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ENGINEERING MATHEMATICS

CE | ME

Date of Test : 02/02/2026

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (c) | 13. (a) | 19. (b) | 25. (c) |
| 2. (d) | 8. (a) | 14. (a) | 20. (c) | 26. (d) |
| 3. (d) | 9. (a) | 15. (b) | 21. (b) | 27. (b) |
| 4. (a) | 10. (a) | 16. (c) | 22. (b) | 28. (c) |
| 5. (b) | 11. (d) | 17. (c) | 23. (c) | 29. (c) |
| 6. (d) | 12. (b) | 18. (d) | 24. (c) | 30. (c) |

DETAILED EXPLANATIONS

1. (b)

$$f(x) = \frac{1}{15-0} = \frac{1}{15}$$

$$P\{5 < x < 9\} = \int_5^9 \frac{1}{15} dx = \frac{4}{15}$$

2. (d)

$$f'(x) = 4x - 5$$

For minima/maxima,

$$f'(x) = 0$$

$$4x - 5 = 0$$

$$x = \frac{5}{4}$$

$$f''(x) = 4 > 0 \Rightarrow \text{Minima}$$

3. (d)

$$1 + \left(\frac{d^3 y}{dx^3} \right)^{7/5-5/7} = 0$$

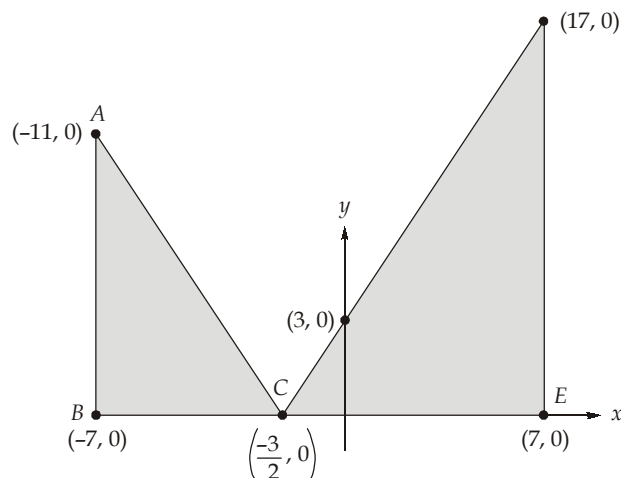
$$\left(\frac{d^3 y}{dx^3} \right)^{24/35} = -1$$

Raising power 35 on both sides.

$$\left(\frac{d^3 y}{dx^3} \right)^{24} = -1$$

From here degree of equation is 24.

4. (a)



The value of integral is equal to area of shaded region.

$$\text{Area} = \frac{1}{2} \times AB \times BC + \frac{1}{2} \times DE \times EC = \frac{1}{2} \times 11 \times \frac{11}{2} + \frac{1}{2} \times 17 \times \frac{17}{2} = \frac{410}{4} = 102.5$$

5. (b)

Let X be the number of rejections

$$n = 8$$

$$p = 0.16$$

$$q = 0.84$$

Probability of at least one rejection

$$= 1 - P(X \leq 0)$$

$$= 1 - P(X_0)$$

$$P(X_0) = {}^nC_r p^r q^{n-r} = {}^8C_0 (0.16)^0 (0.84)^8 = 0.2479$$

$$\text{Probability of at least one rejection} = 1 - 0.2479 = 0.7521$$

6. (d)

$$f_x(X) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$\begin{aligned} E(X^4) &= \int_{-\infty}^{\infty} x^4 \cdot f_x(X) dx \\ &= \int_0^1 x^4 = \frac{x^5}{5} \Big|_0^1 = \frac{1}{5} = 0.2 \end{aligned}$$

7. (c)

Eigen values of $(A + 7I)$ are $\gamma + 7$ and $\delta + 7$

$$\text{Eigen values of } (A + 7I)^{-1} = \frac{1}{\gamma + 7} \text{ and } \frac{1}{\delta + 7}$$

8. (a)

$$D^2 - 4D + 4 = 0$$

$$(D - 2)(D - 2) = 0$$

$$D = 2, 2$$

$$y = (C_1 + C_2 x)e^{2x}$$

$$y(0) = C_1 = 0 \Rightarrow C_1 = 0$$

$$y(1) = e^2 = C_2 \cdot e^2 \Rightarrow C_2 = 1$$

$$y = xe^{2x}$$

$$y(2) = 2e^4 = 109.196$$

9. (a)

$$\text{Probability of drawing a card} = \frac{1}{80}$$

$$E(x_i) = 1 \times \frac{1}{80} + 2 \times \frac{1}{80} + \dots + 80 \times \frac{1}{80} = \frac{1}{80} \times \frac{(80)(80+1)}{2} = \frac{81}{2}$$

