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# ESE 2024 : Mains Test Series

UPSC ENGINEERING SERVICES EXAMINATION

## Electrical Engineering

### Test-1 : Electrical Circuits [All Topics]

### Control Systems [All Topics]

Name : RAJAN KUMAR

Roll No : EE24MTDLA0111

**Test Centres**

Delhi ☑ Bhopal ☐ Jaipur ☐

Pune ☐ Kolkata ☐ Hyderabad ☐

**Student's Signature** Rajan Kumar

**Instructions for Candidates**

1. Do furnish the appropriate details in the answer sheet (viz. Name & Roll No).
2. There are Eight questions divided in TWO sections.
3. Candidate has to attempt FIVE questions in all in English only.
4. Question no. 1 and 5 are compulsory and out of the remaining THREE are to be attempted choosing at least ONE question from each section.
5. Use only black/blue pen.
6. The space limit for every part of the question is specified in this Question Cum Answer Booklet. Candidate should write the answer in the space provided.
7. Any page or portion of the page left blank in the Question Cum Answer Booklet must be clearly struck off.
8. There are few rough work sheets at the end of this booklet. Strike off these pages after completion of the examination.

**FOR OFFICE USE**

| Question No. | Marks Obtained |
|---|---|
| Section-A | |
| Q.1 | 40 |
| Q.2 | 56 |
| Q.3 | |
| Q.4 | 15 |
| Section-B | |
| Q.5 | 37 |
| Q.6 | 25 |
| Q.7 | |
| Q.8 | |
| **Total Marks Obtained** | 173 |

Signature of Evaluator: Nihal Sk.

Cross Checked by

→ Reduce calculation error

→ Handwriting should be improved.

# IMPORTANT INSTRUCTIONS

**CANDIDATES SHOULD READ THE UNDERMENTIONED INSTRUCTIONS CAREFULLY. VIOLATION OF ANY OF THE INSTRUCTIONS MAY LEAD TO PENALTY.**

## DONT'S

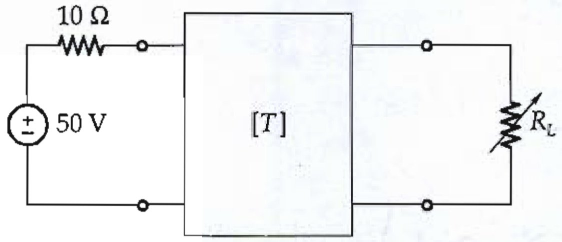
1. Do not write your name or registration number anywhere inside this Question-cum-Answer Booklet (QCAB).
2. Do not write anything other than the actual answers to the questions anywhere inside your QCAB.
3. Do not tear off any leaves from your QCAB, if you find any page missing do not fail to notify the supervisor/invigilator.
4. Do not leave behind your QCAB on your table unattended, it should be handed over to the invigilator after conclusion of the exam.

## DO'S

1. Read the Instructions on the cover page and strictly follow them.
2. Write your registration number and other particulars, in the space provided on the cover of QCAB.
3. Write legibly and neatly.
4. For rough notes or calculation, the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be evaluated.
6. Handover your QCAB personally to the invigilator before leaving the examination hall.

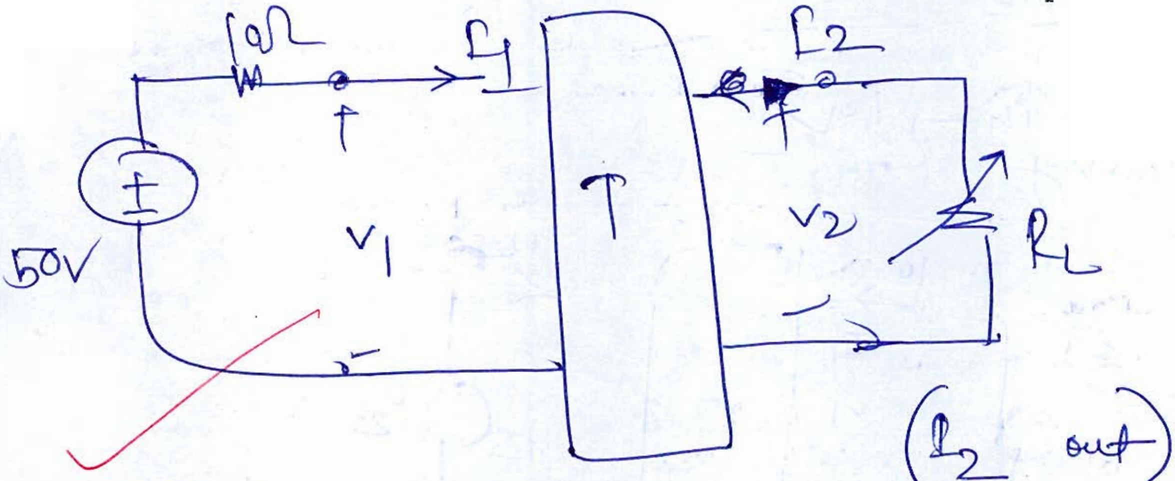
## Section A : Electrical Circuits

(a) The $ABCD$ parameter of the two-port network in figure are $\begin{bmatrix} 4 & 20\,\Omega \\ 0.1S & 2 \end{bmatrix}$.




The output port is connected to a variable load for maximum power transfer. Find $R_L$ and the maximum power transferred.

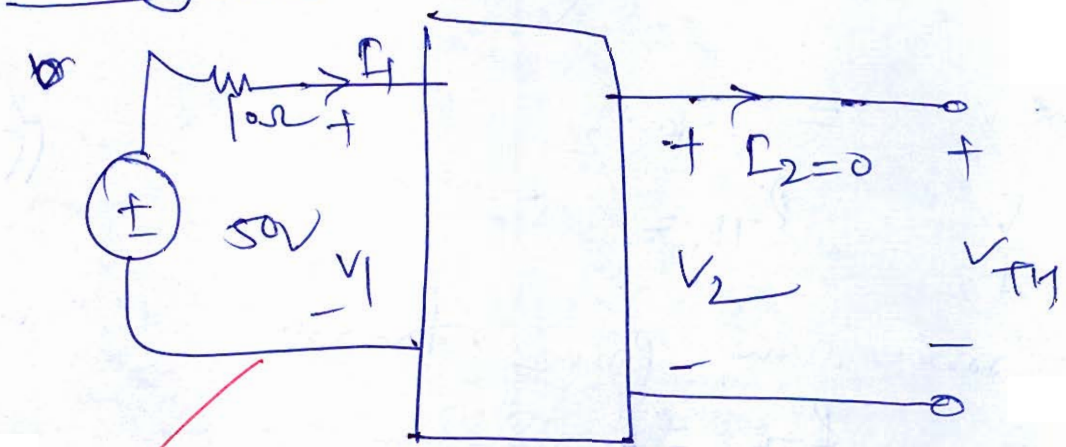
**[12 marks]**




$V_1 = 4V_2 + 20\,I_2$ —— (1)

$I_1 = 0.1\,V_2 + 2\,I_2$ —— (2)

Finding $V_{TH}$ :-




$50 = 10\,I_1 + V_1$ —— (3)

from (1)

$50 - 10 I_1 = 4 V_2$

from (2)

$I_1 = 0.1 V_2$

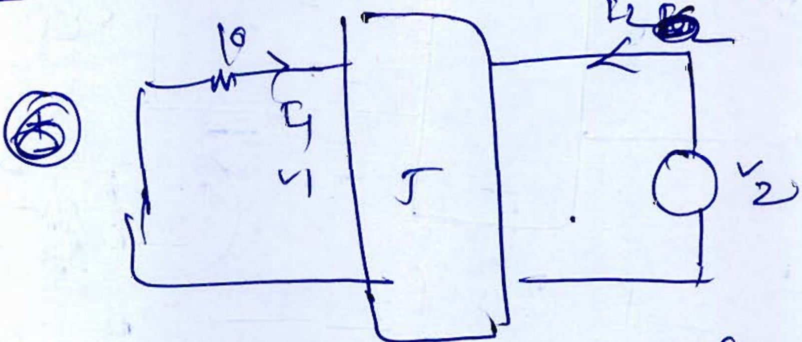
Solving $50 - 10 \times 0.1 V_2 = 4 V_2$

$V_2 = \frac{50}{\;} = 10V$

$V_{TH} = 10V$

10

Finding $R_{TH}$:




$I_2$ (R)

$V_1 = -10 I_1 = 4 V_2 + 20 I_2$ — (4)

$I_1 = 0.1 V_2 - 2 I_2$ — (5)

$\Rightarrow 4V_2 - 20 I_2 = -10(0.1 V_2 - 2 I_2)$

$\Rightarrow 5 V_2 = 40 I_2$

$\frac{V_2}{I_2} = R_{TH} = 8\,\Omega$

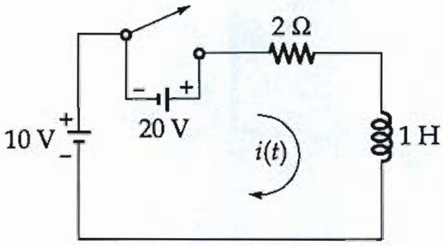
For maximum power transfer

$R_L = R_{TH} = 8\,\Omega$

$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{10^2}{4 \times 8} = 3.125\,W$

Do not write in this margin

**(b)** Determine the current $i(t)$ in the circuit shown in figure at an instant $t$, after opening the switch at $t = 0$, if a current of 1 A had been passed through the circuit at the instant of opening.



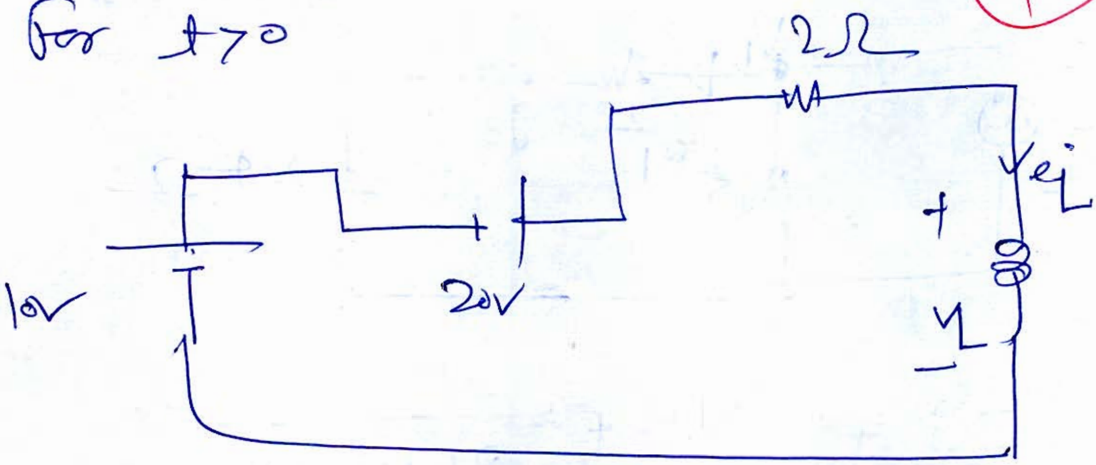

**[12 marks]**

For $t = 0^-$,

the initial current through inductor is $i_L(0^-) = 1$ A

10

For $t > 0$




Using KVL, $30 = 2i_L + \frac{di_L}{dt}$ $\left[v_L = L\frac{di}{dt}\right]$

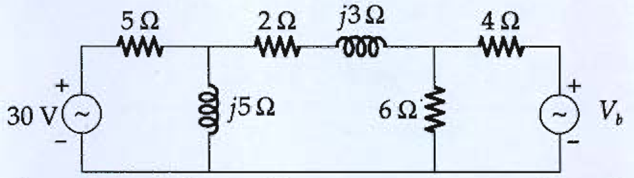
$\Rightarrow \int \frac{di}{i_L - 15} = \int -2dt$

