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STRUCTURE ANALYSIS

CIVIL ENGINEERING

Date of Test : 16/09/2025

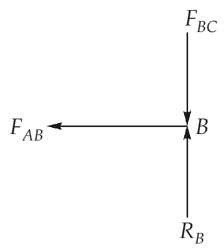
ANSWER KEY

1.	(c)	7.	(a)	13.	(a)	19.	(b)	25.	(0)
2.	(a)	8.	(d)	14.	(a)	20.	(b)	26.	(b)
3.	(d)	9.	(d)	15.	(b)	21.	(c)	27.	(b)
4.	(c)	10.	(a)	16.	(d)	22.	(a)	28.	(b)
5.	(a)	11.	(a)	17.	(a)	23.	(b)	29.	(d)
6.	(a)	12.	(b)	18.	(d)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (c)

For joint B,



Now, $\Sigma F_y = 0 \quad \Rightarrow F_{AB} = 0$

2. (a)

$$D_k = 3j - r_e - m$$

Given that members are not inextensible i.e. members are extensible,

$\Rightarrow \quad m = 0$

$\therefore \quad D_k = 3 \times 4 - (3 + 2)$

$= 12 - 5 = 7$

3. (d)

Moment induced at $A \geq$ Half of moment applied at $B = \frac{60}{2} = 30\,\text{kNm}$

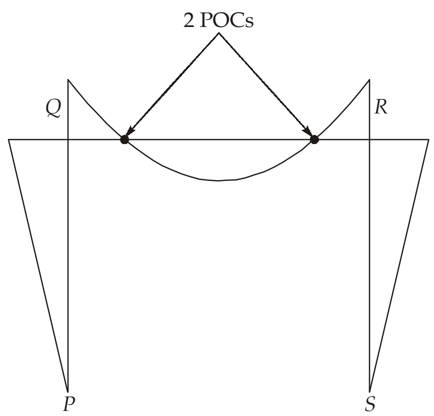
4. (c)

Slope deflection equations can be written for

$M_{AB}, M_{BA}, M_{BC}, M_{CB}, M_{CD}, M_{DC}, M_{CE}$ and M_{EC}

5. (a)

BMD



6. (a)

7. (a)

When there is a rise in temperature, the length of the cable increases and dip increases. The relation between change in length of cable and the dip is

$$\Delta y_c = \frac{3}{16}\frac{l}{y_c}\Delta L$$

Increase in temperature leads to increase in length of cable and hence dip.

8. **(d)**

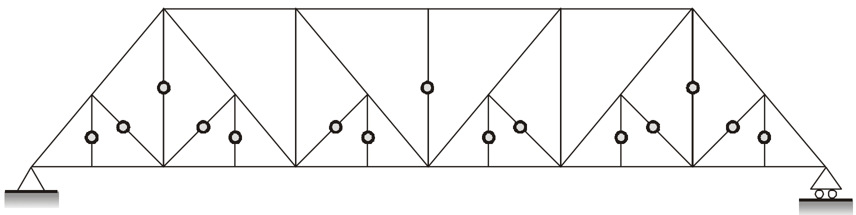
Castingliano's first theorem,

$$\frac{\partial U}{\partial \Delta_j} = P_j$$

Castingliano's second theorem,

$$\frac{\partial U}{\partial P_j} = \Delta_j$$

9. **(d)**



10. **(a)**

B

$_B\sin$

B

C C

$_C\cos$

As length of member will not change because members are inextensible,

$$\Delta_B \sin\theta = \Delta_C \cos\theta$$

$$\Rightarrow \quad \Delta_C = \Delta_B \tan\theta$$

$$\Rightarrow \quad \Delta_C = \frac{4\Delta_B}{3} = \frac{4}{3}\Delta$$

11. **(a)**

Applying, Work done by load = Strain energy stored in beam

Strain energy stored in beam = $U_{RQ} + U_{QP}$

$$= \int_o^2 \frac{(3x)^2\,dx}{2EI} + \int_o^4 \frac{(6)^2\,dx}{2EI} = \frac{9}{2EI}\left.\frac{x^3}{3}\right|_0^2 + \frac{36}{2EI}\left.x\right|_0^4 = \frac{9}{2}\times\frac{8}{3EI} + \frac{36}{2EI}\times 4 = \frac{84}{EI}$$

