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HYDRAULIC MACHINE

MECHANICAL ENGINEERING

Date of Test: 10/09/2025

ANSWER KEY ➤

(a)
(b)
(b)
(c)
(b)
(b)

DETAILED EXPLANATIONS

1. (b)

$$\left(\frac{H}{D^2N^2}\right)_m = \left(\frac{H}{D^2N^2}\right)_p$$

$$\frac{30}{(1)^2 \times N^2} = \frac{20}{(3)^2 \times (600)^2}$$

$$N^2 = \frac{30 \times 3^2 \times 600^2}{20}$$

$$N = 2204.54 \text{ rpm}$$

2. (a)

Maximum height attained =
$$\frac{u^2 \sin^2 \alpha}{2g} = \frac{18^2 \times (\sin 60)^2}{2 \times 9.81} = 12.38 \text{ m}$$

3. (a)

$$(F_r)_m = (F_r)_p$$

$$\left(\frac{V^2}{Lg}\right)_m = \left(\frac{V^2}{Lg}\right)_p$$

$$V_m = V_p \sqrt{\frac{L_m}{L_p}}$$

$$V_m = \frac{1}{\sqrt{36}} = \frac{1}{6} = 0.16 \text{ m/s}$$

4. (d)

Work done by ject on series of plates per second,

$$= F_x \times u$$

= $\rho AV (V - u) \cdot u$

[Where F_x = Force in x direction]

Kinetic energy of jet per second,

$$=\frac{1}{2}\text{mV}^2 = \frac{1}{2}\rho AV \cdot V^2 = \frac{1}{2}\rho AV^3$$

Efficiency of jet =
$$\frac{\rho AV(V-u) \cdot u}{\frac{1}{2}\rho AV^3} = \frac{2(V-u) \cdot u}{V^2}$$

5. (c)

• Specific speed of turbine,
$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

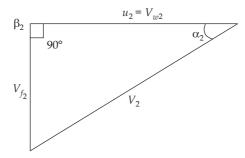
• Specific speed of pump,
$$N_s = \frac{N\sqrt{Q}}{H^{3/4}}$$



•	S.No.	Turbine	Specific speed
	1.	Pelton - Single jet	Upto 30
		- Multijet jet	30 - 60
	2.	Francis	60 - 300
	3.	Propellar	300 - 600
	4.	Kaplan	600 - 1000

- Draft tube is required at the exit of reaction turbine.
- Screw pump is used for pumping viscous oil.

6. (a)



From the outlet velocity triangle

$$V_{w_2} = V_2 \cos \alpha_2 = u_2$$

 $u_2 = \pi D_2 \text{ N}/60 = \pi \times 0.3 \times 1450/60 = 22.78 \text{ m/s}$

Manometric efficiency,

$$\eta_H = \frac{gH}{u_2 \, V_{w_2}} = \frac{gH}{u_2^2}$$

[H = net head developed]

$$0.82 = \frac{9.81 \times H}{\left(22.78\right)^2}$$

$$H = 43.36 \text{ m}$$

7. (c)

Pumps in series

$$Q = Q_1 = Q_2...$$

and $H = H_1 + H_2...$

8. (d)

Depending upon the pump, a critical cavitation number of $\sigma < \sigma_c$ will result in cavitation and cause severe reduction in pump efficiency. Hence for operational purpose it should be seen that $\sigma \ge \sigma_c$

i.e NPSH
$$\geq \sigma_c H$$

Hence minimum NPSH = $\sigma_c \times H$

9. (b)

Power required to run the pump = $P = \rho Qgh$

$$= 1000 \times (20 \times 10^{-3}) \times 9.81 \times 35 = 6.867 \text{ kW}$$

10. (b)

- Rotodynamics machines are those whose functioning depends on the principle of fluid dynamics.
- Positive displacement machines are those whose functioning depends on the change of volume of certain amount of fluid within the machine.

