ESE GATE PSUs

State Engg. Exams

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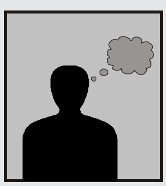
Workbook 2025



Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Theory of Machines

MADE EASY PUBLICATIONS

1 Mechanism

Detailed Explanation of Try Yourself Questions

T1 : Solution

Pair Symbol	Constrained motion	Relative Motion	Degrees of Freedom
Revolute pair	1	Circular	5
Cylindrical pair	2	Cylindrical	4
Screw pair	1	Helical	5
Spherical pair	3	Spherical	3

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3 Gears and Gear Trains

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given data : $T_p = 36$, $T_g = 96$, $\phi = 20°$, m = 10 mm, $a_m = 10$ mm

Pitch circle radius,

$$R = \frac{mT_g}{2} = \frac{10 \times 96}{2} = 480 \text{ mm}$$

Gear Addendum radius,

$$R_a = R + 10 = 490 \text{ mm}$$

$$r = \frac{mT_p}{2} = \frac{10 \times 36}{2} = 180 \text{ mm, pinion}$$

$$r_a = r + 10 = 190 \text{ mm}$$

Path of contact $= \sqrt{R_a^2 - (R\cos\phi)^2} - R\sin\phi + \sqrt{r_a^2 - (r\cos\phi)^2} - r\sin\phi$

or $= \sqrt{490^2 - (480\cos 20°)^2} - 480\sin 20° + \sqrt{190^2 - (180\cos 20°)^2} - 180\sin 20°$

$= 191.446 - 164.17 + 86.54 - 61.56 = \mathbf{52.256 \text{ mm}}$

Arc of contact $= \dfrac{\text{Path of contact}}{\cos 20°} = \dfrac{52.256}{\cos 20°} = \mathbf{55.6 \text{ mm}}$

Contact ratio $= \dfrac{\text{Arc of contact}}{\text{Circular pitch}} = \dfrac{55.6}{\pi \times 10} = \mathbf{1.77}$

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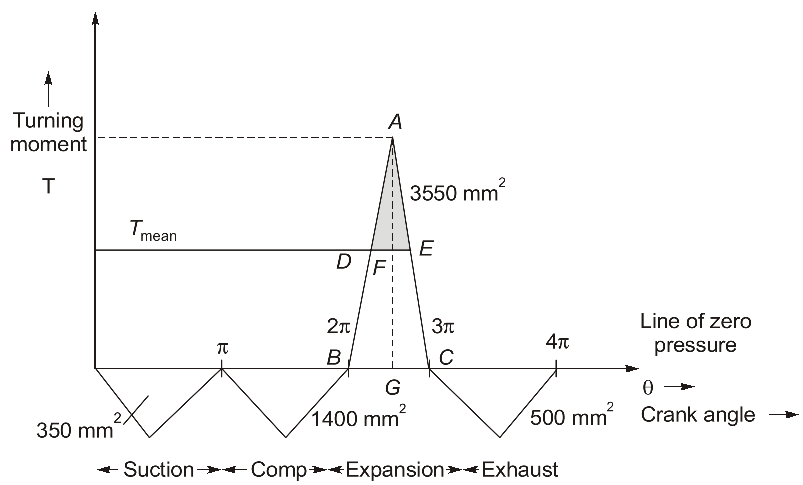
5 Flywheel

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$(T_{mean} \times 4\pi) = 3550 - (1400 + 350 + 500) \qquad [\text{As } 1\ mm^2 = 3\ N\text{-}m]$$

$$= 1300 \times 3$$

$$T_{mean} \times 4\pi = 3900$$

$$T_{mean} = 310.352\ Nm$$

$$\therefore \qquad \frac{1}{2} \times T_{max} \times \pi = 3550 \times 3$$

$$T_{max} = \frac{6 \times 3550}{\pi} = 6780.00\ Nm$$

from similar triangles, ΔADE and ΔABC,

$$\frac{AF}{AG} = \frac{DE}{BC} \Rightarrow \frac{6469.64}{6780} = \frac{DE}{\pi}$$

