$$r_2 = 50 \text{ m (distance of observation well from tubewell)}$$

$$h_1 = 235.6 - 210 = 25.6\ m$$

$$h_2 = 239.8 - 210 = 29.8\ m$$

Given $q = 240 \text{ m}^3/hr$

$$\therefore \quad 240 = \frac{\pi \times k \times (29.8^2 - 25.6^2)}{\ln\left(\frac{50}{0.1}\right)}$$

$$\Rightarrow \quad k = \frac{240 \times \ln(50/0.1)}{\pi(29.8^2 - 25.6^2)}$$

$$\Rightarrow \quad k = 2.040 \text{ m/hour} = 48.96 \text{ m/day}$$

(i) If k is calculated on the basis of radius of influence, $R = 300$ m.

$$r = 0.1 \text{ m} \qquad H = 240.5 - 210 = 30.5 \text{ m}$$

$$h = 235.6 - 210 = 25.6 \text{ m}$$

$$\therefore \quad q = \frac{\pi k(H^2 - h^2)}{\ln\left(\frac{R}{r}\right)}$$

$$\Rightarrow \quad 240 = \frac{\pi \times k \times (30.5^2 - 25.6^2)}{\ln\left(\frac{300}{0.1}\right)}$$

$$\Rightarrow \quad k = \frac{240 \times \ln\left(\frac{300}{0.1}\right)}{\pi(30.5^2 - 25.6^2)}$$

$$\Rightarrow \quad k = 2.225 \text{ m/hour} = 53.4 \text{ m/day}$$

$$\therefore \quad \text{Percent error} = \left(\frac{53.4 - 48.96}{48.96}\right) \times 100 = 9.068\%$$

(ii) Calculation of actual radius of influence

$$\because \quad q = \frac{\pi k(H^2 - h^2)}{\ln\left(\frac{R}{r}\right)}$$

$$\Rightarrow \quad \ln\left(\frac{R}{0.1}\right) = \frac{\pi \times 2.040 \times (30.5^2 - 25.6^2)}{240}$$

$$\Rightarrow \qquad \ln\left(\frac{R}{0.1}\right) = 7.3405$$

$$\Rightarrow \qquad R = 154.15 \text{ m}$$

Q.1 (e) Solution:

(i) Chemical Stabilization

There are many chemicals which are used for soil stabilization.

- **Calcium chloride:** It is used as a water retentive additive in mechanical stabilised bases and surfacing. Being hydroscopic and deliquescent, the salt absorbs moisture from the atmosphere and retains it. It alters the properties of pure water. The vapour pressure gets lowered and the surface tension increases, and thereby the rate of evaporation decreases. Freezing point also gets lowered which prevents frost heave. Calcium chloride acts as a soil flocculant. It facilitates compaction and usually causes a slight inverse in the compacted density.
- **Sodium chloride:** The stabilizing action of sodium chloride is somewhat similar to that of calcium chloride, but it has not been so widely used. It attracts and retains moisture and reduces the rate of evaporation. Another beneficial phenomenon is the crystallisation of the salt in the soil pores near the surface, which retards further evaporation and also reduces the formation of shrinkage cracks.
- **Sodium silicate:** Sodium silicate solution in water, known as water glass, in combination with other chemicals, such as calcium chloride, is used as an injection for stabilising deep deposits of soil. The two chemicals react and precipitate in the form of insoluble silica-gel within the soil pores making the soil imperious the water and increasing its shear strength.

(ii) Stabilisation by heating

Heating a fine grained soil to temperatures of the order of 400 to 600 °C causes irreversible change in clay minerals. The soil becomes non-plastic, less water sensitive and non-expansive. Also the clay clods get converted into aggregates. Soil can be baked in kilns, or in in-situ downward draft slow moving furnaces.

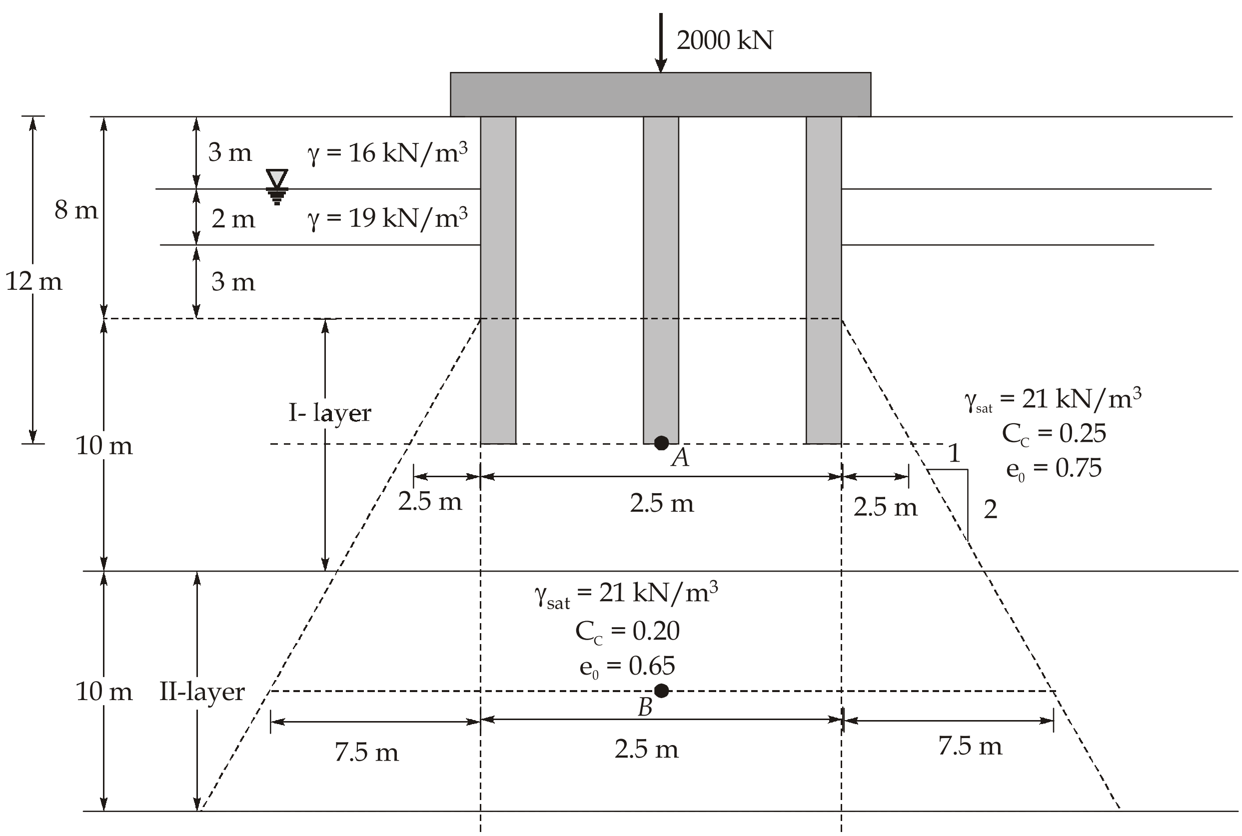
With further increase in temperature, there is some fusion and nitrification and a brick like material is obtained which can be used as an artificial aggregate for mechanical stabilisation.

(iii) Electrical stabilisation

The stability or shear strength of fine-grained soils can be increased by draining them with the passage of direct current through them. This process is also known as electro-osmosis wherein, electrical drainage is accompanied by electro-chemical composition of the electrodes and the deposition of the metal salts in the soil pores. There may also be some changes in the structure of soil. The resulting cementing of soil due to all these reactions, is also known as electro-chemical hardening and for this purpose the use of aluminum anode is recommended.

Q.2 (a) Solution:

(i)



Effective stress σ_0 at point A, middle of layer I.

$$\bar{\sigma}_A = 3 \times 16 + 2 \times (19 - 10) + 8 \times (21 - 10) = 154 \text{ kN/m}^2$$

σ_0 at point B, middle of layer II.

$$\bar{\sigma}_B = 3 \times 16 + 2\,(19 - 10) + 13 \times (21 - 10) + 5 \times (21 - 10)$$

$$= 264 \text{ kN/m}^2$$

$$\text{Width at section } A = 2.5 + 2 \times 5 \times \frac{1}{2} = 7.5 \text{ m}$$

$$\Delta\bar{\sigma}_A = \frac{2000}{7.5\times7.5} = 35.56 \text{ kN/m}^2$$

$$\text{Width at } B = 2.5 + 15 \times 2 \times \frac{1}{2} = 17.5 \text{ m}$$

$$\therefore \quad \Delta\bar{\sigma}_B = \frac{2000}{17.5^2} = 6.53 \text{ kN/m}^2$$

Consolidation settlement is given by

$$\therefore \quad \Delta H = \frac{C_c H_0}{1+e}\log_e\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

$$\text{Consolidation settlement of layer-I}, \Delta H_1 = 0.25 \times \frac{10}{1+0.75}\log\left(\frac{154+35.56}{154}\right)$$

$$= 0.1288 \text{ m}$$

$$\text{Consolidation settlement of layer } II, \Delta H_2 = 0.2\times\frac{10}{1+0.65}\log\left(\frac{264+6.53}{264}\right)$$

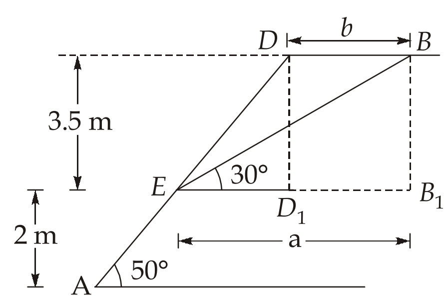
$$\Rightarrow \quad = 0.012862 \text{ m}$$

∴ Total settlement due to consolidation

$$\Delta H = \Delta H_1 + \Delta H_2$$

$$\Rightarrow \quad \Delta H = 0.1288 + 0.012862 = 0.14172 \text{ m}$$

(ii)



$$a = EB_1 = \frac{3.5}{\tan 30°} = 6.062 \text{ m}$$

$$b = EB_1 - ED_1$$

$$b = 6.062 - \frac{3.5}{\tan 50°} = 3.125 \text{ m}$$

$$\text{Weight of triangular wedge } EBD, W = \frac{1}{2}\times3.5\times3.125\times19\times1 \text{ kN}$$

$$\Rightarrow \quad W = 103.91 \text{ kN}$$

∴ Sliding component of W parallel to plane EB

$$T = W \sin 30° = 103.91 \sin 30° = 51.96 \text{ kN}$$

$$\text{Normal component, } N = W \cos 30° = 103.91 \cos 30°$$

$$= 89.99 \simeq 90 \text{ kN}$$

$$\text{Length of plane } EB = \frac{3.5}{\sin 30°} = 7 \text{ m}$$

$\therefore$ Resistance to sliding, $S = CL + N\tan\phi$

$\Rightarrow$ $S = 8 \times 7 + 90 \tan 20°$

$\Rightarrow$ $S = 56 + 32.76 = 88.76$ kN

$\therefore$ Factor of safety $= \dfrac{S}{T} = \dfrac{88.76}{51.96} = 1.71$

Q.2 (b) Solution:

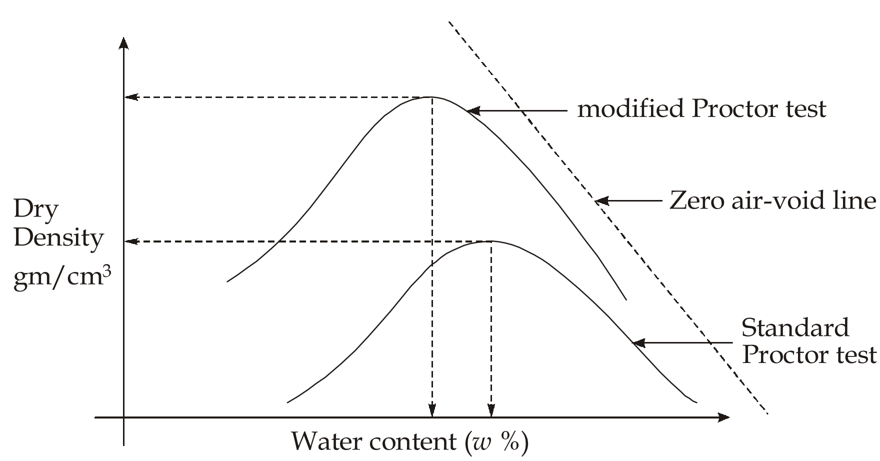
(i) Modified Proctor test

Higher compaction is needed for heavier transport and military aircraft. The modified Proctor test was developed to give a higher standard of compaction. This test was standarised by American Association of State Highway Officials and is known as the modified AASHO test.

