CLASS TEST

Sl.: 02SKME_JKLMNO_02092025

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FLUID MECHANICS

MECHANICAL ENGINEERING

Date of Test : 02/09/2025

ANSWER KEY

1.	(d)	7.	(a)	13.	(a)	19.	(c)	25.	(b)
2.	(b)	8.	(d)	14.	(b)	20.	(b)	26.	(d)
3.	(b)	9.	(b)	15.	(a)	21.	(b)	27.	(a)
4.	(c)	10.	(c)	16.	(b)	22.	(b)	28.	(b)
5.	(d)	11.	(d)	17.	(a)	23.	(d)	29.	(d)
6.	(b)	12.	(b)	18.	(a)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. **(d)**

Total energy of a flowing fluid can be represented in terms of head which is given by $\left(\frac{P}{\rho g}+\frac{V^2}{2g}+Z=\text{Constant}\right)$.

Piezometric head is the sum of pressure head and datum head and it is given by $\left(\frac{P}{\rho g}+Z\right)$.

The pressure at any point in a static fluid is obtained by hydrostatic law which is given by $P = -\rho gh$, where h is the height of the point from the free surface. As we go down h is negative so the pressure gets increased and datum gets decreased.

Therefore, Piezometric head remains constant at all points in the liquid.

2. **(b)**

As for laminar flow,

Boundary layer thickness $(\delta) \propto \frac{1}{\sqrt{\text{Re}}}$

As the free stream, Speed $\uparrow\uparrow$, $\delta\downarrow\downarrow$

For turbulent flow,

Boundary layer thickness $(\delta) \propto \frac{1}{(\text{Re})^{1/5}}$

As the free stream velocity $\uparrow\uparrow$, $\delta\downarrow\downarrow$ and it also depending on the kinematic viscosity $\delta\uparrow\uparrow$ as kinematic viscosity $(\nu)\uparrow$.

3. **(b)**

For parallel pipes, head loss through the pipe is equal,

$$h_{f_1} = h_{f_2}$$

$$\Rightarrow \quad \frac{f_1 L_1 V_1^2}{2gd_1} = \frac{f_2 L_2 V_2^2}{2gd_2}$$

$$\Rightarrow \quad \frac{500\times(0.5)}{0.3\times 800}\times 2\times 9.81\times 0.35 = V_2^2 \qquad \left(\frac{V_1^2}{2g}=0.5\text{ m}\right)$$

$$\Rightarrow \quad V_2 = 2.674\text{ m/s}$$

Discharge through pipe 2,

$$Q = A_2V_2$$

$$= \frac{\pi}{4}(0.35)^2\times 2.476 = 0.2573\text{ m}^3/\text{s}$$

4. **(c)**

Minor due to sudden expansion from 6 cm diameter pipe to 12 cm is given by

$$(h_f)_{\text{expansion}} = \frac{V_1^2}{2g}\left[1-\frac{A_1}{A_2}\right]^2$$

$$= \frac{V_1^2}{2g} \times \left[1 - \frac{d_1^2}{d_2^2}\right]^2 = \frac{V_1}{2g} \times \left(1 - \left(\frac{1}{2}\right)^2\right)^2$$

$$= \frac{9}{16} \frac{V_1^2}{2g}$$

5. (d)

As we know, the average velocity in fully developed laminar pipe flow is

$$V_{avg} = \frac{1}{2} V_{max}$$

$$V_{max} = 2V_{avg} = 2 \times 2 = 4.0 \text{ m/s}$$

6. (b)

Conservation of mass,

$$\dot{m}_{in} - m_{out} = \left.\frac{d\dot{m}}{dt}\right|_{tank}$$

$$\Rightarrow \quad \rho A V_1 - \rho A V_2 = \rho \times \frac{\pi}{4} D^2 \times \frac{dh}{dt}$$

$$\Rightarrow \quad (0.12)^2 \times [2.5 - 1.9] = (0.75)^2 \times \frac{dh}{dt}$$

$$\Rightarrow \quad \frac{dh}{dt} = 0.01536 \text{ m/s}$$

So, time required to fill remaining tank,

$$t = \frac{1 - 0.3}{0.01536} s$$

$$\Rightarrow \quad t = 45.57 s$$

7. (a)