$$\Rightarrow DE = 2.994\ rad$$

$\therefore$ ΔE = Fluctuation of energy

$$\Delta E = \frac{1}{2} \times DE \times AF$$

$$\Delta E = \frac{1}{2} \times 2.994 \times 6469.64$$

$$\Delta E = 9685.05 \text{ Nm}$$

But

$$\Delta E = I K_s \omega_{avg}^2$$

$$\omega_{avg} = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 200}{60} = 20.943 \text{ rad/s}$$

$$9685.05 = I \times 0.04 \times (20.943)^2$$

$$I = 551.98$$

and $I = mk^2$

$\therefore$ $m \times (.75)^2 = 551.98$

$\Rightarrow$ $m = 981.3$ kg

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6 Governor

Detailed Explanation of Try Yourself Questions

T1 : Solution

As per given information, r_1 = 120 mm, r_2 = 80 mm,
ball arm = sleeve arm ($a = b$), m = 2 kg

N_2 = 400 rpm, $\omega_1 = \frac{2\pi \times 400}{60}$, N_1 = 420 rpm, $\omega_2 = \frac{2\pi \times 420}{60}$

Sprint constant?

$$F_1 = mr_1\omega_1^2 = 2 \times 0.120 \times \left(\frac{2\pi \times 420}{60}\right)^2 = 464.266 \text{ N}$$

$$F_2 = mr_2\omega_2^2 = 2 \times 0.80 \times \left(\frac{2\pi \times 400}{60}\right)^2 = 280.735 \text{ N}$$

$$\text{Spring constant, } K = 2\left(\frac{a}{b}\right)^2 \left(\frac{F_1 - F_2}{r_1 - r_2}\right)$$

$$= 2(1)^2 \left(\frac{464.266 - 280.735}{0.040}\right) = 9.176 \times 10^3 \text{ N/m}$$

(ii) $$\text{Spring constant, } K = 9.176 \text{ N/mm}$$

$$F_2 \times a = 0 + \frac{F_{s1}}{2} \cdot b$$

$$F_{s1} = 2F_2 = 2 \times 280.735 \text{ N}$$

(i) $$\text{Initial compression} = \frac{F_{s1}}{K} = \frac{2 \times 280.735}{9.176} \text{N} = 61.1889 \text{ mm}$$

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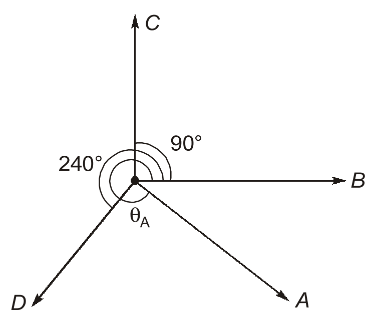
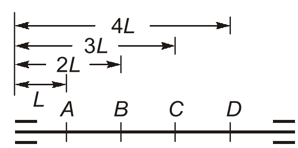
7 Balancing

Detailed Explanation of Try Yourself Questions

T1 : Solution





$\Sigma F_x = 0,\ m_A\, r \cos\theta + m_B\, r\cos 0° + m_C\, r\cos 90° + m_D\, r\cos 240° = 0$

$\Sigma F_y = 0,\ m_A\, r \sin\theta + m_B\, r\sin 0° + m_C\, r\sin 90° + m_D\, r\sin 240° = 0$

$$\Sigma F_x = mr\cos\theta + m_B r - \frac{m_D r}{2} = 0$$

$$m_A \cos\theta + m_B = \frac{m_D}{2}$$

$$m_A \cos\theta + 7 = \frac{m_D}{2}$$

$$\Sigma F_y = 0$$

$$m_A \sin\theta + m_C - \frac{\sqrt{3}}{2} m_D = 0$$

$$m_A \sin\theta + m_C = \frac{\sqrt{3}}{2} m_D$$

Dynamic

$\Sigma M_x = 0$

$m_A\, r\, l \cos\theta + m_B\, r\, 2\, l\cos 0° + m_C\, r\, 3\, l\cos 90° + m_D\, r\, 4\, l \cos 240° = 0$

$$m_A \cos\theta + 2\, m_B = 2\, m_D$$

$\Sigma M_y = 0$

$m_A\, r\, l \sin\theta + m_B\, r\, 2\, l \sin 0° + m_C\, r\, 3\, l \sin 90° + m_D\, r\, 4\, l \sin 240° = 0$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D$$

$$m_A \cos\theta + 7 = \frac{m_D}{2} \quad \text{... (i)}$$

$$m_A \sin\theta + m_C = \frac{\sqrt{3}}{2} m_D \quad \text{... (ii)}$$

$$m_A \cos\theta + 14 = 2\, m_D \quad \text{... (iii)}$$

$$m_A \sin\theta + 3\, m_C = 2\sqrt{3}\, m_D \quad \text{... (iv)}$$

From equation (iii) – (i)