In this test the soil is compacted in the standard Proctor mould (Capacity 945 ml), but in five layers, each layer being given 25 number of blows of a 4.5 kg rammer dropped through a height of 18 inches 45 cms. The compactive energy given to the soil sample is about 2674 kJ/m^3 of soil.

This energy is about 4.55 times that of standard Proctor test.

In the modified Proctor test, the water content-dry density curve lies above the standard Proctor test curve, and has its peak relatively placed towards the left. Thus for a same soil, the effect of heavier compaction is to increase in the maximum dry density and to decrease the optimum moisture content.



(ii)

$$\text{Air content, } a_c = V_a/V_v = 0.06$$

$$\therefore \quad V_a = 0.06V_v$$

Now,
$$V_v = V_a + V_w$$

$$\therefore \quad V_w = 0.94\ V_V$$

Now,
$$V_a = 0.06 \times \left(\frac{V_w}{0.94}\right) = 0.0638\ V_w$$

$$\text{Volume of specimen } (V) = \frac{\pi}{4} \times (15)^2 \times 12.5 = 2208.9 \text{ cm}^3$$

$$= 2208.9 \text{ m}l$$

$$\text{Total volume, } V = V_S + V_V = V_S + V_w + V_a$$

$$\Rightarrow \quad 2208.9 = V_s + V_w + 0.0638\ V_w$$

$$\Rightarrow \quad V_s + 1.0638 V_w = 2208.9\ ml$$

$$\Rightarrow \quad 2208.9 = \frac{M_S}{(2.68 \times 1.0)} + 1.068 \times \frac{M_w}{(1.0)}$$

$$\because \quad \text{Water content, } w = 10\%$$

$$\therefore \quad M_w = 0.1\ M_S$$

$$\therefore \quad 2208.9 = \frac{M_S}{2.68} + 1.0638 \times 0.1\ \text{M}_s$$

$$\therefore \quad M_S = 4606.54 \text{ gm}$$

$$\therefore \quad \text{Mass of wet soil, } M_{\text{wet}} = M_s + M_w = 4606.54 + 0.1(4606.54) = 5067.194\ g$$

$$\text{Bulk density, } \rho = \frac{M}{V} = \frac{5067.194}{2208.9} = 2.294 \text{ gm/cm}^3$$

$$\text{Dry Density, } \rho_d = \frac{\rho}{1+w} = \frac{2.294}{(1+0.10)} = 2.085 \text{ gm/cm}^3$$

$$\therefore \quad \rho_{\text{d}} = \frac{G\rho_w}{1+e}$$

$$\Rightarrow \quad 2.085 = \frac{2.68 \times 1}{1+e}$$

$$\Rightarrow \quad e = 0.285$$

Q.2 (c) Solution:

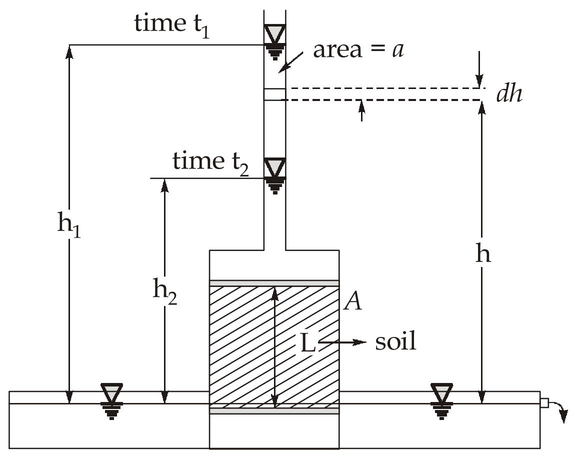
(i)

- The constant head permeability test is used for coarse grained soils only where a reasonable discharge can be collected in a given time.

However falling head test is used for relatively less permeable soil where the discharge is small.

Falling Head Permeability Test

A stand pipe of known cross-sectional area a is fitted to the permeameter and water is allowed to run down. The water level in the stand pipe constantly falls as water flows.



- Observations are started after steady state of flow has reached. The head at any time instant t is equal to difference in the water level in stand pipe and the bottom tank.
- Let h_1 and h_2 be heads at time intervals t_1 and t_2 ($t_2 > t_1$) respectively.

  Let h be the head at any intermediate time interval t, and dh be the change in head in infinitesimal small time interval dt.

Hence from Darcy's law, the rate of flow q is given by

$$q = \frac{(-dh.a)}{dt} = Kia$$

$$\Rightarrow \quad \frac{kh}{L}A = \frac{-dh}{dt}a$$

$$\Rightarrow \quad \frac{Ak}{aL}dt = \frac{-dh}{h}$$

$$\therefore \quad \frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h}$$

$$\Rightarrow \quad \frac{Ak}{aL}(t_2 - t_1) = \ln(h_1) - \ln(h_2)$$

$$\Rightarrow \quad k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right) \quad \text{...(i)}$$

(ii) Given Data

Cross sectional area of stand pipe, $a = 2\ \text{cm}^2$

Cross section area of soil sample, $A = 100\ \text{cm}^2$

Time, t = 5 mins. = 5 × 60 = 300 sec

Head, h_1 = 60 cm, h_2 = 20 cm

Length of soil sample, L = 6 cm

$$\therefore \quad k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right)$$

$$\Rightarrow \quad k = \frac{2\times 6}{100\times 300}\ln\left(\frac{60}{20}\right)$$

$$\Rightarrow \quad k = 4.4 \times 10^{-4}\ cm/s$$

Now, the sample is subjected to a constant head of 18 cm.

$$\therefore \quad Q = kiA \qquad (\text{where } i = h/L)$$

$$\Rightarrow \quad Q = 4.4 \times 10^{-4} \times \frac{18}{6}\times 100$$

$$\Rightarrow \quad Q = 0.132\ \text{cm}^3/\text{s}$$

Q.3 (a) Solution:

Layer AB:

$$k_{p_1} = \frac{1+\sin 35^\circ}{1-\sin 35^\circ} = 3.69$$

$$p_{p_1} = k_{p_1}\sigma_v = k_{p_1}\gamma z$$

At point A (z = 0), $p_{p_1} = 0$

At point B (z = 3 m), $p_{p_1} = 3.69 \times 17 \times 3 = 188.19\ \text{kN/m}^2$

$$\therefore \quad \text{Passive thrust, } P_{p_1} = \frac{1}{2}\times 188.19\times 3 = 282.285\ \text{kN/m}$$

Line of action of P_{p_1}, $\bar{H}_1 = \frac{3}{3}+7 = 8$ m from base

Layer BC:

$$k_{p_2} = \frac{1+\sin 25^\circ}{1-\sin 25^\circ} = 2.46$$

(i) Effect of uniform surcharge due to layer AB

$$q = \gamma_1 H_1 = 17 \times 3 = 51\ \text{kN/m}^2$$

$$p_{p_2} = k_{p_2} q = 2.46\times 51 = 125.46\ \text{kN/m}^2$$

Passive thrust due to portion AB,

$$P_{p_2} = 125.46 \times 3 = 376.38 \text{ kN/m}$$

Line of action of P_{p2}, $\bar{H}_2 = \frac{3}{2} + 4 = 5.5 \text{ m}$ from base

(ii) Effect of soil grains

$$p_{p_3} = k_{p_2}\sigma_v = k_{p_2}\gamma' z$$

At point B ($z = 0$), $p_{p_3} = 0$

At point C ($z = 3$ m), $p_{p_3} = 2.46 \times (20 - 9.81) \times 3 = 75.20 \text{ kN/m}^2$

Passive thrust due to surcharge,

$$P_{p_3} = \frac{1}{2} \times 75.20 \times 3 = 112.8 \text{ kN/m}$$

Line of action of P_{p_3}, $\bar{H}_3 = \frac{3}{3} + 4 = 5 \text{ m}$ from base

(iii) Effect of pore water

$$p_{p_4} = \gamma_w z$$

At point B ($z = 0$), $p_{p_4} = 0$

At point C ($z = 3$ m), $p_{p_4} = 9.81 \times 3 = 29.43 \text{ kN/m}^2$

Passive thrust due to water,

$$P_{p_4} = \frac{1}{2} \times 29.43 \times 3 = 44.145 \text{ kN/m}$$

Line of action of P_{p_4}, $\bar{H}_4 = \frac{3}{3} + 4 = 5 \text{ m}$ from base

Layer CD:

$$k_{p_3} = \frac{1 + \sin 20^\circ}{1 - \sin 20^\circ} = 2.04$$

(i) Effect of uniform surcharge due to layer AB and BC

$$q = \gamma_1 H_1 + \gamma'_2 H_2$$

$$= 17 \times 3 + (20 - 9.81) \times 3$$

$$= 81.57 \text{ kN/m}^2$$

$$p_{p_5} = k_{p_3} q = 2.04 \times 81.57 = 166.40 \text{ kN/m}^2 \text{ (constant)}$$

Passive thrust due to layers AB and BC,

$$P_{p5} = 166.40 \times 4 = 665.6 \text{ kN/m}$$

Line of action of P_{p5}, $\bar{H}_5 = \frac{4}{2} = 2 \text{ m}$ from base

(ii) Effect of soil grains

$$p_{p6} = k_{p3}\sigma_v = k_{p3}\gamma' z$$

At point C ($z = 0$), $p_{p6} = 0$

At point D ($z = 4$ m), $p_{p6} = 2.04 \times (20 - 9.81) \times 4 = 83.15 \text{ kN/m}^2$

Passive thrust due to portion CD,

$$P_{p6} = \frac{1}{2} \times 83.15 \times 4 = 166.3 \text{ kN/m}$$

Line of action of P_{p6}, $\bar{H}_6 = \frac{4}{3} = 1.33 \text{ m}$ from base

(iii) Effect of pore water

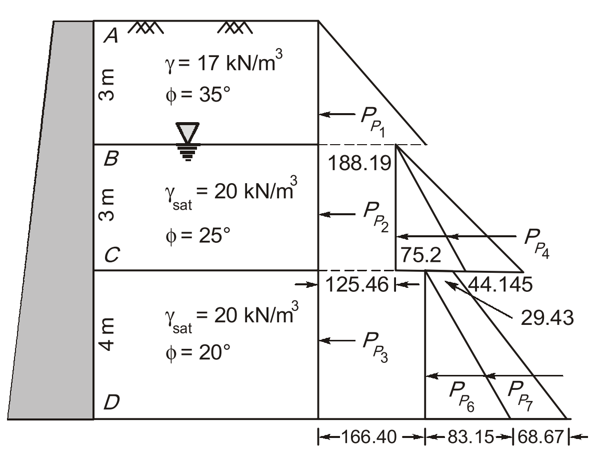
$$P_{p7} = \gamma_w z$$

At point C, ($z = 3$ m), $P_{p7} = \gamma_w \times 3 = 9.81 \times 3 = 29.43 \text{ kN/m}^2$