$$\Rightarrow \quad \frac{1}{2}\times P\times\Delta = \frac{84}{EI}$$

$$\Rightarrow \quad \frac{1}{2}\times 3\times\Delta = \frac{84}{EI}$$

$$\Rightarrow \quad \Delta = \frac{56}{10^4}\times 10^3 = 5.6\text{ mm}$$

12. (b)

$$M_x = \frac{wR^2}{2}(\sin\theta - \sin^2\theta)$$

For maximum bending moment,

$$\frac{dM_x}{d\alpha} = 0$$

$$\frac{wR^2}{2}(\cos\theta - 2\sin\theta\cos\theta) = 0$$

$$\Rightarrow \quad \cos\theta(1 - 2\sin\theta) = 0$$

$$\Rightarrow \quad \cos\theta = 0 \Rightarrow \theta = 90° \text{ and } \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

$$\theta = 90° \text{ (Rejected as BM = 0)}$$

$$\therefore \quad \theta = 30°$$

$$\therefore \quad M_x = \frac{-wR^2}{2}(\sin 30° - \sin^2 30°) = \frac{-wR^2}{8} \quad \text{or} \quad \frac{wR^2}{8} \text{ (Hogging)}$$

13. (a)

Let the applied moment M be clockwise in direction.

$$M_{FAB} = -\frac{12\times 5^2}{12} = -25\text{ kNm}$$

$$M_{FBA} = 25 \text{ kNm}$$

$$M_{FBC} = -\frac{10\times 4}{8} = -5\text{ kNm}$$

$$M_{FCB} = 5 \text{ kNm}$$

Using slope deflection method,

We know that, $M_{BA} + M_{BC} = M$

$$M_{BA} = +25 + \frac{2E(2I)}{5}\left[2\cancel{\theta_B}^{0} + \theta_A\right] \quad \text{where } \theta_A = 0 \text{ (End } A \text{ is fixed)}$$

$$= +25 \qquad \ldots \text{(i)}$$

$$M_{BC} = -5 + \frac{2EI}{4}\left[2\cancel{\theta_B}^{0} + \theta_C\right]$$

$$= -5 + 0.5EI\,[\theta_C] \qquad \ldots \text{(ii)}$$

$$M_{CB} = 0 = 5 + \frac{2EI}{4}\left[2\theta_C + \cancel{\theta_B}^{0}\right]$$

$$\Rightarrow \quad -5 = \frac{2EI}{4}\times 2\theta_C$$

$$\Rightarrow \quad \theta_C = -\frac{5}{EI} \qquad \ldots \text{(iii)}$$

Substituting (iii) in (ii),

$$M_{BA} = 25 \text{ kNm}$$

$$M_{BC} = -5 + 0.5EI\times\left(-\frac{5}{EI}\right) = -5 - 2.5 = -7.5 \text{ kNm}$$

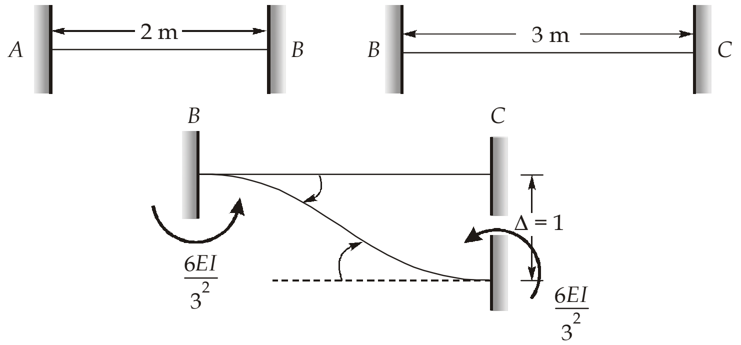
$\therefore \qquad M_{BA} + M_{BC} = M$

$\Rightarrow \qquad 25 + (-7.5) = M$

$\Rightarrow \qquad M = 17.5 \text{ kNm}$

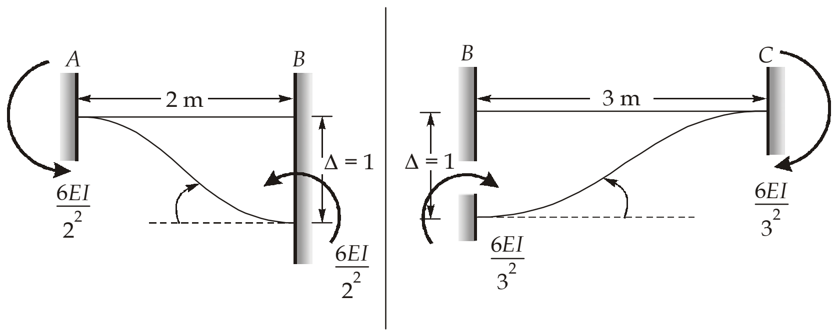
14. (a)