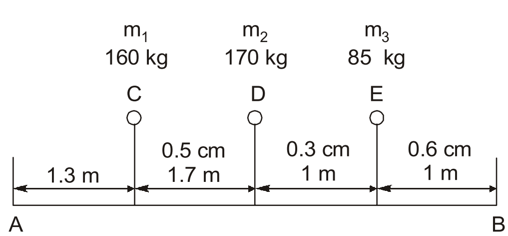
$$m_D = 4.667 \text{ kg}$$
$$m_A \cos\theta + 7 = 2.33$$
$$m_A \sin\theta + m_C = 4.04$$
$$m_A \cos\theta + 14 = 9.332$$
$$m_A \sin\theta + 3\, m_C = 16.14$$
$$m_A \sin 0° + m_C = 4.04$$
$$m_C = 6.0667 \text{ kg}$$
$$m_A \sin\theta = -2$$
$$m_A \cos\theta = -4.66$$
$$m_A = 5.087 \text{ kg}$$
$$\theta = 203.456°$$

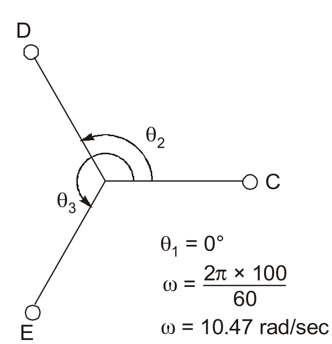
T2 : Solution

No dynamic force at B.

Consider Rerefence Plane passing through A

Couple balancing





X-component

$m_1 r_1 l_1 \cos\theta_1 + m_2 r_2 l_2 \cos\theta_2 + m_3 r_3 l_3 \cos\theta_3 + 0 = 0$

$160 \times 0.5 \times 1.3 \times 1 + 170 \times 0.3 \times 3 \cos\theta_2 + 85 \times 0.6 \times 4 \cos\theta_3 = 0$

$104 + 153 \cos\theta_2 + 204 \cos\theta_3 = 0$ ...(i)

Y-component

$$0 + 153 \sin\theta_2 + 204 \sin\theta_3 = 0$$

$$153 \sin\theta_2 + 204 \sin\theta_3 = 0 \quad ...(ii)$$

Squaring and Adding equation (i) and equation (ii),

$153^2 + 204^2 + 2 \times 153 \times 204 [\cos\theta_2 \cos\theta_3 + \sin\theta_2 \sin\theta_3] = 104^2$

$$\cos(\theta_3 - \theta_2) = -0.8684$$

$$\theta_3 - \theta_2 = 150.27°$$

$$\theta_3 = 150.27° + \theta_2 \quad ...(iii)$$

Putting in equation (ii),

$153 \sin\theta_2 + 204 \sin(150.27 + \theta_2) = 0$

$153 \sin\theta_2 + 204 (0.496 \cos\theta_2 - 8684 \sin\theta_2) = 0$

$153 \sin\theta_2 + 101.184 \cos\theta_2 - 177.15 \sin\theta_2 = 0$

$24.15 \sin\theta_2 = 101.184 \cos\theta_2$

$\tan\theta_2 = \dfrac{101.184}{24.15}$, $\Rightarrow \theta_2 = 76.6°$, From equation (iii) $\theta_3 = 226.85°$

Let the dynamic force at A is F

$\therefore$ Static Balance

X-component

$F.\cos\theta + m_1 r_1 \omega_1^2 \cos\theta_1 + m_2 r_2 \omega_2^2 \cos\theta_2 + m_3 r_3 \omega_3^2 \cos\theta_3 + 0 = 0$