Expected value of the sum of numbers on the ticket drawn:

$$E(x_1 + x_2 + x_3 + \dots) = E(x_1) + E(x_2) + \dots + E(x_{30})$$

$$30 E(x_i) = 30 \times \frac{81}{2} = 1215$$

10. (a)

Let $y = x^x$

Taking logarithm on both sides, we get

$$\log y = x \log x$$

Differentiating w.r.t. x , we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \frac{dy}{dx} = y(1 + \log x) = x^x (1 + \log x)$$

11. (d)

$$y = \frac{2 \ln x}{3x}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{2}{3x} \cdot \frac{1}{x} + \frac{2}{3} \ln x \left(\frac{-1}{x^2} \right) \\ &= \frac{2}{3x^2} (1 - \ln x) \end{aligned}$$

For maxima, $\frac{dy}{dx} = 0$

$$\ln x = 1 \Rightarrow x = e \text{ is a stationary point}$$

$$\frac{d^2y}{dx^2} = \frac{-2}{3x^3} (3 - 2 \ln x)$$

At $x = e$

$$\left(\frac{d^2y}{dx^2} \right)_{x=e} = \frac{-2}{3e^3} < 0$$

Hence maxima at $x = e$

12. (b)

$$y^2 = 2ax$$

$$x = \frac{y^2}{2a}$$

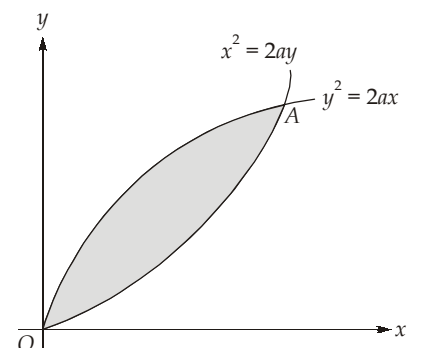
$$x^2 = 2ay$$

Using equation (i)

$$\begin{aligned} \frac{y^4}{4a^2} &= 2ay \\ y^4 - 8a^3y &= 0 \\ y &= 0, y = 2a \end{aligned}$$

The parabolas intersect at $O(0, 0)$ and $A(2a, 2a)$

$$\text{Required area} = \int_0^{2a} \int_{x^2/2a}^{\sqrt{2ax}} dy dx$$



...(i)

$$= \int_0^{2a} \left(\sqrt{2ax} - \frac{x^2}{2a} \right) dx = \left| \sqrt{2a} \frac{2}{3} x^{3/2} - \frac{1}{2a} \cdot \frac{x^3}{3} \right|_0^{2a}$$

$$= \sqrt{2a} \frac{2}{3} (2a)^{3/2} - \frac{1}{2a} \frac{(2a)^3}{3} = \frac{8a^2}{3} - \frac{4a^2}{3} = \frac{4a^2}{3}$$

13. (a)

Parabola given : $x^2 = 4y$... (i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

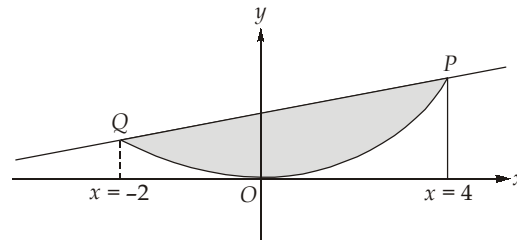
$$\Rightarrow x^2 = 2(x+4)$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x-4) + 2(x-4) = 0$$

$$\Rightarrow x = 4, -2$$



Required area = POQ

$$= \int_{-2}^4 y \, dx \text{ from straight line} - \int_{-2}^4 y \, dx \text{ from parabola}$$

$$= \int_{-2}^4 \left(\frac{x+4}{2} \right) - \int_{-2}^4 \frac{x^2}{4} \, dx$$

$$= \frac{1}{2} \left| \frac{x^2}{2} + 4x \right|_{-2}^4 - \frac{1}{4} \left| \frac{x^3}{3} \right|_{-2}^4$$

$$= \frac{1}{2} \{8 + 16 - (-6)\} - \frac{1}{12} (64 + 8) = \frac{1}{2} \times 30 - \frac{1}{12} \times 72 = 15 - 6 = 9$$

14. (a)

For $f(x)$ to be probability density function = $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\frac{1}{A} \int_3^6 (3x+5) dx = 1 \Rightarrow \frac{1}{A} \left| \frac{3x^2}{2} + 5x \right|_3^6 = 1$$

$$A = \left(\frac{3}{2} 6^2 - \frac{3}{2} 3^2 \right) + 5(6-3) = \frac{3}{2} 27 + 15 = 55.5$$

