CLASS TEST - 2016

CT

Engineering Mathematics

Date : 17/07/2016

ANSWERS

1. (b)	7. (a)	13. (a)	19. (c)	25. (d)
2. (c)	8. (a)	14. (d)	20. (b)	26. (c)
3. (b)	9. (b)	15. (a)	21. (a)	27. (b)
4. (a)	10. (b)	16. (c)	22. (b)	28. (b)
5. (b)	11. (c)	17. (b)	23. (c)	29. (d)
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EXPLANATIONS

1. (b)

Characteristic equation for eigen value is;

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \lambda^2 - (a+d)\lambda + ad - bc = 0$$

$$\Rightarrow \quad \lambda^2 - (a+d)\lambda + 1 = 0 \qquad (\because ad - bc = 1)$$

It has no real eigen value

$\Rightarrow$ no real root exist for λ

$$\Rightarrow \quad (a+d)^2 - 4 < 0$$

$$\therefore \quad (a+d)^2 < 4$$

2. (c)

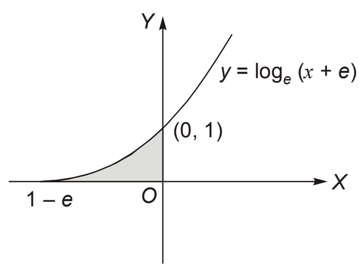
$$A^2 - A + I = 0$$

$$\Rightarrow \quad I = A - A \cdot A$$

Multiplying the equation with A^{-1}

$$\Rightarrow \quad A^{-1} = I - A$$

3. (b)



$$\text{Required area} = \int_{1-e}^{0} \log_e(x+e)\,dx$$

$\Rightarrow$ Let

$$z = x + e$$

$$dz = dx$$

$$= \int_1^e \log_e z \; dz$$

$$= \left[z(\log_e z - 1)\right]_1^e = -1$$

$$\Rightarrow \quad \text{Area enclosed} = 1 \text{ unit}$$

4. (a)

Divergence of curl of any vector is 0.

5. (b)

$$A^2 - B^2 = (A - B)(A + B)$$

$$\Rightarrow \quad A^2 - B^2 = A^2 - BA + AB - B^2$$

$$\Rightarrow \quad BA = AB$$

6. (c)

$$\Delta \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ xy^2 & 2x^2yz & -3yz^2 \end{vmatrix}$$

$$= \bar{i}\left(-3z^2 - 2x^2y\right) - \bar{j}\left(0 - 0\right) + \bar{k}\left(4xyz - 2xy\right)$$

$$\Rightarrow \quad \left(\Delta \times \bar{F}\right)_{(1,-1,1)} = -\bar{i} - 2\bar{k}$$

7. (a)

Method (Homogeneous equation):

Let, $y = vx$, so that $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

We have $v + x\dfrac{dv}{dx} = \dfrac{x + vx}{x} = 1 + v$

$$\Rightarrow \quad x\frac{dv}{dx} = 1$$

$$\Rightarrow \quad dv = \frac{dx}{x}$$

$$\Rightarrow \quad v = ln\,x + ln\,k$$

As $v = \dfrac{y}{x}$ we have y = $x\,ln\,x + (ln\,k)x$

At $x = 1$, $y = 1$ giving

1 = 0 + (ln k)

$\therefore\ ln\,k = 1$, then $y = x\,ln\,x + x$

8. (a)

$$\left(\vec{a} \times \vec{b}\right)^2 = a^2b^2 - \left(\vec{a} \cdot \vec{b}\right)^2 = a^2b^2 - a^2b^2\cos^2\theta$$

$$= (4 \times 2)^2 - (4 \times 2)^2 \cos^2\frac{\pi}{6}$$

$$= 64 \times \sin^2 \frac{\pi}{6} = 64 \times \frac{1}{4} = 16$$

Alternate Solution:

$$\left(\vec{a} \times \vec{b}\right)^2 = (|a|\ |b| \sin \theta)^2$$

$$= \left(4 \times 2 \times \sin\frac{\pi}{6}\right)^2 = \left(4 \times 2 \times \frac{1}{2}\right)^2 = 16$$