At point D, ($z = 7$ m), $P_{p7} = \gamma_w \times 7 = 9.81 \times 7 = 68.67 \text{ kN/m}^2$

Passive thrust due to water

$$= \frac{1}{2}(29.43 + 68.67) \times 4 = 196.2 \text{ kN/m}$$



Line of action, $\bar{H}_7 = \frac{2 \times 29.43 + 68.67}{29.43 + 68.67} \times \frac{4}{3} = 1.733 \text{ m}$

Total passive thrust on wall

$$P_p = \sum P_{p_i}$$
$$= 282.285 + 376.38 + 112.8 + 44.145 + 665.6 + 166.3 + 196.2$$
$$= 1843.71 \text{ kN/m}$$

Line of action of total passive thrust,

$$\bar{H} = \frac{\sum P_{p_i} \bar{H}_i}{\sum P_{p_i}}$$

$$= \frac{282.28 \times 8 + 376.38 \times 5.5 + 112.8 \times 5 + 44.145 \times 5 + 665.6 \times 2 + 166.3 \times 1.33 + 196.2 \times 1.733}{1843.71}$$

$= 3.7997 \text{ m} \simeq 3.8 \text{ m}$ from base

Q.3 (b) Solution:

$$\gamma_d = 17.66 \text{ kN/m}^3 = 1.8 \text{ g/cc}$$
$$G = 2.65$$

$\because$ $$\gamma_d = \frac{G\gamma_w}{(1+e)}$$

$\Rightarrow$ $$1.8 = \frac{2.65 \times 1}{(1+e)}$$

$\Rightarrow$ $$e = 0.472$$

$\therefore$ $$\gamma_{sat} = \frac{(G+Se)\gamma_w}{1+e} = \frac{2.65+0.472}{1.472} \times 9.81 \text{ kN/m}^3 = 20.81 \text{ kN/m}^3$$

Total stress at 2 m below GL = $2 \times 17.66 \text{ kN/m}^2 = 35.32 \text{ kN/m}^2$

Total stress at 6 m below GL = $2 \times 17.66 + 4 \times 20.81 = 118.56 \text{ kN/m}^2$

Neutral stress at 2 m below GL = 0 (Zero)

Neutral stress at 6 m below GL = $4 \times \gamma_w = 4 \times 9.81 \text{ kN/m}^2 = 39.24 \text{ kN/m}^2$

Effective stress at 2 m below GL,

$$\bar{\sigma} = \sigma - u = 35.32 - 0 = 35.32 \text{ kN/m}^2$$

Effective stress at 6 m below GL,

$$\bar{\sigma} = \sigma - u = 118.56 - 39.24 = 79.32 \text{ kN/m}^2$$

The variation in the total, neutral, and effective stresses with depth are shown in Fig (i) below.

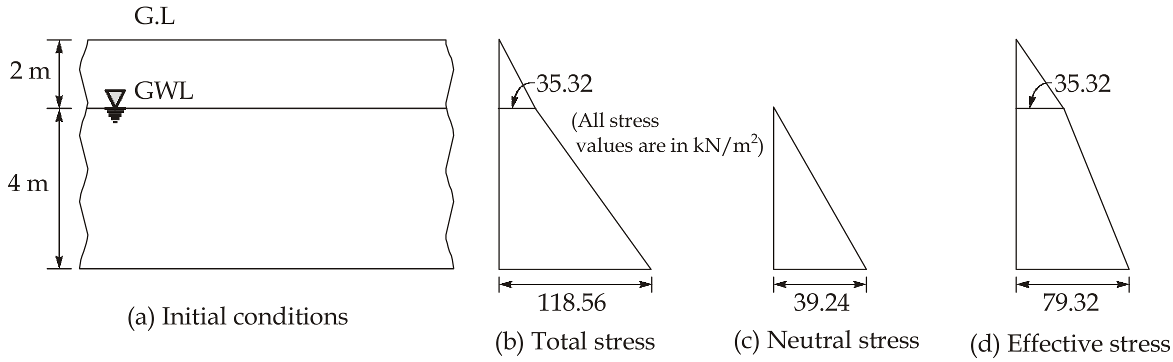


Fig (i) : Soil profile and stress variation diagrams

After the water table gets lowered by 1 m, the conditions are shown in Fig (ii) below.

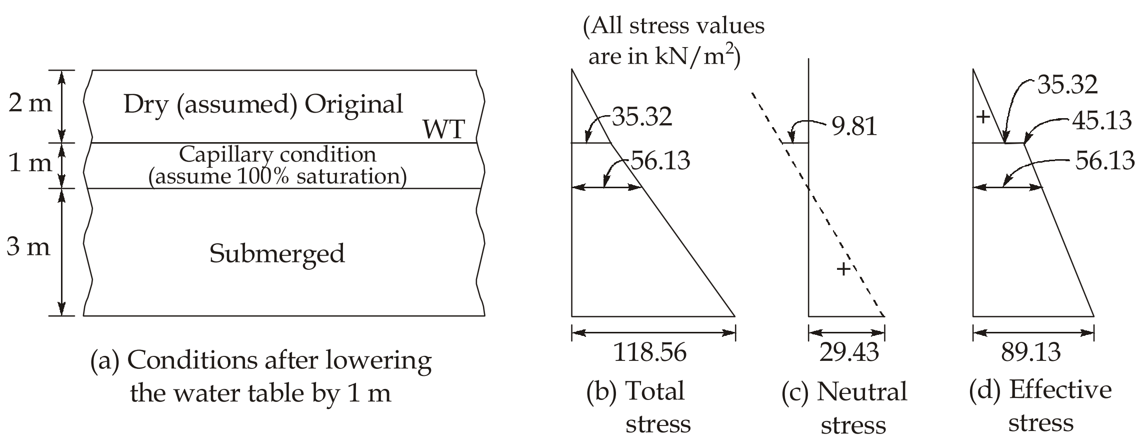


Fig (ii) : Soil profile and stress variation diagrams

The top 2 m depth is assumed to be dry.

The next 1 m depth is under capillary saturation.

The next 3 m depth is submerged.

Total stress:

$$\sigma \text{ at 2 m below GL} = 2 \times 17.66 \text{ kN/m}^2 = 35.32 \text{ kN/m}^2$$

$$\sigma \text{ at 3 m below GL} = (2 \times 17.66 + 1 \times 20.81) = 56.13 \text{ kN/m}^2$$

$$\sigma \text{ at 6 m below GL} = (2 \times 17.66 + 4 \times 20.81) = 118.56 \text{ kN/m}^2$$

Neutral stress:

u at GL is uncertain.

u at 2 m below GL is due to capillary action = $-1 \times 9.81 = -9.81$ kN/m^2

u at 3 m below GL is zero

u at 6 m below GL = $+3 \times 9.81$ kN/m^3 = 29.43 kN/m^2

Effective stress:

$\bar{\sigma}$ at 2 m below GL = 35.32 – (–9.81) = 45.13 kN/m^2

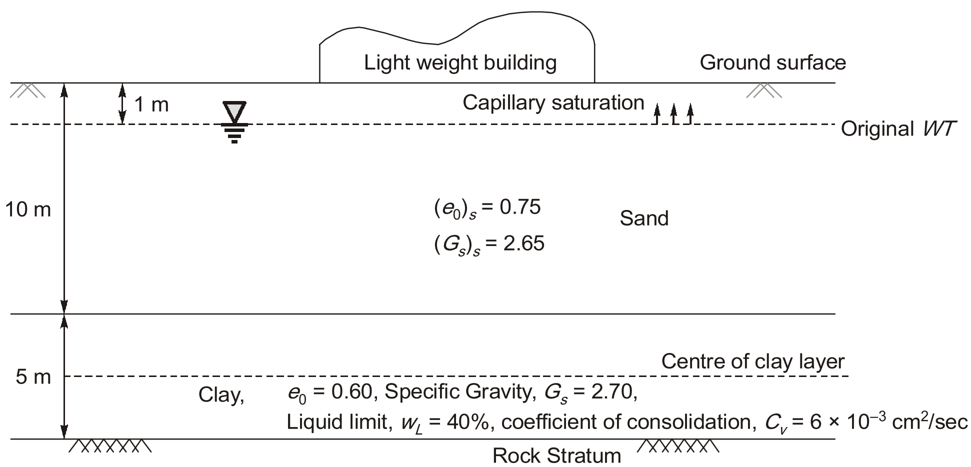
$\bar{\sigma}$ upto 3 m below GL = 56.13 kN/m^2

$\bar{\sigma}$ upto 6 m below GL = 118.56 – 29.43 = 89.13 kN/m^2

The variation of total, neutral, and effective stresses are shown in Fig (ii).

The variation in second case from the surface up to 2 m depth is uncertain because the capillary conditions in this zone cannot be easily assessed.

Q.3 (c) Solution:



The above figure shows the subsoil profile below the light weight building.

$$(\gamma_{sat})_{sand} = \left(\frac{G_s + e}{1+e}\right)\gamma_w = \left(\frac{2.65+0.75}{1+0.75}\right)\times 9.81 = 19.06 \text{ kN/m}^3$$

$$(\gamma_{sat})_{clay} = \left(\frac{2.7+0.60}{1+0.60}\right)\times 9.81 = 20.233 \text{ kN/m}^3$$

The total stress is the same whether the soil is saturated by gravity flow or capillary flow.

The sand is saturated by gravity flow below water table and by capillary flow upto 1 m height above water table.

Hence, initial effective stress at centre of clay layer, $\bar{\sigma}_0$ = 19.06 × (9.0 + 1.0) + 2.5 × 20.233 – 9.81 × 11.5

$$\Rightarrow \qquad \bar{\sigma}_0 = 128.37 \text{ kN/m}^2$$

Now, due to lowering of the water table by 4 m as a result of pumping of water from sand stratum, the effective stress will increase.

Final effective stress at centre of clay layer;

$$\bar{\sigma}_f = (\gamma_d)_{sand} \times 5 + (\gamma_{sat})_{sand} \times 5 + (\gamma_{sat})_{clay} \times 2.5 - \gamma_w \times 7.5$$

$$\because \quad (\gamma_d)_{sand} = \frac{G_s\gamma_w}{1+e} = \frac{2.65\times 9.81}{1+0.75} = 14.855 \text{ kN/m}^3$$

$$\therefore \quad \bar{\sigma}_f = (14.855 \times 5 + 19.06 \times 5 + 20.233 \times 2.50 - 9.81 \times 7.50)$$

$$\Rightarrow \quad \bar{\sigma}_f = 146.583 \text{ kN/m}^2$$

We know, due to change in effective stress in clay,

Final consolidation settlement, $S_f = \frac{C_c H_0}{1+e}\log\left(\frac{\bar{\sigma}_f}{\bar{\sigma}_0}\right)$

where C_c = Coefficient of compression index = $0.009\,(w_L - 10)$

$= 0.009\,(40 - 10) = 0.270$

$H = 5 \text{ m} = 5000 \text{ mm}$

= Length of drainage path (single drainage)

$$\therefore \quad S_f = \frac{0.27\times 5000}{(1+0.60)}\times \log_{10}\left(\frac{146.583}{128.370}\right)$$

$$\Rightarrow \quad S_f = 48.62 \text{ mm}$$

For settlement, $S_t = 25$ mm after time t, we have

$$\text{Degree of consolidation, } U(\%) = \frac{S_i}{S_f} = \frac{25}{48.62} = 51.42\% < 60\%$$

Hence, Time factor, $T_v = \frac{\pi}{4}U^2 = \frac{\pi}{4}(0.5142)^2$

$$\Rightarrow \quad T_v = 0.20766$$

$$\because \quad T_v = \frac{C_v t}{H^2} = 0.20766 \text{ and } C_v = 6 \times 10^{-3} \text{ cm}^2/\text{sec}$$

$$\Rightarrow \quad \frac{6\times 10^{-3}\times t}{(500)^2} = 0.20766$$

$$\Rightarrow \quad t = 8652500 \text{ sec} = 100.14 \text{ days} \simeq 100 \text{ days}$$

Hence, the building will settle by 25 mm in 100 days.

