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Electronics Engineering

CLASS TEST
2016

EC

Signals & Systems

Date : 20/08/2016

ANSWERS

1.	(c)	7.	(c)	13.	(a)	19.	(b)	25.	(c)
2.	(a)	8.	(a)	14.	(b)	20.	(c)	26.	(a)
3.	(a)	9.	(c)	15.	(b)	21.	(a)	27.	(d)
4.	(d)	10.	(a)	16.	(a)	22.	(b)	28.	(c)
5.	(b)	11.	(c)	17.	(b)	23.	(d)	29.	(c)
6.	(b)	12.	(a)	18.	(a)	24.	(d)	30.	(b)

Explanation

1. (c)

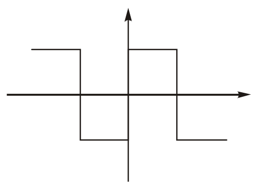
$(1 - z^{-1})\, X(z)$

$\left(\frac{z-1}{z}\right) X(z)$

for z-transform to exist, z should not be equal to '0'. So, the region of convergence will be the intersection of the ROC of $X(z)$ and the entire z-plane accept at $z = 0$

2. (a)

Since the waveform has a hidden symmetry the above signal can be split into an odd signal shifted by a DC level



thus dc term and sine terms will be the only signals present

3 (a)

$$\int_{-\infty}^{\infty} \delta(t)\left(\frac{3t}{2}\right) = f(0) = \cos\left(\frac{3\times 0}{2}\right) = \cos 0 = 1$$

4 (d)

$$y = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

$$y(t - t_0) = \int_{-\infty}^{t-t_0} x(\tau)\cos(3\tau)\,d\tau$$

$y'(t)$ for input $x(t - t_0)$ is

$$y'(t) = \int_{-\infty}^{t} x(\tau - t_0)\cos 3\tau\,d\tau$$

$$y'(t) = \int_{-\infty}^{(t-t_0)} x(\tau)\cos 3(\tau + t_0)\,d\tau$$

$y'(t) \neq y(t - t_0)$ so system is not time invariant

for input $x(\tau) = \cos(3\tau)$ (bounded i/p)

$y(t) = \int_{-\infty}^{t} \cos^2(3\tau) \to \infty$ as $t \to \infty$

So for bounded i/p, o/p is not bounded therefore system is not stable.

6. (b)

$$E = \int_{-\infty}^{\infty} f(t)^2\,dt$$

$$E' = \int_{-\infty}^{\infty} f(2t)^2\,dt = \int_{-\infty}^{\infty} f(p)^2 \frac{dp}{2} \left(2t = p;\ dt = \frac{dp}{2}\right)$$

$$E' = \frac{E}{2}$$

7. (c)

$$\frac{Y(z)}{X(z)} = \frac{\frac{5}{3}[1-z]}{z^2 - z + \frac{2}{9}}$$

$$\Rightarrow \qquad Y(z) = \frac{5}{3}\frac{(-z)}{\left(z-\frac{1}{3}\right)\left(z-\frac{2}{3}\right)}$$

$$Y(z) = \frac{5z}{z-\frac{1}{3}} - \frac{5z}{z-\frac{2}{3}}$$

$$y(n) = 5\left(\frac{1}{3}\right)^n u(n) - 5\left(\frac{2}{3}\right)^n u(n)$$

8. (a)

$$\tilde{x}[n] = 1 + \cos\left(\frac{\pi n}{8}\right)$$

$$= 1 + \frac{e^{j\frac{\pi n}{8}}}{2} + \frac{e^{-j\frac{\pi n}{8}}}{2} \qquad (N = 16)$$

$$= 1 + \frac{e^{j\left(\frac{2\pi}{16}\right)n}}{2} + \frac{e^{-j\left(\frac{2\pi}{16}\right)n}}{2}$$

$$\Rightarrow \qquad a_1 = \frac{1}{2} \text{ and also } a_{31} = a_{15} = a_1^* = \frac{1}{2}$$