$\Rightarrow i_L = 15 + C_0 e^{-2t}$

Hence $i_L(0) = 1$ $\Rightarrow 15 + C_0 = 1$ $C_0 = -14$

Thus $i(t) = i_L(t) = 15 - 14e^{-2t}$ $u(t)$ A

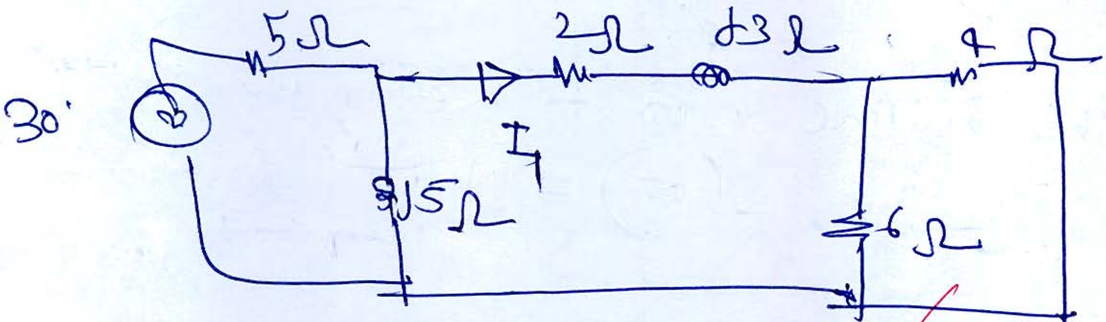
**Q.1 (c)** For the circuit shown below:

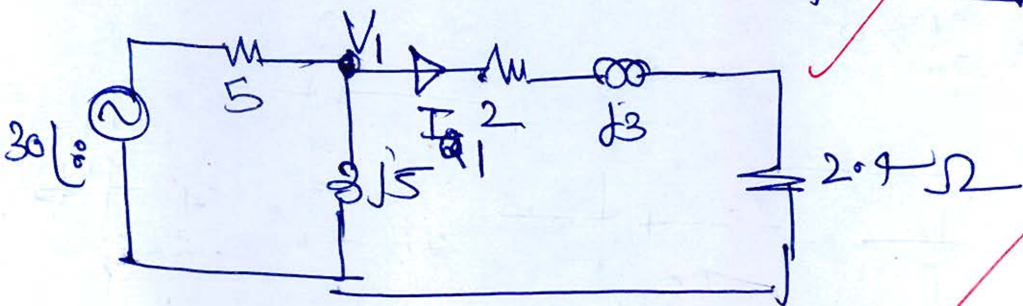



Determine the voltage $V_b$ which results in a zero current through the $(2 + j3)\Omega$ impedance branch. Using superposition theorem.

**[12 marks]**

Assuming only 30 V source active,




$$\frac{V_1 - 30}{5} + \frac{V_1}{j5} + \frac{V_1}{2 + j3 + 2.4} = 0$$

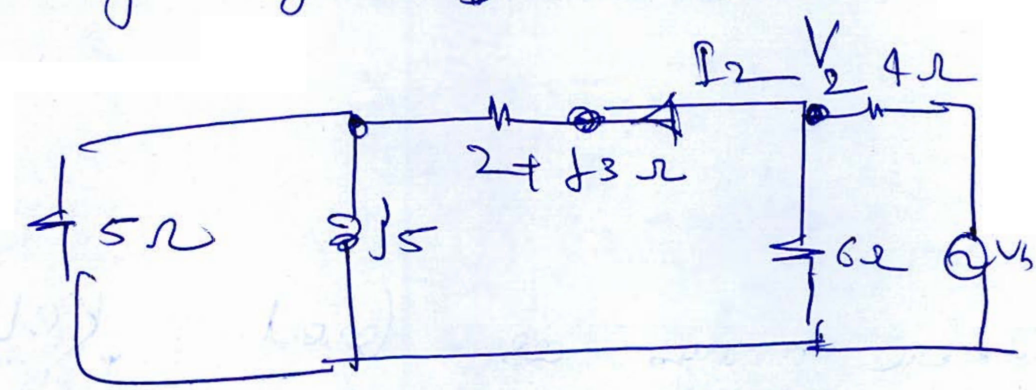
$$V_1 = \frac{6}{\frac{1}{5} + \frac{1}{j5} + \frac{1}{4.4 + j3}} = 12.80\angle 40.728°$$

$$I_1 = \frac{V_1}{4.4 + j3}$$

$$= \frac{12.80\angle 40.728}{4.4 + j3}$$

$= 2.404 \angle 6.440 \; A$

Assuming only $V_b$ active,




$$\frac{V_2 - V_b}{4} + \frac{V_2}{6} + \frac{V_2}{4+j3+5 \| j5} = 0$$

$$V_2 = \frac{\frac{V_b}{4}}{\frac{1}{4} + \frac{1}{6} + \frac{1}{2+j3+\frac{j25}{5+j5}}}$$

$$= 0.4832 \angle 12.15^\circ \; V_b$$

$$I_2 = \frac{V_2}{2+j3+\frac{j25}{5+j5}} = 0.0679 \angle -38.56^\circ \; V_b$$

For $I = 0$

$I_1 + I_2 = 0$

$2.404 \angle 6.44 + 0.0679 \angle -38.56 \; V_b = 0$

$V_b = 0.02828 \angle 135^\circ \; V$

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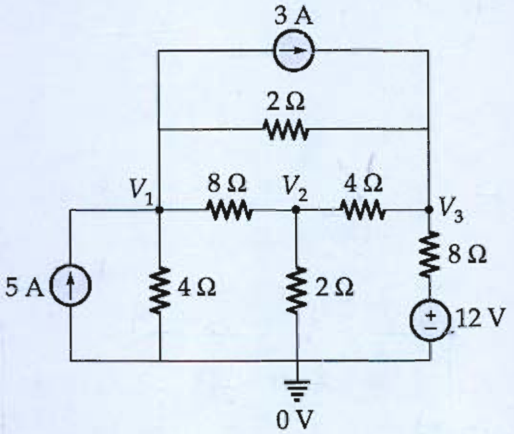
**Q.1 (d)** Use nodal analysis to find $V_1$, $V_2$ and $V_3$ in the circuit of figure.




**[12 marks]**

Nodal analys is based on KCL at node.

Applying KCL at node $V_1$

$\Sigma$ outgoing current = $\Sigma$ incoming current

$$\frac{V_1}{4}+\frac{V_1-V_2}{8}+\frac{V_1-V_3}{2}+3=5$$

$$\frac{7}{8}V_1-\frac{1}{8}V_2-\frac{1}{2}V_3=2 \quad \text{— (1)}$$

Similarly at node $V_2$

$$\frac{V_2}{2}+\frac{V_2-V_1}{8}+\frac{V_2-V_3}{4}=0$$

$$-\frac{1}{8}V_1+\frac{7}{8}V_1-\frac{1}{4}V_3=0 \quad \text{— (2)}$$

and at node $V_3$,

$$\frac{(V_3-12)}{8} + \frac{(V_3-V_2)}{4} + \left(\frac{V_3-V_1}{2}\right) = 3$$

$$-\frac{1}{2}V_1 - \frac{1}{4}V_2 + \frac{7}{8}V_3 = 4.5 \quad -(3)$$

In matrix form

$$\begin{bmatrix} 7/8 & -1/8 & -1/2 \\ -1/8 & 7/8 & -1/4 \\ -1/2 & -1/4 & 7/8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4.5 \end{bmatrix}$$

Solving we get,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ \frac{74}{15} \\ \frac{184}{15} \end{bmatrix}$$

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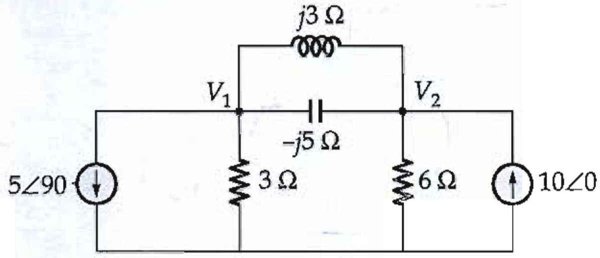
**Q.1 (e)** Use nodal analysis on the circuit to find $V_2$.




**[12 marks]**

Nodal analysys is based on KCL.

Applying KCL at node $V_1$.

$$\frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} = 5\angle 90^\circ$$

$$= j5$$

$$V_1 \frac{1}{3} + (V_1 - V_2)\frac{2}{j15} = j5$$

$$\left(\frac{1}{3} + \frac{2}{j15}\right)V_1 - \frac{2}{j15}V_2 = j5 \quad \text{--- (1)}$$

Similarly at node $V_2$,

$$-\frac{2}{j15}V_1 + \left(\frac{1}{6} + \frac{2}{j15}\right)V_2 = -10 \quad \text{--- (2)}$$

1

or
$$\begin{bmatrix} \frac{1}{3} + \frac{2}{j15} & -\frac{2}{j15} \\ -\frac{2}{j15} & \frac{1}{6} + \frac{2}{j15} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j5 \\ -10 \end{bmatrix}$$

using Cramer Rule,

$$V_2 = \frac{\begin{bmatrix} \frac{1}{3} + \frac{2}{j15} & j5 \\ -\frac{2}{j15} & -10 \end{bmatrix}}{\begin{bmatrix} \frac{1}{3} + \frac{2}{j15} & -2/j15 \\ -2/j15 & \frac{1}{6} + \frac{2}{j15} \end{bmatrix}} = 34.3556\angle{-103.29^\circ}$$

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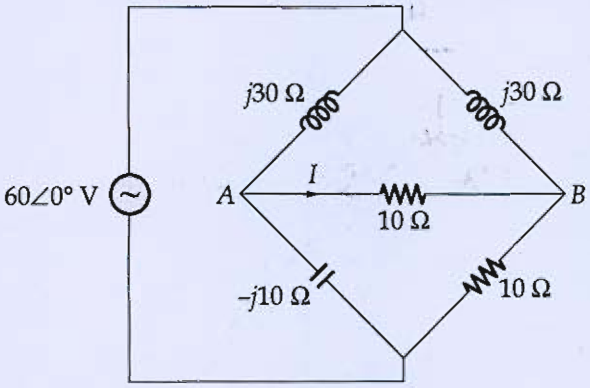
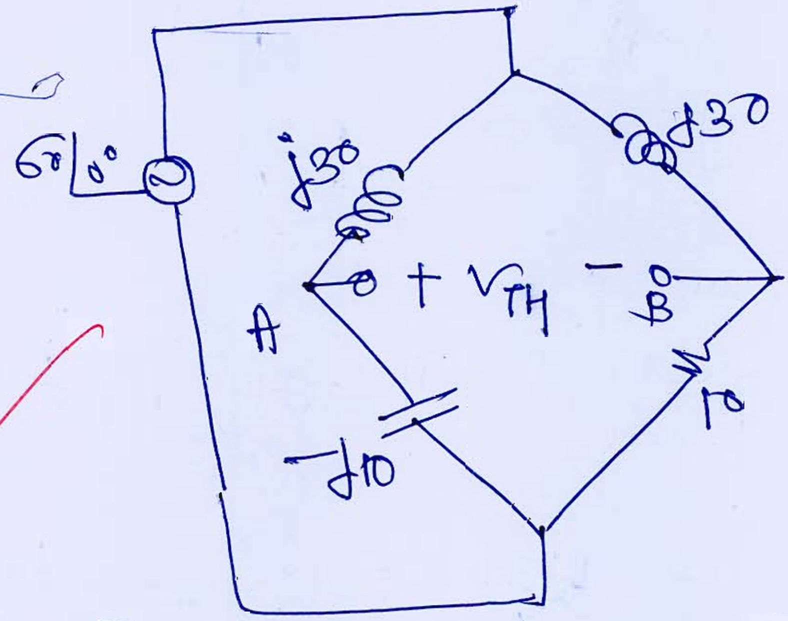
**(a)** Determine the current $I$ through the terminal AB of the network shown below:




