# ESE GATE

# MADE EASY Workbook 2025

## Detailed Explanations of Try Yourself *Questions*

## Electrical Engineering

Electromagnetic Theory

MADE EASY PUBLICATIONS

# 1 Vector Analysis

**T1.** **(d)**

$$\bar{B} = -\rho\hat{a}_\phi + z\hat{a}_z$$

$$\rho = \sqrt{x^2 + y^2}$$

$$z = z$$

$$\begin{bmatrix} a_\phi \\ \hat{a}_\phi \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix}$$

$\Rightarrow$
$$\hat{a}_\phi = -\sin\phi\hat{a}_x + \cos\phi\hat{a}_y$$

$$\tan\phi = \frac{y}{x}$$

$$\hat{B} = -\sqrt{x^2 + y^2}\{-\sin\phi\hat{a}_x + \cos\phi\hat{a}_y\} + z\hat{a}_z$$

$$= -\sqrt{x^2 + y^2}\left\{\frac{-y}{\sqrt{x^2 + y^2}}\hat{a}_x + \frac{x}{\sqrt{x^2 + y^2}}\hat{a}_y\right\} + z\hat{a}_z$$

$$= y\hat{a}_x - x\hat{a}_y + za_z$$

**T2.** **(–3)**

As
$$\int_C \bar{F}\cdot d\bar{l} \equiv \int ydx - \int xdy$$

So,
$$y = x^2 \Rightarrow dy = 2xdx$$

$$\int_C \bar{F}\cdot d\bar{l} = \int x^2dx - \int x\cdot 2xdx$$

$$= \int(x^2 - 2x^2)dx \equiv -\int_{x=-1}^{2} x^2dx$$

$$\equiv -\left.\frac{x^3}{3}\right|_{-1}^{2} \equiv -\left[\frac{8}{3} + \frac{1}{3}\right] \equiv -3$$

**T3. (224)**

$$\bar{A} = \nabla f = 4xyz\hat{a}_x + 2x^2z\hat{a}_y + 2x^2y\hat{a}_z$$

$$(0,0,0) \xrightarrow{dx\hat{a}_x} (2,0,0) \xrightarrow{dy\hat{a}_y} (2,7,0) \xrightarrow{dz\hat{a}_z} (2,7,4)$$

$$\therefore \int \bar{A}\cdot d\bar{l} = \int 4xyzdx \text{ (at } y = 0 \text{ and } z = 0 + \int 2x^2zdy \text{ (at } z = 0 \text{ and } x = 2) + \int_{z=0}^{4} 2x^2ydz \text{ (at } x = 2 \text{ and } y = 7)$$

$$= 224$$

**T4.**

$$\oint \bar{D}\cdot d\bar{S} = \int \frac{5r^2}{4}\cdot r^2 \sin\theta d\theta d\phi \qquad \left(\text{at } \theta = 0, \frac{\pi}{4}, \phi = 0, 2\pi\right)$$

$$= 589.1 \text{ C}$$

$$\int (\nabla\cdot\bar{D})dV = \int (5r)\cdot r^2 \sin\theta d\theta d\phi dr = 589.1 \text{ C}$$

**T5. (d)**

$$\nabla\cdot F = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}[\rho(\sin^2\phi)] + \frac{\partial}{\partial z}(-z)$$

$$= 2 + 2\sin\phi\cos\phi - 1 = 1 + \sin 2\phi$$

**T6. Sol.**

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{1}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(-r^2\sin\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(10\cos\phi)$$

$$= \frac{1}{r^2} - 2r\cos\theta - \frac{10\sin\phi}{r\sin\theta}$$

$$\nabla\cdot\vec{A} \text{ at } \left(2,\frac{\pi}{4},\frac{\pi}{2}\right) = \frac{1}{4} - 4\times\frac{1}{\sqrt{2}} - \frac{10}{2\times\frac{1}{\sqrt{2}}} = \frac{1}{4} - 7\sqrt{2} = -9.65$$

**T7. (b)**

$$\nabla\cdot A = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot Kr^n) = \frac{1}{r^2}K(n+2)\,r^{n+1}$$

$$\nabla\times A = 0$$

Hence, for $n = -2$ given vector is solenoidal and always irrotational.