Q.4 (a) Solution:

$D_f = 1.2$ m, $\gamma_t = 1.8 \text{ t/m}^3$

Unconfined compressive strength = 5.5 t/m^2

$$C = \left(\frac{q_u}{2}\right) = \left(\frac{5.5}{2}\right) = 2.75 \text{ t/m}^2$$

1. By Terzaghi's theory,

$$\text{Ultimate bearing capacity, } q_u = \left(1.3CN_c + \gamma D_f N_q + 0.4B\gamma N_\gamma\right)$$

$$\text{For cohesive soil, } \phi = 0, \quad N_c = 5.7, \quad N_q = 1.0, \quad N_\gamma = 0$$

$$\therefore \quad q_u = (1.3 \times 2.75 \times 5.7) + 1.8 \times 1.2 \times 1 + 0$$

$$= 22.54 \text{ t/m}^2$$

$$\therefore \text{Net ultimate bearing capacity, } q_{nu} = (q_u - \gamma D_f)$$

$$= (22.54 - 1.8 \times 1.2) = 20.38 \text{ t/m}^2$$

$$\text{Safe bearing capacity, } q_s = \left(\frac{q_{nu}}{\text{FOS}}\right) + \gamma D_f$$

$$= \left(\frac{20.38}{2.54} + (1.8 \times 1.2)\right) = 10.18 \text{ t/m}^2$$

2. By Skempton's theory

$$\left(\frac{D_f}{B}\right) = \left(\frac{1.2}{2.5}\right) = 0.48 < 2.5$$

$$\therefore \quad N_c = 6\left[1 + 0.2\left(\frac{D_f}{B}\right)\right] = 6\left[1 + 0.2 \times \frac{1.2}{2.5}\right] = 6.576$$

$$\therefore \quad q_{nu} = CN_c$$

$$= (2.75 \times 6.576) = 18.084 \text{ t/m}^2$$

$$\therefore \quad \text{Ultimate bearing capacity, } q_u = \left(q_{nu} + \gamma D_f\right) = 18.084 + 1.8 \times 1.2 = 20.244 \text{ t/m}^2$$

$$\therefore \quad \text{Safe bearing capacity, } q_s = \left(\frac{q_{nu}}{\text{FOS}}\right) + \gamma D_f = \left(\frac{18.084}{2.54} + 1.8 \times 1.2\right) = 9.279 \text{ t/m}^2$$

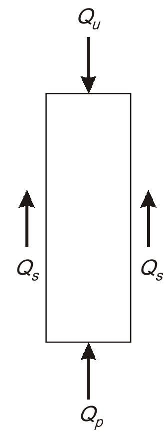
Therefore, Skempton's approach gives more conservative result as compared to Terzaghi's approach.

(ii)

A pile foundation should be safe against shear failure and also the settlement should be within the permissible limits. The methods for estimating the load-carrying capacity of a pile foundation can be grouped into the following four categories.

1. **Static methods:** The static methods give the ultimate capacity of an individual pile, depending upon the characteristics of the soil. The ultimate load capacity is given by

$$Q_u = Q_p + Q_s$$



where Q_u = ultimate failure load, Q_p = point (or base or tip) resistance of the pile, Q_s = shaft resistance developed by friction (or adhesion) between the soil and the pile shaft.

The static formulas give a reasonable estimate of the pile capacity if judiciously applied.

2. **Dynamic Formulae:** The ultimate capacity of piles driven in certain types of soils is related to the resistance against penetration developed during driving operation. The ultimate load capacity formulae are based on the principle that the resistance of a pile to further penetration by driving depends upon the energy imparted to the pile by the hammer. It is tacitly assumed that the load-carrying capacity of the pile is equal to the dynamic resistance during driving. The dynamic formulae are not much reliable.

3. **In-situ Penetration Tests:** The pile capacity can be determined from the results of in-situ standard penetration test. Empirical formulas are used to determine the point resistance and the shaft resistance from the standard penetration number (N). Alternatively, the static formulae can be used after determining the N-value, as this value is related to the angle of shearing resistance (ϕ).

Cone penetration tests are also used to estimate the pile capacity.

4. **Pile Load Tests:** The most reliable method of estimating the pile capacity is to conduct the pile load test. The test pile is driven and loaded to failure. The pile capacity is related to the ultimate load or the load at which the settlement does not exceed permissible limits.

Q.4 (b) Solution:

(i) The worst-case scenario occurs when the ground water rises to the surface.

$\therefore$ Uplift pressure, $p_w = h\gamma_w = (4 + 1)9.81 = 49.05 \text{ kPa} \simeq 49 \text{ kPa}$

Uplift force, P_{up} at the base of the culvert is

$$P_{up} = Bp_w = 4 \times 49 = 196 \text{ kN/meter length}$$

Assume a wall thickness of t m

Weight per unit length, $W = [(4 \times 5) - (4 - 2t)(5 - 2t)] \times 24$

Now, $W = P_{up} \times \text{FOS}$

$\Rightarrow$ $24[(4 \times 5) - (4 - 2t)(5 - 2t)] = 196 \times 1.2$

$\Rightarrow$ $4t^2 - 18t + 9.8 = 0$

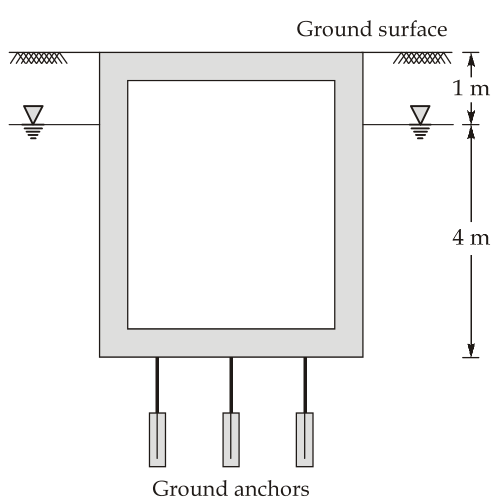
$\therefore$ $t = 0.633677 \text{ m}$

(Neglecting $t = 3.866$ m being very high)

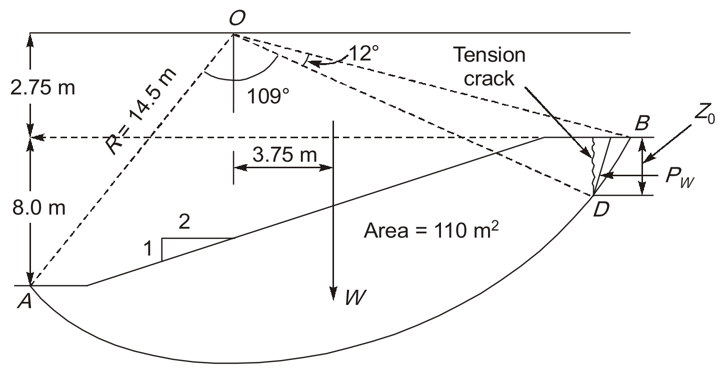
$= 633.67 \text{ mm} \simeq 635 \text{ mm}$

(ii)

One potential method to prevent uplift is to use ground anchors, as shown below. The anchors must support the difference of force between the uplift force and the net downward resistance (weight plus side shear resistance) with sufficient factor of safety (> 1.2).



4. (c) Solution :



A total stress analysis is made to determine the factor of safety against immediate shear failure.

(i) Ignoring the tension crack: Circular slip surface is *AB*.

Radius of slip circle, $R = 14.5$ m

Sector angle (at centre), $\theta = 121°$

Area of sliding mass = 110 m^2

Moment arm of weight of sliding wedge

= 3.75 m

$$\therefore \quad \text{Factor of safey, } F = \frac{C_u R^2 \theta}{Wx}$$

$$= \frac{27 \times 14.5^2 \times 121 \times \left(\frac{\pi}{180}\right)}{110 \times 18 \times 3.75} = 1.61$$

(ii) Development of tension crack reduces the arc length from *AB* to *AD*.

$$Z_0 = \text{Depth of tension crack} = \frac{2C_u}{\gamma} = \frac{2 \times 27}{18} = 3 \text{ m}$$

Reduced sector angle at centre,

$\theta_D = 109°$

Area of sliding mass = 110 – 1.5 = 108.5 m^2

Centroid distance from *O* = 3.60 m

$$\therefore \quad \text{Factor of safety, } F = \frac{C_u R^2 \theta_D}{Wx}$$

$$= \frac{27 \times 14.5^2 \times 109 \times \left(\frac{\pi}{180}\right)}{108.5 \times 18 \times 3.6} = 1.54$$

(iii) When the tension crack is filled with water, the hydrostatic force P_w, will cause an additional disturbing moment.

$$P_w = \frac{1}{2}\gamma_w Z_0^2 = \frac{1}{2}\times 9.81\times 3^2 = 44.145 \text{ kN/m}$$

Moment arm of $P_w O$ i.e., $l = 4.75$ m $\left(\text{i.e., } 2.75+\frac{2}{3}\times 3\right)$

$$\therefore \quad F = \frac{C_u R^2\theta}{Wx + P_w l} = \frac{27\times 14.5^2\times 109\times\left(\frac{\pi}{180}\right)}{(108.5\times 18\times 3.6 + 44.145\times 4.75)} = 1.492$$

Section B : Surveying and Geology

Q.5 (a) Solution:

$$D = Ks + C$$

Given: $D_1 = 100$ m, $D_2 = 300$ m, $s_1 = 0.99$ m, $s_2 = 3.00$ m

Now, $D_1 = Ks_1 + C$

$\Rightarrow \quad 100 = K \times 0.99 + C \quad$ ...(i)

$D_2 = Ks_2 + C$

$\Rightarrow \quad 300 = K \times 3.00 + C \quad$ ...(ii)

From equations (i) and (ii)

$$2.01\,K = 200$$

$$\Rightarrow \quad K = \frac{200}{2.01} = 99.502$$

Substituting the value of K in the eq. (ii)

$$300 = 99.502 \times 3 + C$$

or $\quad C = 300 - 99.502 \times 3 = 1.494 \simeq 1.5$

Hence, the constants of instrument are 99.502 and 1.5.

