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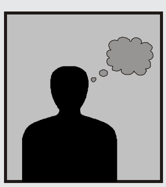
MADE EASY WORKBOOK 2025



Detailed Explanations of Try Yourself *Questions*

Computer Science & IT

Discrete & Engineering Mathematics

MADE EASY PUBLICATIONS

1 Propositional Logic

Detailed Explanation *of* Try Yourself Questions

T1 : Solution

(a)

$$\begin{aligned}[(p\wedge q)\rightarrow(p\vee q)]\vee \sim p\vee q &\equiv \sim(p\wedge q)\vee(p\vee q)\vee\sim p \vee q\\ &\equiv (\sim p\vee\sim q)\vee(p\vee q)\vee\sim p\vee q\\ &\equiv (\sim p\vee p)\vee(\sim q\vee q)\vee\sim p\vee q\\ &\equiv \mathrm{T}\vee\mathrm{T}\vee\sim p\vee q\\ &\equiv \mathrm{T}\end{aligned}$$

T2 : Solution

(d)

$$\begin{aligned}\exists x(P(x)\rightarrow\exists y\, Q(y)) &\cong \neg\,\forall x\;\neg(P(x)\rightarrow\exists y\, Q(y))\\ &\cong \exists x(\neg P(x)\vee\exists y\, Q(y))\\ &\cong \exists x(\neg\exists y\, Q(y)\rightarrow\neg P(x))\\ &\cong \neg\forall x(P(x)\wedge\neg\exists y\, Q(y))\end{aligned}$$

T3 : Solution

(b)

For every person x, if person x is female and person x is a parent, then there exists a person y such that person x is the mother of person y.

$F(x)$: x is female

$P(x)$: x is a parent.

$M(x, y)$: x is the mother of y

$$\forall x\big((F(x)\wedge P(x))\rightarrow\exists y\, M(x, y)\big)$$

$$\cong\ \forall x\exists y\big((F(x)\wedge P(x))\rightarrow M(x, y)\big)$$

T4 : Solution

(b)

Given: $\forall x \in$ R, if $x > 2$ then $x^2 > 4$

Contrapositive is: $\forall x \in$ R, if $x^2 \leq 4$ then $x \leq 2$

Converse is: $\forall x \in$ R, if $x^2 > 4$ then $x > 2$

Inverse is: $\forall x \in$ R, if $x \leq 2$ then $x^2 \leq 4$

T5 : Solution

(c)

$\forall x \forall y \forall z$ ((Apple $(x) \wedge$ Apple $(y) \wedge$ Apple $(z)) \rightarrow (x = y \vee x = z \vee y = z))$

T6 : Solution

(a)

In I_1 both hypothesis are true and conclusion is also true by Modes Ponens.

Socrates is human : p

Socrates is mortal : q

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore \quad q \end{array} \Rightarrow \text{by Modus Ponens}$$

In I_2 both hypothesis are true and conclusion is also true by Modus Tollens.

MADE EASY is closed today : q

It will rain today : p

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \ \sim p \end{array} \Rightarrow \text{by Modus Tollens}$$

T7 : Solution

(d)

p	q	$p \rightarrow q$	$\sim(p \rightarrow q)$	$\sim q$	$\sim(p \rightarrow q) \rightarrow \sim q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

p	q	$p \rightarrow q$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

T8 : Solution

(d)

(*i*) $$\sim(\forall x)\exists y\, P(x, y) \equiv (\exists x)(\forall y)[\sim P(x, y)]$$

$$\exists x \forall y[\sim P(x, y)] \equiv (\exists x)(\forall y)[\sim P(x, y)]$$

Above logic is true.

(*ii*) $$\sim(\forall x)P(x) \equiv \exists x[\sim P(x)]$$

$$\exists x[\sim P(x)] \equiv \exists x[\sim P(x)]$$

Above logic is true.

(*iii*) $$\sim(\exists x)(\forall y)[P(x,y) \vee Q(x,y)] \equiv (\forall x)(\exists x)[\sim P(x,y) \wedge \sim Q(x,y)]$$

$$(\forall x)(\exists x)[\sim P(x,y) \wedge \sim Q(x,y)] \equiv (\forall x)(\exists x)[\sim P(x,y) \wedge \sim Q(x,y)]$$

Above logic is true.

Since all the above option are correct.

T9 : Solution

(c)

$\neg\forall z[P(z) \to (\neg Q(z) \to P(z))]$

$\exists z \neg[\neg P(z)] \vee \neg(\neg Q(z) \vee P(z))]$

$\exists z \neg[\neg P(z) \vee (Q(z) \vee P(z))]$

$\exists z \neg[P(z) \wedge (\neg Q(z) \wedge \neg P(z))]$

$\exists z[P(z) \wedge \neg Q(z) \wedge \neg P(z)]$ $\quad [\therefore\ P \cdot \neg P = 0]$

T10 : Solution

(b)

Option (a) is correct because every valid formula is tautology and every tautology is satisfiable.

Option (b) is incorrect because some satisfiable are tautology.

Option (c) is correct because no contradiction is satisfiable.