15. (b)

$$|A - \lambda I| = \begin{vmatrix} 2 - \lambda & 2 & 6 \\ 2 & 10 - \lambda & 2 \\ 6 & 2 & 2 - \lambda \end{vmatrix} = 0$$

$$\lambda^3 - 14\lambda^2 + 288 = 0$$

From here

$$p + q + r = 14$$

$$pq + qr + rp = 0$$

$$pqr = -288$$

$$pq + qr + rp - pqr = 0 - (-288) \\ = 288$$

16. (c)

$$\begin{aligned} \int_0^5 f(x) \cdot dx &= \frac{h}{2} [(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)] \\ &= \frac{1}{2} [(2 + 0.077) + 2(1 + 0.4 + 0.2 + 0.1176)] \\ &\simeq 2.75 \end{aligned}$$

17. (c)

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$

$$|A| = -8 \times 5 = -40$$

$$|A| (A^{-1}) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-40}{-8}, \frac{-40}{5} = 5, -8$$

18. (d)

$$x = (N)^{1/2N}$$

$$x^{2N} = N$$

$$x^{2N} - N = 0$$

$$f(x) = x^{2N} - N$$

$$f'(x) = 2N \cdot x^{2N-1}$$

According to Newton Raphson method

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^{2N} - N}{2Nx_k^{2N-1}} = \frac{2Nx_k^{2N} - x_k^{2N} + N}{2Nx_k^{2N-1}}$$

$$x_{k+1} = \left(\frac{2N-1}{2N} \right) x_k + \frac{x_k^{1-2N}}{2}$$

19. (b)

$$y^2 = 9x$$

$$2y \frac{dy}{dx} = 9$$

$$\frac{dy}{dx} = \frac{9}{2y} = \text{slope of tangent } (m_1)$$

$$\text{Slope of normal } (m_2) = \frac{-2y}{9} \quad [as \ m_1 m_2 = -1]$$

$$\text{Slope of the given line is } \frac{-2}{3}$$

$$\frac{-2y}{9} = -\frac{2}{3}$$

$$y = 3$$

For $y = 3$

$$3^2 = 9x$$

$$x = 1$$

For $(1, 3)$ to lie on the given line

$$\lambda = 2x + 3y = 2 + 9 = 11$$

20. (c)

$$\text{Probability of hitting target} = \frac{1}{6}$$

$$\text{Probability of missing target} = \frac{5}{6}$$

Probability of hitting target as least once = $1 - (\text{Probability of not hitting } n \text{ times})$

$$= 1 - {}^nC_0 p^0 q^n = 1 - \left(\frac{5}{6}\right)^n$$

$$1 - \left(\frac{5}{6}\right)^n > \frac{2}{5}$$

$$\left(\frac{5}{6}\right)^n < \frac{3}{5}$$

$$n > 2.8$$

$$n_{\text{least}} = 3$$

21. (b)

Since the probability of occurrence is very small, this follows Poisson distribution.

$$\text{mean} = m = np$$

$$= 1500 \times 0.002 = 3$$

Probability that more than 2 will get a hanging problem

$$= 1 - P(0) - P(1) - P(2)$$

$$= 1 - \left[e^{-m} + \frac{e^{-m} \cdot m^1}{1!} + \frac{e^{-m} \cdot m^2}{2!} \right] = 1 - \left[e^{-3} + \frac{e^{-3} \cdot 3}{1} + \frac{e^{-3} \cdot 3^2}{2} \right]$$

$$= 1 - \left[\frac{1}{e^3} + \frac{3}{e^3} + \frac{9/2}{e^3} \right] = 1 - \frac{17}{2e^3}$$

22. (b)

Rearranging the equation,

$$\frac{dy}{dx} - \frac{y}{(2x+1)} = e^{4x} (2x+1)$$

The equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$IF = e^{\int P(x)dx} = e^{\int \frac{-1}{2x+1} dx} = e^{-\frac{\ln(2x+1)}{2}} = \frac{1}{\sqrt{2x+1}}$$

23. (c) $AX = B$

$$\begin{bmatrix} p & q \\ r & s \end{bmatrix} \begin{bmatrix} \frac{9}{2} & -6 \\ 3 & -3 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{15}{2} & 3 \\ -3 & 3 \\ 2 & 2 \end{bmatrix}$$

$$\begin{aligned} \frac{9}{2}p + \frac{3}{2}q &= \frac{15}{2} & -6p - \frac{3}{2}q &= 3 \\ p &= -7 & q &= 26 \end{aligned}$$

$$\begin{aligned} \frac{9}{2}r + \frac{3}{2}s &= -3 & -6r - \frac{3}{2}s &= \frac{3}{2} \\ r &= 1 & s &= -5 \end{aligned}$$

$$\Rightarrow A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -7 & 26 \\ 1 & -5 \end{vmatrix} = 35 - 26 = 9$$

24. (c)