**T8. (d)**

$$\oint \bar{A}\cdot d\bar{l} = \left[\int_A^B + \int_B^C + \int_C^A\right]\bar{A}\cdot d\bar{l}$$

$$\left.\begin{aligned}\bar{A} &= 3x^2y^3\hat{a}_x - x^3y^2\hat{a}_y \\ d\bar{l} &= dx\hat{a}_x + dy\hat{a}_y\end{aligned}\right\} \bar{A}\cdot d\bar{l} = 3x^2y^3dx - x^3y^2dy$$

$$\bar{A}\cdot d\bar{l} = 3x^2y^3dx - x^3y^2dy$$

Path $AB$ : $y = x \Rightarrow dy = dx$

$$\int \bar{A}\cdot d\bar{l} \equiv \int 3x^2y^3dx - x^3y^2dy \equiv \int 3x^5 - x^5dx \equiv \int_{x=1}^{2} 2x^5dx \equiv 2\cdot\frac{x^6}{6}\Big|_1^2 = 21$$

Path $CA$ : $x = 2 \Rightarrow dx = 0$

Path $CA$ : $y = 1 \Rightarrow dy = 0$

$$\int \bar{A}\cdot d\bar{l} = \int 3x^2y^3dx \equiv 3y^3\int_{x=2}^{1} x^2dx \text{ at } y = 1 = 3\cdot\frac{x^3}{3}\Big|_2^1 = -7$$

Path $BC$ : $x = 2 \Rightarrow dx = 0$

$\therefore$
$$\oint \bar{A}\cdot d\bar{l} = 21 + \frac{56}{3} - 7 \equiv \frac{98}{3}$$

$$\int \bar{A}\cdot d\bar{l} = -\int_2^1 x^3y^2dy \text{ at } x = 2$$

$$= \frac{56}{3}$$

$$\oint \bar{A}\cdot d\bar{l} = 21 + \frac{56}{3} - 7 = \frac{98}{3}$$

$\int (\nabla\times\bar{A})\cdot d\bar{S}$ :

$$\nabla\times\bar{A} = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y^3 & -x^3y^2 & 0\end{vmatrix} = -12x^2y^2\hat{a}_z$$

$$d\bar{s} = dxdy(-\hat{a}_z) \qquad \text{(using RH curl)}$$

$\therefore$
$$\int \nabla\times\bar{A}\cdot d\bar{s} = 12\int_{x=1}^{2} x^2dx\int_{y=1}^{x} y^2dy = 12\int_{x=1}^{2} x^2dx\frac{y^3}{3}\Big|_{y=2}^{x} = \frac{12}{3}\int_{x=1}^{2} x^2dx(x^3 - 1)$$

$$= 4\left[\int_1^2 x^5dx - \int_1^2 x^2dx\right] = \frac{98}{3}$$

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# 2 Electrostatics

**T1. Sol.**

$$\psi = Q_{enc} = \int P_V dv = \int (\nabla \cdot \bar{D}) dv$$

$$\Rightarrow \quad \psi = \int_V (y + x + z) dxdydz$$

$$= \int_{y=-2}^{2} ydy \int_{x=1}^{4} dx \int_{z=-1}^{2} dz + \int_{z=-1}^{2} zdz \int_{x=1}^{4} dx \int_{y=-2}^{2} dy$$

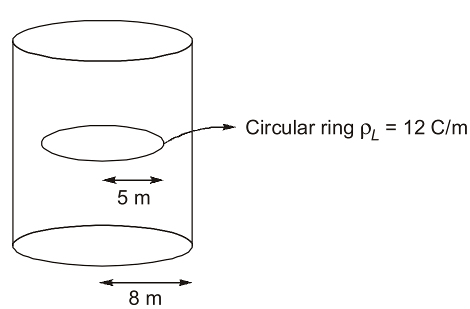
$$\Rightarrow \quad \psi = \frac{y^2}{2}\Big|_{-2}^{2} \cdot x\Big|_1^4 \cdot z\Big|_{-1}^{2} + \frac{x^2}{2}\Big|_1^4 \cdot y\Big|_{-2}^{2} \cdot z\Big|_{-1}^{2} + \frac{z^2}{2}\Big|_{-1}^{2} \cdot x\Big|_1^4 \cdot y\Big|_{-2}^{2}$$

$$\Rightarrow \quad \psi = 0 + \frac{15}{2} \cdot 4 \cdot 3 + \frac{3}{2} \cdot 3 \cdot 4 = 90 + 18 = 108C$$

**T2. (b)**

$$\nabla \cdot D = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho 20 \rho) + \frac{\partial}{\partial \rho}\left(\rho \frac{\rho^2}{3}\right)\right] = \frac{20 \times 2\rho}{\rho} + \frac{1}{3} \cdot \frac{3\rho^2}{\rho} = 40 + \rho$$

**T3. Sol.**




Total charge enclosed $= \rho_L \times 2\pi R = 12 \times 2\pi \times 5 = 120\pi$ C

**T4. (c, d)**

**T5. (d)**

Calculate the distance of the charges from the centre of the sphere and identify whether the charge is inside or outside the sphere.