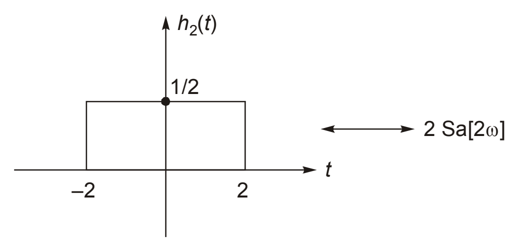
9. (c)

$$\frac{2\cos\omega\sin 2\omega}{\omega} = H(j\omega) \qquad \text{...(1)}$$

$$H(j\omega) = 2\cos\omega\left(\frac{2\sin 2\omega}{2\omega}\right) = H_1(j\omega).H_2(j\omega) \qquad \text{...(2)}$$

where $H_1(j\omega) = 2\cos\omega$

$$H_2(j\omega) = \frac{2\sin 2\omega}{2\omega} = 2\,\text{Sa}(2\omega)$$



$H_1(j\omega) = 2\cos\omega = (e^{j\omega} + e^{-j\omega}) = e^{j\omega 1} + e^{j\omega(-1)}$

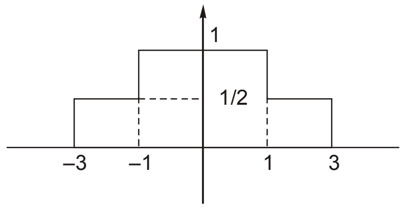
So

$H(j\omega) = 2\cos\omega\, H_2(j\omega) = (e^{j\omega} + e^{-j\omega})\, H_2(j\omega)$

$H(j\omega) = e^{j\omega} H_2(j\omega) + e^{-j\omega} H_2(j\omega)$

$\Rightarrow \quad h(t) = h_2(t+1) + h_2(t-1)$

$h(t) =$



So $h(0) = 1$

10. (a)

$$\dot{y}(t) + 5y(t) = u(t)$$

Taking the Laplace transform of the above equation

$$sY(s) - y(0) + 5Y(s) = \frac{1}{s}$$

$$(s+5)\,Y(s) - 1 = \frac{1}{s}$$

$$(s+5)\,Y(s) = \frac{s+1}{s}$$

$$Y(s) = \frac{(s+1)}{s(s+5)}$$

$$\frac{A}{s} + \frac{B}{(s+5)} = \frac{(s+1)}{s(s+5)}$$

$$\boxed{A = 0.2}$$

$$B = \frac{-4}{-5} = 0.8$$

$$Y(s) = \frac{0.2}{s} + \frac{0.8}{(s+5)}$$

$$\boxed{y(t) = 0.2 + 0.8e^{-5t}}$$

11. (c)

ROC π of a finite duration signal is entire s-plane.

12 (a)

$$H(z) = \frac{z^4}{\left[\left(z - \frac{1}{2}\right)^2 + \frac{1}{4}\right]\left[\left(z + \frac{1}{2}\right)^2 + \frac{1}{4}\right]}$$

$$\text{Poles} = \frac{1}{2} \pm j\frac{1}{2}, -\frac{1}{2} \pm j\frac{1}{2}$$

$$= \frac{z^4}{\left(z^2 - z + \frac{1}{2}\right)\left(z^2 + z + \frac{1}{2}\right)}$$

$$= \frac{z^4}{z^4 + z^2 + \frac{1}{4} - z^2}$$

$$H(z) = \frac{z^4}{z^4 + \frac{1}{4}}$$

$$= Z^4 + \frac{1}{4}\Big)\ Z^4 \Big(\ 1 - \frac{1}{4}z^{-4}\ldots$$

$$\frac{Z^4 + \frac{1}{4}}{\quad}$$

$$-\frac{1}{4}$$

13. (a)

$$x(t) = \cos\left(10\pi t + \frac{\pi}{4}\right)$$

$$X(f) = \frac{1}{2}[\delta(f-5) + \delta(f+5)]\, e^{j\pi/4}$$

After sampling $x(t)$ by 15 Hz, we get

$$X_s(f) = \frac{15}{2}\sum_{n=-\alpha}^{\infty} X(f - 15n)e^{j\pi/4}$$

Given, $$h(t) = \left[\frac{\sin(\pi t)}{\pi t}\right]\cos\left(40\pi t - \frac{\pi}{2}\right)$$

$$H(f) = \frac{1}{2}\left[\text{rect}\frac{(f-20)}{2\pi} + \text{rect}\frac{(f+20)}{2\pi}\right]e^{-j\frac{\pi}{2}}$$

Output $Y(f) = X_s(f) \times H(f)$

$$y(t) = \frac{15}{2}\cos\left(40\pi t - \frac{\pi}{4}\right)$$

14. (b)

$$\text{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s} = \frac{F(s)}{s}$$

with zero initial condition.