Given: $\psi = 2y(x^2 - y^2)$

Since, $$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}\left(2y\left(x^2 - y^2\right)\right)$$

$$v = 2y\,(2x) = 4xy$$

and, $$u = \frac{-\partial \psi}{\partial y} = \frac{-\partial}{\partial y}\left(2y(x^2 - y^2)\right)$$

$$u = -[2y\,(-2y) + (x^2 - y^2) \cdot 2]$$

$$= 4y^2 - 2x^2 + 2y^2$$

$$= 6y^2 - 2x^2$$

Thus, velocity field is

$$\vec{V} = u\hat{i} + v\hat{j}$$

$$\vec{V} = \left(6y^2 - 2x^2\right)\hat{i} + 4xy\,\hat{j}$$

8. (d)

Continuity equation, $Q_1 = Q_2 + Q_3$

Now, $Q_2 = A_2V_2 = 0.008V_2$

$Q_3 = A_3V_3 = 0.004V_3 = 0.004 \times 2V_2$ [Given, $V_3 = 2V_2$]

$= 0.008V_2$

Now, $Q_2 = Q_3$

$\therefore \quad Q_3 = \dfrac{Q_1}{2} = \dfrac{0.3}{2} = 0.15\ \text{m}^3/\text{s}$

9. (b)

By force equilibrium at X-X

$$P_{atm}A + F = (P_{atm} + \rho_L gH)A$$

$$F = \rho_L gHA$$

10. (c)

$$F_V = \rho g V$$

where, V = Volume of gate

As upward force and downward force will cancel out each other and net force is due to the volume of gate

Volume of gate, V = Area of semi circle × Width of gate

$$V = \frac{\pi}{2} \times R^2 \times w$$

$$V = \frac{\pi}{2} \times 1^2 \times 1$$

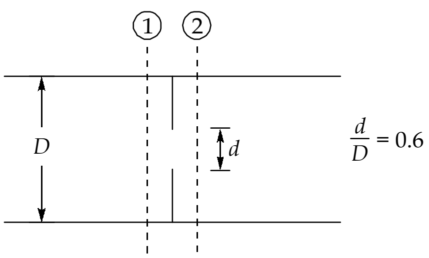
$$V = \frac{\pi}{2}$$

Now, $F_V = 1000 \times 10 \times \dfrac{\pi}{2}$

$= 15707.96$ N

$= 15.707$ kN ≈ 15.71 kN

11. (d)



Applying Bernoulli's equation across the orifice,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

From continuity equation,

$$A_1V_1 = A_2V_2$$

$$\Rightarrow \quad D^2V_1 = d^2V_2$$

$$V_1 = (0.6)^2V_2$$

$$\Rightarrow \quad V_1 = 0.36V_2$$

$$\frac{43.5\times10^3}{10^3\times g} = \frac{V_2^2-(0.36V_2)^2}{2g}$$

$$\Rightarrow \quad V_2^2 = 99.954$$

$$V_2 = 9.997 \text{ m/s}$$

$$Q_{\text{theoretical}} = A_{\text{orifice}} \times V_2$$

$$= \frac{\pi}{4}\times\left(\frac{20}{\sqrt{\pi}}\right)^2\times10^{-6}\times9.997$$

$$= 9.997 \times 10^{-4} \text{ m}^3/\text{s}$$

Discharge coefficient, $c_d = \dfrac{Q_{act}}{Q_{theoretical}} = \dfrac{3\times10^{-4}}{9.997\times10^{-4}}$

$$= 0.3$$

12. (b)

From mass conservation for the control volume.

