CLASS TEST

S.No.: 01_SK_ECEE_10082025

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SIGNAL & SYSTEM

EC-EE

Date of Test : 10/08/2025

ANSWER KEY

1.	(b)	7.	(a)	13.	(b)	19.	(a)	25.	(a)
2.	(c)	8.	(d)	14.	(d)	20.	(d)	26.	(c)
3.	(b)	9.	(a)	15.	(d)	21.	(a)	27.	(d)
4.	(b)	10.	(b)	16.	(b)	22.	(a)	28.	(b)
5.	(d)	11.	(c)	17.	(a)	23.	(d)	29.	(c)
6.	(a)	12.	(c)	18.	(b)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

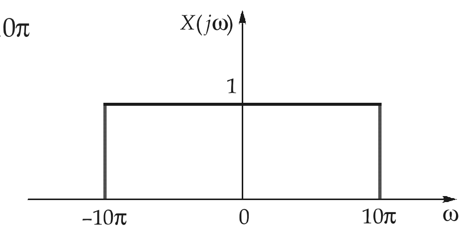
1. **(b)**

Given, $$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or



∴ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

∴ $$T_s < \frac{1}{10}$$

2. **(c)**

We know that,

$$a_n = \frac{2}{T}\int_T x(t)(\cos\omega_0 nt)\,dt = \frac{2}{T}\int_T x(t)\left[\frac{e^{j\omega_0 nt} + e^{-j\omega_0 nt}}{2}\right]dt$$

$$= \frac{1}{T}\left[\int_T x(t)e^{j\omega_0 nt}dt + \int_T x(t)e^{-j\omega_0 nt}dt\right]$$

$a_n = C_n + C_{-n}$ [By the definition of exponential Fourier series coefficient].

3. **(b)**

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} [f(t)\cos\omega t - jf(t)\sin\omega t]\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt$$

$f(t) \Rightarrow$ even signal

$f(t)\cos\omega t \Rightarrow$ even signal

$f(t)\sin\omega t \Rightarrow$ odd signal

$$\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt = 0$$

$$\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt = 2\int_0^{\infty} f(t)\cos\omega t\,dt$$

∴ $$F(\omega) = 2\int_0^{\infty} f(t)\cos\omega t\,dt$$

4. **(b)**

Given, $$y(t) = \frac{1}{2}x\left(\frac{t}{2}\right) * \delta(t-4) = \frac{1}{2}x\left(\frac{t-4}{2}\right) = \frac{1}{2}x\left(\frac{1}{2}t-2\right)$$

at $t = 2$; $$y(2) = \frac{1}{2}x\left(\frac{1}{2}(2)-2\right) = \frac{1}{2}x(-1)$$

Slope of given signal $x(t)$ is 2.

$$x(-1) = -2$$

$$y(2) = \frac{1}{2}(-2) = -1$$

5. **(d)**

The given sequence of $x[n]$ is finite duration. Hence, the region of convergence is $0 < |z| < \infty$.

6. **(a)**

$$r_{xx}(k) = \sum_{k=-\infty}^{\infty} x[n]\,x[n-k]$$

$$\sum_{k=-4}^{k=4} r_{xx}(k) = x[n]\,x[n+3] + x[n]\,x[n+2] + x[n]\,x[n+1] + x[n]\,x[n] + x[n]\,x[n-1]$$

$$+ x[n]\,x[n-2] + x[n]\,x[n-3]$$

$$= -2 - 5 + 2 + 10 + 2 - 5 - 2 = 0$$

7. **(a)**

By Parsevals theorem,

Energy of a signal $x[n]$ is, $E = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})^2\right| d\omega$

where $X(e^{j\omega})$ is discrete time Fourier transform of $x[n]$,

So, $$\frac{\sin(2n)}{\pi n} \xleftrightarrow{\text{DTFT}} X(e^{j\omega}) = \begin{cases} 1\,; & |\omega| \le 2 \\ 0\,; & \text{otherwise} \end{cases}$$

$\therefore$ $$\text{Energy, } E = \frac{1}{2\pi}\int_{-2}^{2} 1 \cdot d\omega = \frac{1}{2\pi}[4]$$

