CLASS TEST

Sl.: 01SK_EE_ABCDEFG_280425

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ELECTROMAGNETICS THEORY

ELECTRICAL ENGINEERING

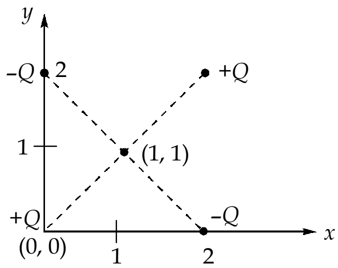
Date of Test : 28/04/2025

ANSWER KEY

1.	(a)	7.	(c)	13.	(c)	19.	(a)	25.	(d)
2.	(c)	8.	(c)	14.	(d)	20.	(c)	26.	(a)
3.	(a)	9.	(a)	15.	(b)	21.	(a)	27.	(b)
4.	(d)	10.	(a)	16.	(c)	22.	(a)	28.	(a)
5.	(d)	11.	(a)	17.	(b)	23.	(c)	29.	(a)
6.	(d)	12.	(d)	18.	(c)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (a)



The two positive charges Q are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produced by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

2. (c)

If the divergence of a given vector is zero, then it is said to be solenoidal.

$$\nabla . \vec{A} = 0$$

By Divergence theorem,

$$\int_V (\nabla . \vec{A}) \text{dv} = \oint_s \vec{A} . \vec{ds}$$

So, for a solenoidal field,

$$\nabla . \vec{A} = 0 \text{ and } \oint_s \vec{A} . \vec{ds} = 0$$

3. (a)

4. (d)

From Biot savart law,

$$\vec{H} = \int_0^{2\pi} \frac{IR d\phi \hat{a}_\phi \times (-\hat{a}_\rho)}{4\pi R^2}$$

$$= \left(\frac{I}{4\pi} \int_0^{2\pi} \frac{R d\phi}{R^2} \right) \hat{a}_z$$

$$\vec{H} = \frac{I}{2R} \hat{a}_z$$

5. (d)

$$\text{Net flux} = \iiint \rho_v \cdot dv = \int_0^{2\pi} \int_0^{\pi} \int_1^{2} \frac{10}{r^2} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 10 \times 1 \times 2 \times 2\pi = 40\pi \text{ mC}$$

6. (d)

$$C = C_1 + C_2$$

$$= 100 + 50$$

$$= 150 \ \mu\text{F}$$

$$\text{Total energy stored} = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(150\times10^{-6})\times10^{6}$$
$$= 75 \text{ Joules}$$

7. **(c)**

Normal component of $\vec{B}$ is continous at the boundary while the normal complenent of $\vec{H}$ is discontinous at the boundary.

i.e. $B_{1n} = B_{2n}$

or $\mu_1 H_{1n} = \mu_2 H_2 n$

Hence (c) is wrong.

8. **(c)**

Two wires carrying current in the same direction attract each other and if the currents are opposite in direction, they repel each other. This can be observed using right hand rule.

9. **(a)**

$$E = -N\frac{d\phi}{dt} = -100(3t^2-2)\times10^{-3}$$
$$= -100(12-2)\times10^{-3} = -1 \text{ V}$$

10. **(a)**

When the spheres are brought into contact with each other, the total charge on them gets redistributed equally since they are identical.

∴ Resultant charge on each sphere,

$$Q = \frac{Q_1+Q_2}{2} = \frac{3.1-0.1}{2} = 1.5 \text{ nC}$$

After the separation the charge distribution remains unchanged.

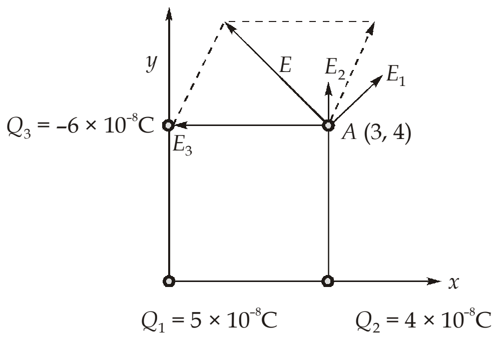
$$|\vec{F}| = \frac{Q_1\times Q_2}{4\pi\varepsilon_0 R^2}$$
$$= \frac{(1.5\times10^{-9})\times(1.5\times10^{-9})}{4\pi\times8.854\times10^{-12}\times(50\times10^{-2})^2}$$
$$|\vec{F}| = 80.89 \text{ nN.}$$