For 2 C, $r_1 = \sqrt{4^2 + 8^2 + 3^2} = 9.43 \rightarrow$ Outside the sphere

For 8 C, $r_2 = \sqrt{2^2 + 1^2 + 3^2} = 3.74 \rightarrow$ Inside the sphere

For –12 C, $r_3 = \sqrt{4^2 + 0^2 + 1^2} = 4.123 \rightarrow$ Inside the sphere

Flux leaving the surface = 8 – 12 = –4 C

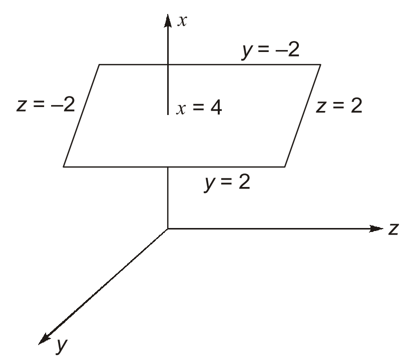
**T6. (c)**

1 nC and 3 nC.

**T7. (a)**

$$\text{Charge enclosed} = \rho_L \times \text{length}_{\text{enclosed}}$$
$$= 15 \times 10 = 150 \text{ nC}$$

**T8. (5.903)**



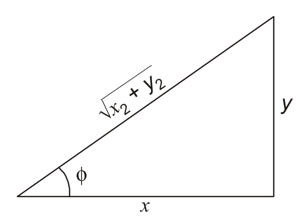

$$\bar{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

$$d\hat{s} = dydz\hat{a}_x$$

$$\therefore \quad \phi = \int \bar{D}\cdot\overline{ds} = \frac{\rho_L}{2\pi\rho}dydz\hat{a}_\rho\cdot\hat{a}_x$$

$$\because \quad \rho = \sqrt{x^2+y^2}\ ;\ \hat{a}_\rho\cdot\hat{a}_x = \cos\phi$$

$$\therefore \quad \phi = \int \frac{\rho_L}{2\pi\sqrt{x^2+y^2}}dydz\cdot\cos\phi$$




$$\because \quad \tan\phi = \frac{y}{x}$$

$$\therefore \quad \cos\phi = \frac{x}{\sqrt{x^2+y^2}}$$

Hence,
$$\phi = \int \frac{\rho_L}{2\pi\sqrt{x^2+y^2}}dy\,dz\cdot\frac{x}{\sqrt{x^2+y^2}}$$

$$= \int \frac{\rho_L x}{2\pi\sqrt{x^2+y^2}}dy\,dz$$

As $x = 4$, $y \in (-2, 2)$, $z \in (-2, 2)$

$$\therefore \quad \phi = \int \frac{\rho_L\cdot 4}{2\pi(16+y^2)}dydz$$

$$= \frac{\rho_L}{2\pi}\int_{-2}^{2}\frac{4dy}{16+y^2}z\Big|_{-2}^{2}$$

$$= \frac{4\rho_L}{2\pi}\cdot\int_{-2}^{2}\frac{4}{(4)^2+y^2}dy$$

$$= \frac{2\rho_L}{2\pi} \cdot 4 \cdot \int_{-2}^{2} \frac{1}{(4)^2 + y^2} dy$$

$$= \frac{8\rho_L}{\pi} \cdot \frac{\tan^{-1}}{4} \left(\frac{1}{2}\right)$$

$$= \frac{2\rho_L}{\pi} \tan^{-1}\left(\frac{1}{2}\right) \cdot 2$$

$$= \frac{4\rho_L}{\pi} \tan^{-1}\left(\frac{1}{2}\right)$$

$$= 5.903 \text{ nC}$$

**T9. Sol.**

$$E \text{ due to line charge} = \frac{\rho_L}{2\pi\epsilon\sqrt{2^2 + 2^2}} \frac{(\hat{a}_x + \hat{a}_y)}{\sqrt{2}} = \frac{\rho_L}{8\pi\epsilon}(\hat{a}_x + \hat{a}_y)$$

$$E \text{ due to sheet charge} = \frac{\rho_S}{2\epsilon}(-\hat{a}_y)$$

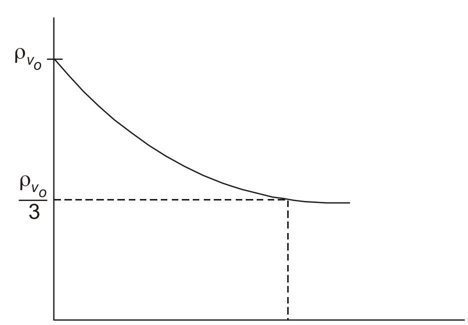
Equating the $y$-components, $\frac{\rho_L}{8\pi\epsilon} = \frac{\rho_S}{2\epsilon}$

$$\rho_L = 4\pi\rho_s$$

**T10. Sol.**

$$Q = \int_1 \rho_s \, ds = \int_{x=-1}^{1} \int_{y=0}^{2} 4x^2 y \, dx \, dy = 4 \cdot \frac{x^3}{3}\bigg|_{-1}^{1} \frac{y^2}{2}\bigg|_{0}^{2} = \frac{16}{3}$$