Now, $\quad s = 2.670 - 1.000 = 1.670$ m and $\theta = 10°$

Horizontal distance, $\quad D = Ks\cos^2\theta + C\cos\theta$

$$= 99.502 \times 1.670 \cos^2 10° + 1.5 \times \cos 10° = 162.63 \text{ m}$$

Vertical distance, $\quad V = \frac{Ks}{2}\sin 2\theta + C\sin\theta$

$$= \frac{99.502\times 1.670}{2}\sin 20° + 1.5\sin 10° = 28.68 \text{ m}$$

$\therefore \quad$ R.L. of B = R.L. of A + H.I – V – h

$$= 450.5 + 1.42 - 28.68 - 1.835 = 421.405 \text{ m}$$

Q.5 (b) Solution:

(i) The following four properties are used for interpretation of remote sensing information:

- **Spectral :** Wavelength or frequency, refractive or emissive properties of objects during intersection of electro magnetic waves.
- **Spatial :** Viewing angle of sensor, shape and size of the object, position size, distribution, texture, etc.
- **Temporal :** Changes in time and position which affect spectral and spatial properties.
- **Polarization :** Object effects in relation to the polarization conditions of the transmitter and receiver.

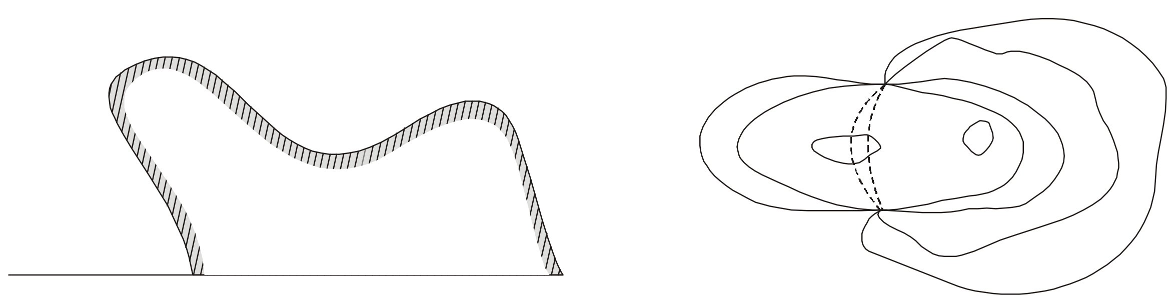
(ii) Sources of error in GIS:

- Error arising from our understanding and modeling of reality.
- Errors due to source data.
- Errors occurring during data input.
- Errors in data analysis and manipulation.

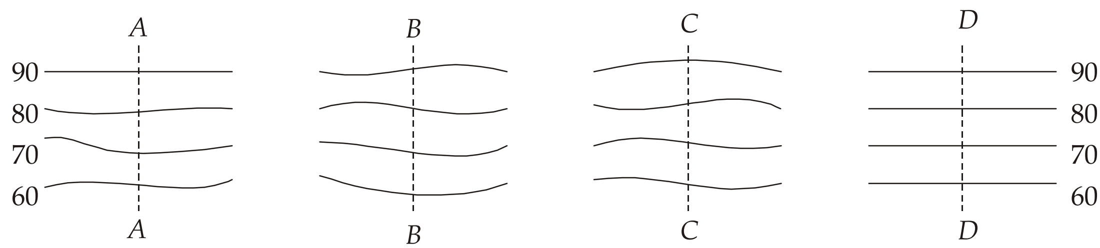
Q.5 (c) (i)Solution:

The following are the characteristics of contours:

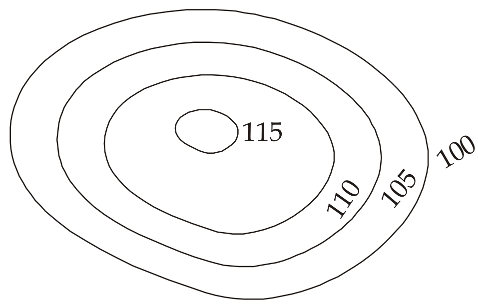
1. Two contour lines of different elevations cannot cross each other. However, contour lines of different elevations can intersect only in case of an overhanging cliff.



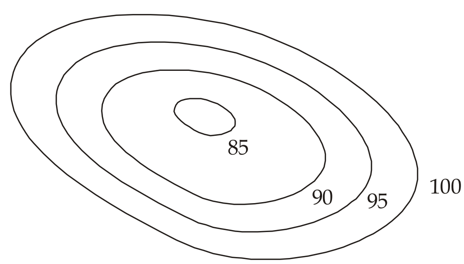
2. Contour lines of different elevations can unite to form one line only in the case of vertical cliff.
3. Contour lines which are close to each other indicates steep slope. They indicate a gentle slope if they are far apart. If they are equally spaced, uniform slope is indicated. A series of straight, parallel and equally spaced contours represent a plane surface. Thus in the figures shown below, steep slope is represented at *A-A*, a gentle slope at *B-B*, a uniform slope at *C-C* and a plane surface at *D-D*.



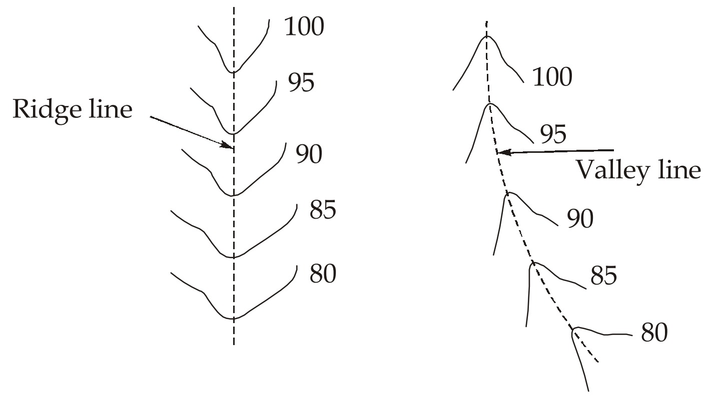
4. A contour passing through any point is perpendicular to the line of steepest slope at that point.
5. A closed contour line with one or more higher ones inside it represents a hill.



6. A closed contour line with one or more lower ones inside it indicates a depression.



7. Two contour lines having the same elevation cannot unite and continue as one line. Similarly a single contour cannot split into two contour lines.
8. A contour line must close upon itself, though not necessarily within the limits of the map.
9. Contour lines cross a watershed or ridge line at right angles. They form curves of U-shape around it with the concave side of the curve towards the higher ground.



10. Contour lines cross a valley line at right angle. They form sharp curves of V-shape around it with convex side of the curve towards the higher ground.
11. The same contour appears on either sides of a ridge or valley, for the highest horizontal plane that intersects the ridge must cut it on both sides. The same is true of the lower horizontal plane that cuts a valley.

Q.5 (c) (ii) Solution:

In the first set, the centre of the bubble has moved $\frac{20-10}{2} = 5$ divisions towards eyepiece end of the tube.

In the second set, the centre of the bubble has moved $\frac{20-10}{2} = 5$ divisions towards objective end.

Thus total number of divisions through which the bubble has moved,

$$n = 5 + 5 = 10$$

The change in staff reading, $S = 1.68 - 1.602 = 0.078$ m

Sensitivity of bubble tube:

$$\alpha = \left(\frac{S}{nD} \times 206265\right)'' \quad (n = 10, D = 80 \text{ m})$$

$$= \left(\frac{0.078}{10 \times 80} \times 26265\right)''$$

$$= 20.11''$$

Now, radius of curvature of bubble tube:

Also, $$\alpha = \left(\frac{l}{R} \times 206265\right)'' \quad (l = 2 \text{ mm})$$

$$20.11'' = \left(\frac{2\times10^{-3}}{R}\times206265\right)''$$

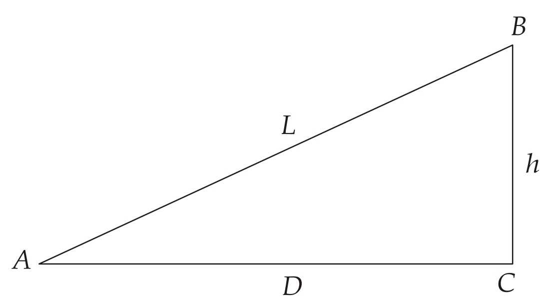
$$R = 20.5 \text{ m}$$

Hence, the radius of curvature of bubble tube, $R = 20.5$ m

Q.5 (d) Solution:

The distance measured along the slope is always greater than the horizontal distance between the points. Therefore, if the distance is measured on the slope, it must be reduced to its corresponding horizontal distance.

$$D = \sqrt{L^2 - h^2}$$



Error occurred during length measurement over slope,

$$E_{sl} = L - D$$

$$\Rightarrow \quad E_{sl} = L - \sqrt{L^2 - h^2}$$

$$\Rightarrow \quad E_{sl} = L - L\left(1 - \frac{h^2}{L^2}\right)^{1/2}$$

$$\Rightarrow \quad E_{sl} = L - L\left(1 - \frac{h^2}{2L^2} - \frac{h^4}{8L^4} + \ldots\right)$$

$$\Rightarrow \quad E_{sl} = L - L + \frac{h^2}{2L} + \frac{h^4}{8L^3} + \ldots.$$

$$\therefore \quad E_{sl} = \frac{h^2}{2L} \qquad \text{(Neglecting the higher order terms)}$$

$$\therefore \quad C_{sl} = -\frac{h^2}{2L}$$

Where, h = difference is elevations of A and B

L = measured length

Slope correction is always subtractive as $L > D$, so we have actually done positive error, which requires negative correction to be applied.

Numerical:

While measuring the distance, the chain was out of alignment as well as the measured distance was on slope. Let us first correct the slope distance for alignment and then correct it for horizontal equivalent.

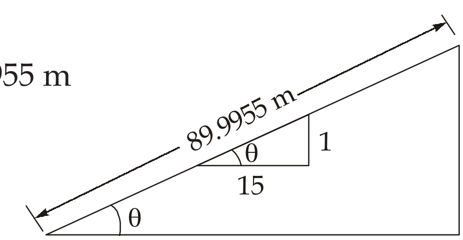
Correction for misalignment,

$$C_{ma} = 90-\sqrt{90^2-0.9^2}$$

$$\Rightarrow \quad C_{ma} = 0.0045 \text{ m}$$

$$\text{Corrected slope length} = 90 - 0.0045 = 89.9955 \text{ m}$$

$$\text{Gradient} = 1 : 15$$



$$\therefore \quad \tan\theta = \frac{1}{15}$$

$$\therefore \quad \theta = 3.814°$$

$$\text{Correction for gradient} = l_o(\cos\theta - 1) = 89.9955\{\cos(3.814°) - 1\} = -0.199 \text{ m}$$

$$\text{Corrected length} = 89.9955 - 0.199 = 89.7965 \text{ m}$$

Q.5 (e) Solution:

Let R be the radius of the curve.

$$ED = ET_1 = R\tan 15°15' = 0.2726\,R$$

$$FD = FT_2 = R\tan 20°15' = 0.3689\,R$$

$$\because \quad EF = ED + FD = EF = 175 \text{ m}$$

$$\Rightarrow \quad 0.2726\,R + 0.3689\,R = 175$$

$$0.6415\,R = 175$$

$$R = 272.79 \text{ m}$$

$$\therefore \quad ET_1 = 0.2726 \times 272.79 = 74.36 \text{ m}$$

$$FT_2 = 0.3689 \times 272.79 = 100.63 \text{ m}$$

Angle, $\theta = 180° - (30°30' + 40°30') = 180° - 71° = 109°$

Applying the sine rule in ΔBEF

$$\frac{BE}{\sin 40°30'} = \frac{BF}{\sin 30°30'} = \frac{EF}{\sin 109°}$$

$\therefore$ $$BE = EF \times \frac{\sin 40°30'}{\sin 109°} = 175 \times \frac{0.6494}{0.9455} = 120.19 \text{ m}$$

$$BF = 175 \times \frac{\sin 30°30'}{\sin 109°} = 93.93 \text{ m}$$

Curve length, $$T_1D = \frac{\pi \times 272.79 \times 30°30'}{180°} = 145.21\text{m}$$

Curve length, $$DT_2 = \frac{\pi \times 272.79 \times 40°30'}{180°} = 192.82 \text{ m}$$

$$\text{Chainage at } T_1 = 1500 - (BE + ET_1)$$
$$= 1500 - (120.19 + 74.36)$$
$$= 1305.45 \text{ m}$$

$$\text{Chainage at } T_2 = \text{Chainage of } T_1 + T_1D + DT_2$$
$$= 1305.45 + 145.21 + 192.82$$
$$= 1643.48 \text{ m}$$

Q.6 (a) Solution:

(i)

1. **Photogrammetry :** Photogrammetry may be defined as the science and art of producing a mosaic or map by compiling the photographs. The word photogrammetry is derived from the Greek words 'Photos', 'grammar' and 'matron' meaning 'light' 'that is drawn or written' and to measure respectively. Therefore, its literal meaning is measuring from photograph and thus may be defined as the science of obtaining reliable measurements by means of photographs in order to primarily determine geometric characteristics such as size, form and position of the object.