$\therefore$ $$E = \frac{2}{\pi} = 0.64 \text{ J}$$

8. **(d)**

Given, DFT signal $x[n]$

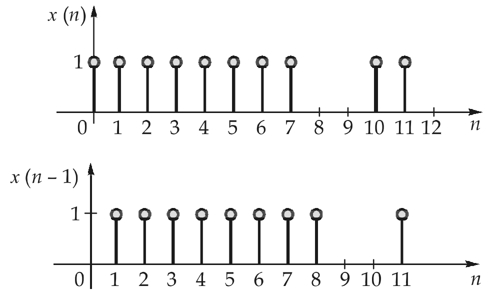
$$\{4, 1+2j, 2-2j, 3+j, -3, 3-j, 2+2j, 1-2j\}$$

$$\sum_{n=0}^{7}(-1)^n x[n] = x[0] - x[1] + x[2] - x[3] + x[4] - x[5] + x[6] - x[7]$$

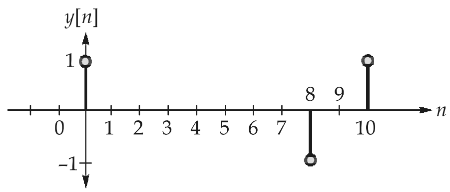
(or)

$$= X\left[\frac{8}{2}\right] = X[4] = -3$$

9. (a)



Subtracting the two signal, we get



$\Rightarrow N = 10$

10. (b)

For a causal system, $h(t) = 0 \quad \text{for } t < 0$

Given, $x(t) = 0 \quad \text{for } t < -1$

$$\therefore \quad y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau$$

$$= \int_{0}^{\infty} h(\tau) x(t-\tau)\, d\tau \qquad \because h(\tau) = 0 \text{ for } \tau < 0$$

$$= \int_{0}^{t+1} h(\tau) x(t-\tau)\, d\tau \qquad \because x(t-\tau) = 0 \text{ for } t-\tau < -1$$

11. (c)

We know that the Laplace transform of

$$\cos(at)\, u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \quad \cos(\pi t)\, u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t-1)]$$
$$= \cos(\pi t)u(t) - \cos\pi t\; u(t-1)$$
$$= \cos\pi t\, u(t) - \cos\pi(t - 1 + 1)u(t-1)$$
$$= \cos\pi t\, u(t) - \cos[\pi(t-1) + \pi]\, u(t-1)$$
$$x(t) = \cos(\pi t)\, u(t) + \cos[\pi(t-1)]\, u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s e^{-s}}{s^2 + \pi^2} \qquad [\because x(t - t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

12. (c)

Given, sinusoidal pulse

$$z(t) = \begin{cases} e^{j10t} ; & |t| < \pi \\ 0 ; & |t| > \pi \end{cases}$$

We may express $z(t)$ as the product of a complex sinusoid e^{j10t} and a rectangular pulse $x(t)$.

Let, $$x(t) = \begin{cases} 1 ; & |t| < \pi \\ 0 ; & |t| > \pi \end{cases}$$

Fourier transform of $x(t)$ is $X(j\omega)$

$$\therefore \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\pi}^{\pi} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\pi}^{\pi}$$

$$= -\frac{1}{j\omega}\left[e^{-j\pi\omega} - e^{+j\omega\pi}\right] = \frac{e^{j\omega\pi} - e^{-j\omega\pi}}{j\omega} = \frac{2}{\omega}\left[\frac{e^{j\omega\pi} - e^{-j\omega\pi}}{2j}\right]$$

$$\therefore \quad X(j\omega) = \frac{2}{\omega}\sin(\omega\pi)$$

By using frequency shift property of Fourier transform, we get,

$$z(t) = e^{j10t}\cdot x(t) \xleftrightarrow{\text{FT}} X(j(\omega - 10))$$

$$\therefore \quad z(t) \xleftrightarrow{\text{FT}} \frac{2}{\omega - 10}\sin((\omega - 10)\pi)$$

13. (b)

Given, $$X(s) = \log(s + 2) - \log(s + 3)$$

Differentiating both the sides with respect to s

$$\frac{d}{ds}X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \text{...(i)}$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