**T11. (b)**




$$\rho_v = \rho_{v_o} e^{-t/T_r}; \; T_r = \frac{\epsilon}{\sigma}$$

$$\therefore \quad \frac{\rho_{v_o}}{3} = \rho_{v_o} e^{-t/T_r}$$

$$\frac{t}{T_r} = 1.0986$$

$$T_r = \frac{15\times10^{-6}}{1.0986} = 1.4\times10^{-5}$$

$$\varepsilon_r = \frac{1.4\times10^{-5}\times2\times10^{-4}}{8.85\times10^{-12}} = 308.56$$

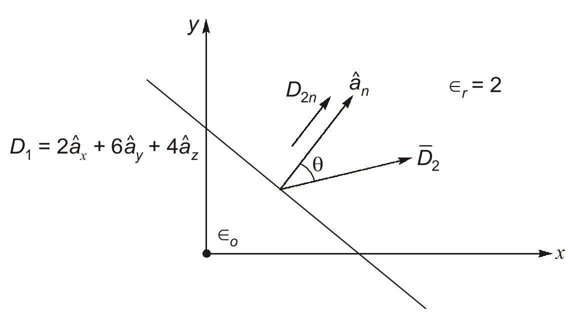
**T12. Sol.**

$$3x + 4y = 12$$

At $x = 0$, $y = 3$

At $y = 0$, $x = 4$

$$\hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{3\hat{a}_x + 4\hat{a}_y}{5}$$




Normal component :

$$\therefore \quad \bar{D}_{\text{ln}} = (\bar{D}_{\text{ln}}\cdot\hat{a}_n)\hat{a}_n = \left(2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z\cdot\frac{3\hat{a}_x+4\hat{a}_y}{5}\right)\left(\frac{3\hat{a}_x+4\hat{a}_y}{5}\right)$$

$$\Rightarrow \quad \bar{D}_{\text{ln}} = 6\left\{\frac{3\hat{a}_x+4\hat{a}_y}{5}\right\} \equiv 3.6\hat{a}_x + 4.8\hat{a}_y \equiv \bar{D}_{2n}$$

Tangential component :

$$\bar{E}_{1t} = \bar{E}_{2t}$$

$$\Rightarrow \quad \frac{\bar{D}_{1t}}{\epsilon_1} = \frac{\bar{D}_{2t}}{\epsilon_2}$$

$$\Rightarrow \quad \bar{D}_{2t} = \frac{\epsilon_2}{\epsilon_1}\bar{D}_{1t} \equiv \frac{2\,\epsilon_0}{\epsilon_0}\{\bar{D}_1 - \bar{D}_{1n}\}$$

$$\equiv 2\{(2, 6, 4) - (3.6, 4.8, 0)\}$$

$$\equiv 2\{-1.6, 1,2, 4\}$$

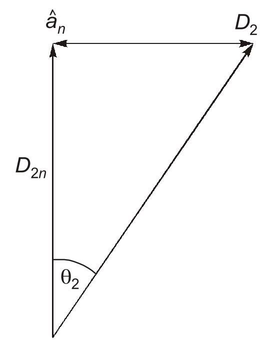
$$\equiv -3.2, 2.4, 8$$

$$\therefore \quad \bar{D}_{2t} = 3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z$$

Hence, $\bar{D}_2 = \bar{D}_{2t} + \bar{D}_{2x} \equiv -3.2\hat{a}_x + 2.4\hat{a}_y + 8\hat{a}_z + 3.6\hat{a}_x + 4.8\hat{a}_y$

$$\Rightarrow \quad \bar{D}_2 = 0.4\hat{a}_x + 7.2\hat{a}_y + 8\hat{a}_z \text{ V/m}$$




$$\cos\theta_2 = \frac{|D_{2n}|}{D_2} = \frac{6}{\sqrt{0.4^2 + 7.2^2 + 8^2}}$$

$$= 56.14^\circ$$

**T12* (d)**

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$\frac{\partial V}{\partial x} = 2y^3 - 3yz^2$$

$$\Rightarrow \quad V = 2y^3x - 3xyz^2 + f(y, z) \quad \text{...(1)}$$

$$\frac{\partial V}{\partial y} = 6xy^2 - 3xz^2$$

$$\Rightarrow \quad V = 2xy^3 - 3xyz^2 + f(x, z) \quad \text{...(2)}$$

$$\frac{\partial V}{\partial z} = -6xyz$$

$$\Rightarrow \quad V = -3xyz^2 + f(x, y) \quad \text{...(3)}$$

Equating equation (1), (2) and (3)

$$V = 2xy^3 - 3xyz^2$$

**T13. Sol.**

$$V = 100(x^2 - y^2)$$

At (2, 1, 1), $V = 100(4 - 1) = 300$

So, $100(x^2 - y^2) = 300$

$\Rightarrow$ $x^2 - y^2 = 3$

**T14. Sol.**

$$\nabla^2 V = 0$$

$$\Rightarrow \quad \nabla^2 V = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3h_1}{h_2}\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