**[20 marks]**

Finding I using thevenin theorem

$V_{TH}$ is:—

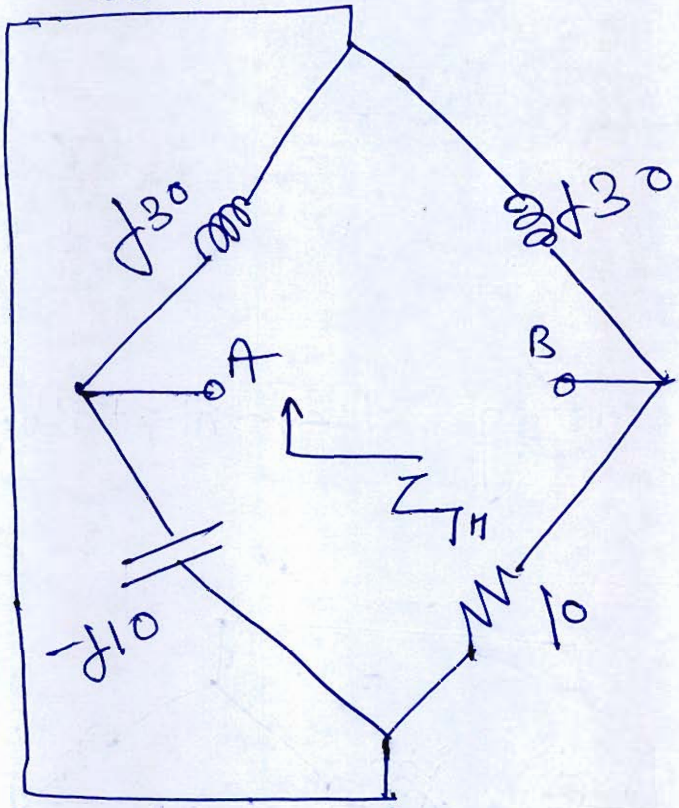



$$V_{TH} = 60\angle 0^\circ \left[ \frac{-j10}{-j10 + j30} - \frac{10}{10 + j30} \right]$$

{using voltage division rule}

$$= 18\sqrt{5} \angle 153.434^0 \ V$$

$Z_{TH}$ is :—




We have,

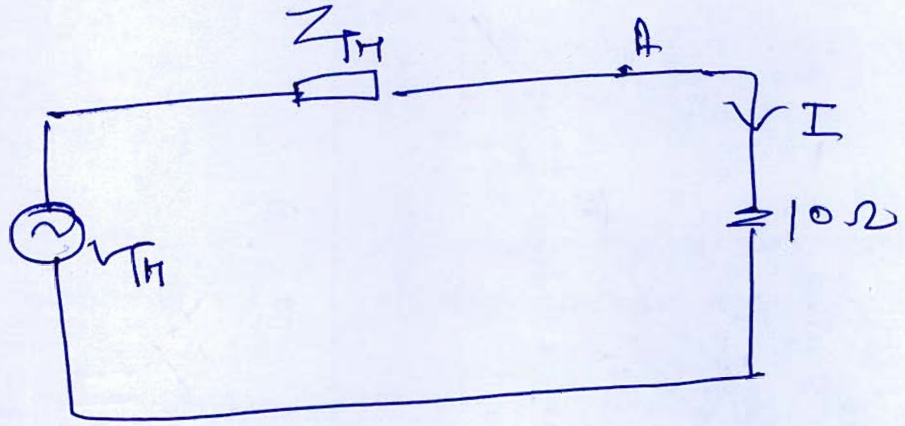
$$Z_{TH} = (j30 \parallel -j10) + (j30 \parallel 10)$$

$$= \frac{j30 \times (-j10)}{j30 - j10} + \frac{j30 \times 10}{j30 + 10}$$

$$= -j15 + \frac{j300}{j30 + 10}$$

$$= 15 \angle -53.13^\circ \ \Omega$$

The Thevenin equivalent circuit is




Thus, $I = \dfrac{V_{TH}}{Z_{TH} + 10}$

$$= \frac{18\sqrt{5}\,\angle 153.434^\circ}{15\angle -53.13^\circ + 10}$$

$$= 1.791\,\angle -174.29^\circ \text{ A}$$

Thus the current, I is,

$$I = 1.791\,\angle -174.29^\circ \text{ A}$$

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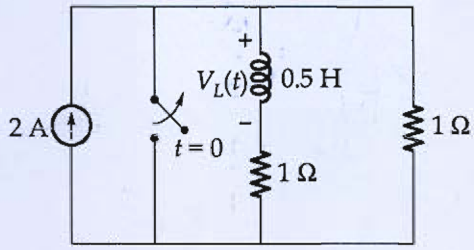
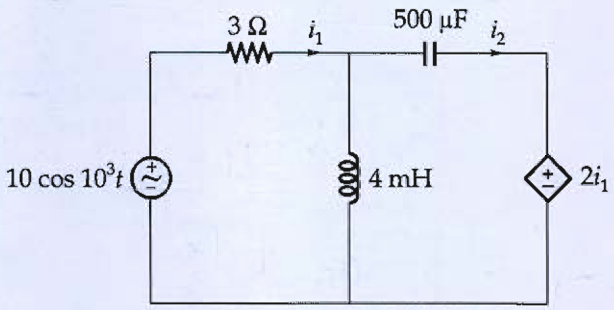
Q.2 (b) (i) For the network shown in figure below, the switch is closed for a long time and at $t = 0$, the switch is opened.




Determine the voltage across inductor for $t > 0$.

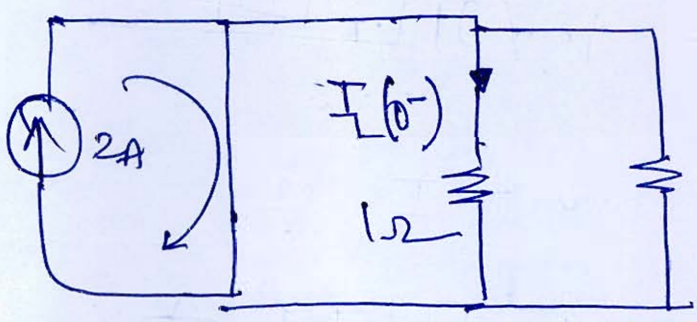
(ii) Obtain expressions for the time domain currents $i_1$ and $i_2$ in the circuit given as figure.




**[10 + 10 marks]**

(i) For a first order circuit,

At $t=0^-$ the current through inductor is,

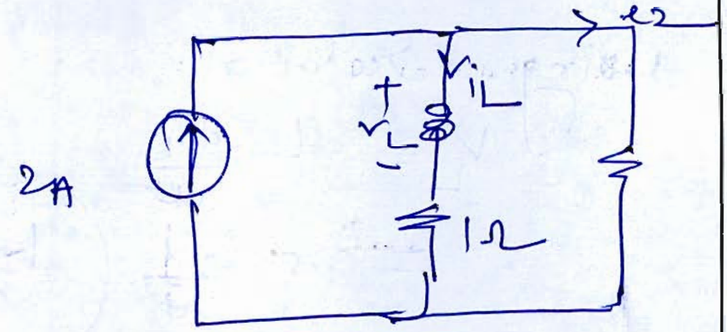



{L- Shorted}

$I_L(0^-) = 0$

$\Rightarrow I_L(0^+) = 0$ {L does not allow sudden change of current}

For $t>0$ :-




using KCL,

$$2 = i_L + i_2$$

$$\Rightarrow 2 = i_L + \frac{V_L + i_L}{1}$$

$$\Rightarrow 2 = 2i_L + 0.5\frac{di_L}{dt} \quad \left[V_L = L\frac{di_L}{dt}\right]$$

$$\Rightarrow 1 - 1 i_L = \frac{di_L}{dt}$$

$$\Rightarrow 1 - i_L = 0.25\frac{di_L}{dt}$$

$$\Rightarrow \int \frac{di_L}{i_L - 1} = -\frac{dt}{0.25}$$

$$\Rightarrow i_L = 1 + C_0 e^{-4t}$$

Using initial condition, $i_L(0) = 0$

$$\Rightarrow 1 + C_0 = 0 \quad \Rightarrow C_0 = -1$$

Thus $i_L(t) = 1 - e^{-4t}$ A

using relation,

$$V_L = L\frac{di_L}{dt}$$

$$= 0.5\frac{d}{dt}\left(1-e^{-4t}\right)$$

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$$= 2e^{-4t}\,u(t)\ V$$

(ii) For Sinusoidal ac analysis

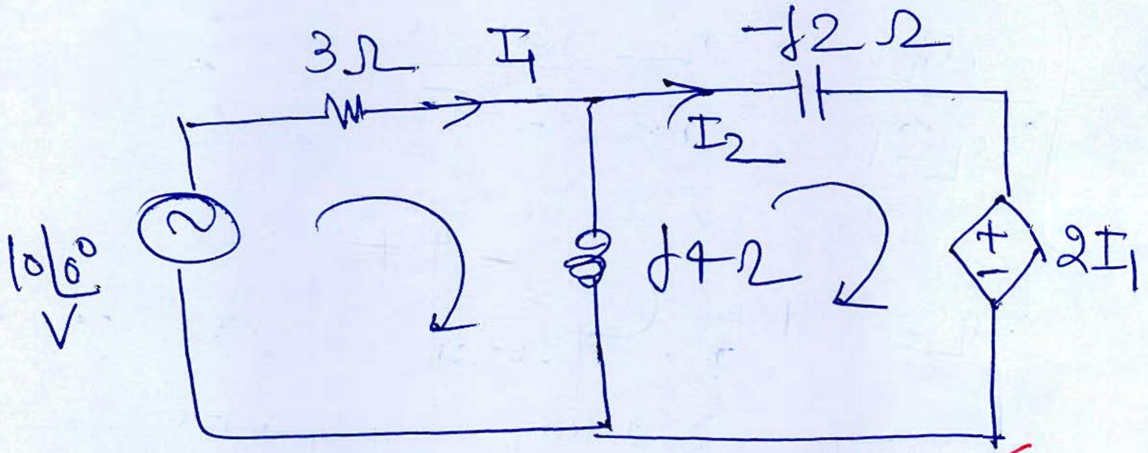
do not write here

$$10\cos 10^3 t\ V \iff 10\angle 0^\circ\ V$$

$$4\,mH \equiv j\,4\times10^{-3}\times10^{3} = j4\,\Omega$$

$$500\,\mu F \equiv \frac{1}{j10^3\times500\times10^{-6}} = -j2\,\Omega$$

The circuit in phasor domain is,




KVL in mesh 1 gives,

$$(3+j4)I_1 - j4I_2 = 10\angle0^\circ \quad \text{—— (1)}$$

Do not write in this margin

writing KVL in mesh 2,

$-j2\, I_2 + 2I_1 + j4(I_2 - I_1) = 0$

$(2 - j4)\, I_1 + j2\, I_2 = 0$

$$I_1 = \left(\frac{-j2}{2-j4}\right) I_2 \quad \text{—— (2)}$$

Substituting into (1)

$$(3+j4)\left[\left(\frac{-j2}{2-j4}\right) I_2\right] - j4\, I_2 = 10\angle 0°$$

10

$$I_2 = \frac{10}{(3+j4)\left[\frac{-j2}{(2-j4)}\right] - j4} = 2.773\angle 56.31° \text{ A}$$

In time domain,

$i_2(t) = 2.773 \cos(10^3 t + 56.31°)$ A

from (2),

$$I_1 = \left(\frac{-j2}{2-j4}\right) \times 2.773\angle 56.31° = 1.24\angle 29.74° \text{ A}$$

In time domain,

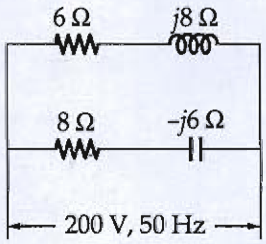
$i_1(t) = 1.24 \cos(10^3 t + 29.74°)$ A

**Q.2 (c)** For the circuit shown below, calculate,

**(i)** Total admittance, total conductance and total susceptance.

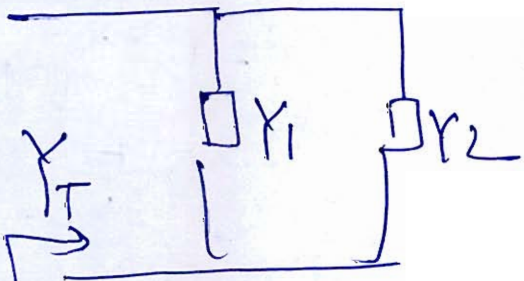
**(ii)** Total current and total power factor (pf).