2. **Map vs Aerial photographs**

   (a) Map is an orthogonal projection, whereas an aerial photograph is a central projection i.e. perspective projection.

   (b) Map has a single constant scale, whereas in an aerial photograph, it varies from point to point depending upon their elevations on an aerial photograph.

   (c) The amount of details on a map are selective but in aerial photograph a plethora of details are there.

   (d) Due to symbolic representation, the clarity of details is more on maps than on a photograph.

(ii)

Except that one of the intermediate sights was taken with an inverted staff, the other sights were taken as usual and since we need only the RLs of the change points.

BS (m)	IS (m)	FS (m)	Height of collimation (m)	RL (m)	Remarks
2.650			126.100	123.450	A (starting BM)
	3.740				
	–2.830			128.930	B (Inverted staff reading)
4.640		4.270	126.47	121.830	C (Change point)
	0.380				
1.640		0.960	127.15	125.510	D (Change point)
	2.840			124.31	
4.680		3.480	128.35	123.670	E (Change point)
		4.260		124.090	F (Bench mark)
ΣBS = 13.610		ΣFS = 12.970		Rise = 0.640	

Arithmetical checks

$$\Sigma BS - \Sigma FS = 13.610 - 12.970$$

$$= 0.640 \text{ m(Rise)} = \text{Last RL} - \text{First RL} = 0.640 \text{ m (Rise)}$$

(OK)

Q.6 (b) Solution:

(i)

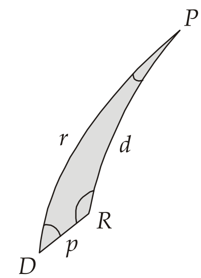
(a) **Equinoctial points :** The points of intersection of the ecliptic circle with the equatorial circle are known as equinoctial points. The point at which the sun transits from Southern to Northern hemisphere is known as first point of aeries and from Northern to Southern hemisphere as first point of Libra.

(b) **Right ascension :** The right ascension of a celestial body is the angular distance along the arc celestial equator measured from the first point of aeries to the foot of the hour circle. It is measured from East to West direction i.e., anti-clockwise in Northern hemisphere.

(ii)

The shortest distance between two stations on the surface of the earth lies along the circumference of the great circle passing through the stations.

Referring to figure, let us consider a great arc RD passing through the Roorkee and Delhi respectively. Thus, arc RD is the shortest distance between the stations. Let P be the pole of the earth and PD and PR are arcs of meridians passing through Delhi and Roorkee stations respectively.



Then, PDR is an astronomical triangle, where

$$P = 77°54' - 77°06' = 48'$$

$$\text{Distance, PD, } r = 90° - 28°34' = 61°26'$$

$$\text{Distance PR, } d = 90° - 29°52' = 60°08'$$

By cosine formula,

$$\cos p = \cos d \cdot \cos r + \cos P \cdot \sin d \cdot \sin r$$

$$= \cos 60°08' \cdot \cos 61°26' + \cos 48' \cdot \sin 60°08' \cdot \sin 61°26'$$

$$= 0.238126 + 0.76154 = 0.99966837$$

Therefore, $p = 1°28'33''$

$$\text{Radius of the earth, } R = 6370 \text{ km}$$

$$\text{Arc distance, } RD = \frac{2\pi}{360} \times 6370 \times 1°28'33'' = 164.079 \text{ km}$$

For the determination of the direction from R to D, the angle R of the spherical triangle is required to be determined.

Using equation,

$$\tan\frac{R+D}{2} = \frac{\cos\left(\frac{r-d}{2}\right)}{\cos\left(\frac{r+d}{2}\right)} \cot\frac{P}{2}$$

$$\Rightarrow \quad \tan\frac{R+D}{2} = \frac{\cos\left(\frac{61°26' - 60°08'}{2}\right)}{\cos\left(\frac{61°26' + 60°08'}{2}\right)} \cot\frac{48'}{2}$$

$$\therefore \quad R + D = 179°36'34.125'' \quad \text{...(i)}$$

Again, using equation,

$$\tan\frac{R-D}{2} = \frac{\sin\left(\frac{61°26 - 60°08'}{2}\right)}{\sin\left(\frac{61°26 + 60°08'}{2}\right)} \cot\frac{48'}{2}$$

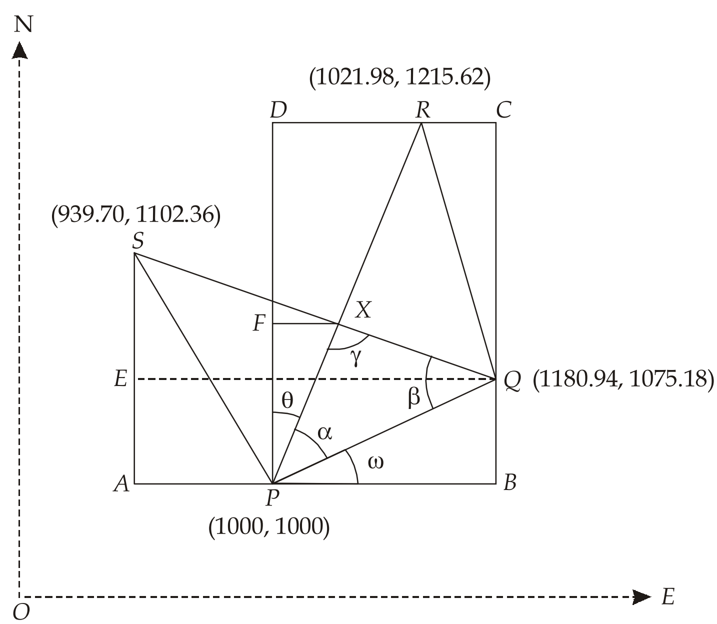
or, $R - D = 123°31'05.20''$ ...(ii)

Solving equation (i) and (ii), we get

$$R = 151°33'49.6''$$

Thus, the azimuth of the line to be set out at station R to proceed along shortest path to the station at Delhi is = 360° – *R* = 208°26′10.3″.

Q.6 (c) Solution:



In above figure,

$$PQ = \sqrt{(QB)^2 + (PB)^2}$$

$$= \sqrt{(1075.18 - 1000)^2 + (1180.94 - 1000)^2}$$

$$= 195.937 \text{ m}$$

$$QR = \sqrt{(QC)^2 + (CR)^2}$$

$$= \sqrt{(1215.62 - 1075.18)^2 + (1180.94 - 1021.98)^2}$$

$$= 212.112 \text{ m}$$

$$PR = \sqrt{(PD)^2 + (DR)^2}$$

$$= \sqrt{(1215.62-1000)^2+(1021.98-1000)^2}$$

$$= 216.737 \text{ m}$$

$$SQ = \sqrt{(SE)^2+(EQ)^2}$$

$$= \sqrt{(1102.36-1075.18)^2+(1180.94-939.70)^2}$$

$$= 242.766 \text{ m}$$

$$SP = \sqrt{(SA)^2+(AP)^2}$$

$$= \sqrt{(1102.36-1000)^2+(1000-939.70)^2}$$

$$= 118.801 \text{ m}$$

From ΔPRQ, we have

$$\cos\alpha = \frac{PR^2+PQ^2-RQ^2}{2PR\times PQ}$$

$$= \frac{216.737^2+195.937^2-212.112^2}{2\times 216.737\times 195.937}$$

$\therefore$ $$\alpha = 61°37'00''$$

From ΔSPQ, we have

$$\cos\beta = \frac{SQ^2+PQ^2-SP^2}{2SQ\times PQ}$$

$$= \frac{242.766^2+195.937^2-118.801^2}{2\times 242.766\times 195.937}$$

$\therefore$ $$\beta = 28°59'28''$$

In ΔPQB, we have

$$\tan\omega = \frac{QB}{PB} = \frac{(1075.18-1000)}{(1180.94-1000)}$$

$\therefore$ $$\omega = 22°33'46''$$

From ΔPQX, we have

$$\Delta PQX = \gamma = 180° - (\alpha+\beta) = 180° - (61°37'00'' + 28°59'28'')$$

$$= 89°23'32''$$

From sine law in ΔPQX, we get

$$\frac{PX}{\sin\beta} = \frac{PQ}{\sin\gamma}$$

$$\Rightarrow \quad PX = \frac{PQ\sin\beta}{\sin\gamma} = \frac{195.937\times\sin(28°59'28'')}{\sin(89°23'32'')} = 94.971 \text{ m}$$

Let bearing of PX be θ,

$$\theta = 90° - (\alpha + \omega)$$

$$= 90° - (61°37'00'' + 22°33'46'') = 5°49'14''$$

$$\text{Departure of } PX = PX\sin\theta = 94.971\sin(5°49'14'') = 9.631 \text{ m}$$

$$\text{Latitude of } PX = PX\cos\theta = 94.971\cos(5°49'14'') = 94.481 \text{ m}$$

Coordinates of X are:

$$\text{Easting of } X = \text{Easting of } P + \text{Departure of } PX = 1000 + 9.631$$

$$= 1009.631 \text{ m}$$

$$\text{Nothing of } X = \text{Northing of } P + \text{Latitude of } PX$$

$$= 1000 + 94.481 = 1094.481 \text{ m}$$

$\therefore$ The coordinates of X are E 1009.631 m and N 1094.481 m

Q.7 (a) Solution:

Given, size of photograph = 23 cm × 23 cm

Average scale of photograph = 1 : 25000

Average elevation of terrain = 335 m

Longitudinal overlap, P_l = 65%

Side overlap, P_s = 28%

Ground speed of aircraft, V = 270 km/hr

Focal length of camera, f = 200 mm

Least count of intervalometer = 0.5 sec

Flying height: $$\frac{f}{H} = \text{Scale of photograph}$$

$$\Rightarrow \quad \frac{0.2}{H} = \frac{1}{25000}$$

$$\Rightarrow \quad H = 5000 \text{ m above ground}$$

∴ Height above datum = 5000 + 335 = 5335 m

Theoretical ground spacing of flight lines:

Ground width covered by each photograph

$$W = (1 - P_w)\, S.W.$$
$$= (1 - 0.28) \times 25000 \times 0.23$$
$$= 4140 \text{ m} = 4.14 \text{ km}$$