9. (b)

S = the sample space = {1, 2, 3, 4, 5, 6}

A = the event of occurrence of an odd number = {1, 3, 5}

B = the event of occurrence of a number greater than 1 = {2, 3, 4, 5, 6}

Here $A \cap B = \{3, 5\}$

so that $$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{n(A \cap B)}{n(A)} = \frac{2}{3} \simeq 0.67$$

10. (b)

$$\frac{d^4y}{dx^4} + 2\sqrt{\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right) + y} = 9$$

After removing radical sign

$$\left(\frac{d^4y}{dx^4} - 9\right)^2 = 4\left[\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right) + y\right]$$

$\therefore$ The order is 4 since highest differential is $\frac{d^4y}{dx^4}$ and the degree is 2 since power of highest differential is 2.

11. (c)

$$x^2y'' + xy' + y = 0$$

$\downarrow$

$\Rightarrow$ $(D(D-1) + D + 1)\,y = 0$

$\Rightarrow$ $(D^2 - D + D + 1)\,y = 0$

$\Rightarrow$ $(D^2 + 1)\,y = 0$

$\Rightarrow$ $m = \pm i$

$\Rightarrow$ $CF = C_1 \cos t + C_2 \sin t$

$\therefore$ Solution is $y = C_1 \cos(\ln x) + C_2 \sin(\ln x)$

12. (c)

$$\text{Standard deviation} = \sqrt{E(X)^2 - \{E(X)\}^2}$$

Mean, $$E(X) = 2\left(\frac{1}{3}\right) + (-1)\left(\frac{2}{3}\right) = 0$$

$$\Rightarrow \quad E(X)^2 = \Sigma X^2 P(X)$$

$$= (2)^2 \times \frac{1}{3} + (-1)^2\left(\frac{2}{3}\right) = 2$$

$$\therefore \quad \text{Standard deviation} = \sqrt{(2) - 0} = \sqrt{2}$$

13. (a)

Let $$f(x) = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\,dx \; (a > 0) \qquad \text{...(i)}$$

We know $$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$$

$$\therefore \quad f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1 + a^x}\,dx \qquad \text{...(ii)}$$

from (i) and (ii)

$$\Rightarrow \quad 2f(x) = \int_{-\pi}^{\pi} \cos^2 x\,dx = 2\int_0^{\pi} \cos^2 x\,dx$$

$$\Rightarrow \quad 2f(x) = 2 \times 2\int_0^{\pi/2} \cos^2 x\,dx$$

$$\Rightarrow \quad 2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$$

$$\left[\text{By using } \int_0^{\pi/2} \cos^n x\,dx = \frac{n-1}{n}\,\frac{n-3}{n-2} \ldots \frac{1}{2} \times \frac{\pi}{2} \text{ if } n \text{ is even}\right]$$

$$f(x) = \frac{\pi}{2}$$

14. (d)

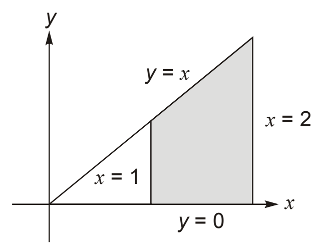
$$A\begin{bmatrix}1\\0\\-1\end{bmatrix} = 3\begin{bmatrix}1\\0\\-1\end{bmatrix} \qquad \therefore \text{ Eigen values of } A \text{ are 1, 2, 3}$$

One of the eigen vector of A is $\begin{bmatrix}1\\-1\\0\end{bmatrix}$ which is eigen vector at $\lambda = 2$.

$$\therefore \quad A^4\begin{bmatrix}1\\-1\\0\end{bmatrix} = \lambda^4\begin{bmatrix}1\\-1\\0\end{bmatrix} = 2^4\begin{bmatrix}1\\-1\\0\end{bmatrix} = 16\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}16.\\-16\\0\end{bmatrix}$$

15. (a)