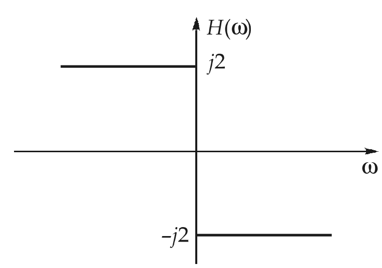
Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or, $$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t}\right]u(t)$$

14. (d)

By redrawing the given frequency response, we get,



We can write $H(\omega) = -j2\,\text{sgn}(\omega)$

We know that,

For $$\text{sgn}(t) \xleftrightarrow{\text{FT}} \frac{2}{j\omega}$$

By duality property

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi\,\text{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi\,\text{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2\,\text{sgn}(\omega)$$

or $= 2(\pi t)^{-1}$

15. (d)

Given $x(t) = \sin(150\pi t)$

Time period, $T = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{150\pi} = \dfrac{1}{75}$ sec

3 time periods $= 3 \times T = 3 \times \dfrac{1}{75} = \dfrac{1}{25}$ sec

∴ The signal sampled at a rate of five samples is $\dfrac{1}{25}$ sec

So, 1 sample in $\dfrac{1}{125}$ sec $= T_s$ [sampling interval]

∴ Sampling frequency $= f_s = \dfrac{1}{T_s} = 125$ samples/sec

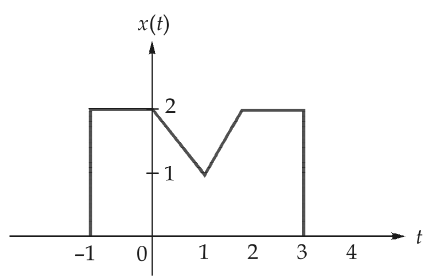
also, Nyquist rate $= f_N = 2f_m = 2 \times 75$ $\quad [\because \omega_m = 150\pi \Rightarrow f_m = 75\text{ Hz}]$

$= 150$ samples/sec

∴ The ratio, $\dfrac{f_s}{f_N} = \dfrac{125}{150} = \dfrac{5}{6} = 0.83$

16. (b)

Given,



By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t}\, d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega$$

$$\therefore \quad \int_{-\infty}^{\infty} X(j\omega)\,d\omega = 2\pi\, x(0) = 2\pi(2) = 4\pi \approx 12.57$$

17. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \dots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

18. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nK}$$

$$g[n] = x[n-2]_{\text{mod N}}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi} X[1]$$

$$G[1] = -X[1] = -7$$

19. (a)

Given,
$$X(z) = \frac{10-8z^{-1}}{2-5z^{-1}+2z^{-2}}$$

$$= \frac{2}{(2-z^{-1})} + \frac{4}{(1-2z^{-1})}$$

$$X(z) = \frac{2z}{2z-1} + \frac{4z}{z-2}$$

$$X(z) = \frac{z}{\left(z-\frac{1}{2}\right)} + \frac{4z}{(z-2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$$\therefore \quad x(1) = \frac{1}{2} = 0.5$$

20. (d)

Given: $x(n) = \{-2, -3, \underset{\uparrow}{-5}, 3, 2\}$

Using Parseval's Theorem,

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega$$

$$\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-2}^{2} |x(n)|^2 = (-2)^2 + (-3)^2 + (-5)^2 + (3)^2 + (2)^2 = 51$$

$$2\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 4\pi \times 51 = 204\pi$$

$$2\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 204\pi$$

21. (a)

Given, signal $x[n] = (0.5)^{2n}\, u[n] = \left(\frac{1}{2}\right)^{2n} u(n) = \left(\frac{1}{4}\right)^n u(n)$

From the definition of DTFT, the signal can be written as,

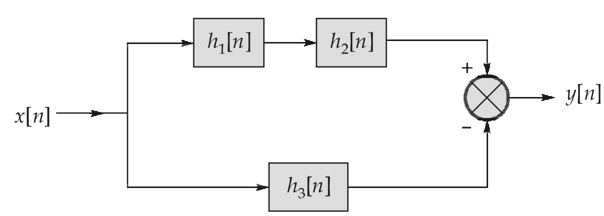
$$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$$