$$m_{\text{in}} = \dot{m}_{out}$$

$$\dot{m}_{AB} = \dot{m}_{BC} + \dot{m}_{AD} + \dot{m}_{CD}$$

$$\dot{m}_{BC} = \rho\times\delta\cdot U_\infty - \rho\times L\times0.1U_\infty - \int_0^\delta \rho\, U_\infty\left[\left(\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3\times dy\right)\right]$$

$$\dot{m}_{BC} = U_\infty\cdot\rho\,[\delta-0.1L]-\rho\, U_\infty\left(\frac{3}{4}\delta-\frac{1}{8}\delta\right)$$

$$= U_\infty\cdot\rho\left[\delta-0.1L-\frac{5\delta}{8}\right]$$

$$= U_\infty\cdot\rho\left[\frac{3\delta}{8}-0.1L\right]$$

13. (a)

For dynamic similarity,

$$(Re)_m = (Re)_p$$

$$\Rightarrow \quad \left(\frac{\rho VD}{\mu}\right)_m = \left(\frac{\rho VD}{\mu}\right)_p$$

$$\Rightarrow \quad \frac{1000\times3\times0.15}{0.001} = \frac{1.2\times V\times2}{1.7\times10^{-5}}$$

$$V = 3.187 \text{ m/s}$$

For dynamic similarity,

C_D will be same.

$$\left(\frac{F_D}{\rho A U^2}\right)_m = \left(\frac{F_D}{\rho A U^2}\right)_p$$

$$\Rightarrow \frac{5}{1000 \times (0.15)^2 \times 3^2 \times \frac{\pi}{4}} = \frac{F_P}{1.2 \times (2)^2 \times \frac{\pi}{4} \times (3.187)^2}$$

$$F_P = 1.203 \text{ N}$$

14. (b)

$$u_{\max} = 1.5 \text{ m/s}$$

$$\bar{u}_{avg} = \frac{2}{3} \times u_{\max} = \frac{2}{3} \times 1.5 = 1 \text{ m/s}$$

$$\bar{u}_{avg} = -\frac{1}{12\mu}\left(\frac{\partial P}{\partial x}\right) B^2$$

$$\frac{\partial P}{\partial x} = \frac{1 \times 12 \times 0.001}{(0.002)^2} = -3000 \text{ N/m}^3$$

$$\text{Wall shear stress } (\tau_\omega) = \left(\frac{-\partial P}{\partial x}\right)\left(\frac{B}{2}\right) = -(-3000)\left(\frac{0.002}{2}\right) = 3 \text{ N/m}^2$$

15. (a)

$$\frac{dH}{dt} = -C\sqrt{t}$$

$$\int_4^H dH = -C\int_0^t \sqrt{t}\, dt$$

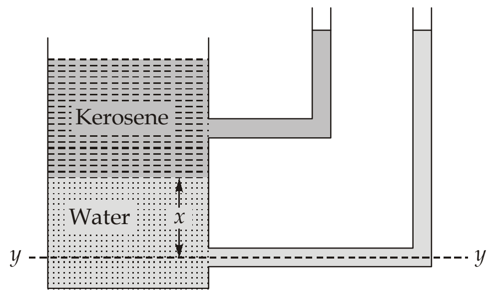
$$(H - 4) = -C\frac{t^{(1/2)+1}}{\frac{1}{2}+1} = -\frac{2C}{3} t^{3/2}$$

$$H = 4 - \frac{2C}{3} t^{3/2}$$

At, t = 0.5 seconds, $H = 4 - \frac{2 \times 0.6}{3} \times 0.5^{3/2} = 3.858 \text{ m}$

16. (b)

Equating pressure on reference y-y

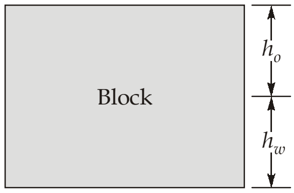


$$(0.5 - x)800g + x \times 1000g = 0.42 \times 1000g$$
$$400 - 800x + 1000x = 420$$
$$200x = 20$$
$$x = 10 \text{ cm}$$