k_{41}:



$$k_{41} = -\frac{6EI}{3^2} = -\frac{6EI}{9}$$

k_{43}:

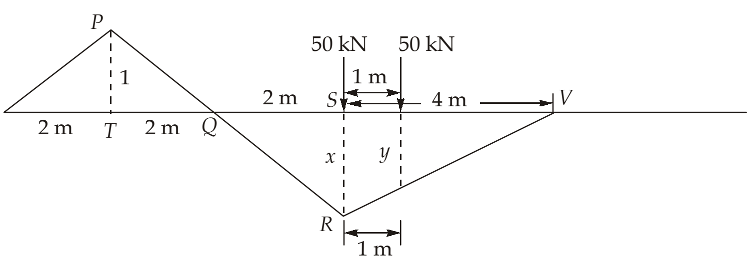


$$k_{43} = \frac{6EI}{9} - \frac{6EI}{4} = \frac{2}{3}EI - \frac{3}{2}EI = -\frac{5EI}{6}$$

$$\text{Ratio} = \frac{k_{41}}{k_{43}} = \frac{-6EI/9}{-5EI/6} = \frac{4}{5} = 0.8$$

15. (b)

As per Muller Breslau principle,



$$\text{BM}_{max} = 50 \times x + 50 \times y$$

From Δs *PTQ* and *QSR*

$$\frac{x}{2} = \frac{1}{2} \quad \Rightarrow \quad x = 1$$

Using ΔSVR $\quad \dfrac{x}{4} = \dfrac{y}{4-1} \Rightarrow \dfrac{1}{4} = \dfrac{y}{3} \Rightarrow y = \dfrac{3}{4}$

$$BM_{max} = 50 \times 1 + 50 \times \frac{3}{4} = 87.5\,\text{kNm}$$

16. (d)

Let l_1 and l_2 be the distances of point C from A and B respectively.

$\therefore \quad \dfrac{l_1}{l_2} = \sqrt{\dfrac{4}{9}}$ [Property of parabola]

$\Rightarrow \quad 3l_1 = 2l_2$

$\Rightarrow \quad l_2 = 1.5l_1$

Also, $\quad l_1 + l_2 = 30$ m

$\Rightarrow \quad l_1 + 1.5l_1 = 30$

$\Rightarrow \quad l_1 = 12$ m

So, $\quad l_2 = 18$ m

Now, $\quad V_A + V_B = 50$ kN

$\Sigma M_B = 0$

$\Rightarrow \quad V_A \times 30 + H_A \times 5 = 50 \times DB$

$DB = BC + CD = 18 + 5 = 23$ m

$\therefore \quad 6V_A + H_A = 230$ kN $\quad$ ...(i)

$\Sigma M_C = 0$ (from left)

$\Rightarrow V_A \times 12 - H_A \times 4 - 50 \times 5 = 0$

$\Rightarrow \quad 3V_A - H_A = 62.5$ kN

$V_A = \dfrac{62.5 + H_A}{3} \quad$ ...(ii)

From eq. (i) and (ii),

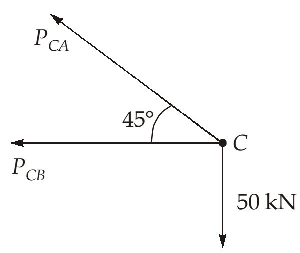
$\Rightarrow \quad 6 \times \dfrac{62.5 + H_A}{3} + H_A = 230$

$\therefore \quad H_A = 35$ kN

17. (a)

P-System of forces

Consider joint C,



$\Sigma F_y = 0$

$\Rightarrow \quad P_{CA} \sin 45° = 50$

$\Rightarrow \quad P_{CA} = \dfrac{50}{(1/\sqrt{2})} = 70.71$ kN

$\Sigma F_x = 0$

$\Rightarrow \quad P_{CA} \cos 45° + P_{CB} = 0$

$\Rightarrow \quad P_{CB} = -P_{CA} \cos 45°$

$$= -70.71 \times \frac{1}{\sqrt{2}} = -50 \text{ kN}$$

As unit load to be applied will be in the same direction of 50 kN.