**(iii)** The value of pure capacitance to be connected in parallel with the above combination to make the total power factor (pf) unity.




**[20 marks]**

(i) Total admittance,




$$Y_T = Y_1 + Y_2$$

$$= \frac{1}{8-j6} + \frac{1}{6+j8}$$

$$= 0.08 + j0.06 + 0.06 - j0.08$$

$$= (0.14 - j0.02)\ \mho$$

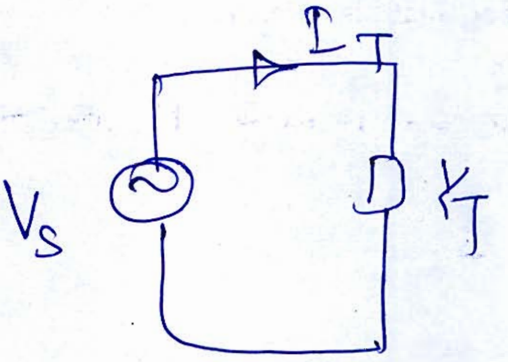
Conductance is the real part of $Y_T$.

Total Conductance $G_T = 0.14\ \mho$

Susceptance is the imaginary part of $Y_T$.

Total Susceptance $B_T = -j0.02\ \Omega$ (conductive)

(ii)



$V_S = 200 \angle 0^\circ$ V (rms)

Total current, $I_T = V_S Y_T$

$= 200 \angle 0^\circ \times (0.14 - j0.02)$

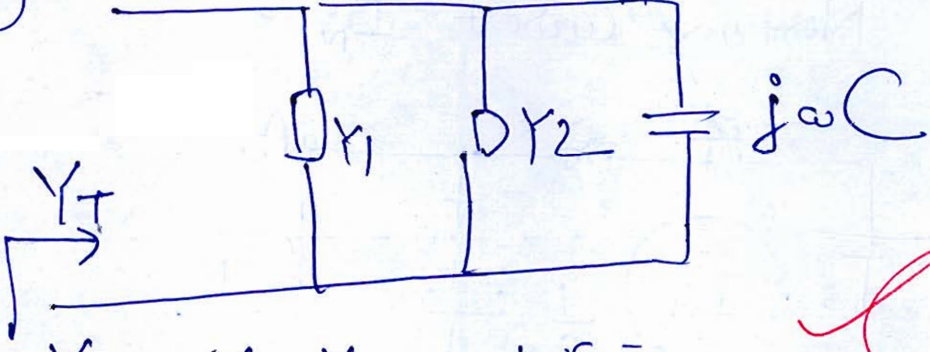
$= 28.28 \angle -8.13^\circ$ A

$I_T$ lags $V_S$ by $8.13^\circ$

Total power factor, $\cos\phi = \cos(8.13^\circ)$

$= 0.9899$ (lagging)

(iii)



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$Y_T = Y_1 + Y_2 + j\omega C =$

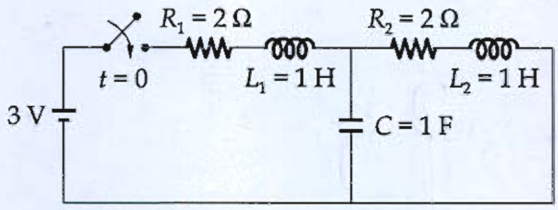
$= 0.14 - j0.02 + j\omega C$

For upf, $Im[Y_T] = 0$

$\Rightarrow (\omega C - 0.02) = 0$

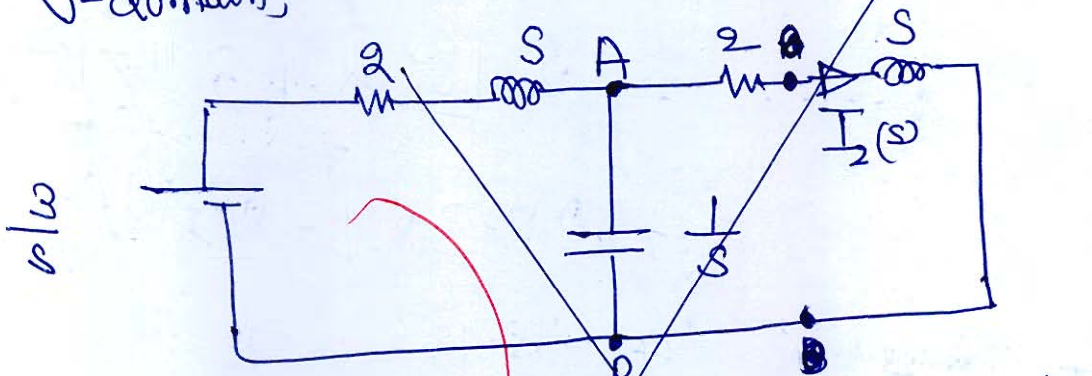
$C = \frac{0.02}{2\pi f} = \frac{0.02}{100\pi} = 63.66$ μF

**Q.3 (a)** In the network shown in figure the switch is closed at time $t = 0$. Assuming all the initial currents and voltages as zero, find the current through the inductor $L_2$ by the use of Norton's theorem.



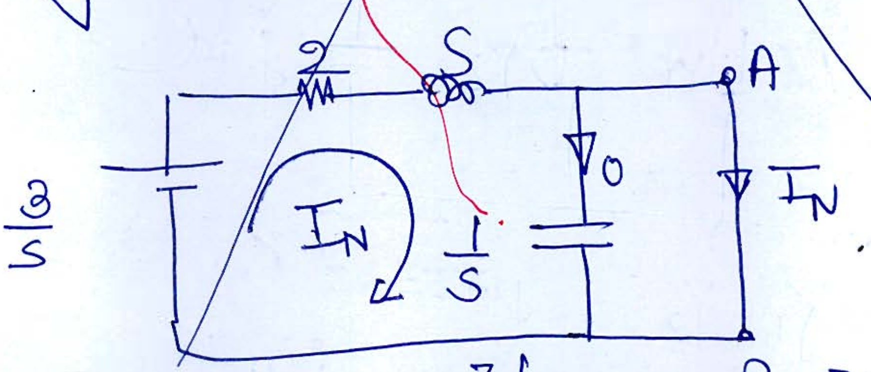

**[20 marks]**

Drawing the equivalent circuit in S-domain,



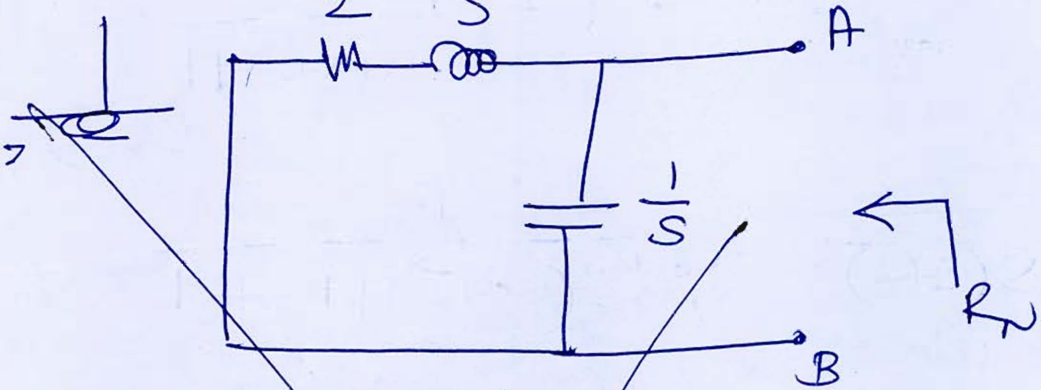

The impedances of L and C are $sL$ and $\frac{1}{Cs}$ respectively.

Finding Norton's current $I_N$ :-



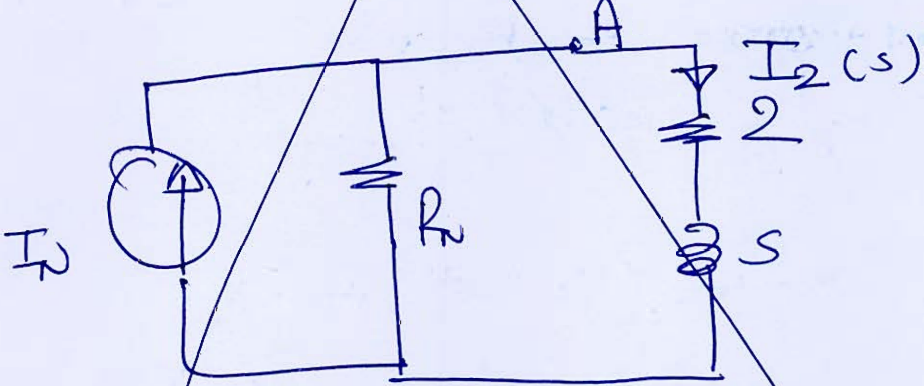

we get, $I_N = \dfrac{3/s}{s+2} = \dfrac{3}{s(s+2)}$

Finding $R_N$ :—




$$R_N = \frac{(S+2)\cdot\frac{1}{S}}{(S+2)+\frac{1}{S}} = \frac{S+2}{S(S+2)+1}$$

The Norton's circuit across A–B




using current division rule,

$$I_2(s) = I_N \times \frac{R_N}{S+2+R_N}$$

$$= \frac{3}{s(S+2)} \times \frac{\frac{S+2}{S(S+2)+1}}{(S+2)+\frac{(S+2)}{s(S+2)+1}}$$

Do not write in this margin

$$= \frac{3}{s(s+2)} \times \frac{\frac{1}{s(s+2)+1}}{1 + \frac{1}{s(s+2)+1}}$$

$$= \frac{3}{s(s+2)} \times \frac{1}{s(s+2)+1+1}$$

$$= \frac{3}{s(s+2)} \times \frac{1}{\quad}$$

**(b)** Show that the resonant frequency $\omega_0$ of a series *R-L-C* circuit is geometric mean of $\omega_1$ and $\omega_2$, i.e., the upper and lower half power frequencies respectively.

**[20 marks]**

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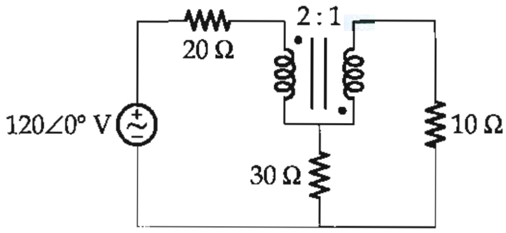
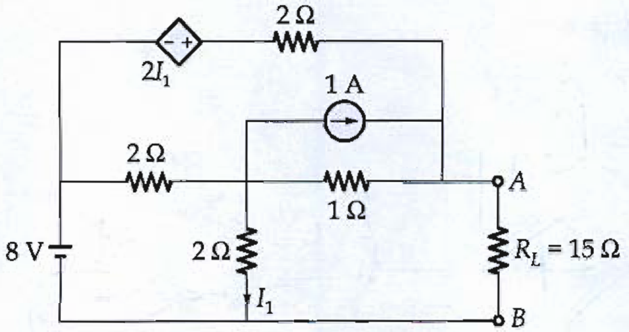
(c) Calculate the power supplied to the 10 Ω resistor in the ideal transformer circuit given in the figure below.