T11 : Solution

(c)

There are atmost two apples : $\forall x\, \forall y\, \forall z\, ((\text{Apple}\,(x) \wedge \text{Apple}\,(y) \wedge \text{Apple}\,(z)) \to (x = y \vee x = z \vee y = z))$

There are exactly two apples : $\exists x\, \exists y\, (\text{Apple}\,(x) \wedge \text{Apple}\,(y) \wedge (x \neq y) \wedge \forall z\, (\text{Apple}\,(z) \to ((z = x) \vee (z = y))))$

There is atmost one apple : $\forall x\, \forall y\, ((\text{Apple}\,(x) \wedge \text{Apple}\,(y)) \to (x = y \vee y = x))$

There is exactly one apple : $\exists x\, (\text{Apple}\,(x) \wedge \forall y\, (\text{Apple}\,(y) \to (x = y)))$

T12 : Solution

(c)

$\forall x \in N\ [x \neq 7 \wedge \text{Prime}(x) \rightarrow \neg\, \text{Divisibleby7}(x)]$

$\cong$

$\forall x \in N\ [x = 7 \vee \neg\, \text{Prime}(x) \vee \neg\, \text{Divisibleby7}(x)]$

$\cong$

$\neg \exists x \in N\ [x \neq 7 \wedge \text{Prime}(x) \wedge \text{Divisibleby7}(x)]$

All represents that “no prime except 7 is divisible by 7”.

T13 : Solution

(a)

Everybody loves Mahesh: $\forall x$ Loves (x, Mahesh)

Everybody loves somebody: $\forall x\ \exists y$ Loves (x, y)

There is somebody whom everybody loves: $\exists y\ \forall x$ Loves (x, y)

There is somebody whom no one loves: $\exists y\ \forall x\ \neg$ Love (x, y)

T14 : Solution

(d)

$\forall x\ P(x) \rightarrow \forall x\ [P(x) \vee Q(x)]$ is valid

$\exists x\ \exists y\ P(x,y) \rightarrow \exists y\ \exists x\ P(x,y)$ is valid

$\exists x\ [R(x) \vee S(x)] \rightarrow \exists x\ R(x) \vee \exists x\ S(x)$ is also valid

T15 : Solution

(b)

$\neg(\neg p \vee q) \vee (r \rightarrow \neg s) \cong (p \wedge \neg q) \vee (\neg r \vee \neg s) \cong (p \vee \neg r \vee \neg s) \wedge (\neg q \vee \neg r \vee \neg s)$

T16 : Solution

(d)

P_1, P_2 and P_3 are equivalent. All are representing the same statement: "there are exactly two apples".

T17 : Solution

(d)

All the statements give true as the truth value. None of them give false as the truth value.

T18 : Solution

(11)

We wish to make

$\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s)) = 1$

$\Rightarrow \quad (p \Rightarrow q) \wedge (\neg r \vee \neg s) = 0$

$\Rightarrow \quad (p \Rightarrow q) = 0$ ...(1)

or $\quad \neg r \vee \neg s = 0$ ...(2)

Now (1) is satisfies only when p = 1 and q = 0.

Equation (2) $\neg r \vee \neg s = 0$, iff $r \wedge s = 1$

i.e. r = 1 and s = 1

i.e. x is a perfect square and x is a prime number. Which is not possible so condition (2) cannot be satisfied by any x.

So condition (1) must be satisfies which is p = 1 and q = 0 i.e. $x \in \{8, 9, 10, 11, 12\}$ and x is not a composite.

Now the only value of x which satisfies this is x = 11.

So correct answer is x = 11.

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2 Combinatorics and Recurrence Relations

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

$$\text{Total number of letters} = 15$$

$$\text{Number of T's} = 3$$

First place 12 letters other than T's at dot places.

X. X. X. X. X. X. X. X. X. X. X. X. X

$$\text{The number of ways} = \frac{12!}{5!3!2!}$$

Since no two T's are together, thus place T's at cross places whose number = 13

$$\text{Their arrangements are} = \frac{{}^{13}P_3}{3!}$$

$$\text{Total number of ways} = \frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}$$

T2 : Solution

(b)

$$\sum_{k=0}^{n} \binom{n}{k} x^k \,.\, y^{n-k} = (x+y)^n$$

$$\sum_{k=0}^{n} \binom{n}{k} (-1)^k \,.\, 3^{n-k} = ((-1)+3)^n = 2^n$$

T3 : Solution

(c)
The candidate is unsuccessful if he fails in 9 or 8 or 7 or 6 or 5 papers.

∴ The number of ways to be unsuccessful

$$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$$
$$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$$
$$= \frac{1}{2}({}^9C_0 + {}^9C_1 + {}^9C_2 + \text{.........} + {}^9C_9)$$
$$= \frac{1}{2}(2^9) = 2^8$$

T4 : Solution

(b)

$$T(n) - 4T(n-1) + 3T(n-2) = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0 \Rightarrow x = 3, 1$$

General Solution: $T(n) = A.3^n + B.1^n$
$$= A.3^n + B$$

Given: $T(0) = 0$ and $T(1) = 2$

By taking $n = 0 \Rightarrow T(n) = A.3^n + B$
$$T(0) = A + B \Rightarrow A + B = 0 \quad \text{...(i)}$$

By taking $n = 1 \Rightarrow T(1) = A.3 + B \Rightarrow 3A + B = 2 \quad \text{...(ii)}$

From (i) and (ii) $\Rightarrow A = 1, B = -1$

$$T(n) = A.3^n + B [\because \text{ Substitute } A = 1 \text{ and } B = -1]$$
$$\Rightarrow \quad T(n) = 3^n - 1$$

T5 : Solution

(a)

$$x + y + z = 17$$
$$x \geq 1, y \geq 1, z \geq 1$$

Put $x = 1 + u, y = 1 + v, z = 1 + w$

$\Rightarrow u + v + w = 14$

Now number of solutions in non-negative integers

$$\binom{14+3-1}{14} = \binom{16}{14} = \binom{16}{2} = 120$$

T6 : Solution

(b)

Example: In how many ways can the pack of 52 cards be partitioned into 4 sets of size B.

$$\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} = \frac{(52)!}{(13!)^4}$$