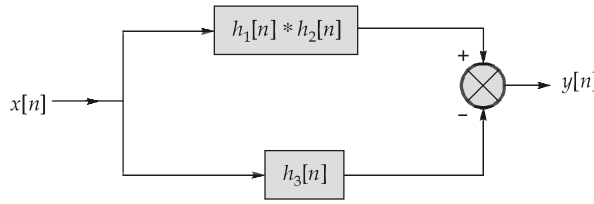
$$X(e^{j\omega}) = \frac{4}{4 - e^{-j\omega}}$$

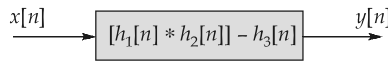
at $\omega = \pi$, $$X(e^{j\pi}) = \frac{4}{4 - \cos\pi} = \frac{4}{4-(-1)} = \frac{4}{5} = 0.8$$

22. (a)

Given,







From the given figure, overall impulse response,

$$\therefore \qquad h[n] = [h_1[n] * h_2[n]] - h_3[n]$$

$$= [u[n+2] * \delta[n-2]] - [u[n] - u[n-2]]$$

$$= u[n] - u[n] + u[n-2] \qquad \{\therefore\ x[n] * \delta[n-n_0] = x[n-n_0]\}$$

$$h[n] = u[n-2]$$

at $n = 2$; $\quad h[2] = u[2-2] = u[0] = 1$

23. (d)

Given, $\quad h(n) = \left(\frac{7}{8}\right)^n u(n)$

Taking DTFT

$$H(e^{j\omega}) = \frac{1}{1-\left(\frac{7}{8}\right)e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{1}{1-\frac{7}{8}\cos\omega + j\frac{7}{8}\sin\omega}$$

$$H(e^{j\omega}) = \frac{1}{\left(1-\frac{7}{8}\cos\omega\right) + j\left(\frac{7}{8}\sin\omega\right)}$$

Let half power frequency bandwidth be ω_0

$$\left|H(e^{j\omega})\right|\Big|_{w=\omega_0} = \frac{1}{\sqrt{2}} H(e^{j\omega})\Big|_{\max(\omega=0)}$$

$$\frac{1}{\sqrt{\left(1-\frac{7}{8}\cos\omega_0\right)^2 + \left(\frac{7}{8}\sin\omega_0\right)^2}} = \frac{1}{\sqrt{2}}\times 8$$

$$1+\left(\frac{7}{8}\cos\omega_0\right)^2 - \frac{7}{4}\cos\omega_0 + \left(\frac{7}{8}\sin\omega_0\right)^2 = \frac{1}{32}$$

$$1+\left(\frac{7}{8}\right)^2 - \frac{7}{4}\cos\omega_0 = \frac{1}{32}$$

$$\cos\omega_0 = \frac{111}{64}\times\frac{4}{7}$$

$$\omega_0 = 0.1337 \text{ radians/sec}$$

24. (d)

Given:
$$x(n) = 3n^3(3)^n\, u(n)$$
$$Y(z) = -\frac{d}{dz}X(z)$$

We know, from the differentiation property of z-transform,

$$nx(n) \xleftrightarrow{Z.T} -z\frac{dX(z)}{dz}$$

Using time shifting property of z-transform,

$$(n-1)x(n-1) \xleftrightarrow{Z.T} \frac{-z^{-1}zdX(z)}{dz}$$

$$y(n) = (n-1)x(n-1) \Longleftrightarrow \frac{-dX(z)}{dz} = Y(z)$$
$$y(n) = (n-1)(3(n-1)^3\, 3^{n-1})u(n-1)$$
$$y(n) = (n-1)^4(3)^{n+1-1}\, u(n-1)$$
$$y(n) = (n-1)^4\,(3)^n u(n-1)$$

25. (a)

The Fourier series representation of signal $x(t)$ is,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where,
$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\left(\frac{\pi}{8}\right)} = 16\text{ rad/sec}$$

When $x(t)$ is passed through given low-pass filter,

the output is,
$$y(t) = \sum_{n=-\infty}^{\infty} C_n H(jn\omega_0)e^{jn\omega_0 t}$$

∴ The largest value of 'n' for which $y(t)$ is non-zero is given by,

$$n\omega_0 \le 170$$
$$16n \le 170$$
$$n \le 10.6$$

∴ maximum harmonics present at the output of filter is 10.