11. (b)

Applying energy equation,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_j}{\rho g} + \frac{V_j^2}{2g} + Z_j + h_L$$

$$\Rightarrow 0 + 0 + 1670 = 0 + \frac{V_j^2}{2g} + 1000 + h_L$$

$$A_{\text{penstock}} \times V_{\text{penstock}} = A_{\text{jet}} \times V_j$$

$$\Rightarrow \frac{\pi}{4} D^2 \times V_p = \frac{\pi}{4} d_j^2 \times V_j$$

$$\Rightarrow V_p = \left(\frac{d_j}{D}\right)^2 \times V_j = \left(\frac{0.18}{1}\right)^2 V_j = 0.0324 V_j$$

$$h_L = \frac{fL V_p^2}{D \times 2g} = \frac{0.015 \times 6000 \times (0.0324)^2 \times V_j^2}{1 \times 2 \times 9.81}$$

$$= 4.815 \times 10^{-3} V_j^2$$

From energy equation,

$$1670 = \frac{V_j^2}{2g} + 1000 + 4.815 \times 10^{-3} V_j^2$$

$$670 = 0.0557 V_j^2$$

$$V_j = 109.675 \text{ m/s}$$

$$V_{\text{bucket}} = \frac{V_j}{2} = \frac{109.675}{2} = 54.8 \text{ m/s}$$

$$V_{\text{bucket}} = 54.8 = \frac{\pi DN}{60}$$

$$N = \frac{54.8 \times 60}{\pi \times 3}$$

$$= 348.86 \text{ rpm} \approx 349 \text{ rpm}$$

12. (a)

Given: Weight flow rate = 230 N/s

Mass flow rate,
$$\dot{m} = \frac{w}{g} = \frac{230}{9.81} = 23.445 \text{ kg/s}$$

$$\dot{Q} = \frac{w}{\rho g} = \frac{230}{10^3 \times 9.81} = 0.02345 \text{ m}^3/\text{s}$$

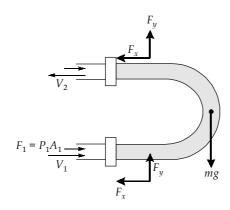
So, average velocity through pipe of water,

$$V = \frac{Q}{A} = \frac{0.02345}{\frac{\pi}{4}(0.05)^2} = 11.94 \text{ m/s}$$

As per second law of motion

$$\Rightarrow \qquad \sum F_{\text{ext}} = \dot{m} (V_f - V_i)$$

Along *x*-direction:



$$-F_x + (P_1 - P_a) \times A_1 + (P_2 - P_a) \times A_2 = m_2 V_2 - m_1 V_1$$

$$-F_{x} = 23.44 \times (-11.94 - 11.94) - \left[(165 - 101) \times 10^{3} + (134 - 101) \times 10^{3} \right] \times \frac{\pi}{4} \times 0.05^{2}$$

$$F_x = 750.206 \text{ N } (\leftarrow)$$

Along y-direction

$$\sum F_y = \dot{m} \left(V_{2y} - V_{1y} \right)$$

$$\Rightarrow F_y - m_{\text{fluid}} \times g = 0$$

$$F_y = m_{\text{fluid}} \times g$$

$$= \frac{\pi}{4} \times (0.05)^2 \times (7.5) \times 1000 \times 9.81$$

$$F_y = 144.46 \text{ N } (\uparrow)$$

So, total force that flange has to withstand is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(750)^2 + (144.46)^2}$$
$$= 763.78 \text{ N} \simeq 764 \text{ N}$$

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13. (b)

$$u_1 = \frac{\pi DN}{60} = \frac{\pi \times 1.2 \times 450}{60} = 9\pi \text{ m/s}$$

From inlet velocity triangle,

$$\frac{V_1}{\sin 120^\circ} = \frac{u_1}{\sin 40^\circ}$$

$$V_1 = 9\pi \times \frac{\sin 120^\circ}{\sin 40^\circ} = 38 \text{ m/s}$$

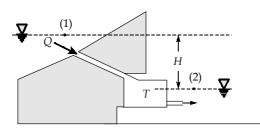
$$V_u = V_1 \cos 20^\circ = 35.7 \text{ m/s};$$

$$V_{w2} = 0 \text{ (Radial exit)}$$

$$V_{f1} = V_1 \sin 20^\circ = 38 \times \sin 20^\circ = 12.99 \approx 13 \text{ m/s}$$
Power developed = $\rho Q(u_1 V_{w1} - u_2 V_{w2})$
= $10^3 \times (0.4 \times 13) \times (9\pi) \times 35.7$
= 5250 kW