11. **(a)**

$Q_1 = +5\times10^{-8}$C, $Q_2 = +4\times10^{-8}$ C and $Q_3 = -6\times10^{-8}$ C

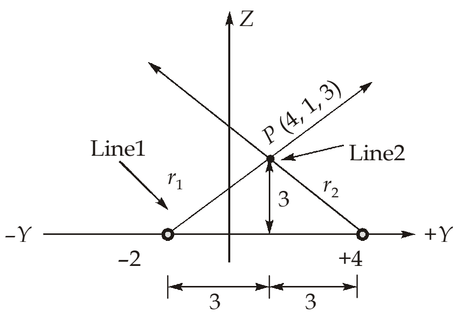
The potential at point A

$$V_A = \frac{Q_1}{4\pi\varepsilon_0 r_1}+\frac{Q_2}{4\pi\varepsilon_0 r_2}+\frac{Q_3}{4\pi\varepsilon_0 r_3}$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-8}}{5}+\frac{4\times10^{-8}}{4}-\frac{6\times10^{-8}}{3}\right]$$
$$= 0$$



12. **(d)**

Let r_1 and r_2 be the directed line segments from the lines 1 and 2 respectively to the point P(in Y-Z plane).



Then, $\quad r_1 = 3\hat{a}_y + 3\hat{a}_z$

and $\quad r_2 = -3\hat{a}_y + 3\hat{a}_z$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r_1}\left(\frac{3\hat{a}_y + 3\hat{a}_z}{r_1}\right) \quad \text{. . . (i)}$$

where, $\quad r_1^2 = 3^2 + 3^2 = r_2^2$ (in magnitude) $= 18 = r^2$

$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}\left(\frac{-3\hat{a}_y + 3\hat{a}_z}{r_2}\right) \quad \text{. . . (ii)}$$

Adding (i) and (ii), we obtain the resultant field

$$E = E_1 + E_2 = \frac{\lambda}{2\pi\varepsilon_0 r^2}(2\times 3\hat{a}_z) \text{ (replacing } r_1 \text{ and } r_2 \text{ by } r)$$

$$= \frac{5\times 10^{-9}}{2\pi\varepsilon_0(18)}(6\hat{a}_z) = 30\hat{a}_z \text{ V/m}$$

13. **(c)**

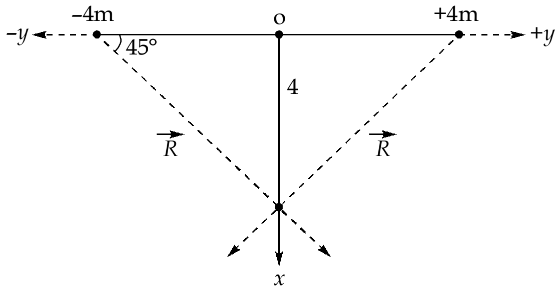
For spherical coordinate systems,

$$\vec{dl} = r\sin\theta d\phi \hat{a}_\phi$$

$$\oint \vec{G}.\vec{dl} = \int_0^{2\pi} 15r\hat{a}_\phi . r\sin\theta d\phi \, \hat{a}_\phi$$

$$= 15 \,.\, r^2 .\sin\theta(2\pi)$$
$$= 15 \,.\, (2)^2 \times \sin 30°(2\pi)$$
$$\oint \vec{G}.\vec{dl} = 60\,\pi$$

14. (d)



The y components of the fields produced by two lines of charge cancel out and only x components will exist in effect.

The resultant field is,

$$\vec{E} = \pm 2 \frac{\lambda}{2\pi\varepsilon_0 |\vec{R}|} . \frac{\vec{R}}{|\vec{R}|}$$

where, $\vec{R} = 4\hat{a}_x$ and $|\vec{R}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ m

$$\vec{E} = \pm 2 \times \frac{4\times 10^{-9}}{2\pi \times 8.854 \times 10^{-12}} \times \frac{4\hat{a}_x}{(4\sqrt{2})^2}$$

$$\vec{E} = \pm 18\hat{a}_x \text{ V/m}$$

15. (b)

According to Gausss's law, the electric flux leaving the surface with R = 5 m is equal to the total flux enclosed by the surface.