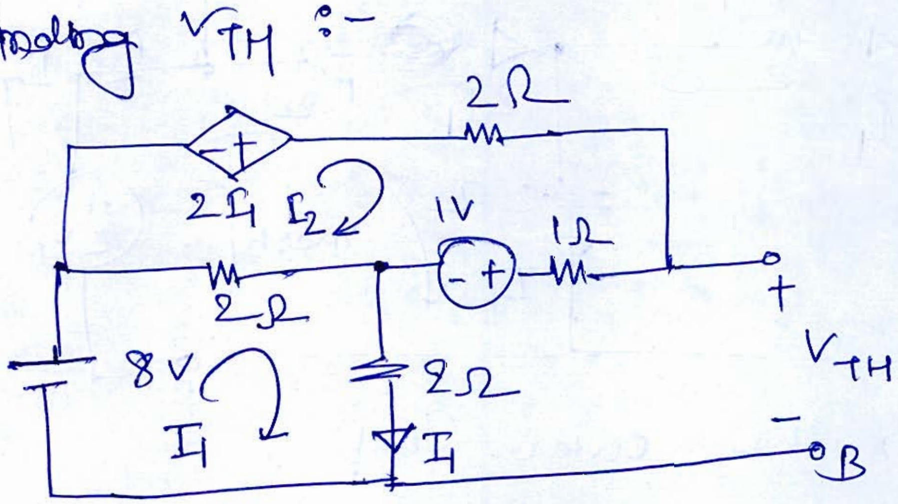

**[20 marks]**

**(a)** Determine the current through the load resistance $R_L = 15\ \Omega$ across the terminal $A$-$B$ of the circuit shown in figure below, using Thevenin's theorem. Also find the maximum power that can be transferred to the load resistance $R_L$.




**[20 marks]**

Finding $V_{TH}$ :-




KVL in mesh 1,

$$8 = 2I_1 + 2(I_1 - I_2)$$

$$\Rightarrow 2I_1 - I_2 = 4 \quad \text{——— (i)}$$

KVL in mesh 2,

$$(2+1)I_2 + 1 + 2(I_2 - I_1) - 2I_1 = 0$$

$$-4I_1 + 5I_2 = -1 \quad \text{——— (ii)}$$

Solving (i) and (ii)

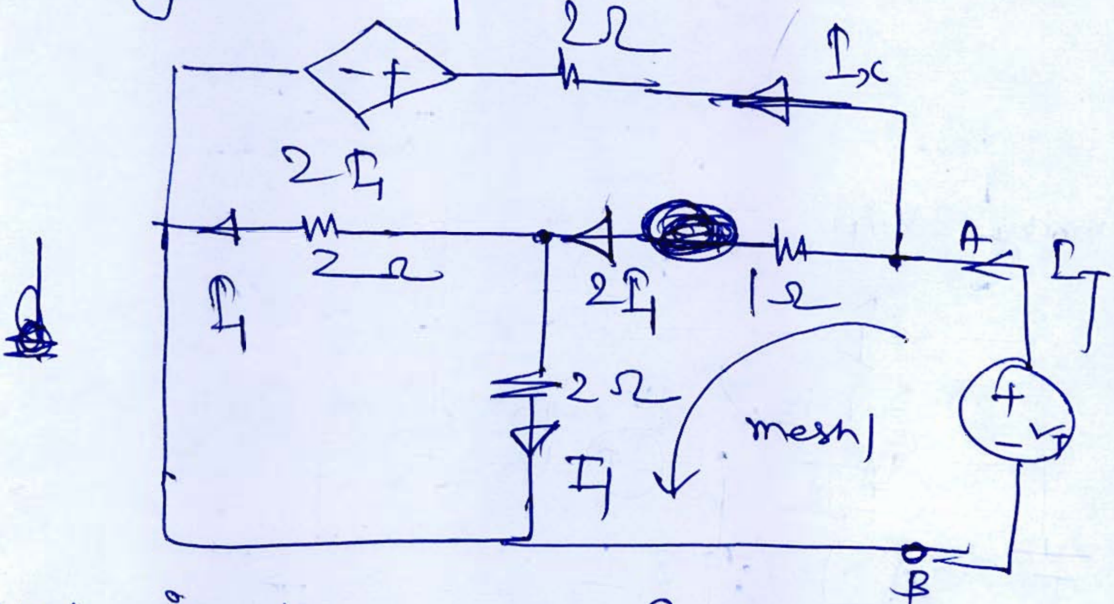
$$I_1 = 3.5A, \quad I_2 = 3A$$

$$V_{TH} = I_2 + 1 + 2I_1$$

$= 3+1+2\times 3.5$

$= 11\ V$

Finding $R_{TH}$




KVL in the outer loop,

$V_T = 2I_x + 2I_1$ —— (3)

KVL in mesh1,

$V_T = 2I_1 + 2I_1 = 4I_1$

$\Rightarrow I_1 = \frac{V_T}{4}$

and $I_T = I_x + 2I_1$

$I_x = I_T - 2I_1$

$= I_T - 2\times\frac{V_T}{4} = I_T - \frac{V_T}{2}$

From (3),

$$V_T = 2I_x + 2I_1$$

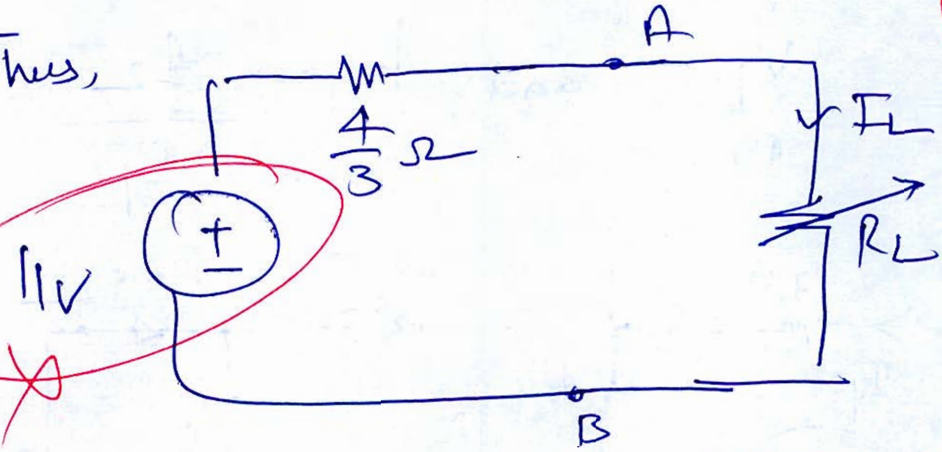
$$V_T = 2\left(I_T - \frac{V_T}{2}\right) + 2 \times \frac{V_T}{4}$$

$$\Rightarrow V_T = 2I_T - V_T + \frac{V_T}{2}$$

$$\frac{V_T}{I_T} = \frac{2}{2 - \frac{1}{2}} = \frac{4}{3}\ \Omega$$

6

Thus,

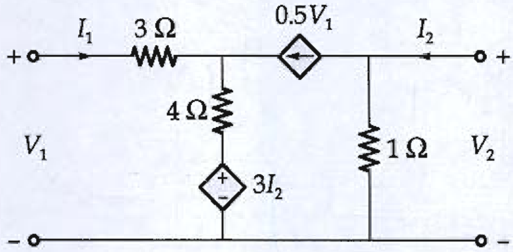



For $R_L = 15\ \Omega$, $I_L = \dfrac{11}{\frac{4}{3}+15} = 0.6734\ A$

For Maximum Power, $R_L = R_{TH}$

$$\boxed{\text{And } P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{11^2}{4 \times \frac{4}{3}} = 22.6875\ W}$$

**Q.4 (b)** Find the $h$-parameters for the two-port network shown



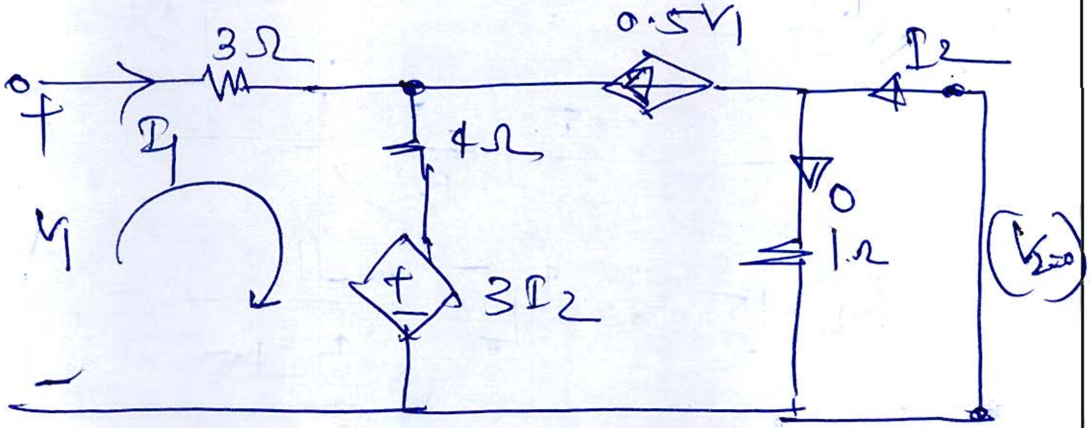

**[20 marks]**

We know,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Let $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1} \quad \text{and} \quad h_{21} = \frac{I_2}{I_1}$$