All partitions are not distinct. Each distinct partition arises in 4! ways. Therefore # ways = $\frac{(52)!}{(13!)^4 \cdot 4!}$

Similarly, $$\frac{\prod_{i=0}^{m-1}\binom{mn-in}{n}}{m!} = \frac{(mn)!}{(n!)^m \cdot m!}$$

T7 : Solution

(a)

General solution: $T(n) = C_1.3^n + C_2.\, 2^n$

$$\begin{bmatrix}\text{Where Homogeneous part : } (x-3) = 0 \Rightarrow x = 3 \\ \text{Particular solution: } C_2.2^n \end{bmatrix}$$

$T(0) = 1,\ T(1) = 3.T(0) + 2' = 3 + 2 = 5$

$T(n) = C_1.3^n + C_2.2^n$

$T(0) = C_1.3^0 + C_2.2^0 \Rightarrow 1 = C_1 + C_2$

$T(1) = C_1.3' + C_2.2' \Rightarrow 5 = 3C_1 + 2C_2$

$C_1 = 3,\ C_2 = -2$

$\therefore \quad T(n) = 3.3^n - 2.2^n = 3^{n+1} - 2^{n+1}$

T8 : Solution

(c)

Each question can be answered in 4 ways.

$\therefore$ The number of ways $= 4^{65}$

T9 : Solution

(c)

$$T(n) = 10.T(n-1) - 25.T(n-2)$$

$\Rightarrow 25T(n-2) - 10.T(n-1) + T(n) = 0$

$\Rightarrow 25 - 10x - x^2 = 0$

$\Rightarrow (x-5)^2 = 0$

$T(n) = C_1 \cdot 5^n + C_2 \cdot n \cdot 5^n$

$T(0) = 5,\ T(1) = 5$

$T(0) = C_1 \cdot 5^\circ + C_2 \cdot 0 \cdot 5^\circ$

$\Rightarrow 5 = C_1$

$$T(1) = C_1 \cdot 5^1 + C_2 \cdot 1 \cdot 5^1$$
$$5 = 5C_1 + 5C_2$$
$$\Rightarrow \quad C_1 + C_2 = 1$$
$$\Rightarrow \quad C_2 = 1 - C_1 = 1 - 5 = 4$$
$$\therefore \quad T(n) = 5.5^n + (-4)n.5^n$$
$$= 5^{n+1} - 4n.5^n$$

T10 : Solution

(10)

We wish to find coefficient of x^{12} in $(x^3 + x^4 + x^5 + ...)^3$

$$= (x^3(1 + x^1 + x^2 + ...))^3$$
$$= x^9(1 + x + x^2 ......)^3$$
$$= \frac{x^9}{(1-x)^3} = x^9 \sum_{r=0}^{\infty} {}^{3-1+r}C_r X^r = x^9 \sum_{r=0}^{\infty} {}^{r+2}C_r X^r$$

Now to make x^{12} we need to put $r = 3$

So coefficient of x^{12} is ${}^{3+2}C_3 = {}^5C_3 = {}^5C_2 = 10$

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3 Set Theory and Algebra

Detailed Explanation of Try Yourself Questions

T1 : Solution

(d)

$$f : N \rightarrow Z$$
$$f(0) = f(2) = 3$$

$\Rightarrow$ f is not injective

Clearly f is not surjective, all numbers in Z do not have preimages in N (example: 0 has no preimage)

f is function which is not injective and not surjective.

T2 : Solution

(d)

Countable sets : Finite Set, $\underbrace{N, O^+, Z^+, Z^-, Z}_{\text{denumerable}}$

Uncountable sets : Real numbers

$\therefore$ Set of real numbers in the interval [0, 1] is uncountable because they can not be enumerated.

T3 : Solution

(b)

(*i*) $|x - y| \leq 2$

If we take $y = (x)$

then $|x - (x)| \leq 2$ True

So it is reflexive.

(*ii*) If we do $|x - y|$ or $|y - x|$, answer will be same. So it is symmetric.

(*iii*) If we do $[x - (x - 2)]$ the $[(x - 2) - (x - 4)]$, the $[x - (x - 4)]$ not follow the condition

So it is not transitive.

T4 : Solution

(c)

$$f(x) = (5x+1)^2$$

$\Rightarrow$ $$y = (5x+1)^2$$

$\Rightarrow$ $$\sqrt{y} = 5x+1$$

$\Rightarrow$ $$\sqrt{y}-1 = 5x$$

$\Rightarrow$ $$x = \frac{\sqrt{y}-1}{5} \quad (\because \text{ swap } x \text{ and } y \text{ for inverse})$$

$\Rightarrow$ $$y = \frac{1}{5}\left(\sqrt{x}-1\right)$$

$\Rightarrow$ $$f^{-1}(x) = \frac{1}{5}\left(\sqrt{x}-1\right)$$

T5 : Solution

(d)

$$x * y = x + y + xy$$

Let e be the identity

$$x * e = x + e + xe \qquad [\text{Put } e = 0]$$

$$x = x$$

$\therefore$ $\boxed{e = 0}$ $\Rightarrow$ 0 is the identity of S

Let y is the inverse of x

Then $$x * y = e \Rightarrow x * y = 0$$

$\Rightarrow$ $$x + y + xy = 0$$

$\Rightarrow$ $$x + y(1 + x) = 0$$

$\Rightarrow$ $$\boxed{y = \frac{-x}{1+x}}$$

$\Rightarrow$ $1 + x \neq 0$ $\Rightarrow$ $x \neq -1$

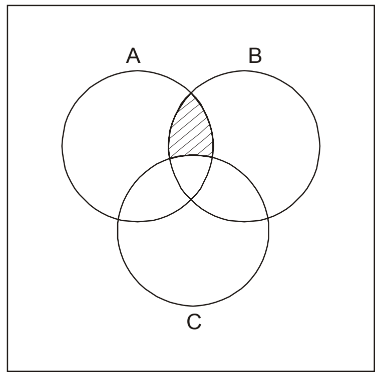
Inverse of $x = \dfrac{-x}{1+x}$; $\forall x \in R \setminus \{-1\}$

$\therefore$ S = R\{–1} is an abelian group.

