CLASS TEST

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ENGINEERING MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 06/10/2025

ANSWER KEY

1.	(b)	7.	(c)	13.	(a)	19.	(c)	25.	(c)
2.	(a)	8.	(a)	14.	(b)	20.	(b)	26.	(b)
3.	(b)	9.	(c)	15.	(b)	21.	(c)	27.	(d)
4.	(b)	10.	(c)	16.	(a)	22.	(c)	28.	(b)
5.	(b)	11.	(b)	17.	(c)	23.	(d)	29.	(d)
6.	(d)	12.	(d)	18.	(c)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

Here $\text{rank}(A) = \text{rank}(AB) <$ Number of variables

So, there will be many solutions possible.

2. (a)

$$A^2 + B^2 = AA + BB$$
$$\downarrow$$
$$= \underline{AB}A + \underline{BA}B \quad \text{(Using values of } A \text{ and } B\text{)}$$
$$= BA + AB$$
$$= \underbrace{A + B}$$

3. (b)

Since matrix addition is symmetric.

And transpose with respect to addition is

$$A' + B' + C' = (A + B + C)'$$

4. (b)

It is a form of $\frac{0}{0}$ so, apply L'Hospital rule

$$\lim_{x \to 0} \frac{e^x - (1 + x)}{3x^2}$$

$$\lim_{x \to 0} \frac{e^x - 1}{6x} \quad \left[\because \frac{0}{0}\right]$$

$$= \frac{e^x}{6} = \frac{1}{6}$$

5. (b)

Lets take C be any skew symmetric matrix of order 2 × 2 i.e. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

Assume, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ then $X^T = [x_1\ x_2]$

So, $$X^T CX = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= [x_2 - x_1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$= [x_1\ x_2 + (-x_1\ x_2)]$$
$$= [0] = \text{Null matrix}$$

6. (d)

$$\text{Mean} = np = \frac{10}{8}$$

Variance $npq = \frac{30}{32}$

$$q = \frac{30}{32} \times \frac{8}{10} = \frac{3}{4}$$

$$p = 1 - \frac{3}{4} = \frac{1}{4}$$

7. (c)

Only first two tosses are heads

So, P(H, H, T, T, T)

And each toss is independent.

So, required probability

$$= P(H) \times P(H) \times (P(T))^3$$

$$= \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^5$$

8. (a)

$$P(x) = \frac{1}{\beta - \alpha} = \frac{1}{1-0} = 1$$

Mean, $\mu = \Sigma x \, P(x)$

$$\Rightarrow \quad = \int_0^1 x = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

Variance, $\sigma^2 = \Sigma x^2 \, P(x) - \mu^2$

$$= \int_0^1 x^2 \frac{1}{4} = \left[\frac{x^3}{3}\right]_0^1 - \frac{1}{4}$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\text{Standard deviation} = \sqrt{\text{variance}} = \frac{1}{\sqrt{12}}$$

9. (c)

Given : $q = 0.4$

X	0	1
$p(X)$	0.4	0.6

$$\text{Required value} = V(X) = E(X^2) - [E(X)]^2$$

$$E(X) = \sum_i X_i p_i = 0 \times 0.4 + 1 \times 0.6 = 0.6$$

$$E(X^2) = \sum_i X_i^2 p_i = 0^2 \times 0.4 + 1^2 \times 0.6 = 0.6$$

$$\therefore \quad V(X) = E(X^2) - [E(X)]^2 = 0.6 - 0.36 = 0.24$$

10. **(c)**

$$\text{Required probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

Favorable outcomes = A false coin is chosen and flipped every time

Probability of selecting a false coin = $\frac{1}{4}$

Probability of getting a tail on every flip of false coin = 1.

$$\therefore \quad \text{Favorable outcomes} = \frac{1}{4} \times 1 = \frac{1}{4}$$

Total possible outcomes = Favourable outcomes + Unfavourable outcomes

Unfavourable outcomes = A fair coin is chosen and flipped every time to get tail

Probability of selecting a fair coin = $\frac{3}{4}$

Probability of flipping a fair coin 4 times and getting

$$\text{tails every time} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\therefore \quad \text{Unfavourable outcomes} = \frac{3}{4} \times \frac{1}{16} = \frac{3}{64}$$

$$\text{Total possible outcomes} = \frac{1}{4} + \frac{3}{64} = \frac{19}{64}$$

$$\therefore \quad \text{Required probability} = \frac{\frac{1}{4}}{\frac{19}{64}} = \frac{16}{19} \approx 0.84$$