14. (c)

So,



Applying Bernoulli's equation point (1) and (2)

$$\frac{P_{1}}{\rho g} + \frac{V_{1}^{2}}{2g} + H = \frac{P_{2}}{\rho g} + \frac{V_{2}^{2}}{2g} + 0 + h_{f} + h_{turbine}$$

$$P_{1} = P_{2} = P_{\text{atm}}$$

$$V_{1} \simeq V_{2} \simeq 0$$
Now,
$$h_{\text{turbine}} = H - h_{f} = H - CQ^{3}$$
So,
$$Power, P = \rho gQh_{\text{turbine}} = \rho gQ (H - CQ^{3})$$

For maximum power,

$$\frac{\partial P}{\partial Q} = \rho g H - 4\rho g Q^3 C = 0$$

$$Q = \sqrt[3]{\frac{H}{4C}} \text{ or } \left(\frac{H}{4C}\right)^{1/3}$$

$$P_{\text{max}} = \rho g Q \left(H - C\left\{\left(\frac{H}{4C}\right)^{1/3}\right\}^3\right)$$

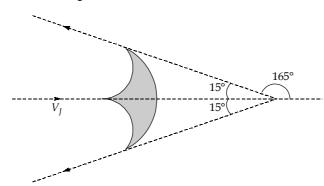
$$P_{\text{max}} = \rho g Q \left(H - \frac{H}{4}\right) = \frac{3}{4}\rho g Q H$$

15.

and

(d) Discharge from jet
$$(Q) = \frac{6.116}{3600} = 1.6989 \times 10^{-3} \text{ m}^3/\text{s}$$
 Now, velocity of jet $= \frac{Q}{\text{Area of jet}}$

$$V_J = \frac{1.6989 \times 10^{-3}}{\frac{\pi}{4} (20 \times 10^{-3})^2} = 5.40 \text{ m/s}$$



For theoretical force it is assumed that velocity is not reduced as water passes around each cup,

$$F_{\text{theoretical}} = \dot{m}(\Delta V) = (\rho \dot{Q})(V_J (1 - \cos 165^\circ))$$
$$= 1000 \times 1.6989 \times 10^{-3} (5.40(1 - \cos 165^\circ)) = 18.035 \text{ N}$$

$$\therefore \frac{\text{Actual force}}{\text{Theoretical force}} = \frac{15}{18.035} = 0.8316$$

If V_1 is the actual velocity at outlet,

Now, Actual force =
$$\dot{m}(\Delta V_a)$$

$$15 = \rho \dot{Q} \left(V_j - V_1 \cos(165^\circ) \right)$$

$$15 = 1000 \times 1.6989 \times 10^{-3} (5.40 - V_1 (\cos 165^\circ))$$

$$V_1 = 3.55 \,\mathrm{m/s}$$

$$\therefore \frac{\text{Outlet velocity}}{\text{Inlet velocity}} = \frac{3.55}{5.40} = 0.6574$$

$$R_1 = 2 \text{ m}, R_2 = 1 \text{ m}$$

$$\therefore$$
 Torque = 0

$$\Rightarrow \qquad T = \dot{m} \left(V_{w_2} R_2 - V_{w_1} R_1 \right) = 0$$

$$\Rightarrow \qquad \qquad V_{w_2}R_2 \;\; = \;\; V_{w_1}R_1 \qquad \qquad ...(\mathrm{i})$$

Now, from velocity triangles,

$$V_{w_1} = V_1 \sin 60^\circ = 20 \left(\frac{\sqrt{3}}{2} \right) = 10\sqrt{3}$$

and

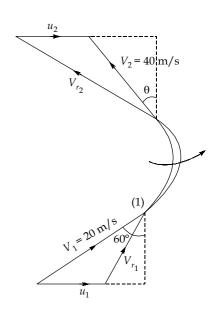
$$V_{w_2} = V_2 \sin \theta = 40 \sin \theta$$