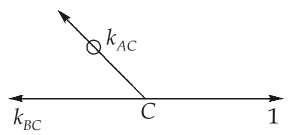
So, $k_{CA} = \sqrt{2}$

$k_{CB} = 1$

Members	P	k	L	PKL
AC	70.71	$\sqrt{2}$	$4\sqrt{2}$	565.68
BC	–50	–1	4	200
				$\Sigma PkL = 765.68$

Hence, $\Delta_{VC} = \frac{765.68}{AE}$

To find horizontal deflection, apply unit load at *C* in the horizontal direction.



$k_{AC} = 0$

$k_{BC} = 1$

So, $\Delta_{HC} = \frac{\Sigma PkL}{AE} = (P_{AC}\, k_{AC}\, L_{AC}) + (P_{BC}\, k_{BC}\, L_{BC})$

$$= 0 + \frac{(-50)\times(1)\times 4}{AE} = -\frac{200}{AE}$$

So, Required ratio $= \frac{\Delta_{VC}}{\Delta_{HC}} = \frac{765.68}{200} = 3.83$

18. (d)

$$M_{FAB} = -\frac{wL^2}{12} = -\frac{16\times 4^2}{12} = -21.33 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{16\times 4^2}{12} = 21.33 \text{ kNm}$$

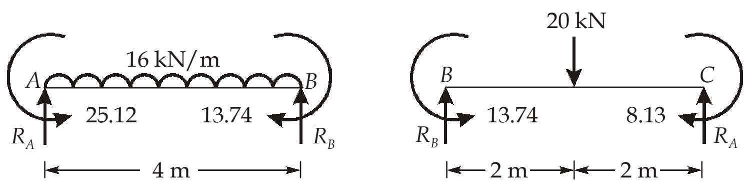
$$M_{FBC} = -\frac{wL}{8} = -\frac{20\times 4}{8} = -10 \text{ kNm}$$

$$M_{FCB} = \frac{wL}{8} = \frac{20\times 4}{8} = 10 \text{ kNm}$$

Consider B as rigid joint.

Joint	Members	Stiffness	Total stiffness	DF
B	BA	$\frac{8EI}{4}$	$3EI$	0.67
	BC	$\frac{4EI}{4}$		0.33

Joint	A	B		C
DF	–	0.67	0.33	–
Fixed end moment	–21.33	21.33	–10	10
Balancing moment	–	–7.59	–3.74	–
Carry over moment	–3.79	–	–	–1.87
Final Moment (kNm)	–25.12	13.74	–13.74	8.13



Considering AB: $\Sigma M_B = 0$

$\Rightarrow R_A \times 4 - 25.12 + 13.74 - 16 \times 4 \times 2 = 0$

$\Rightarrow \quad R_A = 34.845$ kN

Considering BC: $\Sigma M_B = 0$

$-R_C \times 4 + 20 \times 2 + 8.13 - 13.74 = 0$

$\Rightarrow \quad R_C = 8.5975$ kN

So, ΣVertical forces $= 34.845\ (\uparrow) + 8.5975\ (\uparrow) - 16 \times 4\ (\downarrow) - 20\ (\downarrow)$

$= -40.56$ kN

So, an upward force of 40.56 kN ($\uparrow$) is required at B to prevent deflection of B.