Number of flight lines required

$$N_2 = \frac{\text{Width of ground}}{\text{Width covered by one photograph}} + 1$$
$$= \frac{L_2}{W} + 1 = \frac{15}{4.14} + 1 = 4.62 \approx 5$$

Actual spacing of flight lines:

$$W = \frac{15000}{4} = 3750 \text{ m}$$

Against the theoretically calculated value of 4140 m

Length covered by each photograph

$$L = (1 - P_l)\, S.l.$$
$$= (1 - 0.65) \times 25000 \times 0.23$$
$$= 2012.5 \text{ m} = 2.0125 \text{ km}$$

Ground speed of aircraft, $V = 270$ km/hr

∴ Exposure interval, $T = \frac{L}{V} = \frac{2.0125}{270} \times 3600$

$$= 26.83 \text{ sec} \simeq 26.5''$$

(Since least count of intervalometer = 0.5″)

∴ Adjusted ground distance, $L = V \times T = \frac{270 \times 5}{18} \times 26.5 = 1987.5$ m

Number of photographs per flight line

$$N_1 = \frac{L_1}{(1 - P_l) S.l} + 1$$
$$= \frac{L_1}{L} + 1$$
$$= \frac{150 \times 10^3}{1987.5} + 1 = 76.47 = 77$$

∴ Total number of photograph required

$$= N_1 \times N_2$$
$$= 77 \times 5 = 385$$

Q.7 (b) (i) Solution:

Length	Latitude (m)	Departure (m)	Correct Latitude (m)	Correct Departure (m)
AB	0.00	183.79	–0.0565	183.86
BC	128.72	98.05	128.67	98.11
CD	177.76	–140.85	177.69	–140.764
DE	–76.66	–154.44	–76.713	–154.375
EF	–177.09	0.00	–177.14	0.0671
FA	–52.43	13.08	–52.45	13.1
	$\Sigma Lat = 0.3$	$\Sigma Dep = -0.37$		

$$\text{Bearing of } AB = \tan^{-1}\left(\frac{D}{L}\right) = \tan^{-1}\left(\frac{183.79}{0}\right) = 90°$$

$$\text{Bearing of } BC = \tan^{-1}\left(\frac{98.05}{128.72}\right) = 37°17'51.52''$$

$$\text{Bearing of } CD = \tan^{-1}\left(-\frac{140.85}{177.76}\right) = 321°36'29.11''$$

$$\text{Bearing of } DE = \tan^{-1}\left(-\frac{154.44}{-76.66}\right) = 243°36'5.02''$$

$$\text{Bearing of } EF = \tan^{-1}\left(\frac{0.00}{-177.09}\right) = 180°$$

$$\text{Bearing of } FA = \tan^{-1}\left(\frac{13.08}{-52.43}\right) = 165°59'31.36''$$

Alternatively, length can be calculated as:

$$L = \sqrt{(\text{Lat})^2 + (\text{Dep})^2}$$

Length calculation:

$$L_{AB} = 183.79 \text{ m}$$

$$L_{BC} = \frac{98.05}{\sin 37°17'51.52''} = 161.81 \text{ m}$$

$$L_{CD} = \frac{-140.85}{\sin(321°36'29.11'')} = 226.798 \text{ m}$$

$$L_{DE} = \frac{-154.44}{\sin(243°36'5.02'')} = 172.42 \text{ m}$$

$$L_{EF} = 177.09 \text{ m}$$

$$L_{FA} = \frac{13.08}{\sin 165°59'31.36''} = 54.0369 \text{ m}$$

∴ ΣL = Perimeter of traverse = 975.94 m

Length	Correction in latitude $C_L = \Sigma Lat \times \frac{L}{\Sigma L}$	Correction in departure $C_D = \Sigma Dep \times \frac{L}{\Sigma L}$
AB	–0.0565	0.0697
BC	–0.0497	0.0613
CD	–0.0697	0.08598
DE	–0.053	0.0654
EF	–0.0544	0.0671
FA	–0.0166	0.0205

(ii)

Given: L = 30 m, T_0 = 20°C, P_0 = 10 kg, P_m = 15 kg, T_m = 32°C

A = 0.03 cm^2, $\alpha = 11 \times 10^{-6}$ per °C, $E = 2.1 \times 10^6$ kg/cm^2, W = 0.693 kg, ML = 780 m

1. When supported at every 30 m;

Total correction per tape length is to be found out first, Here, n = 1.

$$\text{Temperature correction, } C_t = \alpha(T_m - T_0)L$$
$$= 11 \times 10^{-6}(32 - 20) \times 30$$
$$= 0.00396 \text{ m (+ve)}$$

$$\text{Pull correction, } C_p = \frac{(P_m - P_0)L}{A \times E}$$
$$= \frac{(15-10)\times 30}{0.03 \times 2.1 \times 10^6} = 0.00238 \text{ m (+ve)}$$

$$\text{Sag correction, } C_s = \frac{LW^2}{24n^2P_m^2}$$
$$= \frac{30 \times (0.693)^2}{24 \times (15)^2} = 0.00267 \text{ m (–ve)}$$

$$\text{Total correction} = +0.00396 + 0.00238 - 0.00267$$
$$= +0.00367 \text{ m (too long)}$$

so $L' = L + \text{total correction} = 30.00367 \text{ m}$

$$\text{True length} = \frac{L'}{L} \times ML$$

$$\Rightarrow \quad = \frac{30.00367}{30} \times 780 = 780.094 \text{ m}$$

2. When supported at every 15 m:

Here, span, $n = 2$

Let us find out the correction per tape length.

(i) Temperature correction = 0.00396 m (+ve) as before

(ii) Pull correction = 0.00238 m (+ve) as before

(iii) Sag correction $= \dfrac{LW^2}{24n^2P_n^2}$

$$= \frac{30 \times (0.693)^2}{24 \times 2^2 \times (15)^2} = 0.00067 \text{ m (–ve)}$$

Total correction = +0.00396 + 0.00238 – 0.00067

= +0.00567 m (too long)

so, $L' = L + \text{total correction} = 30.00567 \text{ m}$

$$\text{True length} = \frac{30.00567}{30} \times 780 = 780.147 \text{ m}$$

Q.7 (c) Solution:

(i)

Objectives of triangulation surveys: The triangulation surveys are carried out :

(i) To establish accurate control for plane and geodetic surveys of large areas, by terrestrial methods.

(ii) To establish accurate control for photogrammetric surveys of the large areas.

(iii) To assist in the determination of the size and shape of the earth by making observations for latitude, longitude and gravity.

(iv) To determine accurate locations of points in engineering works such as fixing of centreline and abutments of long bridges over large rivers. etc.

Criteria for selection of layouts of triangles: The criteria while deciding and selecting a suitable layout of triangles are as follows:

1. Triangles should preferably be equilateral.
2. Braced quadrilaterals should preferably be equilateral.

3. Centered polygons are regular.
4. The arrangements should be such that computations can be done through two or more independent routes.
5. The arrangements should be such that at least one route and preferably two routes form well conditioned triangles.
6. No angle of the figure opposite to the known side should be small.
7. The sides of figures should be of comparable lengths. Very long and very short lengths should be avoided.
8. Angles of similar triangles should not be less than 45° and in case of quadrilaterals no angle should be less than 30°.

Well conditioned triangles:

- The triangles of such a shape, in which any error in angular measurement has minimum effect upon the computed lengths is known as well-conditioned triangles.
- The best shape of a well-conditioned triangle is isosceles triangle i.e. triangle, whose base angles are 56°14′ each. However, from practical considerations, an equilateral triangle may be treated as well conditioned triangle. In actual practice, the triangles having any angle less than 30° or more than 120° should not be considered.

Strength of figure: The strength of figure is a factor to be considered in establishing a triangulation system to maintain the computations within the desired degree of precision. It also plays an important role in deciding layout of a triangulation system.

(ii)

Since the error is in seconds only and thus the degrees and minutes of the quantities have not been included in the tabulation. The computations are arranged in the tabular form below:

S.No.	Value	Weight(w)	v	v^2	wv^2
1.	20″	2	1	1	2
2.	18″	2	–1	+1	2
3.	19″	3	0	0	0
		$\Sigma w = 7$			$\Sigma wv^2 = 4$

Weighted arithmetic mean of the seconds readings of the observed angles

$$= \frac{20'' \times 2 + 18'' \times 2 + 19'' \times 3}{2 + 2 + 3} = 19''$$

$\therefore$ Weighted arithmetic mean of the angle = 30° 24′ 19″

$\therefore$
$$v_1 = 20'' - 19'' = 1''$$
$$v_2 = 18'' - 19'' - 1''$$
$$v_3 = 19'' - 19'' = 0$$

1. Probable error of single observation of unit weight is given by

$$E_s = \pm 0.6745\sqrt{\frac{\Sigma wv^2}{n-1}}$$
$$= \pm 0.6745\sqrt{\frac{4}{3-1}} = \pm 0.95'' \qquad ...(i)$$

2. Probable error of the weighted arithmetic mean is given by

$$E_m = \pm 0.6745\sqrt{\frac{\Sigma wv^2}{\Sigma w\,(n-1)}} = \pm 0.6745\sqrt{\frac{4}{7(3-1)}} = \pm 0.36''$$

3. Probable error of single observation of weight 3 is given by

$$E_w = \frac{E_s}{\sqrt{w}} = \pm 0.6745\sqrt{\frac{\Sigma wv^2}{w(n-1)}}$$
$$= \pm\frac{0.95}{\sqrt{3}} \text{(substuting } E_s \text{ from (i))} = \pm 0.55''$$

Q.8 (a) Solution:

(i)

(a) **Least count:** It is the smallest possible value of one division of any scale. For a vernier scale, least count is thus equal to the value of the smallest division on the main scale divided by the total number of divisions on the vernier.

If S is smallest division of main scale and n is smallest division of vernier scale then,

$$\text{Least count} = \frac{S}{n}$$

(b) **Closing error:** In a completely closed traverse, the sum of north latitudes must be equal to that of south latitudes and the sum of eastings must be equal to that of westings, if linear as well as angular measurements of the traverse and/or their computations are correct. If not, the distance between the starting station and the position of the last station obtained by calculation is known as closing error.

The closing error is generally expressed as a ratio of the closing error(e) and the perimeter of the traverse (P). In that form, it is called relative closing error. Thus,

$$\text{Relative closing error} = \frac{\text{Error of closure}}{\text{Perimeter of traverse}} = \frac{e}{P}$$

It is conventional to express the relative closing error with the numerator as unity.

Thus, $$\text{Relative closing error} = \frac{1}{P/e}$$

(c) Arithmetic check: These are simple arithmetic calculations which are made to check the correctness of entries in height of instrument method or rise and fall method in levelling. The arithmetic calculations can be checked by using the following equation:

$$\Sigma\text{Back sight} - \Sigma\text{Fore sight} = \text{Last } RL - \text{First } RL = \Sigma\text{Rise} - \Sigma\text{Fall}$$

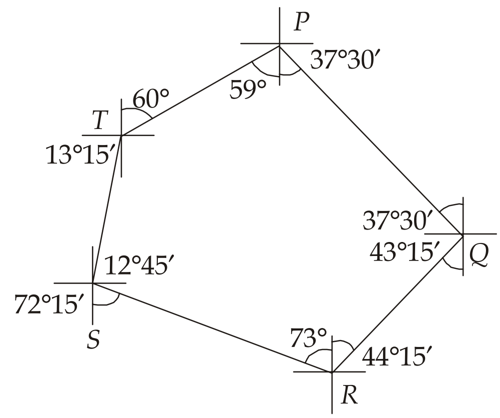
Thus, if the calculations on left hand side as well as on right hand side are equal, then the calculations made for filling the entries in the level book are correct.