11. **(b)**

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$, $R_3 \leftarrow R_3 - 4R_1$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$$

$R_3 \leftarrow R_3 - \frac{3}{2}R_2$

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 2

So, Rank of A = 2

12. (d)

Given function is continuous at $x = 0$

$$\therefore \quad (\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0)$$

Now, $$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-\cos 4x}{8x^2}$$

Put, $x = 0 - h = -h$ when $x \to 0$, $h \to 0$

So, $$\lim_{h\to 0} \frac{1-\cos(-4h)}{8h^2}$$

$$\lim_{h\to 0} \frac{2\sin^2 2h}{8h^2}$$

$$\lim_{h\to 0} \left(\frac{\sin 2h}{2h}\right)^2 = 1$$

At, $x = 0$, $f(0) = K$

Also LHL $= f(0)$, therefore $K = 1$

13. (a)

$$y = 3x^2 + 3x + 1$$

$$\frac{dy}{dx} = 6x + 3, \quad \frac{dy}{dx} = 0$$

$$\Rightarrow \quad 6x + 3 = 0$$

$$x = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} = 6$$

Also, $$\frac{d^2y}{dx^2} = 6 > 0 \text{ so minimum}$$

So maximum value of y in [–2, 0] is maximum $\{f(-2), f(0)\}$ i.e. max {7, 1} = 7.

Minimum value of y in [–2, 0]

$$\min\left\{\begin{matrix} f(-2), & f(0), & f\left(-\frac{1}{2}\right) \\ \downarrow & \downarrow & \downarrow \\ 7 & 1 & \frac{1}{4} \end{matrix}\right\} = \frac{1}{4}$$

So, maximum value 7 and minimum value $\frac{1}{4}$.

14. (b)

$$\lambda = np = \frac{1}{100}\times 100 = 1$$

$$P(X > 2) = 1 - (P(X = 0) + P(X = 1))$$

$$P(X = 0) = \frac{e^{-\lambda}\cdot\lambda^0}{0!} = e^{-\lambda}$$

$$P(X=1) = \frac{e^{-\lambda}\lambda'}{1!} = e^{-\lambda}\cdot\lambda$$

$$P(X>2) = 1-e^{-1}(2) = \frac{1-2}{e} = \frac{e-2}{e}$$

15. (b)

Since $f(1) \neq f(-1)$, Roll's mean value theorem does not apply.

By Lagrange mean value theorem

$$f'(x) = \frac{f(1)-f(-1)}{1-(-1)} = \frac{2}{2} = 1$$

$$-2x + 3x^2 = 1$$

$$x = 1, -\frac{1}{3}$$

x lies in $(-1, 1) \Rightarrow$ $$x = -\frac{1}{3}$$

16. (a)

$$\lim_{x\to 0}\frac{\sin\left(\frac{2}{3}x\right)}{x} = \lim_{\frac{2}{3}x\to 0}\frac{\sin\left(\frac{2}{3}x\right)}{\frac{2}{3}x}\cdot\frac{2}{3} = (1)\left(\frac{2}{3}\right) = \frac{2}{3}$$

17. (c)

$$A = \begin{bmatrix}2\\-4\\7\end{bmatrix}\begin{bmatrix}1 & 9 & 5\end{bmatrix}$$

$$= \begin{bmatrix}2 & 18 & 10\\-4 & -36 & -20\\7 & 63 & 35\end{bmatrix}$$

$$R_2 = -2\times R_1$$

Two rows are dependent. Hence, the determinant of matrix A is 0. Since the product of eigen values is determinant of the matrix. Hence, answer is 0.

18. (c)

Using Crout's method

$$A = \begin{bmatrix}l_{11} & 0\\l_{21} & l_{22}\end{bmatrix}\begin{bmatrix}1 & u_{12}\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2 & 4\\6 & 3\end{bmatrix} = \begin{bmatrix}l_{11} & 0\\l_{21} & l_{22}\end{bmatrix}\begin{bmatrix}1 & u_{12}\\0 & 1\end{bmatrix}$$

$$l_{11} = 2 \qquad l_{11}u_{12} = 4$$

$$u_{12} = \frac{4}{2} = 2$$

$$l_{21} = 6 \qquad l_{21}u_{12} + l_{22} = 3$$

$$6 \times 2 + l_{22} = 3$$
$$l_{22} = 3 - 12$$
$$l_{22} = -9$$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

19. **(c)**

Let be the event that a purchased product breaks down in the third year. Also, let B be the event that a purchased product does not break down in the first two years.