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3$$

Ψ_1 = Electric flux leaving the spherical surface with R = 1m

$$= 20 \times 10^{-9} \times (4\,\pi R^2)$$
$$= 20 \times 10^{-9} \times 4\,\pi = 80\,\pi\text{nC}$$
$$\Psi_2 = -9 \times 10^{-9} \times (4\,\pi\,(2)^2)$$
$$= -144\,\pi\text{nC}$$
$$\Psi_3 = 2 \times 10^{-9} \times (4\pi \times 9) = 72\,\pi\text{ nC}$$
$$\Psi = 8\,\pi\text{ nC}$$

16. (c)

The total dielectric flux $= \Psi = Q = CV$

$$= 2 \times 10^{-4} \times 10^{-6} \times 20 \times 10^3 = 4\,\mu\text{C}$$

Potential gradient $= \dfrac{V}{t} = \dfrac{20\times 10^3}{2\times 10^{-3}}$

$$= 10 \text{ MV/m} = 100 \text{ kV/cm.}$$

17. (b)

$$\nabla \times \vec{H} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & (z^2 - x^2) & 3y \end{vmatrix}$$

$$= \hat{i}(3 - 2z) + \hat{k}(-2x - 2)$$

$$= (3 - 2z)\hat{i} - (2x + 2)\hat{k}$$

At the origin , $x = 0$, $z = 0$

$$\nabla \times \vec{H} = 3\hat{i} - 2\hat{k}$$

$$|\nabla - \vec{H}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

18. (c)

The flux Φ_1, at $i_1 = 5$ A is

$$\Phi = B \times A$$

$$= 1 \times (30 \times 10^{-4})$$

$$\Phi_1 = 30 \times 10^{-4} \text{ Wb}$$

$$\Phi_2 \text{ at } i_2 = 10 \text{ A is,}$$

$$\Phi_2 = 1.5 \times 30 \times 10^{-4}$$

Increase of flux when current is increased from 5 to 10 A

$$= 0.5 \times 30 \times 10^{-4} \text{ Wb}$$

$$\frac{d\phi}{dt}(\text{average}) = \frac{0.5 \times 30 \times 10^{-4}}{5}$$

$$\frac{d\phi}{dt}(\text{average}) = 3 \times 10^{-4} \text{ Wb/A}$$

∴ Mean value of inductance,

$$L = N\frac{d\phi}{di}$$

$$L = 2000 \times 3 \times 10^{-4}$$

$$L = 0.6 \text{ Henry.}$$

19. (a)

$$\mu_1 = 2\,\mu_0;$$

$$\mu_2 = 5\,\mu_0$$

$$B_2 = 10\hat{a}_\rho + 15\hat{a}_\phi - 20\hat{a}_z \text{ mWb/m}^2$$

$$B_{1n} = B_{2n} = 15\hat{a}_\phi$$

$$H_{1t} = H_{2t}$$

$$B_{1t} = \frac{\mu_1}{\mu_2}B_{2t} = \frac{2}{5}\left(10\hat{a}_\rho - 20\hat{a}_z\right)$$

$$B_{1t} = (4\hat{a}_\rho - 8\hat{a}_z)\text{mWb/m}^2$$

$$W_{m1} = \frac{1}{2}B_1 \cdot H_1 = \frac{B_1^2}{2\mu_1}$$

$$= \frac{(4^2 + 15^2 + 8^2)\times 10^{-6}}{2\times 2\times 4\pi\times 10^{-7}}$$

$$= \frac{305}{16\pi}\times 10 = 60.68 \text{ J/m}^3$$

20. (c)

$$E = \int (v\times B)\cdot dl$$

$$v \times B = -0.15 \sin 10^3 t\, u_x \text{ V/m}$$

$$E = \int_0^{0.25} -0.15 \sin 10^3 t\, dx$$

$$E = -0.15 \sin 10^3 t [x]_0^{0.25}$$

$$E = -0.0375 \sin 10^3\, t \text{ V}$$

21. (a)