[**Note:** S is Commutative and Associative over *]

T6 : Solution

(c)



Same Venn diagram can be produced for both S1 and S2.

$\therefore$ S1 = S2

T7 : Solution

(c)

$$-1 \le \sin x \le +1$$
$$-5 \le 5 \sin x \le 5$$
$$-2 \le 3 + 5 \sin x \le 8$$
$$-2 \le f(x) \le 8$$

$\Rightarrow$ Range is: [–2, 8]

T8 : Solution

(d)

R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)}

R is not reflexive: (3, 3) $\notin$ R

R is symmetric: if $(x + y) \le 5 \Rightarrow (y + x) \le 5$

R is not antisymmetric: (1, 2) and (2, 1) in R

R is not transitive: (3, 1) and (1, 3) in R, but (3, 3) $\notin$ R

$\therefore$ R is symmetric

T9 : Solution

(d)

X= {{ }, {a}}

Power set = {{ }, {{ }} {{a}}, {{ }, {a}}}

power set contain 2^n element of original set.

T10 : Solution

(d)

$\mathbb{N}$ is countable set.

Hence, Subset of any countable set is also countable and product of two countable sets is also countable.

T11 : Solution

(c)

Commutative for multiplication of matrices does not hold.

$$AB \neq BA$$

If AB is possible to multiply that does not mean that BA can also be multiplied. Moreover the result will not be the same except for the case when the 2 matrices are same.

T12 : Solution

(2)

$$f(n) = f\left(\frac{n}{2}\right) \text{ if } n \text{ is even}$$

$$f(n) = f(n+5) \text{ if } n \text{ is odd}$$

$f : N^+ \to N^+$

Now $\quad f(2) = f\left(\frac{2}{2}\right) = f(1)$

$$f(3) = f(3+5) = f(8) = f\left(\frac{8}{2}\right) = f(4)$$

$$= f\left(\frac{4}{2}\right) = f(2) = (1)$$

So $\quad f(1) = f(2) = f(3) = f(4) = f(8)$

Now let us find $f(5) = f(5+5) = f(10) = f\left(\frac{10}{2}\right) = f(5)$ so $f(5) = f(10)$

Now let us find $f(9)$

$$f(9) = f(9+5) = f(14) = f\left(\frac{14}{2}\right) = f(7)$$

$$= f(7+5) = f(12) = f\left(\frac{12}{2}\right) = f(6)$$

$$= f\left(\frac{6}{2}\right) = f(3)$$

So $\quad f(9) = f(7) = f(6) = f(3) = f(1) = f(2) = f(4) = f(8)$

For $n > 10$, the function will be equal to one of $f(1), f(2) \ldots. f(10)$

So the maximum number of distinct values f takes is only 2.

First is $\quad f(1) = f(2) = f(3) = f(4) = f(8) = f(9) = f(7) = f(6)$

Second is $\quad f(5) = f(10)$

All other n values will give only one of these two values.

T13 : Solution

(d)

If every subset of a lattice has LUB and GLB, then such a lattice is called as complete lattice. All of the given lattices are complete lattices, since all the lattices are having GLB and LUB.

T14 : Solution

(d)

$$\begin{aligned}(A-B)-C &= (A\cap\bar{B})-C = A\cap\bar{B}\cap\bar{C}\\ (A-C)-(B-C) &= (A\cap\bar{C})-(B\cap\bar{C})\\ &= (A\cap\bar{C})\cap(\bar{B}\cup C)\\ &= (A\cap\bar{C}\cap\bar{B})\cup(A\cap\bar{C}\cap C)\\ &= (A\cap\bar{C}\cap\bar{B})\cup(A\cap\phi)\\ &= (A\cap\bar{C}\cap\bar{B})\cup\phi\\ &= A\cap\bar{C}\cap\bar{B}\end{aligned}$$

T15 : Solution

(c)

A distributive lattice need not be a complemented lattice. A complemented lattice may or may not be distributive lattice. However a complemented has to be bounded because the complement property requires 0 and 1 which when present will make the lattice bounded.

T16 : Solution

(b)

$(a, b)\ R(c, d)$ if $a \le c$ or $b \le d$

P : R is reflexive

Q : R is transitive

Since, $(a, b)\ R(a, b) \Rightarrow a \le a$ or $b \le b$

$\Rightarrow$ t or t

Which is always True, R is reflexive.

Now let us check transitive property

Let $(a, b)\ R(c, d) \Rightarrow a \le c$ or $b \le d$

and $(c, d)\ R(e, f) \Rightarrow c \le e$ or $d \le f$

Now let us take a situation

$a \le c$ (True) or $b \le d$ (false)

and $c \le e$ (False) or $d \le f$ (True)

Now we can get neither $a \le e$ nor $b \le f$

So, $(a, b)\ R(c, d)$ and $(c, d)\ R(e, f) \not\Rightarrow (a, b)\ R(e, f)$. So clearly R is not transitive.

So P is true and Q is false. Choice (b) is correct.