19. (b)

$$\text{Stiffness of beam} = \frac{3EI}{L^3} = \frac{3\times2.5\times10^3}{5^3} = 60 \text{ kN/m}$$

As spring and beam are in parallel,

$$\therefore \quad k_{eq} = k + k + \frac{3EI}{L^3} = 40 + 40 + 60 = 140 \text{ kN/m}$$

$$\text{Mass,} \quad m = 10 \text{ kN} = \frac{10000}{9.081} = 1019.37 \text{ kg}$$

$$\text{Therefore, angular frequency,} \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{140\times10^3}{1019.37}} = 11.72 \text{ rad/s}$$

Now, $\quad V = A\omega \cos(\omega t + \theta)$

Here, $A = \sqrt{y_0^2 + \left(\frac{V_0}{\omega}\right)^2} = \sqrt{5^2 + \left(\frac{135}{11.72}\right)^2} = 12.56 \text{ mm}$

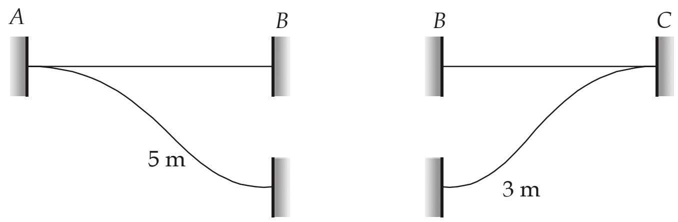
$$\tan\theta = \frac{y_o}{\frac{V_0}{\omega}} = \frac{5\times 11.72}{135}$$

$\Rightarrow$ $\theta = 0.409$ rad

Hence, velocity, $V = A\omega\cos(\omega t + \theta) = 12.56 \times 11.72 \times \cos(11.72 \times 2 + 0.409)$

$= 41.68$ mm/s

20. **(b)**



$$k = \frac{12EI}{(5)^3} + \frac{12EI}{(3)^3} = \frac{12\times 3240}{125} + \frac{12\times 3240}{27}$$

$= 311.04 + 1440 = 1751.04$ kN/m

$$T = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{20}{1751.04\times 10^3}}$$

$= 0.0212 \text{ sec} \simeq 0.02 \text{ sec}$

21. **(c)**

Applying Betti's theorem to system (I) and (III)

$20(0.001) + 10(0.008) = 10\Delta_A + 20(0.003)$

$\Rightarrow$ $\Delta_A = 0.004$ rad

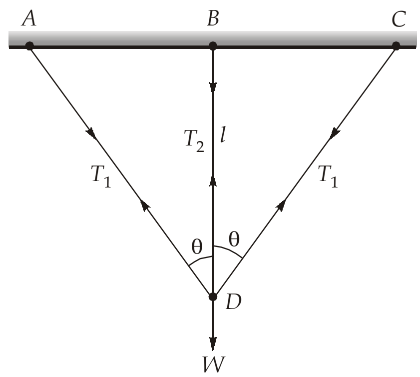
Applying Betti's theorem to system (II) and (III)

$5(0.004) + 20(0.008) = 10(0.0015) + 20\Delta_B$

$\Rightarrow$ $\Delta_B = 0.00825$ rad

$\therefore$ $\Delta_A + \Delta_B = 0.01225$ rad

22. **(a)**



$2T_1\cos\theta + T_2 = W$

$$\Rightarrow \quad T_1 = \frac{W - T_2}{2\cos\theta}$$

Strain energy stored in the wires,

$$U_1 = \sum\frac{P^2L}{2AE} = \frac{T_2^2 l}{2AE} + 2T_1^2\frac{l}{\cos\theta}\frac{1}{2AE}$$

$$= \frac{T_2^2 l}{2AE} + \left(\frac{W - T_2}{2\cos\theta}\right)^2\frac{l}{AE\cos\theta}$$

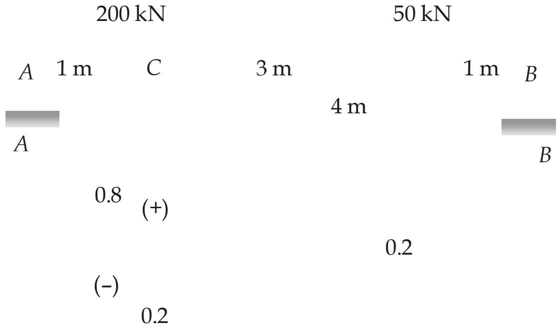
$$= \frac{T_2^2 l}{2AE} + \frac{(W - T_2)^2 l}{4AE\cos^3\theta}$$

$$\frac{\partial U_i}{\partial T_2} = 0, \qquad 0 = \frac{T_2 l}{AE} - \frac{2(W - T_2)l}{AE\cos^3\theta}$$

$$T_2 = \frac{W}{1 + 2\cos^3\theta}$$

23. (b)

The ILD for shear force (S_c)