We have,

$$I_2 = 0.5V_1 \quad \text{———} \quad (1)$$

using KVL

$$V_1 = 3I_1 + 4(I_1 + 0.5V_1) + 3I_2$$

$\Rightarrow$

$$\Rightarrow V_1 = 7I_1 + 2V_1 + 3I_2$$

$$\Rightarrow 7I_1 + V_1 + 3I_2 = 0$$

$$\Rightarrow 7I_1 + V_1 + 3 \times 0.5 V_1 = 0$$

$$\Rightarrow 7I_1 + 2.5 V_1 = 0$$

$$h_{11} = \frac{V_1}{I_1} = -\frac{7}{2.5} = -2.8\ \Omega$$

Similarly,

$$h_{21} = \frac{I_2}{I_1}$$

$$= 0.5\left(\frac{V_1}{I_1}\right)$$

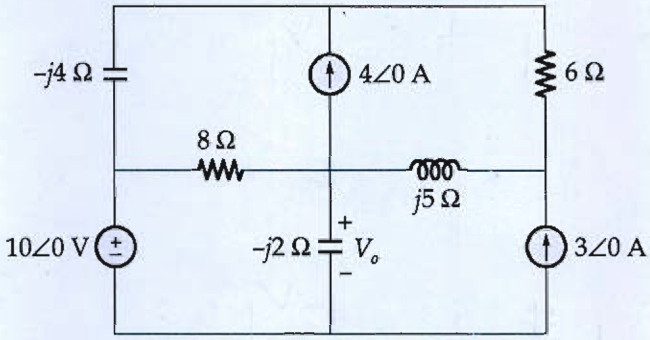
$$= 0.5 \times -2.8$$

$$= -1.4$$

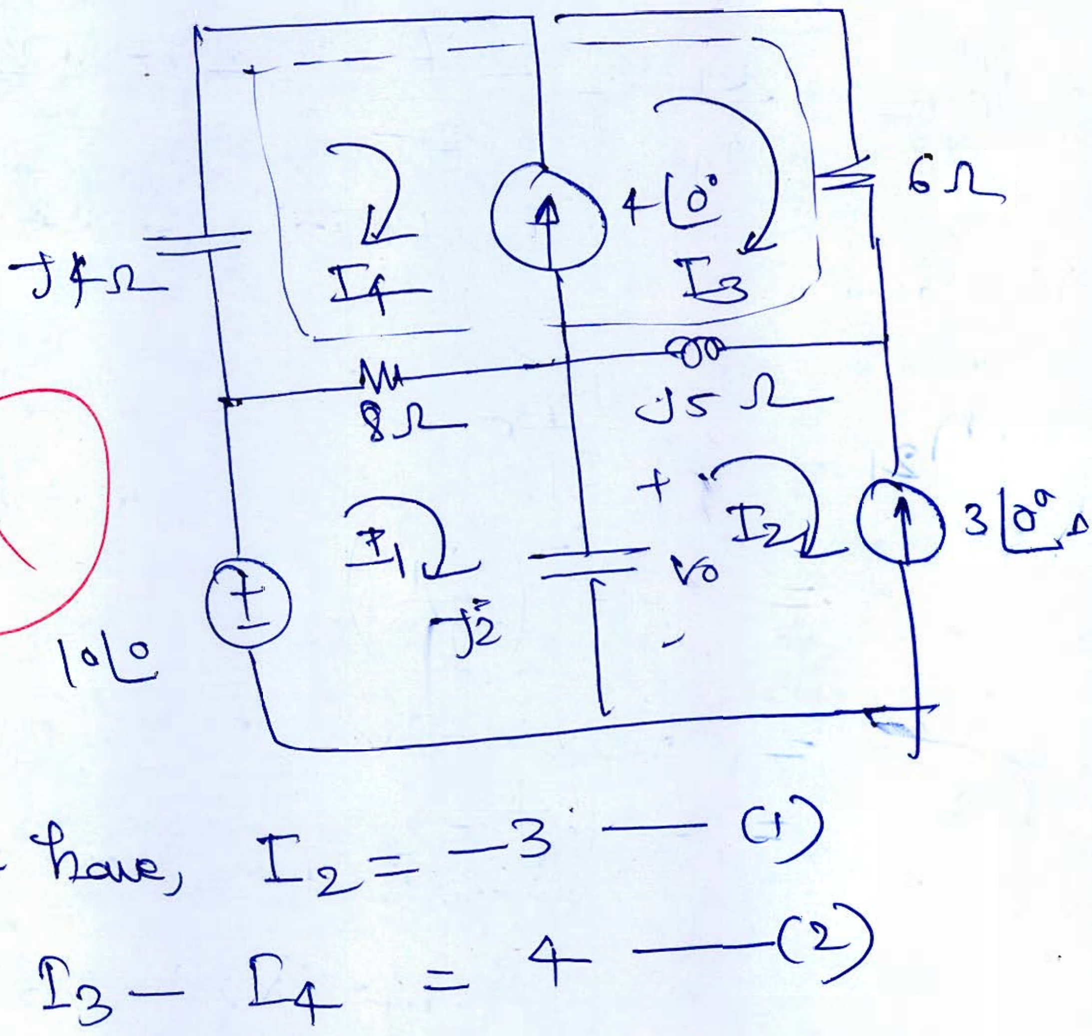
8

$h_{12}$ and $h_{22}$

**Q.4 (c)** Solve for $V_o$ in the circuit of figure using mesh analysis.




**[20 marks]**



1

we have, $I_2 = -3$ —— (1)

$I_3 - I_4 = 4$ —— (2)

KVL in mesh 1,

$(8-j2) I_1 - 8 I_4 = 10$ —— (3)

$-j2(-I_2)$

KVL in ~~supermesh gives~~,

$$-j4\, I_4 + 6\, I_3 + j5\,(I_3 - I_2) + 8\,(I_4 - I_1) = 0$$

$$-8\, I_1 - j5\, I_2 + (6+j5)\, I_3 + (8-j4)\, I_4 = 0$$

$$-8\, I_1 + j15 + (6+j5)\, I_3 + (8-j4)\, I_4 = 0 \quad -(4)$$

from (3) $I_4 = \dfrac{(8-j2)\, I_1 - 10}{8}$

$$= (1 - j0.25)\, I_1 - 1.25$$

from (2) $I_3 = I_4 + 4$

$$= (1 - j0.25)\, I_1 + 2.75$$

Substituting in (4)

$$-8\, I_1 + j15 + (6+j5)\left[(1-j0.25)\, I_1 + 2.75\right] + (8-j4)\left[(1-j0.25)\, I_1 - 1.25\right] = 0$$

$$\Rightarrow I_1 = \frac{-2.75\,(6+j5) + 1.25\,(8-j4) - j15}{-8 + (6+j5)(1-j0.25) + (8-j4)(1-j0.25)}$$

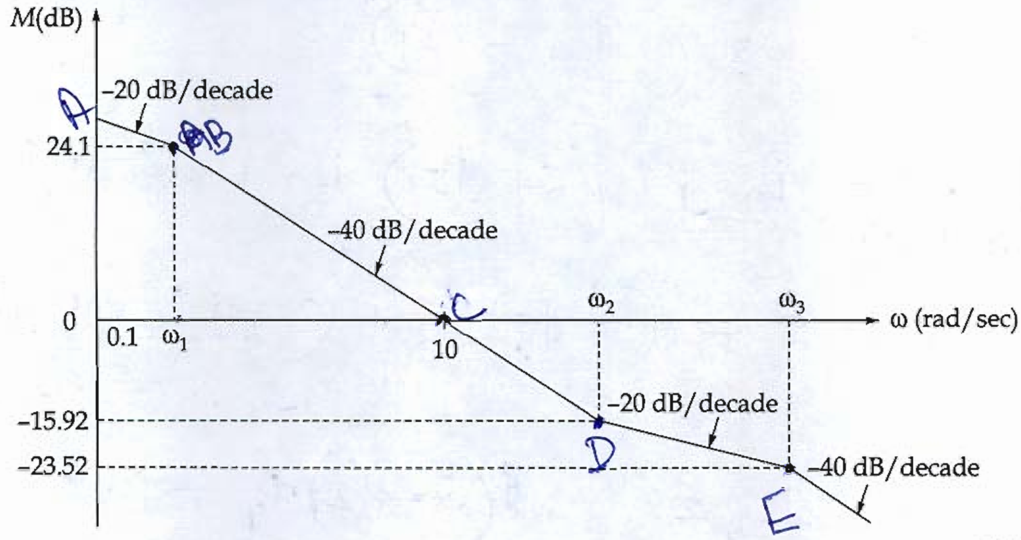
$$= 5.10\angle -79^\circ \text{ A}$$

Thus $V_o = -j2\,(I_1 - I_2) = -j2\,(5.10\angle -79^\circ - (-3))$

$$= 12.784\angle -141.65^\circ$$

## Section B : Control System

**Q.5 (a)** Obtain the open loop transfer function for a unity negative feedback system whose bode magnitude plot is shown below:




**[12 marks]**

observing the slope,

Poles :— $s = 0$, $s = -\omega_1$, $s = -\omega_3$

Zeros :— $s = -\omega_2$

Thus, $G(s) = \dfrac{K\left(\dfrac{s}{\omega_2}+1\right)}{s\left(\dfrac{s}{\omega_1}+1\right)\left(\dfrac{s}{\omega_3}+1\right)}$

~~Let $\omega < \omega_1$ :—~~

~~$M = 20 \log \left(\frac{K}{\omega}\right)$~~

~~$24.1 = 20 \log \left(\frac{K}{0.1}\right)$~~

~~$K = 1.6$~~

For section BC, $-40 = \dfrac{24.1 - 0}{\log(\omega_1) - \log 10}$

$\Rightarrow \omega_1 = 2.5 \ \dfrac{rad}{sec}$

for section CD,

$-40 = \dfrac{-15.92 - 0}{\log \omega_2 - \log 10}$

$\omega_2 = 25 \ \dfrac{rad}{sec}$

for section DE

$-20 = \dfrac{-23.52 + 15.92}{\log \omega_3 - \log 25}$

$\omega_3 = 60 \ \dfrac{rad}{sec}$

for section AB,

$M = 20 \log K - 20 \log \omega$

$\Rightarrow 24.1 = 20 \log K - 20 \log 2.5$

$K = 40$

Thus $G(S) = \dfrac{40\left(\dfrac{S}{25} + 1\right)}{S\left(\dfrac{S}{2.5} + 1\right)\left(\dfrac{S}{60} + 1\right)}$

10

$= \dfrac{240\,(S+25)}{S\,(S+2.5)\,(S+60)}$

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**Q.5 (b)** A servo mechanism is represented by the equation :

$$\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144E$$

where $E = C - 0.5y$ is the actuating signal. Find the value of damping ratio, damped and undamped frequency of oscillation. Draw the block diagram of the system described by the above equation.

**[12 marks]**

We have, $\frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} = 144(C - 0.5y)$

$\Rightarrow \frac{d^2y}{dt^2} + 4.8\frac{dy}{dt} + 72y = 144C$

Taking laplace

$(s^2 + 4.8s + 72)Y(s) = 144C(s)$

$\frac{C(s)}{Y(s)} = \frac{144}{s^2 + 4.8s + 72}$

7

Comparing,

$s^2 + 4.8s + 72 = s^2 + 2\xi\omega_n s + \omega_n^2$

$\omega_n = \sqrt{72} = 8.485$ rad/sec

$\xi = \frac{4.8}{2\times\sqrt{72}} = 0.2828$ rad/sec

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 8.485\times\sqrt{1-0.2828^2} = 8.1386$ rad/sec

Block diagram:—

$y \longrightarrow$

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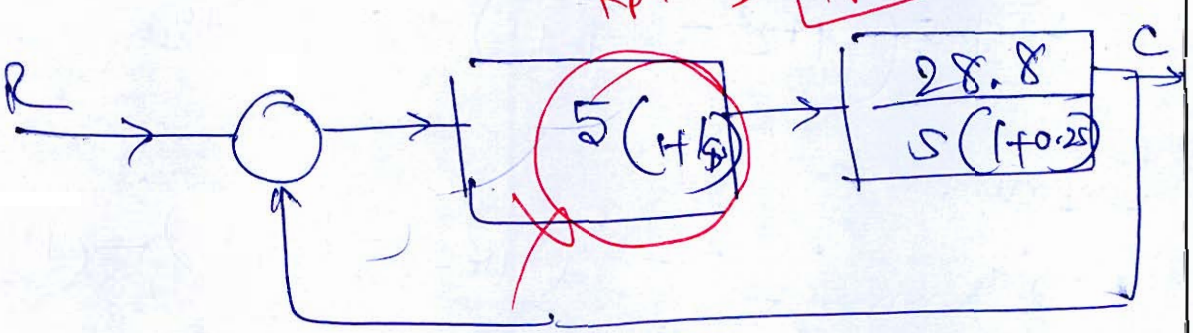
(c) *Closed loop system with unity feedback has the forward loop transfer function as :*

$$G(s) = \frac{28.8}{s(1 + 0.2s)}$$

Modify the design using cascaded compensation to satisfy the optimum performance criterion, so that the transient response to unit step input reaches its final steady state value in minimum time without having any overshoot. Take gain of proportional controller equal to 5.

**[12 marks]**

$K_p + sK_D$ $\boxed{K_p = 5}$




$$\frac{C(s)}{R(s)} = \frac{\dfrac{28.8}{s(1+0.2s)} \times 5(1+K_D s)}{1 + \dfrac{28.8}{s(1+0.2s)} \times 5(1+K_D s)}$$

$$= \frac{1}{s(1+0.2s) + 28.8\times5(1+K_D s)}$$

$$= \frac{1}{0.2s^2 + (142.5K_D + 1)s + 142.5}$$

The response reaches its final steady state value in minimum time without overshoot of the

System is underdamped.

$(\xi = 1)$

$q(s)$ :—

$0.2\ s^2 + (1+2.5\ K_D+1)\ s + 1+2.5$

$s^2 + 5(1+2.5\ K_D+1)\ s + 7\cdot 12.5$

Comparing,

$2\xi\omega_n = 5\ (1+2.5\ K_D+1)$ — (1)

$\omega_n^2 = 7\cdot 12.5$

$\xi = 1$

1

From (1)

$2 \times 1 \times \sqrt{7\cdot 12.5} = 5\ (1+2.5\ K_D+1)$

$\Rightarrow K_D = 0.0679$

Thus controller transfer function is,

$G_c(s) = 5\ (1 + K_D s)$

$= 5\ (1 + 0.0679\ S)$

[PD controller]

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.5 (d) A unity negative feedback system has open loop transfer fucntion, $G(s) = \frac{K}{s(1+sT)}$, where $K$ and $T$ are positive constants. Determine the factor by which the amplifier gain $K$ be reduced so that peak overshoot of the unit step response is reduced from 80% to 50%?

**[12 marks]**

Peak overshoot is a function of $\xi$ only.