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4 Graph Theory

Detailed Explanation of Try Yourself Questions

T1 : Solution

(d)

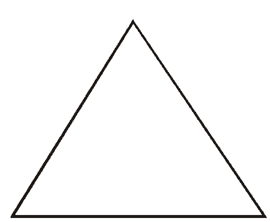
To find maximum number of edges in the disconnected graph with n-vertices, form a complete graph with $(n-1)$ vertices and 1 vertex is isolated. So the graph will be disconnected and addition of any edge will make the graph as connected.

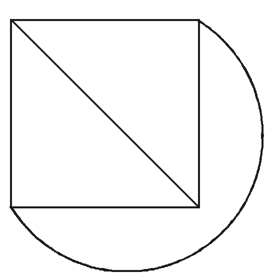
$\therefore$ Maximum number of edges in disconnected graph can present:

$$^{(n-1)}C_2 = \frac{(n-1)(n-2)}{2}$$

T2 : Solution

(a)

K_3 :  $\rightarrow \chi(K_3) = 3$

K_4 :  $\rightarrow \chi(K_4) = 4$

Any planar graph $G \Rightarrow \chi(G) \leq 4$

Every face is bordered by exactly 3 edges $\Rightarrow \chi(G) \geq 3$

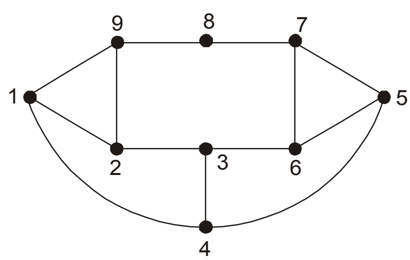
$\therefore \quad \chi(G) = 3 \text{ or } 4$

So, $\chi(G)$ can never have the value 2

T3 : Solution

(a)

G_1 :



is Hamiltonian [$\because$ Hamiltonian cycle exists]

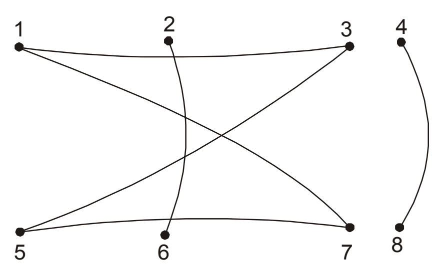
G_2 : not Hamiltonian graph [$\because$ Hamiltonian cycle does not exists in G_2]

$\therefore$ Only G_1 is Hamiltonian graph.

T4 : Solution

(d)

Let $n = 2 \Rightarrow$ # vertices = 8 [$\because$ # vertices in $G = 4n$]



$\Rightarrow$ 3 components [**Note:** For any n, the #components in $G = 3$]

T5 : Solution

(b)

$$\left.\begin{aligned} V(C_1) &= \{1, 3, 5, 7\} \Rightarrow m_1 = 4 \\ V(C_2) &= \{2, 6\} \Rightarrow m_2 = 2 \\ V(C_3) &= \{4, 8\} \Rightarrow m_3 = 2 \end{aligned}\right\} \max = 4$$

In general, 'G' with $4n$ vertices has 3 components

$$V(C_1) = \{1, 3, 5, 7, 9, 11, 13, 15, \ldots\ldots\} \Rightarrow m_1 = 2n$$

$$V(C_2) = \{2, 6, 10, 14, 18, \ldots\ldots\} \Rightarrow m_2 = n$$

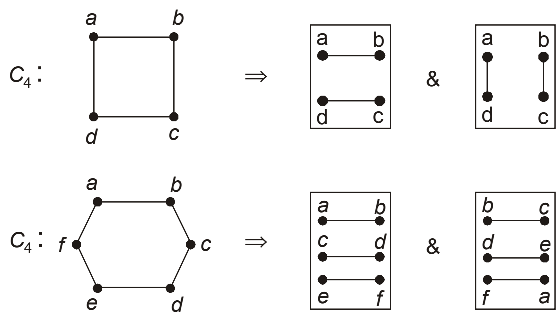
$$V(C_3) = \{4, 8, 12, 16, 20, \ldots\ldots\} \Rightarrow m_3 = n$$

$\therefore \quad \text{Max}\,(m_1, m_2, m_3) = 2n$

T6 : Solution

(b)

Example: $C_{2n} \Rightarrow n = 2 \Rightarrow C_4$



A cycle graph with even vertices has 2 perfect matchings.

T7 : Solution

(c)

S_1 is true but converse of S_1 is not true. If a graphs is Hamiltonian that does not mean that $d(v) \geq n/2$ for each vertex in G.

S_2 is true and converse of S_2 is also true because G is connected graph. If g is Eulerian then every vertex has to have even degree.

T8 : Solution

(a)

G_1 and G_2 are isomorphic

$$a \to 1$$
$$b \to 3$$
$$c \to 4$$
$$d \to 5$$
$$e \to 6$$
$$f \to 2$$

T9 : Solution

(c)

G is a planar graph. Every planar graph is 4-colorable. Every face is bordered by 3 edges.

So graph has possibilities of 3 or 4 colors.

k_3 colored with 3 and k_4 colored with 4 colors.

T10 : Solution

(c)

S_1 : The maximum number of edges $= \dfrac{(n-1)(n-2)}{2}$ when a graph has disconnected into two components where one component with a single vertex and other component is complete graph on $(n-1)$ vertices.

$\therefore$ S_1 is true.

S_2 : G is a forest if and only if G has $(n-k)$ edges. If G is a forest, then each connected component is a tree.

Example: G has 10 vertices and 3 components. Two components are having a single vertex. Third component must be a tree with 7 edges i.e., G has 3 components with 7 edges where two components with no edge and third component with 7 edges.

$\therefore$ S_2 is true.