As V = $f(\phi)$

$$\Rightarrow \quad \nabla^2 V = \frac{1}{\rho}\left[\frac{\partial}{\partial\phi}\left(\frac{1}{\rho}\frac{\partial V}{\partial \phi}\right)\right] = 0$$

$$\Rightarrow \quad \frac{\partial^2 V}{\partial \phi^2} = 0 \Rightarrow \frac{\partial V}{\partial \phi} = A$$

$$\Rightarrow \quad V = A\phi + B$$

At $\phi = 0°$, $V = 10\text{ V} \Rightarrow 10 = B$

At $\phi = \frac{\pi}{6}$, $V = 150\text{ V}$

$$150 = \frac{A\pi}{6} + 10$$

$$\Rightarrow \quad A = \frac{840}{\pi}$$

$$\therefore \quad V = \left(\frac{840}{\pi}\phi + 10\right)\text{V}$$

As $\vec{E} = -\nabla V = \frac{-1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi$

$$\Rightarrow \quad \vec{E} = \left(\frac{-840}{\pi\rho}\hat{a}_\phi\right)\text{ V/m}$$

$$\therefore \quad \vec{D} = \varepsilon_o\vec{E} = \frac{-840}{\pi\rho}\varepsilon_o\hat{a}_\phi$$

As the conductor is a plane surface.

$$\therefore \quad \rho_s = |D|$$

So, $\rho_s = \frac{840\varepsilon_o}{\pi\rho}$

$$\Rightarrow \quad Q = \int \rho_s \vec{ds}$$

$$\vec{ds} = ds\hat{a}_\phi = d\rho dz\hat{a}_\phi$$

$$\therefore \quad Q = \int \frac{840\varepsilon_o}{\pi\rho} d\rho dz = \frac{840}{\pi}\varepsilon_o \ln\rho\Big|_1^2 \; z\Big|_0^1$$

$$Q = \frac{840\varepsilon_o}{\pi}\ln(2) \Rightarrow Q = 1.64\text{ nC}$$

**T15. Sol.**

$$Q = C_oV_o = C_{eq}V'$$

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.6$$

$$V' = \frac{24}{1.6} = 15\text{ V}$$

**T16.** **(c)**

For air filled parallel plate capacitor

$$C_o = \frac{\varepsilon_o A}{d}$$

$$Q = C_o V$$

The equivalent arrangement is parallel capacitor

$$C_{eq} = \frac{4\varepsilon_o A}{2d} + \frac{\varepsilon_o A}{2d} = \frac{5\varepsilon_o A}{2d} = \frac{5}{2}C_o$$

$$Q_T = C_{eq}V'$$

$$2.5Q = 2.5C_o V'$$

$$Q = C_o V'$$

But $\quad Q = C_o V$

$\therefore \quad V' = V$

**T17.** **Sol.**

$$C_1 = \frac{\epsilon_1 A/2}{d}$$

$$C_2 = \frac{\epsilon_2 A/2}{d}$$

$$W_E = \frac{1}{2}CV^2$$

$$\frac{W_{E_1}}{W_{E_2}} = \frac{1}{2} = 0.5$$

**T18.** **Sol.**

**T21.** **(b)**

Concept : Method of images

For $\vec{F}_1$ :

$$\bar{r} = \bar{r}_f - \bar{r}_i = (1, 0, 1) - (1, 0, -1) = 2\hat{a}_z$$

$$\therefore \quad \hat{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{2\hat{a}_z}{2} = \hat{a}_z$$

$$\therefore \quad \bar{F}_1 = \frac{-Q^2}{4\pi\varepsilon_o(r)}\hat{a}_z = \frac{-Q^2}{16\pi\varepsilon_o}\hat{a}_z$$

For $\bar{F}_2$ :

$$\bar{r} = \bar{r}_f - \bar{r}_i = (1, 0, 1) - (0, 0, -1) = \hat{a}_x + 2\hat{a}_z$$

$$|\bar{r}| = \sqrt{5}$$

$$\therefore \quad \hat{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{\hat{a}_x + 2\hat{a}_z}{\sqrt{5}}$$

$$\therefore \quad \bar{F}_2 = \frac{Q^2}{4\pi\varepsilon_o(5)} \cdot \frac{\hat{a}_x + 2\hat{a}_z}{\sqrt{5}}$$