We parameterize the curve using $t = y$

$$x = 2 - 3t^2 \quad -1 \leq t \leq 1$$

$$y = t$$

Then

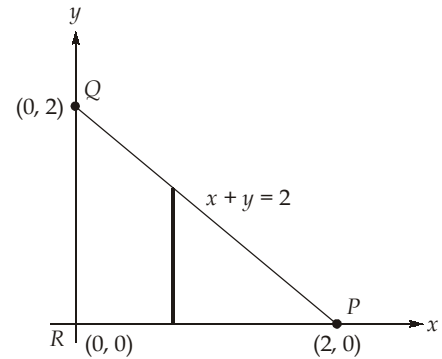
$$dx = -6t \, dt$$

$$dy = dt$$

$$\begin{aligned} \int_c 2y^3 dx + 3x^2 dy &= \int_{-1}^1 \left[2t^3(-6t) + 3(2-3t^2)^2 \right] dt = \int_{-1}^1 (15t^4 - 36t^2 + 12) dt \\ &= \left[\frac{15t^5}{5} - \frac{36t^3}{3} + 12t \right]_{-1}^1 = [3t^5 - 12t^3 + 12t]_{-1}^1 = 3 + 3 = 6 \end{aligned}$$

25. (c)

$$\begin{aligned} I &= \int_0^2 \int_0^{2-x} 5y \, dy \, dx = \int_0^2 \left[\frac{5y^2}{2} \right]_0^{2-x} dx \\ &= \int_0^2 \frac{5}{2} (2-x)^2 dx = -\frac{5}{2} \frac{(2-x)^3}{3} \bigg|_0^2 \\ &= -\frac{5(2-x)^3}{6} \bigg|_0^2 = -\frac{5}{6}(-8) = \frac{20}{3} \end{aligned}$$



26. (d)

$$\frac{d^2 y}{dx^2} + y = \cos x$$

$$(D^2 + 1)y = \cos x$$

$$PI = \frac{\cos x}{D^2 + 1}$$

Putting

$$D^2 = -1$$

$$PI = \frac{\cos x}{-1 + 1} \quad [\text{Makes denominator zero}]$$

∴ Differentiating numerator and denominator

$$\begin{aligned} PI &= x \cdot \frac{\cos x}{2D} \\ &= \frac{1}{2} x \int \cos x \, dx = \frac{1}{2} x \sin x \end{aligned}$$

27. (b)

$$\begin{aligned} \int_0^4 f(x) \, dx &= \frac{h}{3} [(y_0 + y_4) + 4(y_1 + y_3) + 2y_2] \\ &= \frac{1}{3} [(0.5 + 0.029) + 4(0.25 + 0.05) + 2 \times 0.1] \approx 0.64 \end{aligned}$$

28. (c)

$$S = 160t - 20t^2$$

For maximum height

$$\frac{dS}{dt} = 160 - 40t = 0$$

and

$$t = 4 \text{ sec}$$

$$\frac{d^2 S}{dt^2} = -40 < 0 \Rightarrow \text{Maxima}$$

$$S_{\max} = 160 \times 4 - 20 \times 4^2$$

$$\text{Maximum height} = 320 \text{ m}$$

29. (c)

Output produced by A = 40%

$$\therefore P(A) = 0.4$$

Output produced by B = 60%

$$\therefore P(B) = 0.6$$

Let, $P\left(\frac{D}{A}\right)$ = probability that item produced by A is defective

$$\therefore P\left(\frac{D}{A}\right) = \frac{9}{1000} = 0.009$$

similarly, $P\left(\frac{D}{B}\right) = \frac{1}{250} = 0.004$

$P\left(\frac{A}{D}\right)$ = Probability that product is produced by A given that it is defective.

$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \times P\left(\frac{D}{A}\right) + P(B) \times P\left(\frac{D}{B}\right)}$$

$$= \frac{0.4 \times 0.009}{0.4 \times 0.009 + 0.6 \times 0.004}$$

$$P\left(\frac{A}{D}\right) = \frac{0.0036}{0.0036 + 0.0024} = \frac{0.0036}{0.006} = 0.6$$

$$\therefore P\left(\frac{A}{D}\right) = 0.6$$

30. (c)

Given, $z = f(u, v); u = x^2 - 2xy - y^2, v = a$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

and

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

Also, $\frac{\partial u}{\partial x} = 2x - 2y$ $\frac{\partial u}{\partial y} = -2x - 2y$

$$\frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial z}{\partial x} = (2x - 2y) \left(\frac{\partial z}{\partial u} \right) \quad \dots (i), \quad \frac{\partial z}{\partial y} = -(2x + 2y) \left(\frac{\partial z}{\partial u} \right) \quad \dots (ii)$$

From equation (i) and (ii), we get

$$(x + y) \frac{\partial z}{\partial x} = (y - x) \frac{\partial z}{\partial y}$$

Hence, option (c) is correct.