■■■■

5 Probability and Statistics

Detailed Explanation of Try Yourself Questions

T1 : Solution

(b)

P(X = 2) + P(X = 4) + P(X = 6) + ...

$= [P.(1–P)] + [(1 – P).(1 – P).(1 – P).P] + [(1 – P).(1 – P) (1 – P) (1 – P) (1 – P).P] + \ldots$

$= P (1 – P) + (1 – P)^3. P + (1 – P)^5.P + \ldots$

$= P (1 – P) [1 + (1 – P)^2 + (1 – P)^4 + \ldots]$

[Since $1 + x + x^2 + x^3 + \ldots = \dfrac{1}{1-x}; x < 1$]

$$= P(1-P)\left[\frac{1}{1-(1-P)^2}\right] = \frac{1-P}{2-P}$$

T2 : Solution

(c)

$$E[(X-c)^2] = E[X^2] - 2cE[X] + c^2$$

$$= \text{Var}(X) + [E[X]]^2 - 2c\mu + c^2 \qquad [\because \ E[X^2] = \text{Var}(X) + [E(X)]^2]$$

$$= \sigma^2 + \mu^2 - 2c\mu + c^2$$

$$= (\mu - c)^2 + \sigma^2$$

T3 : Solution

(b)

In any connected, planar, simple graph $\Rightarrow$ $e \leq 3n - 6$. This theorem goes only one way from left to right.

Choice (a) is same as this theorem and hence valid.

Choice (c) is the contrapositive of this theorem and hence is also valid.

Choice (d) if $e = 3n - 5$ then surely $e > 3n - 6$ and so the contrapositive of the theorem says that the graph is not planar. So, choice (d) is valid.

Choice (b) is invalid because it is the converse of the one way theorem.

T4 : Solution

(b)

Exact weight cannot be written but there will be limit to measure the weight.

Therefore it is continuous.

Number of questions in a test is finite and can be find easily that number of questions attempted.

Hence it is discrete.

T5 : Solution

(c)

A. Binomial distribution (discrete)

$$P(x) = \binom{n}{x} \cdot P^x \cdot (1-P)^{n-x}, \quad x \geq 0$$

B. Poisson distribution (discrete)

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x \geq 1$$

C. Exponential distribution (continuous)

$$f(x) = \lambda \cdot e^{-\lambda x}, 0 < x < \infty$$

D. Uniform distribution (continuous)

$$f(x) = \frac{1}{b-a} \text{ for } a < x < b$$

T6 : Solution

(c)

$$P(x = r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!}, \text{ where } \lambda = np$$

$$\lambda = 500 \times 0.006 = 3$$

$$P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= \frac{e^{-\lambda}\cdot\lambda^0}{0!} + \frac{e^{-\lambda}\cdot\lambda^1}{1!}$$

$$= e^{-3} + e^{-3}\cdot 3$$

$$= 4e^{-3}$$

T7 : Solution

(b)

Probability density function:

$$f(x) = \lambda\cdot e^{-\lambda x}, x > 0$$

$$E(X) = \int_0^\infty x\cdot f(x)\cdot dx$$

$$= \int_0^\infty x\lambda\cdot e^{-\lambda x}\cdot dx = \frac{1}{\lambda}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2\cdot f(x)\cdot dx$$

$$= \int_0^\infty x^2\cdot\lambda\cdot e^{-\lambda x}\cdot dx = \frac{2}{\lambda^2}$$

$$\Rightarrow \quad \text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

T8 : Solution

(d)

$$\begin{aligned} E[6X] &= 6.E[X] = 6 \\ Var[6X] &= 6^2\, Var[X] = 36 \times 2 = 72 \\ E[1-X] &= 1 + (-1)\, E[X] = 1 - 1 = 0 \\ Var[1-X] &= (-1)^2\, Var[X] = Var[X] = 2 \end{aligned}$$

$\therefore$ $\quad Var[1-X] \neq 3$

T9 : Solution

(6)

$\boxed{\int_{-\infty}^{\infty} f(x)dx = 1}$ is density function

$$f(x) = \begin{Bmatrix} \lambda(-x^2+3x-2) & ; & 1 \le x \le 2 \\ 0 & ; & \text{otherwise} \end{Bmatrix}$$

$$= \int_{-\infty}^{0} f(x)\cdot dx + \int_{1}^{2} f(x)\cdot dx + \int_{3}^{\infty} f(x)\cdot dx$$

$$= 0 + \int_{1}^{2} f(x)\cdot dx + 0$$

$\therefore$ $$\int_{1}^{2} \lambda(-x^2+3x-2)dx = 1$$

$\Rightarrow$ $$\lambda\left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x\right]_1^2 = 1$$

$\Rightarrow$ $$\lambda\left[-\left(\frac{8}{3} - \frac{1}{3}\right) + \frac{3}{2}(4-1) - 2(2-1)\right] = 1$$

$\Rightarrow$ $$\lambda\left[-\frac{7}{3} + \frac{9}{2} - 2\right] = 1$$

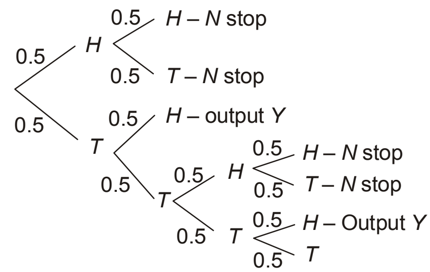
$\Rightarrow$ $$\lambda\left[\frac{-14+27-12}{6}\right] = 1$$

$\Rightarrow$ $$\lambda = \frac{6}{1} = 6$$

$\therefore$ $$\lambda = 6$$

T10 : Solution

(0.33)



The tree diagram for the problem is given above.