17. (a)

By force equilibrium of block,

$$\text{Weight of block} = \text{Net buoyancy force}$$
$$\rho_b V g = (F_B)_{\text{water}} + (F_B)_{\text{oil}}$$
$$\rho_b A H g = \rho_w A.\, h_w g + \rho_o A,\, h_o g$$
$$800H = 1000h_w + \frac{2}{3} \times 1000h_o$$
$$0.8H = (H - h_o) + \frac{2}{3}h_o$$
$$h_o - \frac{2}{3}h_o = H - 0.8H = 0.2H$$
$$\frac{h_o}{3} = 0.2H$$
$$h_o = 0.6H$$



18. (a)

$$m_{\text{flow}} = \int \rho u\, dA = \int_o^h \rho \times \frac{\rho g \sin\theta}{\mu}\left(hy - \frac{y^2}{2}\right) 1 \cdot dy$$
$$m_{\text{flow}} = \frac{\rho^2 g \sin\theta}{\mu} \int_o^h \left(hy - \frac{y^2}{2}\right) dy$$
$$m_{\text{flow}} = \frac{\rho^2 g \sin\theta}{\mu}\left(\frac{h^3}{2} - \frac{h^3}{6}\right)$$
$$m_{\text{flow}} = \frac{\rho^2 g \sin\theta h^3}{3\mu}$$

19. (c)

$$(\text{Re}) = \frac{v_a D_a}{\upsilon_a} = \frac{v_w D_w}{\upsilon_w}$$

$\therefore$
$$\frac{v_a}{v_w}\left(\frac{D_a}{D_w}\right) = \frac{\upsilon_a}{\upsilon_w} \qquad \text{... (i)}$$

$$\text{Head loss, } h_f = \frac{fLv^2}{2gD}$$

As the pressure drop are same and the pipes are horizontal,

$$h_f = \frac{f_a L_a v_a^2}{2gD_a} = \frac{f_w L_w V_w^2}{2gD_w}$$

As, $L_a = L_w$ and Reynolds number is same $f_a = f_w$

$\therefore$
$$\left(\frac{v_a}{v_w}\right)^2 = \left(\frac{D_a}{D_w}\right) \qquad \text{... (ii)}$$

Substituting in equation (ii)

$$\left(\frac{v_a}{v_w}\right)^3 = \left(\frac{v_a}{v_w}\right)$$

$$\Rightarrow \qquad \left(\frac{v_a}{v_w}\right) = \left(\frac{v_a}{v_w}\right)^{1/3}$$

20. (b)

For the limiting case, apply Bernoulli's equation between these two points:

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

For horizontal pipe, $Z_1 = Z_2$

$$\frac{P_{atm}-\rho g h}{\rho g}-\frac{V_1^2}{2g} = \frac{P_{atm}}{\rho g}+\frac{V_1^2}{2g}$$

$$\frac{V_1^2-V_2^2}{2g} = h$$

$$\Rightarrow \qquad V_1^2 - V_2^2 = 2gh$$

From continuity equation for incompressible flow,

$$A_1V_1 = A_2V_2$$

$$D_1^2V_1 = D_2^2V_2$$

$$V_1^2 = \left(\frac{D_2}{D_1}\right)^4 V_2^2$$

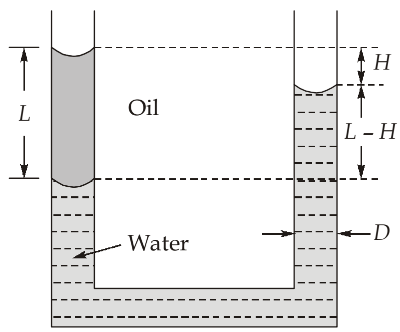
$$V_2^2\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right] = 2gh$$

$$V_2 = \frac{\sqrt{2gh}}{\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]^{1/2}}$$

For velocity, $$V_2 \geq \frac{\sqrt{2gh}}{\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right]^{1/2}}$$

The reservoir liquid will rise in the tube upto the section 1.