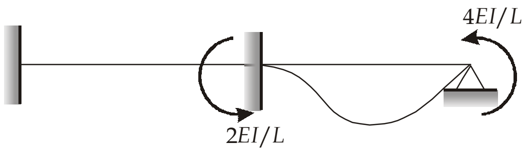
$$\frac{0.8}{4} = \frac{x}{1} \qquad \Rightarrow x = 0.2$$

Maximum shear force at C,

$$= 200(0.8) + 50(0.2) = 170 \text{ kN}$$

24. (c)

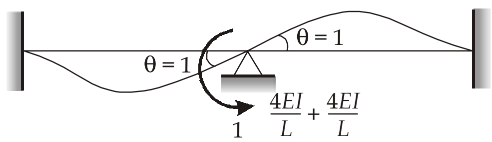
Give unit displacement in the direction (1)



$$k_{11} = \frac{4EI}{L}$$

$$k_{21} = \frac{2EI}{L}$$

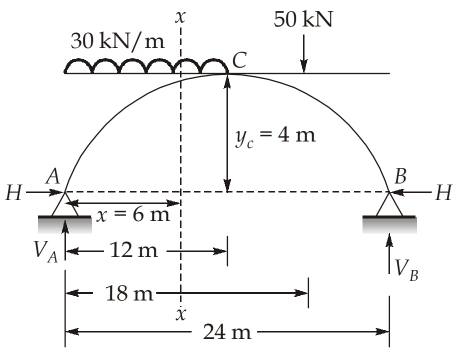
Give unit displacement in the direction (2)



$$k_{12} = \frac{2EI}{L}; \qquad k_{22} = \frac{8EI}{L}$$

$$\therefore \text{ The stiffness matrix } [k] = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{8EI}{L} \end{bmatrix}$$

25. (0)



The arch is shown in figure above,

Taking moments about B,

$\Sigma M_B = 0$

$V_A \times 24 - 30 \times 12 \times 18 - 50 \times 6 = 0$

$\Rightarrow \quad V_A = 282.5$ kN

$\therefore \quad V_B = 30 \times 12 + 50 - 282.5 = 127.5$ kN

$\Sigma M_C = 0$ (From right)

$V_B \times 12 - H \times 4 - 50 \times 6 = 0$

$$\Rightarrow \quad H = \frac{127.5 \times 12 - 50 \times 6}{4} = 307.5 \text{ kN}$$

Now, at 6 m from left support, $y = \frac{4y_c}{L^2} x(L - x) = \frac{4 \times 4}{24^2} \times 6(24 - 6) = 3 \text{ m}$

Also, $\frac{dy}{dx} = \frac{4y_c}{L^2}(L - 2x)$

At $x = 6$ m, $\frac{dy}{dx} = \tan\theta = \frac{4 \times 4}{24^2}(24 - 2 \times 6) = \frac{1}{3}$

$\therefore \quad \cos\theta = \frac{3}{\sqrt{10}}; \quad \sin\theta = \frac{1}{\sqrt{10}}$

Radial shear at a section 6 m from left support,

$Q = V\cos\theta - H\sin\theta$

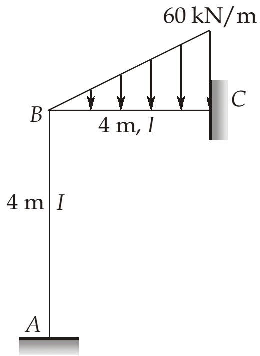
V = Vertical shear at x-x = $V_A - wx = 282.5 - 30 \times 6$

= 102.5 kN

$$\therefore \quad Q = 102.5 \times \frac{3}{\sqrt{10}} - 307.5 \times \frac{1}{\sqrt{10}} = 0$$

26. (b)

Here degree of freedom is 3 i.e. the rotations θ_B θ_C and θ_D. But due to symmetry, we have $\theta_B = -\theta_D$ and $\theta_C = 0$. For analysis of the frame as shown in figure above, only half as of the given structure is considered and $\theta_C = 0$, therefore only θ_B is the only unknown.