$$M = X_m H$$

$$= X_m \frac{B}{\mu} = \frac{(\mu_r - 1)}{\mu_r \mu_0} B = \left(1 - \frac{1}{4.5}\right)\times \frac{4y}{\mu_0}\times 10^{-3}\hat{a}_z$$

$$= \frac{28y}{9\mu_0}\times 10^{-3}\hat{a}_z$$

$$J = \nabla \times M$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{28y}{9\mu_0} \end{vmatrix} = \frac{28}{9\mu_0}\times 10^{-3}\hat{a}_z = 2475.7 \text{ A/m}^2$$

22. (a)

$$E = \frac{\rho_s}{2\epsilon_0}\hat{a}_n$$

$$E = E_1 + E_2 + E_3$$

$$= \left(\frac{10}{2\epsilon_0}\hat{a}_x + \frac{(-20)}{2\epsilon_0}\hat{a}_y + \frac{30}{2\epsilon_0}(-\hat{a}_z)\right)\times 10^{-6}$$

$$= \frac{10}{2\epsilon_0}\left[\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z\right]\times 10^{-6}$$

$$|E| = \frac{10}{2\in_0}\sqrt{1+2^2+3^2}\times 10^{-6}$$

$$= \frac{10}{2\in_0}\sqrt{14} = \frac{18.71}{\in_0}\times 10^{-6} = 2.11 \times 10^6 \text{ V/m}$$

23. (c)

$$H = H_x + H_y$$

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$H_x = \frac{5}{4\pi\times 2}(\cos 0° - \cos 90°)(-\hat{a}_x\times\hat{a}_z) = \frac{5}{8\pi}\hat{a}_y \text{A/m}$$

$$H_y = \frac{5}{4\pi\times 2}(\cos 0° - \cos 90°)(\hat{a}_y\times\hat{a}_z) = \frac{5}{8\pi}\hat{a}_x$$

$$\vec{H} = \frac{5}{8\pi}(\hat{a}_x + \hat{a}_y) \text{ A/m}$$

24. (b)

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$\rho = \sqrt{5^2+5^2} = \sqrt{50}\text{ m}$$

$$= \frac{2}{4\pi\sqrt{5^2+5^2}}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$\cos\alpha_2 = \frac{10}{\sqrt{50+100}} = \frac{10}{\sqrt{150}}$$

$$\cos\alpha_1 = \cos 90° = 0$$

$$\hat{a}_\phi = \hat{a}_l\times\hat{a}_\rho = \hat{a}_z\times\left(\frac{5\hat{a}_x+5\hat{a}_y}{5\sqrt{2}}\right) = \left(\frac{-\hat{a}_x+\hat{a}_y}{\sqrt{2}}\right)$$

$$\vec{H} = \frac{2}{4\pi\times 5\sqrt{2}}\times\left(\frac{10}{\sqrt{150}} - 0\right)\times\left(\frac{-\hat{a}_x+\hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{1}{20\pi}\left(-\hat{a}_x+\hat{a}_y\right)\times\frac{10}{5\sqrt{6}}$$

$$= \frac{1}{10\pi\sqrt{6}}\left(-\hat{a}_x+\hat{a}_y\right)\text{A/m}$$

25. (d)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = 0$$

$$= \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) = 0$$

$$\frac{dV}{d\rho} = \frac{A}{\rho}$$

$$V = A \ln \rho + B$$

$$V(\rho = 4) = A \ln 4 + B = 0$$

$$V(\rho = 12) = A \ln 12 + B = V_0$$

$$V_0 = A \ln 12 - A \ln 4 = A \ln 3$$

$$E = -\nabla V = -\left[\frac{dV}{d\rho}\hat{a}_\rho\right] = -\frac{A}{\rho}\hat{a}_\rho$$