Putting values of V_{w_1} and V_{w_2} in equation (i), we get

$$(40\sin\theta)(1) = 10\sqrt{3}(2)$$

$$\sin\theta = \frac{20\sqrt{3}}{40} = \frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^{\circ}$$



17. (a

Given:
$$Q = 0.05 \text{ m}^3/\text{s}$$
, $H = 30 \text{ m}$, $d = 15 \text{ cm} = 0.15 \text{ m}$, $l = 150 \text{ m}$, $f' = 0.014$, $\eta_o = 0.75$.

$$\therefore$$
 \Rightarrow

$$Q = A \times V$$
$$V = 2.83 \text{ m/s}$$

Manometric head,

$$H_{m} = H + \frac{4fLV^{2}}{2gD} + \frac{V^{2}}{2g} = 30 + \frac{4 \times 0.014 \times 150 \times (2.83)^{2}}{2 \times 9.81 \times 0.15} + \frac{(2.83)^{2}}{2 \times 9.81}$$

$$= 53.267 \text{ m}$$

$$\eta_{o} = \frac{\text{Water power}}{\text{Shaft power}}$$

or,
$$0.75 = \frac{\rho g Q H_m}{SP}$$

$$SP = \frac{1000 \times 9.81 \times 0.05 \times 53.267}{0.75} = 34.84 \text{ kW}$$

18. (a)

:.

Power =
$$T_1 \omega_1$$

$$75000 = T_1 \times \frac{2\pi \times 210}{60}$$

$$T_1 = 3410.463 \text{ Nm}$$

As we know,

Unit torque,
$$T_u = \frac{T}{H}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{H_2}{H_1}$$

$$\frac{T_2}{3410.463} = \frac{10}{5}$$

$$T_2 = 6820.926 \text{ Nm}$$

19. (c)

$$\text{Manometric efficiency, } \eta_{\text{m}} = \frac{gH_{m}}{V_{w2}u_{2}}$$

$$\Rightarrow \qquad 0.75 = \frac{g \times 30}{V_{w2} u_2}$$

Total head,
$$\frac{V_{w2}u_2}{g} = \frac{30}{0.75} = 40 \text{ m}$$

Pressure rise through the impeller $\left(\frac{p_2 - p_1}{\rho g}\right)$ is 65% of the total head developed by the pump.

Thus,
$$\frac{p_2 - p_1}{\rho g} = 0.65 \times \frac{V_{w2} u_2}{g} = 0.65 \times 40 = 26 \text{ m}$$

Energy equation between the inlet and outlet tip of the impeller,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + \frac{V_{w2}u_2}{g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow \frac{p_2 - p_1}{\rho g} = \frac{V_f^2}{2g} + \frac{V_{w2}u_2}{g} - \frac{V_2^2}{2g}$$

$$\Rightarrow 26 = \frac{3^2}{2 \times 9.81} + 40 - \frac{V_2^2}{2g}$$
(: $z_1 = z_2$; $V_1 = V_f$)

$$\Rightarrow \frac{V_2^2}{2g} = \frac{3^2}{2 \times 9.81} + 40 - 26$$

$$V_2 = 16.8428 \text{ m/s} = \sqrt{V_{f2}^2 + V_{w2}^2} = \sqrt{3^2 + V_{w2}^2}$$

 $V_{\omega 2} = 16.5735 \text{ m/s}$

20. (a) Given,

$$H = 24.5 \text{ m}$$

 $Q = 10.1 \text{ m}^3/\text{s}$
 $N = 4 \text{ rev/sec} = 4 \times 60 = 240 \text{ rpm}$
 $\eta_0 = 0.90$

Power generated

$$= \rho gH \times 0.9 \times Q$$

$$= 1000 \times 9.81 \times 10.1 \times 24.5 \times 0.9 = 2184.74 \text{ kW}$$

$$N_S = \frac{N\sqrt{P}}{H^{5/4}} = \frac{240\sqrt{2184.7}}{(24.5)^{5/4}} = 205.80$$

Types of turbine Specific speed (S.I.)