21. (b)



Given: D = 6.35 mm, V_{oil} = 3.25 cm^3, SG = 0.827

From pressure equilibrium equation,

$$\rho_{oil} \times g \times L = \rho_{water} \times g \times (L - H)$$

$$H = L(1 - SG_{oil})$$

$$H = L(1 - 0.827)$$

$$V_{oil} = \frac{\pi}{4}D^2 \times L$$

$$L = \frac{V_{oil} \times 4}{\pi D^2} = \frac{3.25 \times 10^3 \times 4}{\pi \times 6.35^2} = 102.623 \text{ mm}$$

$$H = 102.623\ (1 - 0.827) = 17.75 \text{ mm}$$

22. (b)

From dynamic similarity:

$$(\text{Re})_m = (\text{Re})_p$$

$$\Rightarrow \quad \frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\Rightarrow \quad \frac{1000 \times V_m \times D_p}{2 \times 1.01 \times 10^{-3}} = \frac{1.2 \times 60 \times D_p}{1.86 \times 10^{-5}}$$

$$V_m = 7.819 \text{ m/s} = 7.82 \text{ m/s}$$

$$\frac{(F_D)_p}{(F_D)_m} = \frac{C_D \times \frac{1}{2} \times (\rho A U^2)_p}{C_D \times \frac{1}{2} \times (\rho A U^2)_m} = \frac{\rho_p \times D_p^2 \times V_p^2}{\rho_m \times D_m^2 \times V_m^2}$$

$$= \frac{1.2}{1000} \times 4 \times \left(\frac{60}{7.82}\right)^2 = 0.2826$$

$$(F_D)_p = 0.2826 \times 540 = 152.60 \text{ N}$$

23. (d)

Rate of change of density is given by

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}$$

$$= 0 + u_0 e^{-x/L} \frac{\partial}{\partial x}\left(\rho_0 e^{-2x/L}\right)$$

$$= u_0 e^{-x/L} \cdot \rho_0 e^{-2x/L}\left(\frac{-2}{L}\right)$$

$$= \frac{-2\rho_0 u_0}{L} e^{-3x/L}$$

$$\left.\frac{D\rho}{Dt}\right|_{x=\frac{L}{2}} = \frac{-2\rho_0 u_0}{L} e^{-3(L/2)/L} = \frac{-2\rho_0 u_0}{L} e^{-1.5}$$

24. (d)

The cylindrical polar co-ordinate system,

$$a_r = \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}$$

$$a_\theta = \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}$$

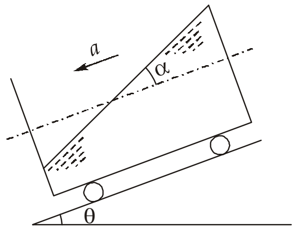
$$a_z = \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z}$$

25. **(b)**

Given, $a = 3\ m/s^2$; $\theta = 30°$

$a_x = a \cos 30° = 2.6\ m/s^2$

$a_y = a \sin 30° = 1.5\ m/s^2$



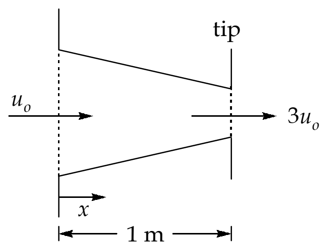
when tank moves with an acceleration down the inclined plane,

$$\tan\alpha = \frac{a_x}{g - a_y} = \frac{2.6}{9.81 - 1.5} = 0.3128$$

$$\Rightarrow \quad \alpha = \tan^{-1}(0.3128) = 17.37°$$

26. **(d)**

At a distance x from the base,



$$u(x) = u_0 + \frac{3u_0 - u_0}{l}(x)$$

$$u(x) = u_0\left(1 + \frac{2x}{l}\right)$$

Now, $\bar{a} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}$ (for 1 – D flow)