$F\cos\theta + 160 \times 0.005 \times (10.47)^2 \cos 0° + 170 \times 0.003 \times (10.47)^2 \cos 76.6° + 85 \times (0.006) \times (10.47)^2 \times \cos(226.85°)$

$F\cos\theta = -62.4178$

Y-component

$F\sin\theta + 160 \times 0.005 \times (10.47)^2 \times \sin 0° + 170 \times 0.003 \times 10.47^2 \sin 76.6° + 85 \times 0.006 \times 10.47 \times \sin 226.85°$

$F\sin\theta = -13.6$

$$\therefore \quad F = \sqrt{(62.4178)^2 + (13.6)^2}$$

$$F = 63.88 \text{ N}$$

At angle (with C)

$$\tan\theta = \frac{-13.6}{-62.4178}$$

$$\theta = 180 + 12.3$$

$$\theta = 192.3°$$

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10 Vibration

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given; d = 50 mm = 0.05 m; l = 300 mm = 0.03 m; m = 100 kg; E = 200 GN/m^2 = 200 × 109 N/m^2

We knowt hat cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6} \text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l.}{A.E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6} \text{ m} ; \quad \omega_n = \sqrt{g/\delta}$$

∴ Frequency of longitudinal vibration,

$$f_n = \frac{\omega_n}{2\pi} = \left(\frac{\sqrt{g}}{2\pi}\right) \times \frac{1}{\sqrt{\delta}}$$

⇒ $$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz}$$

Frequency of transverse vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l^3}{3E.I} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

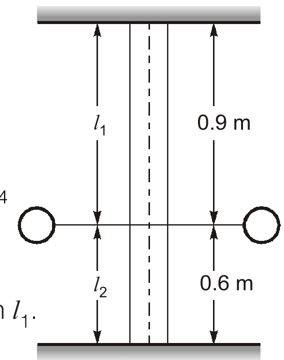
$$f_n = \frac{\omega_n}{2\pi} = \left(\sqrt{\frac{g}{2\pi}}\right) \times \frac{1}{\sqrt{\delta}}$$

⇒ $$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{Hz}$$

T2 : Solution

Given: d = 50 mm = 0.05 m; m = 500 kg; E = 200 GN/m^2 = 200 × 10^9 N/m^2

We know that cross-sectional area of shaft,



$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3}\text{m}^2$$

and moment of inertia of shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\text{m}^4$$

Natural frequency of longitudinal vibration

Let m_1 = Mass of flywheel carried by the length l_1.

∴ $m - m_1$ = Mass of flywheel carried by length l_2.

We know that extension of length l_1

$$= \frac{W_1 \cdot l_1}{A.E} = \frac{m_1.g.l_1}{A.E} \quad \text{...(i)}$$

Similarly, compression of length l_2

$$= \frac{(W - W_1)l_2}{A.E} = \frac{(m - m_1)g.l_2}{A.E} \quad \text{...(ii)}$$

Since extenson of length l_1 must be equal to compression of length l_2, therefore equating equations (i) and (ii),

$$m_1.l_1 = (m - m_1)l_2$$

$$m_1 \times 0.9 = (500 - m_1)0.6 = 300 - 0.6\, m_1 \quad \text{or} \quad m_1 = 200 \text{ kg}$$

∴ Extension of length l_1,

$$\delta = \frac{m_1.g.l_1}{A.E} = \frac{200 \times 9.81 \times 0.9}{1.96 \times 10^{-3} \times 200 \times 10^9} = 4.5 \times 10^{-6}\text{m}$$

We know that natural frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{4.5 \times 10^{-6}}} = 235 \text{ Hz}$$

Alternate: Natural freuency of longitudinal vibration

$$\text{Axial stiffness} = \frac{AF}{l} \Rightarrow s_1 = \frac{AE}{l_1}; s_2 = \frac{AE}{l_2}$$

The 2-stiffness are in parallel

$$\Rightarrow \quad s = s_1 + s_2 = 1.0908 \times 10^9$$

$$w_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.0908 \times 10^9}{500}} = 1477.04$$

$$f = \frac{w_n}{2\pi} = 235 \text{ Hz}$$

Natural frequency of tranverse vibration

We know that the static deflection for a shaft fixed at both ends and carrying a point load is given by

$$\delta = \frac{Wa^3b^3}{3EIl^3} = \frac{500 \times 9.81(0.9)^3(0.6)^3}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6}(1.5)^3} = 1.24 \times 10^{-3}\text{m}$$

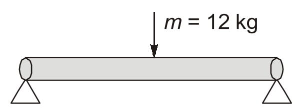
...(Substituting $W = m.g$; $a = l_1$, and $b = l_2$)

We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{1.24\times10^{-3}}} = 14.15 \text{ Hz}$$

T3 : Solution

Given:



Since it is a short bearing
∴ It is simply supported.