Pelton wheel with single jet 8.5 to 30 Pelton wheel with two or more jets 30 to 51

Francis turbine 51 to 225

Kaplan or propeller turbine 255 to 860

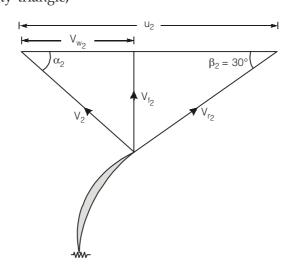
Hence, turbine is Francis.

21. (a)

 $u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1200}{60} = 18.85 \text{ m/s}$ At the outlet

$$V_{f2} = 2.0 \text{ m/s} \text{ and } \beta_2 = 30^{\circ}$$

From the outlet velocity triangle,



$$\tan \beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 30^{\circ} = \frac{2.0}{18.85 - V_{w_2}}$$

18.85 –
$$V_{w_2}$$
 = 3.464; and hence V_{w_2} = 15.386 m/s

Manometric efficiency

$$\eta_m = \frac{gH}{u_2 V_{w_2}}$$

Head developed,

$$H = \eta_m \frac{u_2 V_{w_2}}{g} = \frac{0.85 \times 18.85 \times 15.386}{9.81} = 25.13 \text{ m}$$

22. (d)

Net available head, H = 500 (1 - 0.05) = 475 m

Power per jet =
$$\frac{1500}{2}$$
 = 750 kW

Specific speed
$$(N_s) = \frac{N\sqrt{P}}{H^{5/4}}$$

$$\Rightarrow \qquad 15 = \frac{N\sqrt{750}}{(475)^{5/4}}$$

$$\Rightarrow$$
 $N = 1214.58 \text{ rpm}$

23. (a)

$$\sigma_c = \frac{(NPSH)_{min}}{H}$$

$$0.12 = \frac{(\text{NPSH})_{\text{min}}}{30}$$

Minimum NPSH = 3.6 m

NPSH =
$$\frac{(P_{\text{atm}})_{\text{abs}}}{\gamma} - \frac{P_v}{\gamma} - z_s - h_L$$

where, z_s = Elevation of the pump above the rump water surface, $(z_s)_{\rm max}$ corresponds to σ_c .

Hence,

$$(z_s)_{\text{max}} = \frac{(P_{\text{atm}})}{\gamma} - \frac{P_v}{\gamma} - h_L - (\text{NPSH})_{\text{min}}$$

= $\frac{96.0}{9.79} - \frac{3.0}{9.79} - 0.3 - 3.6 = 5.6 \text{ m}$

24. (d)

$$N_S = \frac{N\sqrt{P}}{(H)^{5/4}} = \frac{100\sqrt{6400}}{(10)^{5/4}} = 450$$

$$N_S(450) > 250$$

Turbine is Kaplan.

25. (a)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L + h_T$$

$$0 + 0 + 95 = 0 + \frac{(1.5)^2}{2 \times 10} + 0 + 20 + h_T$$

$$h_T = 95 - 0.1125 - 20 = 74.8875 \text{ m}$$

The work done by the fluid on a turbine is positive. The power extracted is

 $\Delta x = 62.62 \text{ mm} = 6.262 \text{ cm}$

$$P = \rho g Q h_T = 1000 \times 10 \times 25 \times 74.8875$$

= 18.72185 MW

26.

The force on the jet is upward and to the left. The leftward component is supplied by the compression spring.

$$F_x = \rho Q[(V_x)_2 - (V_x)_1]$$

$$= 1000 \times \left(\frac{\pi}{4} \times (0.035)^2 \times 20\right) \times (20\cos 45^\circ - 20)$$

$$= -112.7185 \text{ N}$$

$$F_x = -k\Delta x$$

$$-112.7185 = -1.8 \times 1000 (\Delta x)$$

By Hooke's law,

27. (b)