For steady flow

$$\Rightarrow \quad \bar{a} = u\frac{du}{dx}$$

$$\Rightarrow \quad \bar{a} = u_0\left(1 + \frac{2x}{l}\right)\left(\frac{d}{dx}u_0\left(1 + \frac{2x}{l}\right)\right)$$

$$\Rightarrow \quad \bar{a} = u_0\left(1 + \frac{2x}{l}\right)\left(\frac{2u_0}{l}\right)$$

$$\bar{a} = \frac{2u_0^2}{l}\left(1 + \frac{2x}{l}\right)$$

at the tip of a nozzle, $x = 1$ m

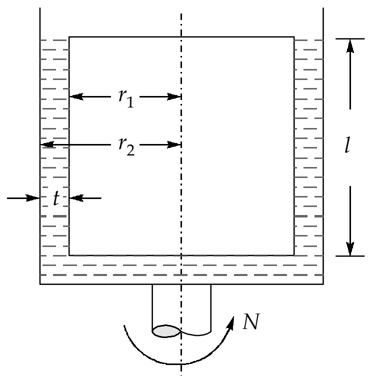
$$\bar{a} = \frac{2(10)^2}{1}\left(1 + \frac{2(1)}{1}\right)$$

$$\bar{a} = 600\ m/s^2$$

27. **(a)**

Given: $r_1 = 4$ cm $= 0.04$ m, $r_2 = 4.2$ cm $= 0.042$ m, $N = 200$ rpm, $l = 8$ cm $= 0.08$ m, $T = 10^{-4}$ Nm

Now, for given cylinder speed, tangential velocity is,



$$u = \frac{2\pi r_2 N}{60} = \frac{2\pi(0.042)(200)}{60}$$

$$u = 0.8796\ m/s$$

Since, linear velocity distribution is given by,

$$\frac{du}{dy} = \frac{u}{t} = \frac{0.8796}{(r_2 - r_1)} = \frac{0.8796}{(0.042 - 0.04)}$$

$$\Rightarrow \quad \frac{du}{dy} = 439.823 \text{ per second}$$

and, $\tau = u\dfrac{du}{dy} = \mu(439.823)$

then, shear force, $F = \tau * \text{area}$

$= \mu(439.823)\ (2\pi r_1 l)$

$\Rightarrow F = \mu(439.823)(2\pi\ (0.04)(0.08))$

$\Rightarrow F = 8.843\ \mu\text{N}$

Viscous torque $(T) = F * \text{radius}$

$T = 8.843\mu * r_1$ (on inner cylinder)

$\Rightarrow 10^{-4} = 8.843\mu * 0.04$

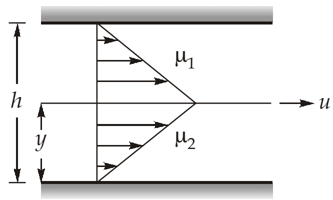
$$\Rightarrow \mu = \frac{10^{-4}}{0.04 \times 8.843}$$

$\mu = 2.827 \times 10^{-4}\ \text{Ns/m}^2$

$\mu = 28.27 \times 10^{-4}$ poise

28. **(b)**

Force on lower side of plate,



$$F_1 = \mu_2 \frac{u}{y} A$$

Force on upper side of plate,

$$F_2 = \mu_1 \frac{u}{(h-y)} A$$

So, total drag force, $F = F_1 + F_2 = \mu_2 \dfrac{u}{y} A + \mu_1 \dfrac{u}{h-y} A$

For drag force to be minimum,

$$\frac{dF}{dy} = 0$$

$$\Rightarrow \frac{-\mu_2 u}{y^2} A + \frac{\mu_1 u A}{(h-y)^2} = 0$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{(h-y)^2}{y^2} = \frac{h^2}{y^2} + 1 - \frac{2h}{y}$$

$$\Rightarrow \frac{h^2}{y^2} - 2\frac{h}{y} + \left(1 - \frac{\mu_1}{\mu_2}\right) = 0$$