(d) Local attraction: Local attraction is the attraction of magnetic needle to a local magnetic field other than earth's magnetic field. The local magnetic field is caused by iron fences, iron pipes, steel bars, vehicles, steel doors and windows, iron deposits, etc. Even small items made of iron or steel such as the wrist watch, pen, belt buckle, tapping arrows and steel tapes cause local attraction. DC power lines also develop a local magnetic field. A freely suspended magnetic needle takes the direction of the earth's magnetic field only if there is no local attraction. The magnetic needle will deviate from the magnetic meridian under the local magnetic field (forces).

(e) Whole to the part: It is the first principle of surveying. The surveyor should first establish accurately large main framework consisting of widely spaced control points. Between the large main framework, subsidiary small frameworks can be established by relatively less accurate surveys. The errors in small framework are thus localised and are not magnified and the accumulation of errors is controlled. In the reverse process of working from the part to whole, the small frameworks will be expanded to the large framework and the errors will get magnified.

(ii)

Interior angle at P = $37°30' + 59°00' = 96°30'$



Interior angle at Q = $180° - (37°30' + 43°15')$

$= 180° - 80°45' = 99°15'$

Interior angle at R = $44°15' + 73°00' = 117°15'$

Interior angle at S = $180° - (12°45' + 72°15') = 95°$

Interior angle at T = $13°15' + 90°00' + (90° - 60°) = 133°15'$

Sum of interior angles = $541°15°$

Theoretical sum of all interior angles = $(2n - 4) \times 90°$

$= (2 \times 5 - 4)(90) = 540°$

Hence Error = $541°15' - 540° = 1°15'$

Distributing the error equally in all angles, correction at each angle

$= -(1°15')/5 = -15'$

∴ The corrected angles are as below:

$\angle P = 96°30' - 15' = 96°15'$

$\angle Q = 99°15' - 15' = 99°00'$

$\angle R = 117°15' - 15' = 117°00'$

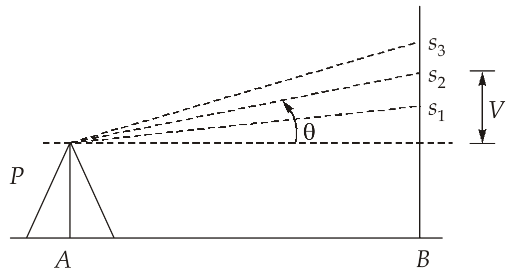
$\angle S = 95°00' - 15' = 94°45'$

$\angle T = 133°15' - 15' = 133°00'$

Sum = $540°00'$ (OK)

Q.8 (b) Solution:

Instrument P at station A and staff held vertical at B



$$s = s_3 - s_1$$

$\Rightarrow$ $$s = 1.795 - 1.090 = 0.705 \text{ m}$$

$$\theta = 5°44'$$

$\therefore$ $$AB = \text{Distance between A and B}$$

$$= D \cos \theta$$

$$= (Ks \cos\theta + C) \cos\theta$$

$$= Ks \cos^2\theta + C\cos\theta$$

$$= 100 \times 0.705 \cos^2 5°44' + 0.3 \cos 5°44'$$

$$= 70.095 \text{ m}$$

Now, $$V = Ks \cos\theta \sin\theta + C\sin\theta$$

$$= 100 \times 0.705 \times \cos 5°44' \times \sin 5°44' + 0.3 \times \sin 5°44'$$

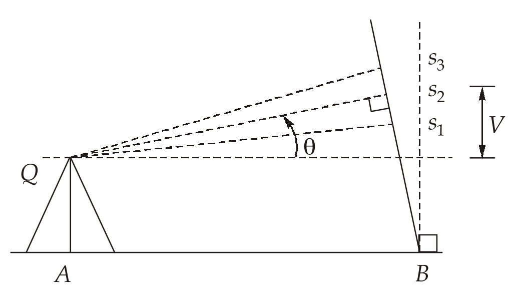
$$= 7.038 \text{ m}$$

$\therefore$ $$\text{RL of } B = \text{RL of A} + \text{HI} + \text{V} - s_2$$

$\Rightarrow$ $$\text{RL of } B = 100.00 + 1.400 + 7.038 - 1.440$$

$\Rightarrow$ $$\text{RL of } B = 106.998 \text{ m}$$

Instrument Q at station A and staff held normal at B



$$AB = (Ks + C) \cos \theta + s_2 \sin \theta$$

$\Rightarrow$ $$70.095 = (95s + 0.45) \cos 5°44' + s_2 \sin 5°44'$$

$\Rightarrow$ $$69.647 = 94.525s + 0.0999s_2 \quad \text{...(i)}$$

Now, $V = (Ks + C)\sin\theta$

$= (95s + 0.45)\sin 5°44'$

$\therefore$ RL of B = RL of A + HI + V – $s_2 \cos\theta$

$\Rightarrow \quad 106.998 = 100 + 1.450 + (95s + 0.45)\sin 5°44' - s_2\cos 5°44'$

$\Rightarrow \quad 9.4904s - 0.995s_2 = 5.50305$ ...(ii)

From equation (i) and (ii),

$$s = 0.7352 \text{ m}$$

$$s_2 = 1.4821 \text{ m}$$

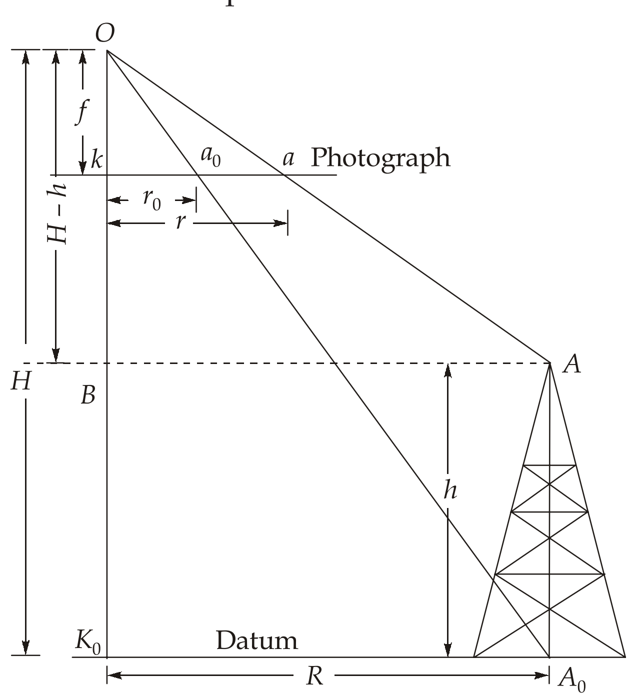
$\therefore$ Lower stadia wire reading $= s_2 - \frac{s}{2} = 1.4821 - \frac{0.7352}{2} = 1.1145$ m

Upper stadia wire reading $= s_2 + \frac{s}{2} = 1.4821 + \frac{0.7352}{2} = 1.8497$ m

$\therefore$ Staff readings with instrument Q are 1.1145, 1.4821 and 1.8497.

Q.8 (c) Solution:

(i) **Relief displacement:** When the ground is not horizontal, the scale of the photograph varies from point to point and is not constant. Since the photograph is the perspective view, the ground relief is shown in perspective on the photograph. Every point on the photograph is therefore, displaced from their true orthographic position. This displacement is called relief displacement.



As is clear from the figure, the point a is displaced outward from its datum photograph position, the displacement being along the corresponding radial lines from the principal point. The radial distance aa_0 is the relief displacement of A. The point k has not been displaced since it coincides with the principal point of the photograph. To calculate the amount of relief displacement,

Let r = Radial distance of a from k.

r_0 = Radial distance of a_0 from k.

R = Horizontal distance of A or A_0 on ground from K_0

Then, from similar triangles Oka and OBA

$$\frac{f}{H-h} = \frac{r}{R}, \quad \Rightarrow r = \frac{Rf}{H-h} \quad \text{...(1)}$$

Also, from similar triangles Oka and OK_oA_o

$$\frac{f}{H} = \frac{r_0}{R} \quad \Rightarrow r_0 = \frac{Rf}{H} \quad \text{...(2)}$$

Hence the relief displacement (d) is given by

$$\therefore \quad d = r - r_0 = \frac{Rf}{H-h} - \frac{Rf}{H}$$

$$\Rightarrow \quad d = \frac{Rfh}{H(H-h)} \quad \text{...(3)}$$

But, $R = \frac{r(H-h)}{f} = \frac{r_0 H}{f}$ (From eq. 1 and eq. 2)

Substituting the values of R in (3), we get,

$$d = \frac{r(H-h)}{f} \cdot \frac{fh}{H(H-h)} = \frac{rh}{H}$$

Also, $d = \frac{r_0 H}{f} \cdot \frac{fh}{H(H-h)} = \frac{r_0 h}{H-h}$

(ii)

Temporary adjustments are the adjustments which are done before taking the observations at every setup of the instrument. They consists of setting up the theodolite, levelling it, and focussing the eye piece and objective.

- **Setting up the theodolite:** It consists of centring the theodolite on the station and its approximate levelling by tripod legs. Centring involves setting the theodolite exactly on the station mark while approximate levelling involves levelling the instrument with the legs of the tripod so as to bring the small circular bubble provided on the tribrach in the centre.

- **Levelling:** After setting up the theodolite, levelling is done. It is done to ensure that as the instrument is swung about the vertical axis, the horizontal plate moves in a horizontal plane. It is done with the help of the foot screws.
- **Focussing:** It consists of focussing the eye piece and the objective. The operation of focusing the eye piece is done to make the cross-hairs appear clearly visible while the later operation is done to bring the image of the object in the plane of the cross-hairs.

(ii) Errors and mistakes in chaining may arise from any one or more of the following sources such as erroneous length of chain, variation of pull, displacement of arrows, miscounting chain lengths, misreading and erroneous looking.

Compensating Errors: These are the errors which are liable to occur in both the directions and tend to compensate. Compensating errors are proportional to the square root of the length of the line. They do not affect the results much. The compensating errors are caused by the following:

1. Incorrect holding and marking of the arrows.
2. Fractional parts of the chain may not be correct, i.e., the chain may not be calibrated uniformly.
3. Plumbing may be incorrect while chaining on slopes.
4. In settling chain angles with a chain.

Cumulative Errors: These are the errors which are liable to occur in the same direction and tend to accumulate. The errors thus considerably increase or decrease the actual measurements. The cumulative errors are proportional to the length of the line and may be positive or negative.

Positive Cumulative Errors: These are the errors which make the measured lengths more than the actual. Therefore, the actual length can be found by subtracting the correction from the measured length. These errors may be caused due to the following reasons:

1. The length of chain is shorter than the standard length like taking measurements when temperature is less than the standard temperature.
2. Bending of links, knots in links, removal of rings during adjustment of the chain, clogging of rings with mud, etc.
3. Not applying slope correction to the length measured along slopes.
4. Not applying sag correction.

Negative Cumulative Errors: These are the errors which make the measured length less than the actual. Therefore, the actual length can be found by adding the correction to the measured length. These errors may be caused due to the following reasons:

1. Length of the chain is more than its standard length, which may be due to flattening of rings, opening of joints, etc.
2. Not applying the temperature correction when temperature during measurements is more than the standard temperature.

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