For $\bar{F}_3$ :

$$\bar{r} = \bar{r}_f - \bar{r}_i = (1, 0, 1) - (0, 0, 1) = \hat{a}_x$$

$$|\bar{r}| = 1$$

$$\therefore \quad \hat{a}_r = \frac{\hat{r}}{|\bar{r}|} = \hat{a}_x$$

$$\therefore \quad \bar{F}_3 = \frac{-Q^2}{4\pi\varepsilon_o(1)}\hat{a}_x$$

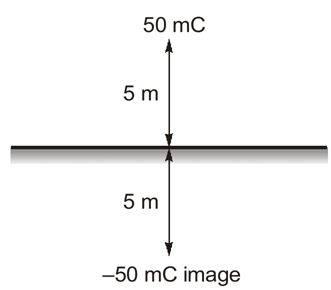
Hence,

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = -\frac{Q^2}{16\pi\varepsilon}\hat{a}_z + \frac{Q^2}{4\pi\varepsilon_o(5\sqrt{5})}(\hat{a}_x + 2\hat{a}_z) - \frac{Q^2}{4\pi\varepsilon_o}\hat{a}_x$$

$$\bar{F} = \frac{Q^2}{4\pi\varepsilon_o}\left[\frac{-\hat{a}_z}{4} + \frac{2\hat{a}_z}{5\sqrt{5}} + \frac{\hat{a}_x}{5\sqrt{5}} - \hat{a}_x\right]$$

$$\bar{F} = \frac{-Q^2}{4\pi\varepsilon_o}[0.91\hat{a}_x + 0.071\hat{a}_z]\text{N}$$

**T22. Sol.**




The induced surface charge density is the normal flux density at the point

$$D_{\text{normal}} = \rho_s$$

$$E \text{ due charge} = \frac{50\times10^{-3}\times9\times10^{9}}{25} = 18\times10^{6} \text{ V/m}$$

$$E \text{ due to image} = 18\times10^{6} \text{ V/m (same direction)}$$

$$\text{Total } E = 36\times10^{6}$$

$$D = 36\times10^{6}\times\frac{1}{36\pi\times10^{9}} = \frac{1}{\pi}\text{ mC}$$

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# 3 Magnetostatics

**T1. Sol.**

$$H \propto \frac{I}{d} \text{ for a square loop}$$

$$\frac{H_1}{H_2} = \frac{I_1}{I_2}\frac{d_2}{d_1} = \frac{20}{5}\frac{d}{d/3} = 12$$

**T2. (b)**

$$B \text{ at centre } = \frac{\mu_o I}{2R}\left[1 - \frac{1}{2} + \frac{1}{4} \ldots\ldots\right]\hat{a}_z$$

$$= \frac{\mu_o I}{2R}\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right]\hat{a}_z = \frac{\mu_o I}{3R}\hat{a}_z$$

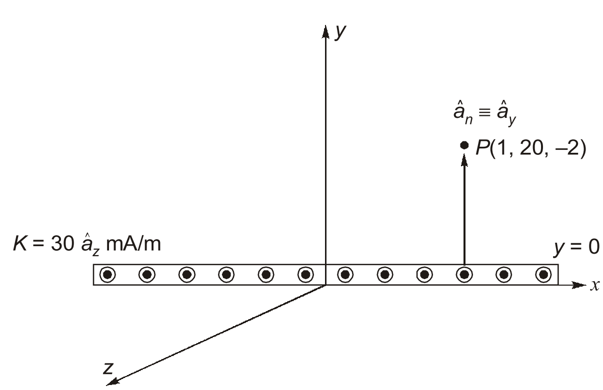
**T3. (d)**

$$\hat{a}_H = \hat{a}_L \times \hat{a}_r$$

1. For $xy$-plane wire : $\hat{a}_H = \hat{a}_x \times -\hat{a}_y = -\hat{a}_z$

2. For $yz$-plane wire : $\hat{a}_H = \hat{a}_y \times -\hat{a}_z = -\hat{a}_x$

**T4. Sol.**




$$\bar{K} = 30\hat{a}_z \text{ mA/m at } y = 0, \text{ i.e., } xz\text{-plane.}$$

$P(1, 20, -2)$ has $y = 20$ which lies above $y = 0$.