26. (c)

$$x(t) = \underbrace{4\cos\left(\frac{2\pi}{3}t + 40^\circ\right)}_{x_1(t)} + \underbrace{3\sin\left(\frac{4\pi}{5}t + 20^\circ\right)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{3} \quad \Rightarrow \quad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

$$\omega_2 = \frac{4\pi}{5} \quad \Rightarrow \quad T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$$

$$T = \text{LCM of } (T_1, T_2)$$

$$T = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\Rightarrow \quad T = \frac{3 \times 5}{1} = 15 \text{ sec}$$

27. (d)

Given, $x[n]$ is a causal sequence,

$\therefore$ $x[n]$ will be zero for $n < 0$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

and $$h[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$h[n] = \begin{cases} \left(\frac{1}{4}\right)^n ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$$

$\therefore$ $h[0] = 1, \quad h[1] = \frac{1}{4}, h[2] = \frac{1}{16}$

$$y[0] = x[0]\, h[0] + x[1]\, h[-1] + ...$$

$$\frac{1}{2} = x[0]\,(1) + x[1] \times 0$$

$$\therefore \quad x[0] = \frac{1}{2}$$

$$y[1] = x[0]\, h[1] + x[1]\, h[0] + x[2]\, h[-1] + ...$$

$$1 = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + x[1]\,[1] + x[2]\,[0]$$

$$1 - \frac{1}{8} = x[1]$$

$$\therefore \quad x[1] = \frac{7}{8} = 0.875$$

28. (b)

Given, $$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Now, we may write $h[n] - Ah[n-1] = \delta[n]$

as $$\left(\frac{1}{2}\right)^n u[n] - A\left(\frac{1}{2}\right)^{n-1} u[n-1] = \delta[n]$$

Substitute $n = 1$, we get,

$$\left(\frac{1}{2}\right)^1 u[1] - A\left(\frac{1}{2}\right)^{1-1} u[1-1] = \delta[1]$$

$$\Rightarrow \quad \frac{1}{2} - A = 0 \qquad (\because \delta[1] = 0)$$

$$\therefore \quad A = \frac{1}{2}$$

29. **(c)**

Using Parseval's theorem, the energy of signal $x(n)$ is,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{N}\sum_{K=0}^{N-1} |X(K)|^2$$

Here, $X(0) = 3, X(1) = 2 + j; X(2) = j$

Using conjugate property of DFT,

$$X(K) = X^*(N-K)$$
$$N = 4$$
$$X(3) = X^*(4-3)$$
$$= X^*(1)$$
$$X(3) = 2 - j$$

$$E = \frac{1}{4}\left[\left|X(0)^2\right| + |X(1)|^2 + |X(2)|^2 + |X(3)|^2\right]$$

$$= \frac{1}{4}\left[3^2 + (4+1) + 1 + (4+1)\right]$$

$$= \frac{1}{4}[9+5+1+5]$$

$$E = 5 \text{ Joules}$$

30. **(c)**

$$X(z) = \frac{z\left(z-\frac{1}{3}\right)}{(z-1)(z+2)}$$

$$\frac{X(z)}{z} = \frac{\left(z-\frac{1}{3}\right)}{(z-1)(z+2)}$$

Using partial fraction expansion,

$$\frac{X(z)}{z} = \frac{\frac{2}{3}}{3(z-1)} + \frac{\frac{7}{3}}{3(z+2)}$$

$$X(z) = \frac{2}{9}\left(\frac{z}{z-1}\right) + \frac{7}{9}\left(\frac{z}{z+2}\right)$$

ROC : $|z| > 2$

$\therefore$ $x(n)$ is a right-sided signal.

Taking inverse z-transform of $X(z)$,

$$x(n) = \frac{2}{9}u(n) + \frac{7}{9}(-2)^n u(n)$$

At $n = 2$, $$x(2) = \frac{2}{9} + \frac{7}{9}(-2)^2 = \frac{2}{9} + \frac{7}{9} \times 4 = \frac{30}{9}$$

$$x(2) = 3.33$$

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