The closed loop transfer function,

$$T(s) = \frac{G}{1+G} = \frac{\frac{K}{s(1+sT)}}{1+\frac{K}{s(1+sT)}} = \frac{K}{s(1+sT)+K}$$

$$= \frac{K}{Ts^2+s+K}$$

$$= \frac{K/T}{s^2+\frac{1}{T}s+K/T} \equiv \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n = \sqrt{\frac{K}{T}}$ and $\xi = \frac{1}{2}\sqrt{\frac{T}{K}}$

$$\Rightarrow K = \frac{T}{4\xi^2}$$

Since, $M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$

$$\xi = \sqrt{\frac{(\ln M_p)^2}{(\ln M_p)^2+\pi^2}}$$

for, $M_p = 0.80$,

$$\xi_1 = \sqrt{\frac{(\ln 0.8)^2}{(\ln 0.8)^2 + \pi^2}} = 0.0708$$

for $M_{p1}$, $\xi = \xi_1$ and $K = K_1$

for $M_p = 0.50$,

$$\xi_2 = \sqrt{\frac{(\ln 0.50)^2}{(\ln 0.50)^2 + \pi^2}} = 0.2154$$

for $M_{p2}$, $\xi = \xi_2$ & $K = K_2$

~~The~~ since $K = \frac{I}{4\xi^2}$

$$\frac{K_2}{K_1} = \frac{\xi_1^2}{\xi_2^2} = \frac{0.0708^2}{0.2154^2}$$

$$= 0.108$$

The factor by which K should be reduced is $\left(\frac{1}{0.108} = 9.259\right)$

10

.5 (e) The open loop transfer function of a unity negative feedback system is given as, $G(s) = \dfrac{K}{2s(1+0.1s)(1+s)}$. Determine the value of 'K' for which the gain margin of the system is 14 dB.

**[12 marks]**

$$GM = 20 \log \left[ \frac{1}{|G(j\omega_{pc})|} \right]$$

$$\Rightarrow 14 = 20 \log \left[ \frac{1}{|G(j\omega_{pc})|} \right]$$

$$|G(j\omega_{pc})| = 0.2$$

where $\omega_{pc}$ is phase crossover frequency. $\rightarrow$ at $\omega_{pc}$, $\phi = \angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{pc}} = -180^\circ$

$$\phi = -90^\circ - \tan^{-1}(0.1\omega) - \tan^{-1}\omega$$

$$\Rightarrow -180^\circ = -90^\circ - \tan^{-1}(0.1\omega_{pc}) - \tan^{-1}(\omega_{pc})$$

$$\Rightarrow \tan^{-1}(0.1\omega_{pc}) + \tan^{-1}\omega_{pc} = 90^\circ$$

Taking tan

$$\Rightarrow \frac{0.1\omega_{pc} + \omega_{pc}}{1 - 0.1\omega_{pc} \times \omega_{pc}} = \infty$$

$$\Rightarrow \omega_{pc} = \sqrt{10}$$

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$$\left| G(j\omega_{pc}) \right| = \frac{K}{2\,\omega_{pc}\sqrt{(0.1\omega_{pc})^2+1}\,\sqrt{\omega_{pc}^2+1}}$$

$$\Rightarrow 0.9 = \frac{K}{2\times\sqrt{10}\times\sqrt{0.01\times10+1}\,\sqrt{10+1}}$$

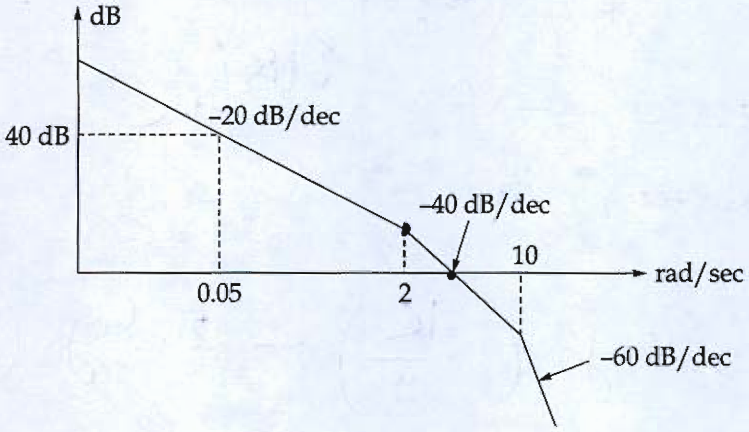
$$\Rightarrow 0.9 = \frac{K}{2\sqrt{10}\,\sqrt{1.1}\,\sqrt{11}}$$

$$K = 19.9$$

9

Write Steps

Q.6 (a) The open loop transfer function of a unity feedback system is given by $G(s)H(s) = e^{-Ts}G_1(s)$, where $G_1(s)$ is minimum phase system. The approximate bode magnitude plot of the open loop transfer function is shown in the figure below. If the phase margin of the system is –18.19°, determine the transportation lag $T$.




[20 marks]

Finding $G_1(s)$ :-

$e^{-Ts}$ has no effect on Magnitude plot.

We have three poles at

$s = 0$, $s = -2$ and $s = -10$

Thus

$$G_1(s) = \frac{K}{s\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}$$

for $\omega < 2$ rad/sec

$$M_{dB} = 20\log\left(\frac{K}{\omega}\right)$$

$$\Rightarrow 40 = 20\log\left(\frac{K}{0.05}\right) \quad [\text{from graph}]$$

$$K = 0.05 \times 10^2 = 5$$

For $2 < \omega < 10$ rad/sec

6

~~$M_{dB} = 20 \log \left(\frac{K}{\omega}\right)$~~

Slope $= -40$ dB/dec

The equation is,

$$M = 20 \log\left(\frac{K}{\omega}\right) - 20 \log\left(\frac{\omega}{2}\right)$$

At $\omega = \omega_{gc}$, $M = 0$

$$0 = 20 \log\left(\frac{5}{\omega_{gc}}\right) - 20 \log\left(\frac{\omega_{gc}}{2}\right)$$

$$\Rightarrow 0 = \log\left(\frac{5}{\omega_{gc}} \times \frac{2}{\omega_{gc}}\right)$$

$$\Rightarrow 0 = \log\left(\frac{10}{\omega_{gc}^2}\right)$$

$$\omega_{gc} = \sqrt{10} \text{ rad/sec}$$

The phase of the system is,

$$GH = e^{-Ts} G_1(s)$$

$$G(j\omega)H(j\omega) = e^{-j\omega T} \frac{5}{(j\omega)\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{10}+1\right)}$$

The phase $\phi(\omega) = \left[ 90^\circ - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) \right.$

Phase Margin,

$$PM = 180^\circ + \phi(\omega_{gc})$$

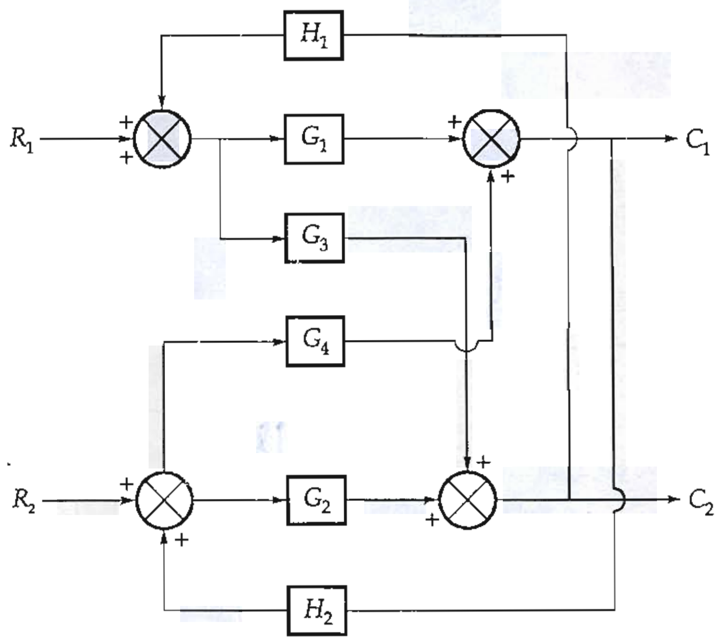
$$\Rightarrow -18.19^\circ = 180^\circ - 90^\circ - \frac{180}{\pi}(\omega_{gc}T) - \tan^{-1}\left(\frac{\omega_{gc}}{2}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{10}\right)$$

$$\Rightarrow -18.19^\circ = 90^\circ - (\sqrt{10}\,T)\frac{180}{\pi} - \tan^{-1}\left(\frac{\sqrt{10}}{2}\right) - \tan^{-1}\left(\frac{\sqrt{10}}{10}\right)$$

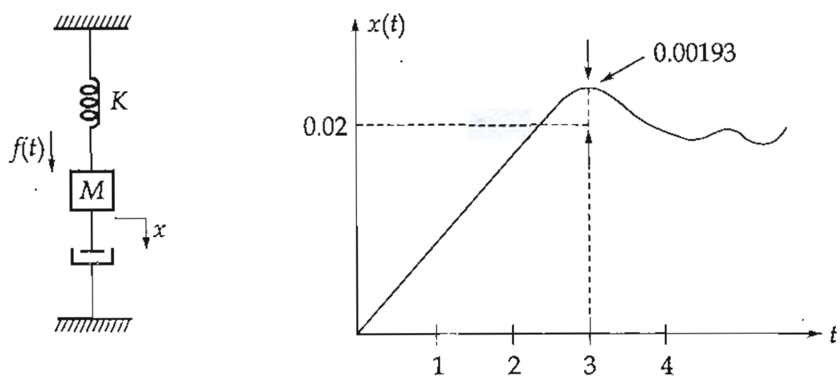
$$T = \frac{90^\circ + 18.19^\circ - \tan^{-1}\left(\frac{\sqrt{10}}{2}\right) - \tan^{-1}\left(\frac{\sqrt{10}}{10}\right)}{(\sqrt{10}) \times \frac{180}{\pi}}$$

$$= 0.1818 \text{ Sec}$$

**Q.6 (b)** (i) Evaluate $\frac{C_2}{R_1}$ for the system whose block diagram representation is shown in figure below. (Use block diagram reduction technique to solve).

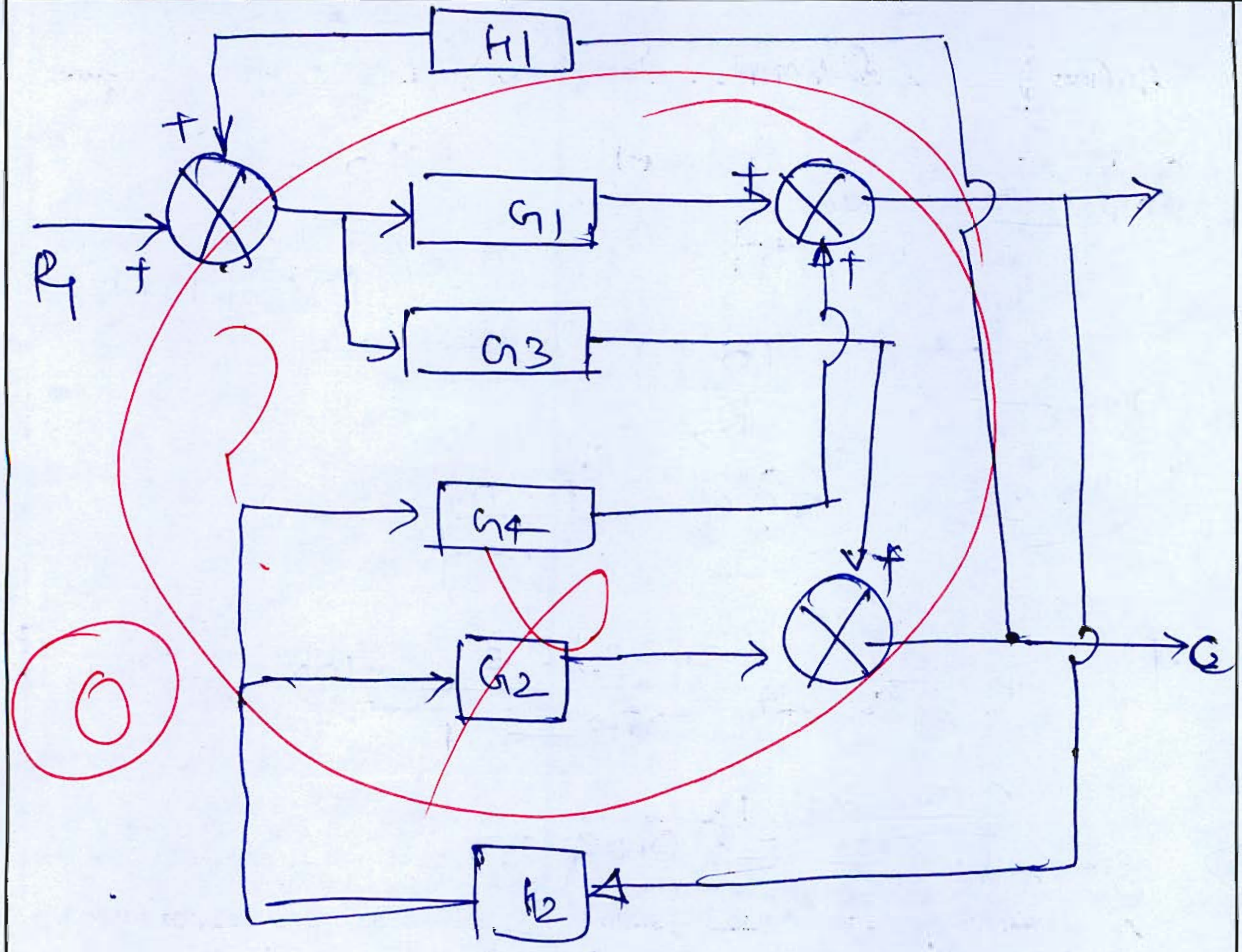



(ii) Figure below shows a mechanical system and the response when 10 N of force is applied to the system. Determine the values of M, F, K. The dimension '$x$' is in meter.