Hydraulic efficiency,
$$\eta_h = \frac{v_{\omega_1}u_1 - v_{\omega_2}u_2}{gH}$$
 [1, 2 \rightarrow Inlet and outlet respectively]
$$0.83 = \frac{4.25\sqrt{H}u_1 - 0.35\sqrt{H}u_2}{9.81H}$$

$$u_1 = u$$

$$u_2 = 0.7u$$

$$0.83 = \frac{4.25\sqrt{H}u - 0.35\sqrt{H}(0.7)u}{9.81H}$$

$$u = \frac{0.83 \times 9.81\sqrt{H}}{4.25 - 0.35 \times 0.7} = 2.03\sqrt{H} = u_1$$

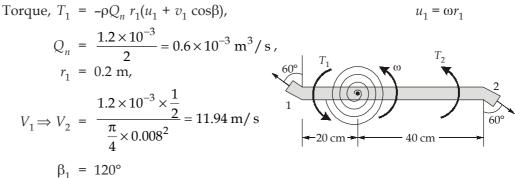
From inlet velocity triangle, we have

 $u_2 = 1.423\sqrt{H}$

$$\tan\theta = \frac{V_{f1}}{(V_{w1} - u_1)} = \frac{1.05\sqrt{H}}{4.25\sqrt{H} - 2.03\sqrt{H}} = 0.47297$$

 $\theta = 25.31^{\circ}$

28 (c)



$$\beta_1 = 120^{\circ}$$

$$T_1 = -10^3 \times 0.6 \times 10^{-3} \times 0.2 \times [\omega \times 0.2 + 11.94 \cos 120^{\circ}]$$

$$T_1 = -0.12 [0.2\omega - 5.97]$$

and

$$\begin{split} T_2 &= -\rho Q_n r_2 [u_2 + V_2 \mathrm{cos}\beta_2] \\ &= -10^3 \times 0.6 \times 10^{-3} \times 0.4 \; [\omega \times 0.4 + 11.94 \; \mathrm{cos}120^\circ] \\ &= -0.24 [0.4\omega - 5.97] \end{split}$$

Total torque,
$$T=T_1+T_2=-\{0.12(0.2\omega-5.97)+0.24(0.4\omega-5.97)\}$$

= $-(0.024\omega-0.7164+0.096\omega-1.4328)$
 $T=-(0.12\omega-2.1492)$

Now,

$$\omega = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

$$T = 2.1492 - 0.12 \times 10.47 \Rightarrow 0.8926 \text{ N-m}$$

$$\theta = \frac{0.8926 (N-m)}{2(N-m/rad)} = 0.4463 \text{ rad/s} = 25.57^{\circ}$$

29. (b)

> Power developed $P \propto Q \propto d^2$ where, d = diameter of the jet Required jet diameter,

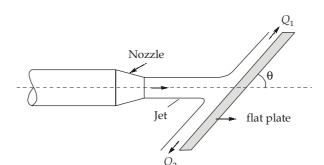
$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

$$\frac{P}{(1-0.64)P} = \frac{(150)^2}{d_2^2}$$

$$\frac{1}{0.36} = \left(\frac{150}{d_2}\right)^2$$

$$d_2 = 150\sqrt{0.36}$$

$$d_2 = 90 \text{ mm}$$



Since the plate surface is smooth, there can be no force exerted by the plate on the fluid jet in the tangential direction.

$$F_{t} = \rho Q_{1}v - \rho Q_{2}v - \rho Qv \times \cos\theta = 0$$

$$Q_{1} - Q_{2} = Q\cos\theta \qquad ...(1)$$

$$Q = Q_{1} + Q_{2} \qquad ...(2)$$

Solving eq. (1) and (2), one jets

if $\theta = 26^{\circ}$

$$Q_1 = \frac{Q}{2}(1 + \cos \theta)$$
$$Q_2 = \frac{Q}{2}(1 - \cos \theta)$$

$$Q_2 = \frac{2}{2}(1 - \cos\theta)$$

$$\frac{Q_1}{Q_2} = \frac{1 + \cos \theta}{1 - \cos \theta}$$
$$= \frac{1 + \cos 26^{\circ}}{1 + \cos 26^{\circ}}$$

$$\frac{Q_1}{Q_2} = 18.76$$