From the given integral, the region of integration is bounded by



$$I = \int_1^2\int_0^x \frac{1}{\left(x^2+y^2\right)^{3/2}}\,dy\,dx$$

The region shown in the diagram can be represented in polar form as θ varies 0 to $\frac{\pi}{4}$.

$$r\cos\theta = 1, \qquad r\cos\theta = 2$$
$$r = \sec\theta, \qquad r = 2\sec\theta$$

$$\therefore \quad I = \int_0^{\pi/4}\int_{\sec\theta}^{2\sec\theta}\frac{1}{r^2}\,dr\,d\theta$$

$$a = \frac{\pi}{4}, \quad b = \sec\theta, \quad c = 2\sec\theta$$

16. (c)

$$F(x) = \int_1^x \frac{\ln t}{1+t}dt + \int_1^{1/x}\frac{\ln t}{1+t}dt$$

In second integral replace 't' by '$1/t$'

$$\Rightarrow \quad F(x) = \int_1^x\left(\frac{\ln t}{1+t}+\frac{\ln t}{(1+t)t}\right)dt = \int_1^x \frac{\ln t}{t}dt = \frac{1}{2}(\ln x)^2$$

$$\therefore \quad F(e) = \frac{1}{2}$$

17. (b)

$$P(A_1) = 0.6, \qquad P(A_2) = 0.8, \qquad P(A_1 \cup A_2) = 0.9$$

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$$

$$= 0.6 + 0.8 - 0.9 = 0.5$$

18. (b)

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$$

Let $x = e^z$

$\Rightarrow$ $z = \log x$

For this we have,

$$x\frac{dy}{dx} = Dy$$

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

$\Rightarrow$ $[D(D-1) - D - 3]y = e^{2z} \,.\, z$

For complementary function,

$$(D^2 - 2D - 3) = 0$$

$\therefore$ Roots $= -1, 3$

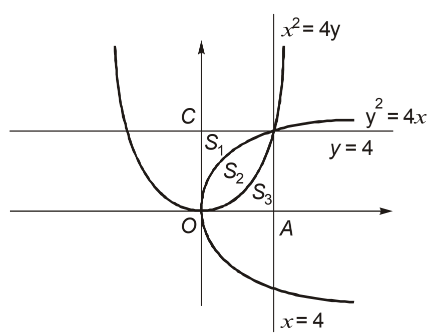
$$\text{C.F.} = c_1e^{3z} + c_2e^{-z}$$

$$= c_1e^{3 \log x} + c_2e^{-\log x}$$

$$= c_1x^3 + c_2x^{-1}$$

19. (c)

$$\text{Total area} = 4 \times 4 = 16 \text{ sq. units}$$



$$\text{Area of } S_3 = \int_0^4 \frac{x^2}{4}dx = \frac{16}{3} = \text{Area of } S_1$$

$\Rightarrow$ $S_2 = 16 - \frac{16}{3} \times 2 = \frac{16}{3}$

$\therefore$ $S_1 : S_2 : S_3$ is 1 : 1 : 1

20. (b)

$$\nabla\cdot\bar{F} = 4x-2x-0 = 2x$$

$$\therefore \quad \int_V \nabla\cdot\bar{F}\,dV = \int_V 2x\,dV$$

$$= \int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y} 2x\,dz\,dy\,dx$$

$$= \int_0^2\int_0^{2-x} 2x(4-2x-2y)\,dy\,dx$$

$$= \int_0^2\left[4\left(2x-x^2\right)y-4x\frac{y^2}{2}\right]_0^{2-x} dx$$

$$= 4\int_0^2\left[\left(2x-x^2\right)(2-x)-\frac{x}{2}(2-x)^2\right]dx$$

$$= 2\int_0^2\left(x^3-4x^2+4x\right)dx = \frac{8}{3} \simeq 2.67$$

21. (a)

$$P = \frac{1}{2} = 0.5$$

$$Q = \frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4} = \frac{7}{16} = 0.4375$$

$$R = \frac{1}{8}+\frac{7}{8}\cdot\frac{1}{8}+\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{1}{8}+\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{1}{8}+\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{7}{8}\cdot\frac{1}{8}$$

$$= \frac{8^4+8^3\cdot 7+8^2\cdot 7^2+8\cdot 7^3+7^4}{8^5} = 0.487$$

$\therefore$ P first, R second, Q third.