Solving quadratic equation for $\dfrac{h}{y}$ we get,

$$\frac{h}{y} = \frac{2 \pm \sqrt{4 - 4(1)\left(1 - \dfrac{\mu_1}{\mu_2}\right)}}{2(1)}$$

$$\Rightarrow \frac{h}{y} = \frac{2 \pm \sqrt{4 - 4 + 4\dfrac{\mu_1}{\mu_2}}}{2} = \frac{2 \pm 2\sqrt{\dfrac{\mu_1}{\mu_2}}}{2}$$

$$\Rightarrow \qquad \frac{h}{y} = 1 \pm \sqrt{\frac{\mu_1}{\mu_2}}, \text{ since } \frac{h}{y} \text{ can not be less than unity,}$$

then
$$\frac{h}{y} = 1 + \sqrt{\frac{\mu_1}{\mu_2}}$$

$$\Rightarrow \qquad y = \frac{h}{1 + \sqrt{\mu_1/\mu_2}}$$

29. (d)

$$U = U_{max}\left(1 - \frac{r}{r_o}\right)^k \qquad \text{...(given)}$$

We know, K.E. correct factor,

$$\alpha = \frac{1}{U_a^3 A}\int_o^{r_o} U^3 dA$$

where, U_a = average velocity

$$U_a = \frac{1}{A}\int_o^{r_o} UdA = \frac{U_{max}}{\pi r_o^2}\int_o^{r_o}\left(1 - \frac{r}{r_o}\right)^k 2\pi r dr \qquad \text{...(i)}$$

Let $1 - \frac{r}{r_o} = z$ so, $\quad dz = \frac{-dr}{r_o}$ & $\frac{r}{r_o} = 1 - z$

Rewriting equation (i),

$$\frac{U_a}{U_{max}} = \frac{r_o^2}{\pi r_o^2}\int_1^o z^k \times 2\pi\left(\frac{r}{r_o}\right)\times\left(\frac{dr}{r_o}\right)$$

$$= 2\int_1^o z^k(1-z)(-dz) = 2\int_o^1 z^k - z^{k+1}.dz$$

$$U_a = U_{max}2\left(\frac{z^{k+1}}{k+1} - \frac{z^{k+2}}{k+2}\right)\Bigg|_o^1 = \frac{2U_{max}}{(k+1)(k+2)}$$

Now,
$$\alpha = \frac{U_{max}^3}{U_a^3 A}\int_o^{r_o}\left(1 - \frac{r}{r_o}\right)^{3k} 2\pi r dr$$

Similarly converting $r \to z$ variable

$$\alpha = \frac{U_{max}^3 2\pi \times r_o^2}{U_a^3 A}\int_o^1 z^{3k} - z^{3k+1}dz$$

$$= \frac{U_{max}^3 2}{U_a^3}\int_o^1 z^{3k} - z^{3k+1}dz = \frac{2U_{max}^3}{U_a^3}\left|\frac{z^{3k+1}}{3k+1} - \frac{z^{3k+2}}{3k+2}\right|_o^1$$

$$= \frac{2U_{max}^3}{U_a^3}\left(\frac{3k+2-3k-1}{(3k+1)(3k+2)}\right) = \frac{2U_{max}^3}{8U_{max}^3}\times\frac{(k+1)^3\times(k+2)^3}{(3k+1)(3k+2)}$$

$$\alpha = \frac{(k+1)^3(k+2)^3}{4(3k+1)(3k+2)}$$

30. **(c)**

Stream function and velocity potential : For a two dimensional, incompressible, irrotational flow, the velocity field can be expressed in terms of both ψ and φ.

$$u = -\frac{\partial \psi}{\partial y};\ v = +\frac{\partial \psi}{\partial x};\ u = -\frac{\partial \phi}{\partial x} \text{ and } v = -\frac{\partial \phi}{\partial y}$$

According to irrotationality condition,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\Rightarrow \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

According to continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

In a flow net, equipotential lines and streamlines are mutually perpendicular to each other.

■■■■