So, $$\hat{a}_x \equiv \hat{a}_y$$

Hence, $$\bar{H} = \frac{1}{2}\bar{K} \times \hat{a}_x$$

$$= \frac{1}{2} \times 30\left[\hat{a}_z \times \hat{a}_y\right]$$

$$= -15\hat{a}_x$$

$\therefore$ $$\bar{H} = -15\hat{i} \text{ mA/m}$$

**T5. Sol.**

Net flux through the loop is zero considering opposite directions of flux as interpreted from arrow shown on the length of loop

$$\iint B \cdot ds = \psi_1 - \psi_2 = 0$$

$$\psi_1 - \psi_2 = \int_0^z \int_a^b \frac{I}{2\pi p} dp dz = \int_0^z \int_b^c \frac{I}{2\pi p} dp dz$$

$$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{c}{b}\right)$$

$$b = \sqrt{ac}$$

**T6. Sol.**

$$I \text{ flow direction} = \text{Vector potential direction}$$

$$\text{A direction at } \rho = \hat{a}_y$$

$$B \text{ direction} = \hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

**T7. Sol.**

$$\nabla^2 A = -\mu J$$

$$\nabla^2 z^2\, \hat{a}_\rho + \nabla^2 2\rho^2 \cos\phi\, \hat{a}_\phi = -\mu J$$

$$\nabla^2 z^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial z^2}{\partial \rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{1}{\rho}\frac{\partial z^2}{\partial \phi}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho \frac{\partial z^2}{\partial z}\right) = 2$$

$$\nabla^2 2\rho^2 \cos\phi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}(2\rho^2 \cos\phi)\right) + \frac{1}{\rho}\frac{\partial}{\partial \phi}\left(\frac{1}{\rho}\frac{\partial}{\partial \phi}(2\rho^2 \cos\phi)\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho \frac{\partial}{\partial z}(2\rho^2 \cos\phi)\right)$$

$$= \frac{1}{\rho}\cos\phi \cdot 4 \cdot 2 \cdot \rho - 2\cos\phi = 6\cos\phi$$

At the origin, $$\nabla^2 A = 2\hat{a}_\rho + 6\hat{a}_\phi = -\mu J$$

$$J = \frac{-1}{\mu}(2\hat{a}_\rho + 6\hat{a}_\phi)$$

**T8.** **(a)**

**T9.** **Sol.**

$$f = \mu N = \mu_{mg} = 0.1 \times 1 \times 10 = 1\text{ N}$$

$$\bar{F} = i\bar{l} \times \bar{B} = 10(0.5)\hat{a}_y \times -1\hat{a}_z \equiv -5\hat{a}_x \quad \Rightarrow \quad |F| = 5$$

$$F_{net} = F - f = 5 - 1 = 4$$

Also, $F_{net} = ma$

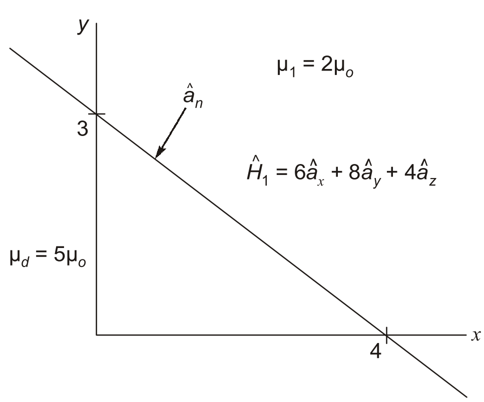
$\therefore$ $ma = 4 \quad \Rightarrow \quad a = \dfrac{4}{m} = \dfrac{4}{1} \equiv 4\text{ m/s}^2$

Now, $V^2 = u^2 + 2as$

$\Rightarrow$ $V^2 = 2as$

$\Rightarrow$ $V = \sqrt{2as} \equiv \sqrt{2*4*1} \equiv \sqrt{8} \equiv 2.8\text{ m/s}$

**T10.** **(b)**




$$3x + 4y = 12$$

At $x = 0$, $y = 3$

At $y = 0$, $x = 4$

As $\bar{H}_1 = 6\hat{a}_x + 8\hat{a}_y + 4\hat{a}_z$

$\Rightarrow$ $\bar{H}_{\text{ln}} = (\bar{H}_1 \cdot \hat{a}_n)\hat{a}_n = \left\{(6,8,4)\left(\dfrac{3,4,0}{5}\right)\right\}\left(\dfrac{3,4,0}{5}\right)$

$$= \left\{\frac{18+32}{25}\right\}(3\hat{a}_x + 4\hat{a}_y) \equiv 6\hat{a}_x + 8\hat{a}_y$$

$\therefore$ $\bar{H}_{1t} = \bar{H}_1 - \bar{H}_{\text{ln}} = (6, 8, 4) - (6, 8, 0) \equiv 4\hat{a}_z$

Now, at $\bar{K} = 0$, $\bar{H}_{1t} = \bar{H}_{2t} \quad \Rightarrow \quad \bar{H}_{2t} = 4\hat{a}_2$