22. (b)

$$\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\frac{x}{2}\right][1-\sin x]}{\left[1+\tan\frac{x}{2}\right](\pi-2x)^3} = \lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)[1-\sin x]}{(\pi-2x)^3}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\left[1-\cos\left(\frac{\pi}{2}-x\right)\right]}{64\left(\frac{\pi}{4}-\frac{x}{2}\right)^3}$$

$$= \frac{1}{64} \lim_{x \to \frac{\pi}{2}} \left\{ \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)}{\left(\frac{\pi}{4} - \frac{x}{2}\right)} \right\} \left\{ \frac{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{\left(\frac{\pi}{4} - \frac{x}{2}\right)^2} \right\}$$

$$\Rightarrow \quad \frac{1}{64} \times 1 \times 2 = \frac{1}{32} \qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = \frac{\sin x}{x} = 1\right]$$

Applying L' Hospital rule

$$= \frac{1}{64} \lim_{x \to \frac{\pi}{2}} \left[\frac{\sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{\frac{-1}{2}} \right] \left[\frac{2\sin\left(\frac{\pi}{2} - x\right)\left(\frac{-1}{2}\right)}{2\left(\frac{\pi}{4} - \frac{x}{2}\right)\left(\frac{-1}{2}\right)} \right]$$

Applying L' Hospital rule

$$= \frac{1}{64} \lim_{x \to \frac{\pi}{2}} (-1) \left[\frac{2\cos\left(\frac{\pi}{2} - x\right)}{2\left(\frac{-1}{2}\right)} \right]$$

$$= \frac{1}{32}$$

23. (c)

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

$$r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$$

we know that $\nabla^2 [f(r)] = f''(r) + \frac{2}{r} f'(r)$

$$\Rightarrow \quad \nabla^2 [\log(r)] = -\frac{1}{r^2} + \frac{2}{r}\left(\frac{1}{r}\right) = \frac{1}{r^2}$$

24. (c)

$$F_1 = 2xy^3 + y, \qquad F_2 = 3x^2y^2 + 2x$$

$$\frac{\partial F_1}{\partial y} = 6xy^2 + 1, \qquad \frac{\partial F_2}{\partial x} = 6xy^2 + 2$$

By Green's theorem,

$$\int_C F_1\,dx + F_2\,dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{dy} \right) dx\,dy$$

$$= \iint \left(6xy^2 + 2 - 6xy^2 - 1\right) dx\, dy$$

$$= \iint dx\, dy$$

$$= \text{Area of the circle } x^2 + y^2 = 1$$

$$= \pi$$

25. (d)

The above system of equations can be written as $AX = B$

Where, $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 3 \\ 3 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 6 \\ 4 \\ -1 \end{bmatrix}$

$$\Rightarrow \quad [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 2 & 1 & 3 & 6 \\ 3 & -1 & 2 & 4 \\ 1 & 1 & 1 & -1 \end{array}\right]$$

Applying, $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ and $R_4 \to R_4 - R_1$

$$\Rightarrow \quad [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & -7 & -4 & -11 \\ 0 & -1 & -1 & -6 \end{array}\right]$$

Applying, $R_3 \to 3R_3 - 7R_2$ and $R_4 \to 3R_4 - R_2$

$$\Rightarrow \quad [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -2 & -14 \end{array}\right]$$

Applying, $R_4 \to 5R_4 - 2R_3$

$$\Rightarrow \quad [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & -3 & -1 & -4 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & 0 & -60 \end{array}\right]$$

$\therefore$ Rank $(A) = 3 \neq \text{rank}(A|B) = 4$

$\therefore$ The system is inconsistent and has no solution.