15. (b)
by using Taylor series we can expand the sin(z) into polynomial components

i.e. $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots$$

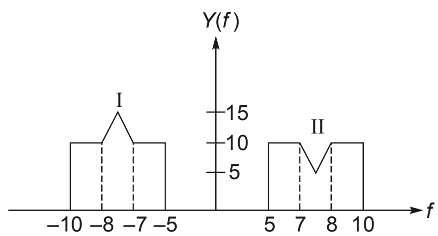
thus $$\sin(z^2) = z^2 - \frac{z^6}{3!} + \frac{z^{10}}{5!} - \ldots\ldots$$

now, from the above equation, we can deduce that $x(-10) = \frac{1}{5!}$

which is nothing but the coefficient of z^{10}

16. (a)
Since the Fourier series coefficient of real and even signal are real and even.

17. (b)
we know $y(0)$ = Area under $Y(f)$ curve
when we multiply the two figures i.e. $X(f) \cdot H(f)$ we get



now area under the curve will be area under curve I and area under curve II

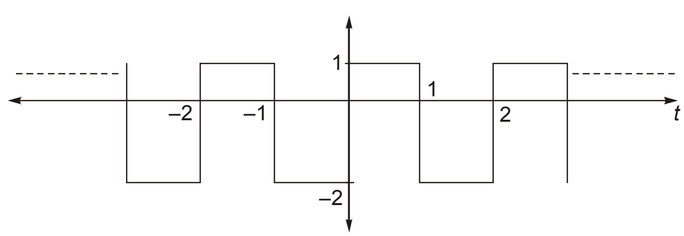
$\therefore$ Area under curve I = Area of rectangle + Area of triangle

Area under curve II = Area of rectangle – Area of triangle

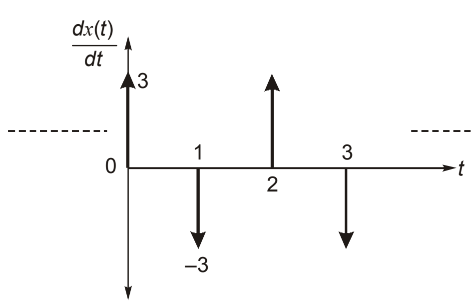
$\therefore$ Total Area = 2 Area of rectangle

= 2 × 10 × 5 = 100

18. (a)
The above signal can be represented as



Then differentiating the signal we get



$$\frac{dx(t)}{dt} = 3g(t) - 3g(t-1)$$

thus
$$A_1 = 3,$$
$$A_2 = -3$$
$$T_1 = 0$$
$$T_2 = 1$$

19. (b)

The relation between DTFT of output and it's input is given by

$$x[n] \overset{F}{\longleftrightarrow} X(e^{j\omega})$$

the
$$n \cdot x[n] \overset{F}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}$$

∴
$$E = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega = \sum_{n=\infty}^{\infty}|n \cdot x[n]|^2$$
$$= [1 + (2 \times 2)^2 + (3 \times 3)^2 + (4 \times 4)^2]$$
$$= [1 + 4^2 + 9^2 + 16^2]$$
$$= 354$$

20. (c)

Given
$$X(z) = ln\left(\frac{\alpha}{\alpha - z^{-1}}\right); \text{ROC}; |z| > \frac{1}{\alpha}$$
$$= -ln\left(1-(\alpha z)^{-1}\right)$$

now expanding it by Taylor series, we get

$$X(z) = \left[(\alpha z)^{-1} + \frac{(\alpha z)^{-2}}{2} + \frac{(\alpha z)^{-3}}{3} + \ldots\ldots\ldots\right]$$
$$= \sum_{k=1}^{\infty}\frac{\left[(\alpha z)^{-1}\right]^k}{k}$$