**[10 + 10 marks]**

Let $R_2 = 0$ :—




i) The differential equation is,

$$f(t) = M\frac{d^2x}{dt^2} + F\frac{dx}{dt} + Kx$$

Thus $$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Fs + K}$$

$$\left\{ \omega_n^2 = \frac{K}{M},\ 2\xi\omega_n = \frac{F}{M} \right\}$$

Let $F(s) = \dfrac{10}{s}$

$$X(s) = \frac{10}{s(Ms^2 + Fs + K)}$$

$$x(\infty) = \lim_{s\to 0} s X(s)$$

$$0.02 = \lim_{s\to 0} s \frac{10}{s(Ms^2+Fs+K)}$$

$$0.02 = \frac{10}{K}$$

$$K = 500 \ \frac{Nm}{rad}$$

$$\% M_p = \frac{0.00193}{0.02} \times 100$$

3

$$\Rightarrow e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.0965$$

$$\xi = \sqrt{\frac{(\ln 0.0965)^2}{\pi^2 + (\ln 0.0965)^2}} = 0.6$$

$$t_p = \frac{\pi}{\omega_d} = 3 \Rightarrow \omega_d = 0.954 \ \frac{rad}{sec}$$

$\omega_d = \pi/3$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{0.954}{\sqrt{1-0.6^2}} = 1.1925 \ \frac{rad}{sec}$$

$$\Rightarrow \sqrt{\frac{K}{M}} = 1.1925$$

$$M = \frac{K}{1.1925^2} = \frac{500}{1.1925^2} = 351.6 \ Kg$$

$$F = 2\xi\omega_n M = 2 \times 0.6 \times 1.1925 \times 351.6$$

$$= 503.1396 \ \frac{N}{rad/sec}$$

Do not write in this margin

(i)

The differential equation is,

$$f(t) = M\frac{d^2x}{dt^2} + F\frac{dx}{dt} + Kx$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Fs + K}$$

$$= \frac{1/M}{s^2 + F/M\, s + K/M}$$

From the response,

**Q.6 (c)** Derive the expression for the transfer function of an ac servomotor and obtain the same in respect of a servomotor having following data :

(i) Starting torque = 0.166 N-m

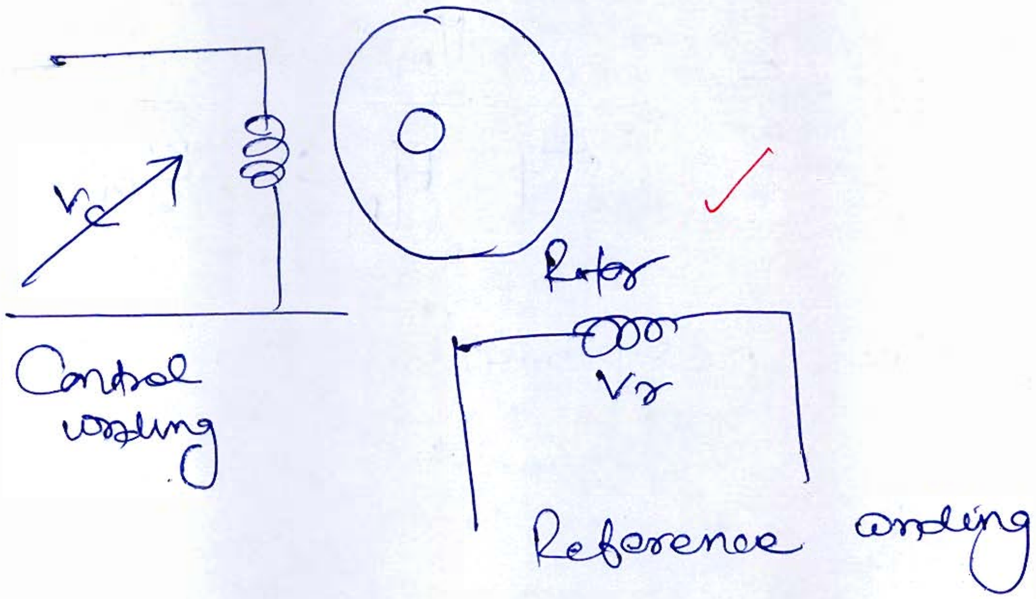
(ii) Moment of inertia, $J = 1 \times 10^{-5}$ $kgm^2$

(iii) Supply voltage = 115 Volts

(iv) No load speed = 2904 rpm

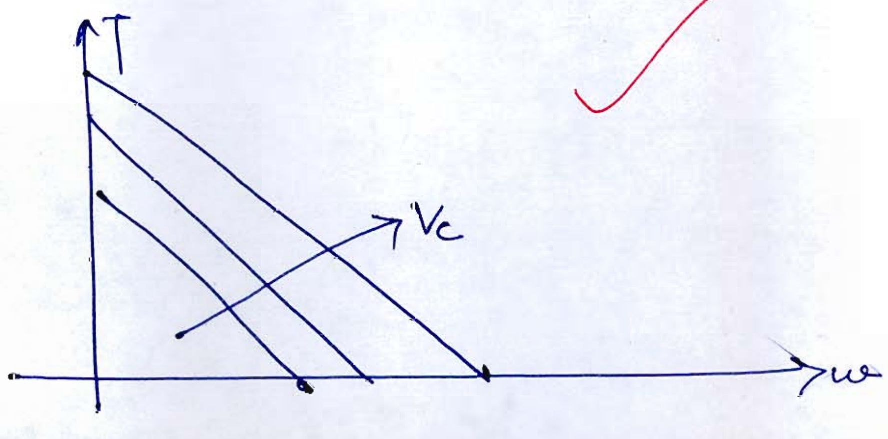
(Assume friction to be zero)

**[15 + 5 = 20 marks]**

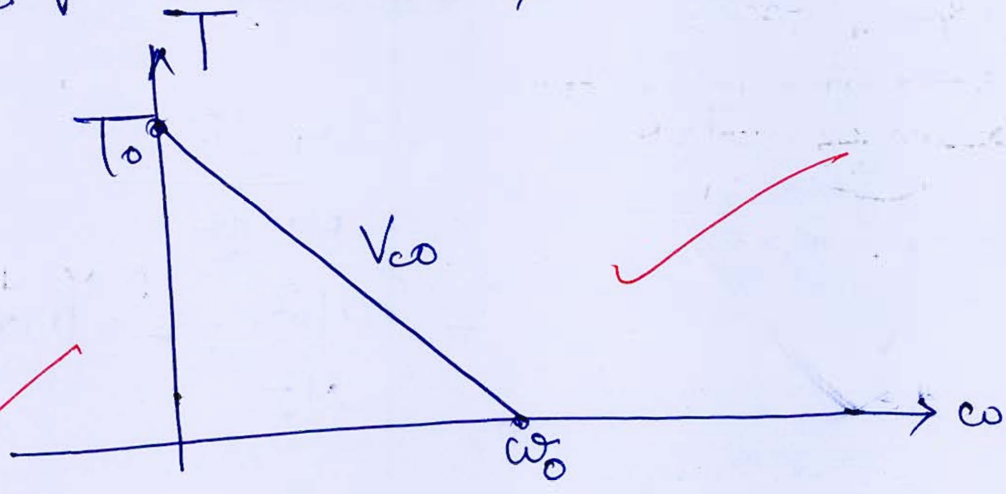



Ac Servomotor is a 2-phase induction motor. The control phase winding is variable.

The torque-speed curve is,



Let, for $V_c = V_{co}$,




$T_0 \propto V_c$, Let $T_0 = K V_{co}$

Also, let the slope is $-M$.

$$T = -M\omega + K V_c$$

$$T(s) = -Ms\,\theta(s) + K V_c(s) \quad \text{---(1)}$$

$$T(s) = (Js^2 + Bs)\,\theta(s) \quad \text{---(2)}$$

$$(Js^2 + Bs)\theta = -Ms\,\theta + K V_c$$

$$\frac{\theta(s)}{V_c(s)} = \frac{K}{Js^2 + (B+M)s}$$

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we have,

$$K = \frac{T_0}{V_C} = \frac{\text{Starting Torque}}{\text{voltage}}$$

$$= \frac{0.166}{115} = 1.4434 \times 10^{-3}$$

$$M = \frac{T_0}{\omega_s} = \frac{0.166}{2\pi \times \frac{2904}{60}} = 5.4586 \times 10^{-3}$$

$10^{-4}$

$$J = 10^{-5}$$

For $B=0$,

$$\frac{\theta(s)}{V_C(s)} = \frac{K}{Js^2 + Ms}$$

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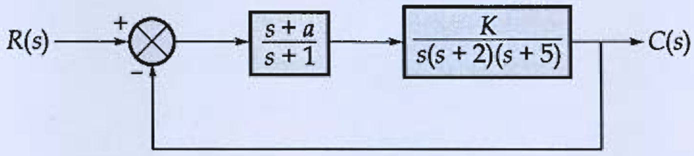
$$= \frac{1.4434 \times 10^{-3}}{10^{-5} s^2 + 5.4586 \times 10^{-3} s}$$

$$= \frac{144.34}{s(s + 545.86)}$$

**Q.7 (a)** **(i)** A position control system is shown in figure below :




$K$ and $a$ are the parameters of the system. Determine the range of $K$ and $a$ for which system is stable.

**(ii)** Sketch the root-locus of $G(s) = \dfrac{K(s+1)}{s^2(s+2)}$.

**[10 + 10 marks]**

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**Q.7 (b)** Sketch the polar plot of the transfer function given below. Determine whether the plot crosses the real axis. If so, determine the frequency at which the plot cross the real axis and the corresponding magnitude $|G(j\omega)|$.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

**[20 marks]**

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Q.7 (c) Construct the state model for a system characterised by the differential equation :

$$\frac{d^3y}{dt^3} + \frac{6d^2y}{dt^2} + \frac{11dy}{dt} + 6y = u$$

Give the block diagram representation of the state model.

**[15 + 5 = 20 marks]**

**Q.8 (a)** The open-loop transfer function of a unity feedback control system is given below:

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

Plot the root locus and determine the value of $K$ at the breakaway point.

**[20 marks]**

(b) The open loop transfer function of a feedback control system is

$$G(s)H(s) = \frac{K(1+2s)}{s(1+s)(1+s+s^2)}$$

Find the restriction on $K$ for stability. Find the value of $K$ for the system to have a gain margin of 3 dB. With this value of $K$, find the gain cross over frequency and phase margin. Use Nyquist Approach.

**[20 marks]**

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(c) The state space model of a second order system given below is designed using feedback control system.

$$\dot{x} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

**(i)** What are the conditions for the desired response? Also check whether desired response is possible or not.

**(ii)** Design an observer system such that the above system has settling time of 0.5 sec and damping frequency of 6 rad/sec.

**[8 + 12 marks]**

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Space for Rough Work

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