(i) Deflection at mid span,
$$\Delta = \frac{(mg)L^3}{48EI}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{48EI}{L^3\times m}} = \sqrt{\frac{48\times200\times10^9\times\pi\times(.01)^4}{64\times0.4^3\times12}}$$

$$\omega_n = 78.332 \text{ rad/s} \Rightarrow 748 \text{ rpm}$$

(ii)
$$\Delta = \frac{Fl^3}{48EI}+\frac{5\omega l^4}{384EI}$$

Where
$$\omega = \frac{W}{L} = \frac{\rho\times V_g}{L} = \rho\times\frac{\pi}{4}d^2\times g$$

∴
$$\Delta = 1.5987\times10^{-3} + 1.962\times10^{-5}$$

$$\omega_n = \sqrt{\frac{g}{\Delta}} = 77.85 \text{ rad/s}$$

$$N = \frac{\omega_n\times60}{2\pi} = 743.48 \text{ rpm} \approx 744 \text{ rpm}$$

T4 : Solution

Given: $f_d = 90/\text{min} = 90/60 = 1.5$ Hz

We know that time period,

$$t_p = 1/f_d = 1/1.5 = 0.67 \text{ s}$$

Let
x_1 = Initial amplitude, and
x_2 = Final amplitude after one complete vibration
$= 20\%\, x_1 = 0.2\, x_1$ ...(Given)

We know that
$$\log_e\left(\frac{x_1}{x_2}\right) = a.t_p \quad \text{or} \quad \log_e\left(\frac{x_1}{0.2x_1}\right) = a\times0.67$$

∴ $\log_e 5 = 0.67\, a$ or $1.61 = 0.67\, a$ or $a = 2.4$ ...($\because \log_e 5 = 1.61$)

We also know that frequency of free damped vibration

$$f_d = \frac{1}{2\pi}\sqrt{(\omega_n)^2 - a^2}$$

or
$$(\omega_n)^2 = (2\pi\times f_d)^2 + a^2 \quad \text{...(By squaring and arranging)}$$
$$= (2\pi\times1.5)^2 + (2.4)^2 = 94.6$$
$$\omega_n = 9.726 \text{ rad/s}$$

We know that frequency of undamped vibration,

$$f_n = \frac{\omega_n}{2\pi} = \frac{9.726}{2\pi} = 1.55 \text{ Hz}$$

Alternate

Damped frquency, $f_d = 1.5 \text{ /s}$

Given $n_1 = 0.2\, x_0$

$$\Rightarrow \quad \frac{x_0}{x_1} = \frac{x_1}{x_2} = \ldots \frac{x_{n-1}}{x_n} = 5 = e^{\delta} \qquad \text{where } \delta = \text{logarithmic decrement}$$

$$\Rightarrow \quad d = In\, 5 = 1.609$$

$$\Rightarrow \quad \frac{2\pi_{\xi}}{\sqrt{1-\xi^2}} = 1.609$$

$$\Rightarrow \quad \frac{\xi^2}{1-\xi^2} = 0.066$$

$$\Rightarrow \quad \frac{1}{\xi^2} - 1 = 15.241$$

$$\Rightarrow \quad \xi = 0.248$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$\Rightarrow \quad \frac{\omega_d}{2\pi} = \sqrt{1-\xi^2}\,\frac{\omega_n}{2\pi}$$

$$\Rightarrow \quad f_d = \sqrt{1-\xi^2}\, f$$

$$\Rightarrow \quad f_n = \frac{f_d}{\sqrt{1-\xi^2}} = \frac{1.5}{\sqrt{1-0.248^2}} = 1.55 \text{ Hz}$$

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