∴
$$X(z) = \sum_{k=1}^{\infty}\frac{\alpha^{-k}}{k} \cdot z^{-k}$$

Taking the inverse z-transform, we get

$$x[n] = \sum_{k=1}^{\infty}\frac{\alpha^{-k}}{k}\delta(n-k)$$

$[\because \delta[n-k] \leftrightarrow z^{-k}]$

∴
$$x[n] = \left(\frac{\alpha^{-n}}{n}\right)u[n-1]$$

21. (a)

the above system consists of two linear systems whose outputs can be represented as

$$y(t) = y_1(t) + y_2(t)$$
$$= [x_1(t)\cos(\omega_c t) + \cos(\omega_c t)] + [x_1(t)\cos(\omega_c t) + k\cos(\omega_c t)]$$
$$= [x_1(t) + x_1(t)]\cos(\omega_c t) + (1 + k)\cos\omega_c t$$

for $k = -1$

$$y(t) = 2x_1(t)\cos(\omega_c t)$$

The system will become linear system.

22. (b)

$$\because \quad f_s = \frac{2f_2}{m}$$

where m = largest integer less than $\frac{f_2}{B} = \frac{30}{8} = 3.75$

$$\therefore \quad m = 3$$

$$\therefore \quad f_s = 2 \times \frac{30}{3} = 20 \text{ kHz}$$

23. (d)

$$|H(e^{j\omega})|^2 = H(e^{j\omega}) \cdot H(e^{-j\omega})$$

thus if we replace $j\omega$ by $-j\omega$ then the magnitude plot will not change.

Hence $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ will have the same magnitude plot as $H(e^{j\omega})$

The transfer function $H_3(e^{j\omega})$ can be viewed as $H(e^{j\omega})$ in cascade with an all pass filter.

thus its magnitude plot will also be equal to $H(e^{j\omega})$.

24. (d)

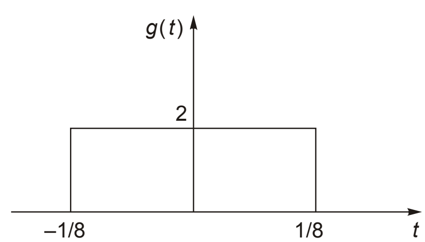
$$g(t) = \text{rect}(4t) * 4\delta(-2t)$$
$$= 4\,\text{rect}(4t) * \delta(-2t)$$

$(\because\ \delta(-t) = \delta(t))$

$$= 2\,\text{rect}(4t)$$

$$\left(\because \delta(at) = \frac{1}{|a|}\delta(t)\right)$$

thus $g(t)$ is given as



now,

$$\text{rect}(t) \xleftrightarrow{F.T} \text{sinc}(f)$$

$$2\text{rect}(t) \xleftrightarrow{F.T} 2\,\text{sinc}(f)$$

$$2\text{rect}(4t) \xleftrightarrow{F.T} 2 \cdot \frac{1}{4}\text{sinc}\left(\frac{f}{4}\right) \quad \text{(scaling property)}$$

$$\therefore \quad 2\text{rect}(4t) \xleftrightarrow{F.T} \frac{1}{2}\text{sinc}\left(\frac{f}{4}\right)$$

25. (c)

$$\because \quad \int_{-\infty}^{\infty} x(-t)\delta(t_0 - t)dt = \int_{-\infty}^{\infty} x(-t)\delta(t - t_0)dt$$

(since, $\delta(t) = \delta(-t)$)

$$= \int_{-\infty}^{\infty} x(-t_0)\delta(t - t_0)dt$$

(since, $x(t)\,\delta(t - t_0) = x(t_0)$)

$$= x(-t_0)\int_{-\infty}^{\infty} \delta(t - t_0)dt$$

$$= x(-t_0)$$

26. (a)

$$F_2(s) = F_1(s)\cdot e^{-s\tau}$$

$$\therefore \quad G(s) = e^{-s\tau}\cdot\frac{F_1(s)\cdot F_1^*(s)}{|F_1(s)|^2} = e^{-s\tau}\cdot\frac{|F_1(s)|^2}{|F_1(s)|^2}$$