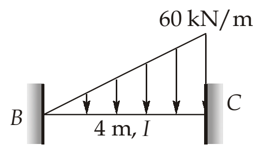
Joint equilibrium equation,

$$M_{BA} + M_{BC} = 0$$

Fixed end moment: Span *AB*

$$M_{FAB} = M_{FBA} = 0$$

Span *BC*:



$$M_{FBC} = -\frac{wL^2}{30} = \frac{60 \times 4^2}{30} = 32 \text{ kNm} \qquad (\theta_A = 0, \Delta = 0)$$

$$\therefore \quad M_{BC} = M_{FBC} + \frac{2EI}{4}\left(2\theta_A + \theta_B - \frac{3\Delta}{4}\right)$$

$$= 32 + \frac{EI\theta_B}{2} \quad \dots (\alpha)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{4}\left(2\theta_B + \theta_A - \frac{3\Delta}{4}\right)$$

$$\Rightarrow \quad M_{BA} = EI\theta_B \quad \dots (\beta)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{4}\left(2\theta_B + \theta_C - \frac{3\Delta}{4}\right) \qquad (\theta_C = 0, \Delta = 0)$$

$$= -32 + EI\theta_B \quad \dots (\gamma)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{4}\left(2\theta_C + \theta_B - \frac{3\Delta}{4}\right)$$

$$= 48 + \frac{EI\theta_B}{2} \quad \dots (\delta)$$

Now, from equation (i), $M_{BA} + M_{BC} = 0$

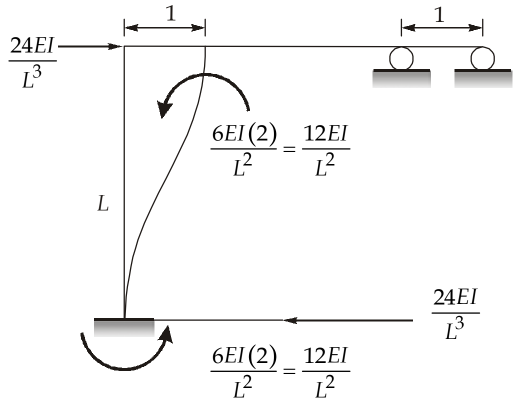
$$\Rightarrow \quad EI\theta_B - 32 + EI\theta_B = 0$$

$$\Rightarrow \quad EI\theta_B = 16$$

Now, from equation (δ)

$$M_{CB} = 48 + \frac{16}{2} = 56 \text{ kNm}$$

27. **(b)**



$$\therefore \quad k_{11} = \frac{24EI}{L^3}$$

$$\therefore \quad \alpha = 24$$

28. **(b)**

Using slope deflection method

Equilibrium equation at joint B is:

$$M_{BA} + M_{BC} + 10 \times 3 + 12 \times 3 \times 1.5 = 0$$

$$\Rightarrow \quad \frac{2EI}{4}(2\theta_B) + \frac{2EI}{4}(2\theta_B) + 84 = 0$$

$$\Rightarrow \quad \theta_B = \frac{-42}{EI}$$

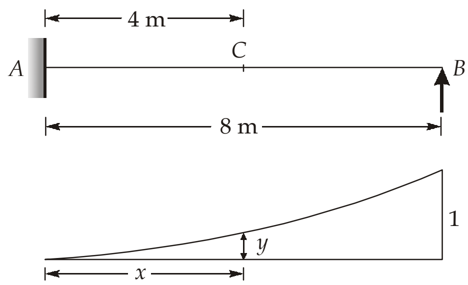
29. **(d)**

Area under free moment diagram = Area under fixing moment

$$\Rightarrow \quad \frac{1}{2}\times 2\times 370.02 + \frac{1}{2}\times 2\times(370.02+332.18) + \frac{1}{2}\times 2\times 332.18 = \frac{1}{2}\times 6\times(M_A + 228.29)$$

$$\Rightarrow \quad M_A = 239.843 \text{ kNm} \simeq 239.84 \text{ kNm}$$

30. **(c)**



$$\Delta_B = 0$$

$$\Rightarrow \quad \frac{x^3}{3EI} + \frac{x^2}{2EI}(L-x) = \frac{R_B L^3}{3EI} \qquad (x = 4 \text{ m}, L = 8 \text{ m})$$

$$\Rightarrow \qquad \frac{4^3}{3} + \frac{4^2}{2}(8-4) = \frac{R_B \times 8^3}{3}$$

$$\Rightarrow \qquad R_B = 0.3125 = \text{Ordinate of ILD for } R_B \text{ at } C.$$

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