Also, $\bar{B}_{1n} = B_{2n}$

$\Rightarrow$ $\mu_1\bar{H}_{1n} = \mu_2\bar{H}_{2n}$

$$\Rightarrow \quad \vec{H}_{2n} = \frac{\mu_1}{\mu_2}\bar{H}_{1n} = \frac{2}{5}\{6\hat{a}_x + 2\hat{a}_y\} \equiv 2.4\hat{a}_x + 0.8\hat{a}_y$$

$$\therefore \quad \bar{H}_2 = \bar{H}_{2t} + \bar{H}_{2n} = 2.4\hat{a}_x + 0.8\hat{a}_y + 4\hat{a}_z \text{ A/m}$$

**T11. Sol.**

$$H_{t_1} = 4\hat{a}_x - 5\hat{a}_z$$

$$H_{t_2} = 8\hat{a}_x - 5\hat{a}_z$$

$$H_{t_2} = H_{t_1} + \bar{K}\times\hat{a}_y$$

$$(H_{t_2} - H_{t_1}) = \bar{K}\times\hat{a}_y$$

$$\bar{K} = \hat{a}_y \times (H_{t_2} - H_{t_1}) = \hat{a}_y \times (8\hat{a}_x - 5\hat{a}_z - 4\hat{a}_x + 5\hat{a}_z) = \hat{a}_y \times 4\hat{a}_x = -4\hat{a}_z$$

$$\bar{K} = -4\hat{a}_z$$

**T12. (d)**

Magnetic energy, $$W_m = \frac{1}{2}\int \bar{B}\cdot\bar{H}dV$$

$$W_m = \frac{1}{2}\mu\int |H|^2\, dV$$

$$\oint \bar{H}\cdot d\bar{l} = I_{enc} = \int \bar{J}\cdot d\bar{s}$$

$$I_{enc} = \frac{I}{\pi R^2}\pi r^2 = \frac{Ir^2}{R^2}$$

$$\oint \bar{H}\cdot d\bar{l} = H2\pi r$$

$$H2\pi r = \frac{Ir^2}{R^2}$$

$$\Rightarrow \quad H = \frac{Ir}{2\pi R^2}$$

$$\therefore \quad W_{in} = \frac{\mu}{2}\int \frac{I^2 r^2}{4\pi^2 R^4}\cdot r dr d\theta dz \equiv \frac{\mu}{2}\cdot\frac{I^2}{4\pi^2 R^4}\int_{r=0}^{R} r^3 dr \int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{l} dz$$

$$\Rightarrow \quad W_{in} = \frac{\mu I^2}{8\pi^2 R^4}\cdot\frac{R^4}{4}\cdot 2\pi l \equiv \frac{\mu I^2 l}{16\pi}$$

$$\therefore \quad W_{in} = \frac{\mu I^2 l}{16\pi}$$

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# 4 Time Varying Fields

**T1. (c)**

**T2. (200)**

$$V_{emf} = \int (\bar{v} \times \bar{B}) \overline{dl}$$

$$\bar{v} = \rho\omega \hat{a}_\phi$$

$$\bar{B} = B_o(-\hat{a}_z)$$

$$\bar{v} \times \bar{B} = B_o \rho\omega (\hat{a}\rho)$$

$$\overline{dl} = d\rho \hat{a}\rho$$

∴ $$V_{emf} = \int_0^R -B_o \rho\omega d\rho = \frac{-B_o \omega R^2}{2}$$ ; –ve sign Þ for 0 → $R$ $V$↓

∴ $$i_{ind} = \frac{B_o \rho^2 \omega}{2R} = 0.1 \text{ A}$$

∴ $$I_{p-P} = 2 \times 0.1 = 0.2 \text{ A} = 200 \text{ mA}$$

**T3. (d)**

**T4. (a, b, c)**

Region has no charge, i.e., $\rho_v = 0$ and no current, i.e., $\bar{J} = 0$.

(a) $$\nabla \cdot \bar{D} = \rho_v$$

at $\rho_v = 0$, $$\nabla \cdot \bar{D} = 0 \Rightarrow \nabla \cdot \bar{E} = 0$$

(b) $$\nabla \cdot \bar{B} = 0$$

(c) $$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

(d) $$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_d = \bar{J}_d$$

⇒ $$\nabla \times \frac{\bar{B}}{\mu} = \frac{\partial \bar{D}}{\partial t}$$

⇒ $$\nabla \times \bar{B} = \mu \in \frac{\partial \bar{E}}{\partial t}$$

$$\Rightarrow \qquad \nabla \times \bar{B} = \frac{1}{C^2} \cdot \frac{\partial E}{\partial t}$$

$$\Rightarrow \qquad \nabla \times \bar{B} - \frac{1}{C^2} \cdot \frac{\partial \bar{E}}{\partial t} = 0$$

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