The desired output is Y.

Now by rule of total probability

$$p(\text{output} = Y) = 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 \times 0.5 + \ldots$$

Infinite geometric series with

$$a = 0.5 \times 0.5$$

and

$$r = 0.5 \times 0.5$$

So

$$p(\text{output} = Y) = \frac{0.5 \times 0.5}{1 - 0.5 \times 0.5} = \frac{0.25}{0.75}$$

$$\frac{1}{3} = 0.33 \text{ (upto 2 decimal places)}$$

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6 Linear Algebra

Detailed Explanation of Try Yourself Questions

T1 : Solution

(a)

The augmented matrix [A | B] is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & K & 4 \end{array}\right]$$

$$\Rightarrow \begin{array}{l}(R_2 - 2R_1) \to R_2 \\ (R_3 - 3R_1) \to R_3\end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & -1 & K-3 & -2 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 3-K & 2 \end{array}\right]$$

$$\Rightarrow (R_3 - R_2) \to R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -K & 1 \end{array}\right]$$

For unique solution rank [A] = rank [A | B] = 3

$\therefore \quad K \neq 0$

T2 : Solution

(a)

$$\begin{aligned} P^2 + 2P + I &= P^2 + 2PI + I^2 \\ &= (P + I)^2 \end{aligned}$$

Eigen values of P are $-1, \frac{1}{2}, 3$

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \text{eigen values of } I_{3\times3} \text{ are } 1, 1, 1$$

Eigen values of $(P + I)$ are $-1 + 1, \frac{1}{2} + 1, 3 + 1$

$$= 0, \frac{3}{2}, 4$$

Eigen values of $(P + I)^2$ are $(0)^2, \left(\frac{3}{2}\right)^2, (4)^2 = 0, \frac{9}{4}, 16$

T3 : Solution

(c)

$$A = \begin{bmatrix} 1 & 2 & K \\ 3 & -1 & 1 \\ 5 & 3 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & K \\ 0 & -7 & 1-3K \\ 0 & -7 & -5-5K \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & K \\ 0 & -7 & 1-3K \\ 0 & 0 & -6-2K \end{bmatrix}$$

If $-6 - 2K \neq 0$ then A is non-singular

If $-6 - 2K = 0$ then A is singular

$\Rightarrow$ $K = -3$

T4 : Solution

(a)

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 2 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -\frac{3}{2} & 0 \\ 0 & 1 & 2 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -\frac{3}{2} & -8 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$\therefore \qquad A^{-1} = \begin{bmatrix} 1 & -\frac{3}{2} & -8 \\ 0 & \frac{1}{2} & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

T5 : Solution

(d)

1, –1, 2, –2 are eigens of A

$\therefore$ characteristic equation is

$$(\lambda - 1)(\lambda + 1)(\lambda - 2)(\lambda + 2) = 0$$
$$(\lambda^2 - 1)(\lambda^2 - 4) = 0$$
$$\lambda^4 - 5\lambda^2 + 4 = 0$$

By Cayley Hamilton theorem,

$$A^4 - 5A^2 + 4I = 0$$
$$A^4 - 5A^2 = -4I$$
$$B = A^4 - 5A^2 + 5I$$
$$\det B = \det(A^4 - 5A^2 + 5I)$$
$$= \det(-4I + 5I)$$
$$= \det I = 1$$

Since – 1, 1, 2, –2 are eigen values of A

– 1 + 1, 1 + 1, 2 + 1, – 2 + 1 are eigens of A + I

0, 2, 3, –1 are eigens of A + I

$$\text{Hence } \det(A + B) = \det(A + A^4 - 5A^2 + 5I)$$
$$= \det(A - 4I + 5I)$$
$$= \det(A + I)$$
$$= (0)(2)(3)(-1) = 0$$
$$\text{Trace of } (A + B) = \text{trace of } (A + I) = 0 + 2 + 3 - 1 = 4$$

T6 : Solution

(c)

If all diagonal elements of a square matrix are equal then such matrix is scalar matrix.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is scalar matrix. A diagonal matrix is a matrix where diagonal elements are non-zero.

Therefore, it is both diagonal and scalar matrix.

T7 : Solution

(a)

$$|\lambda - AI| = (1-\lambda)(\lambda^2-2)+(2-\lambda)-\lambda$$
$$= -\lambda^3+\lambda^2$$
$$\Rightarrow \quad -\lambda^3+\lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda-1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_2 + R_1}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$
$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$
$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x^3$$

$$\therefore \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

T8 : Solution

(c)

Identity matrix is scalar matrix and hence it is diagonal matrix.

$I_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is scalar, because all of its diagonal elements are same.

T9 : Solution

(c)

$$A^m \cdot A^n = A^{m+n} \text{ and } (A^m)^n = A^{mn}$$

T10 : Solution

(c)

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Given $AB = I$

$$\therefore \quad \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -a+e & -b+f \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$-b + f = 0$$

$$\Rightarrow \quad f = b$$

T11 : Solution

(b)

If we form a matrix using these n-tuples it will look like this:

$$\begin{bmatrix} a_1 & a_2 & \cdots & \cdots & a_n \\ b_1 & b_2 & \cdots & \cdots & b_n \\ c_1 & c_2 & \cdots & \cdots & c_n \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Now the maximum rank of this matrix can be only n. Since the rank represents the number of linearly independent rows, the maximum number of such linearly independent n-tuples is n.