$$= e^{-s\tau}$$

$$\therefore \quad g(t) = \delta(t - \tau)$$

27. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

Frequency of signal $\omega_{sig} = 50\pi$

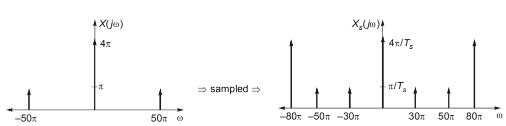
$T_s = 0.025$ sec

$\therefore$ sampling frequency $\omega_s = \dfrac{2\pi}{T_s} = 80\,\pi$ rad/sec

then, $X(j\omega) = 4\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

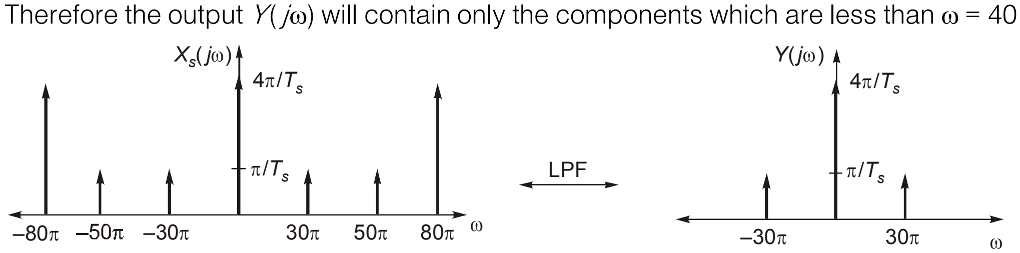
$$X_s(j\omega) = \frac{1}{T_s}\sum_{m=-\infty}^{\infty} X(j(\omega - n\omega_s))$$



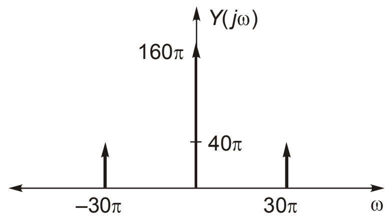
$X_s(j\omega) =$

$$40\sum_{m=-\infty}^{\infty}[4\pi\,\delta(\omega - 80\pi)] + \pi\,\delta(\omega - 50\pi - 80\pi n) - \pi\,\delta(\omega + 50\pi - 80\pi n)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$. Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\,\pi$.



Now by putting $T_s = 0.025$, we will get



28. (c)

Given

$$X(z) = ln\left(\frac{\alpha}{\alpha - z^{-1}}\right); \text{ROC}; |z| > \frac{1}{\alpha}$$

$$= -ln\left(1-(\alpha z)^{-1}\right)$$

now expanding it by Taylor series, we get

$$X(z) = \left[(\alpha z)^{-1} + \frac{(\alpha z)^{-2}}{2} + \frac{(\alpha z)^{-3}}{3} + ...\right]$$

$$= \sum_{k=1}^{\infty} \frac{\left[(\alpha z)^{-1}\right]^k}{k}$$

$$\therefore \quad X(z) = \sum_{k=1}^{\infty} \frac{\alpha^{-k}}{k} \cdot z^{-k}$$

Taking the inverse z-transform, we get

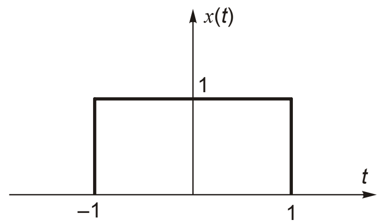
$$x[n] = \sum_{k=1}^{\infty} \frac{\alpha^{-k}}{k} \delta(n-k)$$

$[\because \delta[n-k] \leftrightarrow z^{-k}]$

$$\therefore \quad x[n] = \left(\frac{\alpha^{-n}}{n}\right) u[n-1]$$

29. (c)

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt$$



$\therefore$ $X(0)$ = Area under $x(t)$ curve

$\therefore$ $X(0) = 2$

30. (b)

$$X(e^{j\omega})\big|_{\omega=\pi} = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\pi} = \sum_{n=-\infty}^{\infty} (-1)^n x[n]$$

for $x[n] = \left\{2, -1, \underset{\uparrow}{3}, -1, 2\right\}$

$$X(e^{j\omega})\big|_{\omega=\pi} = (-1)^{-2}(2) + (-1)^{-1}(-1) + (3)(-1)^0 + (-1)^1(-1) + (-1)^2(2)$$

$\therefore$ $X(e^{j\omega})\big|_{\omega=\pi} = 9$

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