$$E(\rho = 8) = -6\,\hat{a}_\rho \text{ kV/m}$$

$$A = 48$$

$$V_0 = 48 \ln 3 \text{ V}$$

26. (a)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\frac{\rho dV}{d\rho} + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = \frac{-\rho_v}{\in} = \frac{-\rho_v}{\in_0 \in_r}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{dp}\right) = \frac{-\rho_v}{\in} = \frac{-10}{\rho} \times \frac{10^{-12}}{3.6 \times 1} \times 36\pi \times 10^9$$

$$\nabla^2 V = \frac{-\rho_v}{\in}$$

$$\frac{d}{d\rho}\frac{\rho dV}{d\rho} = -0.1\,\pi$$

$$\frac{\rho dV}{d\rho} = -0.1\,\pi\rho + A$$

$$V = -0.1\,\pi\rho + A \ln \rho + B$$

$$-0.1\,\pi \times 2 + A \ln 2 + B = 0$$

$$A \ln 2 + B = 0.2\,\pi$$

$$-0.1\,\pi \times 5 + A \ln 5 + B = 60$$

$$A \ln 5 + B = 60 + 0.5\,\pi$$

$$A \ln 2.5 = (60 + 0.3\,\pi)$$

$$A = \left(\frac{60 + 0.3\pi}{\ln 2.5}\right) = 66.51$$

$$E = -\nabla V$$

$$= -\left(-0.1\pi + \frac{A}{\rho}\right)\hat{a}_\rho$$

At $\rho = 1$,

$$E = -\left(-0.1\pi + \frac{66.51}{1}\right)\hat{a}_\rho = -66.19\hat{a}_\rho \text{ V/m}$$

27. (b)

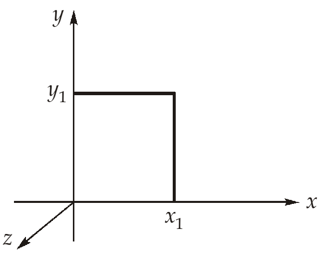
$$x_1 = y_1 = 1 \text{ m}$$

$$B_0 = \sin \pi x \sin \pi y \; T$$

$$B = B_0 \cos \omega_0 t$$

$$E(t) = \iint_s \frac{-dB}{dt} \cdot \hat{n}\, dS$$

$$= \iint_s \omega_0 B_0 \sin\omega t\, \hat{n}\, dS$$



$$E_{max} = \omega_0 \int_{y=0}^{1} \int_{x=0}^{1} \sin\pi x \sin\pi y\, dx\, dy$$

$$E_{max} = 1000 \times 2\pi \times \frac{4}{\pi^2} = \frac{8000}{\pi}\,\text{V/turns}$$

$$E_{rms} = \frac{1}{\sqrt{2}} \times \frac{8000}{\pi} \times 10 = 18\text{ kV}$$

28. **(a)**

Total electric flux over a sphere of 5 m radius

$= Q_1 + Q_2 + Q_3$

$= (5 + 4 - 6) \times 10^{-8}$

$= 3 \times 10^{-8}$ C

29. **(a)**

$$\nabla \times H = \frac{1}{R^2 \sin\theta} \begin{vmatrix} a_r & Ra_\theta & R\sin a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & R^2 \sin^2\theta \end{vmatrix}$$

$$= \frac{1}{R^2 \sin\theta}\left[2R^2 \sin\theta\cos\theta\, \hat{a}_r - 2R\sin^2\theta \cdot R\hat{a}_\theta\right]$$

$$= \left(2\cos\theta\, \hat{a}_r - 2\sin\theta\, \hat{a}_\theta\right)$$

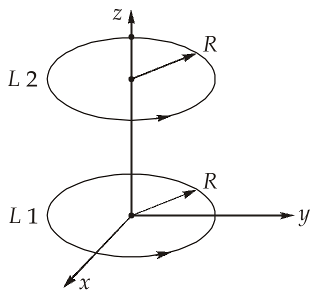
At origin $|\nabla \times H| = 2$

30. **(a)**

$$B = \frac{\mu_0 I R^2}{(R^2 + h^2)^{3/2}} \hat{u}_z$$

$$B = \frac{4\pi \times 10^{-7} \times 150 \times 0.12^2}{(0.12^2 + 0.125^2)^{3/2}}$$

$= 0.522$ mT



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