We are interested in $P(A|B)$.

We have $\quad P(B) = P(T \geq 2)$
$$= e^{-2/5}$$

We also have $\quad P(A) = P(2 \leq T \leq 3)$
$$= P(T \geq 2) - P(T \geq 3)$$
$$= e^{-2/5} - e^{-3/5}$$

Finally, since $A \subset B$, we have $A \cap B = A$.

Therefore, $\quad P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)} = \dfrac{e^{-2/5} - e^{-3/5}}{e^{-2/5}} = 0.1813$

20. **(b)**

For uniform random variable,

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{else} \end{cases}$$

$$\text{Variance }(X) = \frac{(\beta - \alpha)^2}{12} = \frac{(6-2)^2}{12} = \frac{4}{3} = 1.33$$

21. **(c)**

$$x_1 + 2x_2 = b_1 \quad \text{...(i)}$$
$$2x_1 + 4x_2 = b_2 \quad \text{...(ii)}$$
$$3x_1 + 7x_2 = b_3 \quad \text{...(iii)}$$
$$3x_1 + 9x_2 = b_4 \quad \text{...(iv)}$$

From equations (ii) and (i) we can write

$$b_2 = 2[x_1 + 2x_2]\ 2b_1$$

From option (b):

$3b_1 - 6b_3 + b_4$

$$= 3[x_1 + 2x_2] - 6[3x_1 + 7x_2] + 3x_1 + 9x_2 \neq 0$$

From option (c):

$$b_2 = 2b_1 \text{ and } b_1 - 3b_3 + b_4$$
$$= 6[x_1 + 2x_2] - 3[3x_1 + 7x_2] + [3x_1 + 9x_2] = 0$$
$$6b_1 - 3b_3 + b_4 = 0$$

So, option (c) is correct answer.

22. (c)

$$f(x) = \sin^4 x$$

Also, $$f(-x) = \sin^4(-x) = \sin^4 x = f(x)$$

So, $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x\, dx = 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos^2 2x - 2\cos 2x)dx$$

$$\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1-\cos 2x + \frac{1+\cos 4x}{2}\right)dx$$

$$\Rightarrow \frac{1}{4}\int_0^{\frac{\pi}{2}} (3-4\cos 2x+\cos 4x)dx$$

$$\Rightarrow \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

On solving we get $\frac{3\pi}{8}$.

23. (d)

$$f(x) = x^4 + 16 - 8x^2$$
$$= (x^2 - 4)^2$$
$$f'(x) = 2(x^2 - 4) \times 2x$$
$$= 4x(x^2 - 4) = 0$$
$$x = 0, x = 2, x = -2$$

So, $f(x)$ has 3 stationary points 0, 2 and –2.

Now, $$f''(x) = 12x^2 - 16$$
$f''(0) = -16 < 0$ (It is maxima points)
$f''(2) = 32 > 0$ (It is minima points)
$f''(-2) = 32 > 0$ (It is minima points)

So, $f(x)$ has 1 maxima and 2 minima points.

24. (a)

Augmented matrix:

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$$R_3 \leftarrow R_3 - R_2$$
$$R_2 \leftarrow 4R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 0 & 9 & 22 & 28 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$ and $\mu \neq 9$, then system has no solution because Rank$[A \mid B] \neq$ Rank [4].

25. (c)

$$\text{Variance of X} = \text{Var}[X]$$
$$\text{Expectation} = E[X]$$
$$= \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx = \int_0^1 x \cdot (2x) \cdot dx = 2 \cdot \int_0^1 x^2 \cdot dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$
$$E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f(x) \cdot dx = \int_0^1 x^2 \cdot (2x) \cdot dx = 2 \cdot \int_0^1 x^3 \cdot dx = 2 \cdot \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$
$$\therefore \quad \text{Var}[X] = E[X^2] - E[X]^2$$
$$= \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$$

26. (b)

Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd and 4th attempt respectively, Now,

$$P_1 = \frac{1}{24}; \quad P_2 = \frac{1}{24}[1+0.5]$$
$$P_2 = \frac{1}{24} \times \frac{3}{2}$$
$$P_3 = \frac{1}{24} \times \frac{3}{2}[1+0.5] = \frac{1}{24} \times \left(\frac{3}{2}\right)^2$$
$$P_4 = \frac{1}{24} \times \left(\frac{3}{2}\right)^3$$

Now let A_i be selection in on attempt and $\overline{A_i}$ be unsuccessful attempt,

So,

$$P_{\text{selection}} = A_1 + \overline{A_1} A_2 + \overline{A_1}\,\overline{A_2}\, A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}\, A_4$$
$$= \frac{1}{24} + \frac{23}{24} \times \frac{1}{24} \times \frac{3}{2} + \frac{23}{24}\left(1 - \frac{3}{48}\right) \times \frac{1}{24} \times \left(\frac{3}{2}\right)^2 + \frac{23}{24}\left(1 - \frac{3}{48}\right)\times\left(1 - \frac{9}{96}\right) \times \frac{1}{24} \times \left[\frac{3}{2}\right]^3 \simeq 0.3$$

27. (d)

Every matrix satisfies it's own characteristic equation.

$|P - \lambda I|$ gives the characteristic equation.

$$= \begin{vmatrix} 2-\lambda & -2 & 3 \\ 1 & 1-\lambda & 1 \\ 1 & 3 & -1-\lambda \end{vmatrix} = \lambda^3 - 2\lambda^2 - 5\lambda + 6 = 0$$

$\therefore$ Option (d) is correct.

28. **(b)**

$$f(x) = x + \ln x$$

$$f'(x) = 1+\frac{1}{x} \qquad \ldots (1)$$

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{f(e)-f(1)}{e-1}$$

$$= \frac{e+\ln e-(1+\ln 1)}{e-1} = \frac{e+1-1+0}{e-1} = \frac{e}{e-1}$$

$$\Rightarrow \quad f'(c) = \frac{e}{e-1} \quad \text{[from equation (1)]}$$

$$\Rightarrow \quad 1+\frac{1}{c} = \frac{e}{e-1}$$

$$\Rightarrow \quad \frac{1}{c} = \frac{e}{e-1}-1$$

$$\Rightarrow \quad \frac{1}{c} = \frac{e-e+1}{e-1}$$

$$\Rightarrow \quad c = e-1$$

29. **(d)**

$$\lim_{y\to\infty}(y-(y^2+y)^{1/2})$$

This of the form $\infty - \infty$ (indeterminate form)

$$\lim_{y\to\infty}\left(y-\sqrt{y^2+y}\right)\times\frac{\left(y+\sqrt{y^2+y}\right)}{y+\sqrt{y^2+y}} = \lim_{y\to\infty}\left(\frac{y^2-y^2-y}{y+\sqrt{y^2+y}}\right)$$

$$= \lim_{y\to\infty}\left(\frac{-y}{y+\sqrt{y^2+y}}\right)$$

Dividing numerator and denominator by 'y' we get

$$= \lim_{y\to\infty}\left[\frac{-1}{1+\sqrt{1+\frac{1}{y}}}\right] = \frac{-1}{1+\sqrt{1}} = \frac{-1}{2}$$

30. (c)

$$\text{Required probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

Favorable outcomes = A false coin is chosen and flipped every time

Probability of selecting a false coin $= \frac{1}{4}$

Probability of getting a tail on every flip of false coin = 1.

$$\therefore \quad \text{Favorable outcomes} = \frac{1}{4} \times 1 = \frac{1}{4}$$

Total possible outcomes = Favourable outcomes + Unfavourable outcomes

Unfavourable outcomes = A fair coin is chosen and flipped every time to get tail

Probability of selecting a fair coin $= \frac{3}{4}$

Probability of flipping a fair coin 4 times and getting

$$\text{tails every time} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\therefore \quad \text{Unfavourable outcomes} = \frac{3}{4} \times \frac{1}{16} = \frac{3}{64}$$

$$\text{Total possible outcomes} = \frac{1}{4} + \frac{3}{64} = \frac{19}{64}$$

$$\therefore \quad \text{Required probability} = \frac{\frac{1}{4}}{\frac{19}{64}} = \frac{16}{19} \approx 0.84$$

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