26. (c)

Let, $x + 3y = t$

So, that $1 + 3\dfrac{dy}{dx} = \dfrac{dt}{dx}$

The given differential equation becomes $\dfrac{1}{3}\left(\dfrac{dt}{dx} - 1\right) = \sin^2 t + 5$

$$\frac{dt}{dx} = 3\sin^2 t + 16$$

$$\Rightarrow \int \frac{dt}{3\sin^2 t + 16} = \int dx$$

$$\int \frac{dt/\cos^2 t}{3\sin^2 t/\cos^2 t + 16/\cos^2 t} = \int \frac{\sec^2 t\, dt}{3\tan^2 t + 16\sec^2 t} = \int \frac{\sec^2 t\, dt}{3\tan^2 t + 16(1 + \tan^2 t)}$$

Let, $u = \tan t = \tan(x + 3y)$

$du = \sec^2 t\, dt$

$$\Rightarrow x = \int \frac{du}{19u^2 + 16}$$

$$= \frac{1}{19}\int \frac{du}{u^2 + \frac{16}{19}} = \frac{1}{19}\frac{\sqrt{19}}{4}\tan^{-1}\frac{\sqrt{19}\,u}{4} + c$$

$$= \frac{1}{4\sqrt{19}}\tan^{-1}\frac{\sqrt{19}\tan(x + 3y)}{4} + c$$

27. (b)

[By trapezoidal rule using 4 subintervals]

$$h = \frac{1 - 0}{4} = 0.25$$

x	0	0.25	0.5	0.75	1
$y = \frac{1}{1+x}$	1	0.8	0.667	0.5714	0.5

$$I = \frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right] = 0.697$$

28. (b)

$$t^2 \frac{dy}{dt} = ty + y^2$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{y}{t} + \left(\frac{y}{t}\right)^2 \quad \text{...}(i)$$

Assuming, $u = \frac{y}{t}$

$$\Rightarrow \quad \frac{du}{dt} = -\frac{y}{t^2} + \frac{1}{t}\frac{dy}{dt}$$

$$\Rightarrow \quad \frac{dy}{dt} = t\frac{du}{dt} + u \quad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$\frac{y}{t} + \left(\frac{y}{t}\right)^2 = \frac{tdu}{dt} + u$$

$$\Rightarrow \quad t\frac{du}{dt} = u^2$$

$$\Rightarrow \quad \int \frac{du}{u^2} = \int \frac{dt}{t}$$

$$\Rightarrow \quad \frac{1}{u} = -\ln|t| + c$$

$$\therefore \quad y = \frac{t}{\left[c - \ln|t|\right]}$$

29. (d)

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & 0-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -\lambda\left[(1-\lambda)^2 - 1\right] = 0$$

$$\Rightarrow \quad \lambda\left(\lambda^2 - 2\lambda + 1 - 1\right) = 0$$

$$\Rightarrow \quad \lambda\left(\lambda^2 - 2\lambda\right) = 0$$

$$\Rightarrow \quad \lambda^2(\lambda - 2) = 0$$

$$\therefore \quad \lambda = 0, 0, 2$$

30. (d)

$$\int \frac{1}{a\cos x - b\sin x}\,dx \quad \text{where, } a = b = 1$$

Let

$$a = r\cos\theta = 1$$
$$b = r\sin\theta = 1$$

$\therefore$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x}\,dx = \frac{1}{\sqrt{2}}\int \frac{1}{\cos\left(x + \frac{\pi}{4}\right)}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sin\left(\frac{\pi}{2} + x + \frac{\pi}{4}\right)}\,dx = \frac{1}{\sqrt{2}}\int \frac{1}{2\sin\left(\frac{x}{2} + \frac{3\pi}{8}\right)\cos\left(\frac{x}{2} + \frac{3\pi}{8}\right)}\,dx$$

$$= \frac{1}{2\sqrt{2}}\int \frac{\sec^2\left(3\frac{\pi}{8} + \frac{x}{2}\right)}{\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)}\,dx = \frac{1}{2\sqrt{2}} \times 2\log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$$

$$= \frac{1}{\sqrt{2}} \times \log\left|\tan\left(\frac{x}{2} + \frac{3\pi}{8}\right)\right| + C$$

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