T12 : Solution

(c)

$$A = LU$$

Given $$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{ij} = j, \text{ if } i = 1$$
$$= 3 + a_{(i-1)j}; \text{ otherwise}$$

$$\therefore \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A = LU$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

(*i*) $a = 1$, (*ii*) $b = 2$ (*iii*) $c = 3$

(*iv*) $4b + d = 5 \Rightarrow 4 \times 2 + d = 5 \Rightarrow d = -3$

(*v*) $4c + e = 6 \Rightarrow 4 \times 3 + e = 6 \Rightarrow e = -6$

(*vi*) $7c + 2e + f = 9 \Rightarrow 7 \times 3 + 2 \times (-6) + f = 9 \Rightarrow f = 0$

$$\therefore \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

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7 Calculus

Detailed Explanation *of* Try Yourself Questions

T1 : Solution

(d)

Left hand limit: $\lim_{x\to1^-} f(x) = \lim_{h\to0} (1-h) - [1-h]$

$= \lim_{h\to0} (1-h) - 1 = 0$

Right hand limit: $\lim_{x\to1^+} f(x) = \lim_{h\to0} (1+h) - [1+h]$

$= \lim_{h\to0} (1+h) - 1$

$= \lim_{h\to0} h = 0$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1^-} f(x)$$

$f(x) = x - [x]$ is continuous at $x = 1$

Both Assertion and Reason are false.

T2 : Solution

(a)

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$$

$$= e\cdot 1$$

$$= e$$

T3 : Solution

(d)

$$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{x}\times\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}+\sqrt{3-x}} = \lim_{x\to 0}\frac{(3+x)-(3-x)}{x(\sqrt{3+x}+\sqrt{3-x})}$$

$$= \lim_{x\to 0}\frac{2\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt{3-x})} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

T4 : Solution

(b)

$$f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$$

$$\frac{\partial f}{\partial x} = 8x - 8$$

$$\frac{\partial f}{\partial y} = 12y - 4$$

Putting, $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} =$

$$8x - 8 = 0 \text{ and } 12y - 4 = 0$$

Given, $x = 1$ and $y = \frac{1}{3}$

$\left(1, \frac{1}{3}\right)$ is the only stationary point.

$$r = \left[\frac{\partial^2 f}{\partial x^2}\right]_{\left(1,\,\frac{1}{3}\right)} = 8$$

$$s = \left[\frac{\partial^2 f}{\partial x \partial y}\right]_{\left(1,\,\frac{1}{3}\right)} = 0$$

$$t = \left[\frac{\partial^2 f}{\partial y^2}\right]_{\left(1,\,\frac{1}{3}\right)} = 12$$

Since, $rt = 8 \times 12 = 96$ and $s^2 = 0$

We have either a maximum or minimum at $\left(1, \frac{1}{3}\right)$

Since, $r = \left[\frac{\partial^2 f}{\partial x^2}\right]_{\left(1,\,\frac{1}{3}\right)} = 8 > 0$, the point $\left(1, \frac{1}{3}\right)$ is a point of minimum.

T5 : Solution

(c)

The minimum value is

$$f\left(1, \frac{1}{3}\right) = 4 \times 1^2 + 6 \times \left(\frac{1}{3}\right)^2 - 8 \times 1 - 4 \times \frac{1}{3} + 8$$

$$= 4 + \frac{6}{9} - 8 - \frac{4}{3} + 8 = \frac{10}{3}$$

T6 : Solution

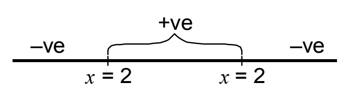
(d)

$$f(x) = \frac{x}{x^2 + 4}$$

$$\Rightarrow \quad f'(x) = \frac{(x^2+4) \cdot 1 - x(2x)}{(x^2+4)^2}$$

$$= \frac{4 - x^2}{(x^2+4)^2} = \frac{(2-x)(2+x)}{(x^2+4)^2}$$

$$\therefore \quad f'(x) = 0 = \frac{(2-x)(2+x)}{(x^2+4)^2} = 0$$



$\therefore$ f is increasing for $-2 < x < 2$

i.e., $x > -2$ and $x < 2$

Note: f is decreasing for $x < -2$ and $x > 2$.

T7 : Solution

(b)

$$f(x) = x + ln\, x$$

$$f'(x) = 1 + \frac{1}{x} \qquad \text{...(1)}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(e) - f(1)}{e - 1}$$

$$= \frac{e + ln\, e - (1 + ln\, 1)}{e - 1} = \frac{e + 1 - 1 + 0}{e - 1} = \frac{e}{e - 1}$$

$$\Rightarrow \quad f'(c) = \frac{e}{e-1} \text{ [from equation (1)]}$$

$$\Rightarrow \quad 1 + \frac{1}{c} = \frac{e}{e-1}$$

$$\Rightarrow \quad \frac{1}{c} = \frac{e}{e-1} - 1$$

$$\Rightarrow \quad \frac{1}{c} = \frac{e-e+1}{e-1}$$

$$\Rightarrow \quad c = e-1$$

T8 : Solution

(d)

$$z = x \sin y - y \sin x$$

$$\frac{\partial z}{\partial x} = \sin y - y \cos x$$

$$\frac{\partial z}{\partial y} = x \cos y - \sin x$$

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$$

$$= (\sin y - y \cos x)\, dx + (x \cos y - \sin x)\, dy$$

T9 : Solution

(c)

S_1: $$f(x) = x^5 + 3x - 1$$

$$f'(x) = 5x^4 + 3 > 0, \text{ for all values of } x$$

$\therefore$ f is increasing function.

S_2: $$f(x) = 1 - x^3 - x^9$$

$$f'(x) = -3x^2 - 9x^8 < 0, \text{ for all values of } x$$

$\therefore$ f is decreasing function.

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