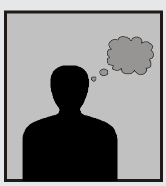
ESE GATE PSUs

State Engg. Exams

MADE EASY Workbook 2025



Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

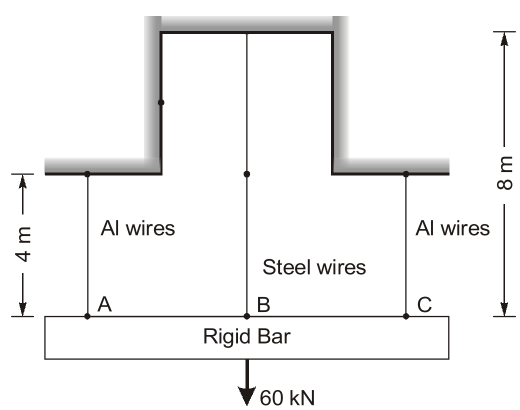
Strength of Materials

MADE EASY PUBLICATIONS

1 Simple Stress-Strain and Elastic Constants

Detailed Explanation of Try Yourself Questions

T1 : Solution



Let suffix 1 be used for Aluminium and 2 for steel

$A_1 = 300 \text{ mm}^2$ $\qquad E_1 = 0.667 \times 10^5 \text{ N/mm}^2$

$A_2 = 200 \text{ mm}^2$ $\qquad E_2 = 2 \times 10^5 \text{ N/mm}^2$

$l_1 = 4 \text{ m}$

$l_2 = 8 \text{ m}$

Now, $\delta l_1 = \delta l_2$, change in depth, line ABC will remain straight

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$P_2 = P_1\left(\frac{l_1}{l_2}\right)\left(\frac{A_2 E_2}{A_1 E_1}\right)$$

$$P_2 = P_1\left(\frac{4}{8}\right)\left(\frac{200 \times 2 \times 10^5}{300 \times 0.667 \times 10^5}\right)$$

$$P_2 = P_1$$

Also $\quad 2P_1 + P_2 = 60$

Since $\quad P_1 = P_2$

$$2P_1 + P_1 = 60$$

$$3P_1 = 60$$

$$P_1 = 20 \text{ kN, in aluminium wires}$$

$$P_2 = 20 \text{ kN, in steel wire}$$

T2 : Solution

(a) Fall in temperature $(\Delta T) = 80 - 22 = 58°C$

$$\text{Strain } (\epsilon) = \alpha\Delta T = 11 \times 10^{-6} \times 58 = 638 \times 10^{-6}$$

$$\text{Stress } (\sigma) = E\epsilon = 200 \times 10^9 \times 638 \times 10^{-6} = 127.6 \text{ MN / m}^2$$

$$\text{Pull exerted (P)} = \sigma A = 127.6 \times 10^6 \times \pi/4 \times 6.25 \times 10^{-4} = 62.635 \text{ kN}$$

(b) Length at 22° C $= l(1 - \alpha\Delta T) = 6(1 - 11 \times 10^{-6} \times 58) = 5.996172$ m

$$\text{Decrease in length} = 6 - 5.996172 = 0.003828 \text{ m}$$

Due to yielding of walls by 1.5 mm, the actual decrease in length

$$= 0.003828 - 0.0015 = 0.002328 \text{ m} = 2328 \times 10^{-6} \text{ m}$$

$$\text{Strain } (\epsilon) = \frac{2328 \times 10^{-6}}{6} = 388 \times 10^{-6}$$

Pull exerted, $\quad P = 200 \times 10^9 \times 388 \times 10^{-6} \times \pi/4 \times 6.25 \times 10^{-4}$

$$= 38.092 \text{ kN}$$

T3 : Solution

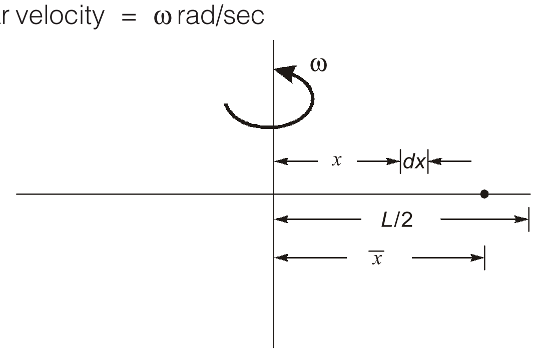
As per given information,

$$\text{Length} = L$$

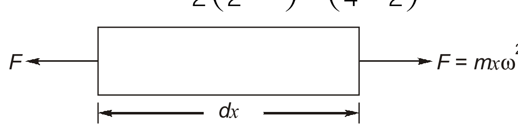
$$\text{Mass density} = \rho$$

$$\text{Modulus elasticity} = E$$

$$\text{Angular velocity} = \omega \text{ rad/sec}$$



$$\bar{x} = x + \frac{1}{2}\left(\frac{L}{2} - x\right) = \left(\frac{L}{4} + \frac{x}{2}\right)$$



$$F = \rho.A\left(\frac{L}{2}-x\right)\cdot\overline{x}\cdot\omega_2$$

$$\text{Stress} = \frac{F}{A} = \rho\left(\frac{L}{2}-x\right)\overline{x}\omega^2$$

$$\text{Strain} = \frac{\text{Stress}}{\text{E}}$$

$$\frac{\delta x}{dx} = \frac{\rho}{E}\left(\frac{L}{2}-x\right)\overline{x}\omega^2$$

$$\text{Change in length} = \int_0^{L/2}\delta x = \int_0^{L/2}\frac{\rho}{E}\left(\frac{L}{2}-x\right)\frac{1}{2}\left(\frac{L}{2}+x\right)\omega^2\cdot dx$$

$$= \frac{\rho\omega^2}{2E}\int_0^{L/2}\left(\frac{L^2}{4}-x^2\right)\cdot dx = \frac{\rho\omega^2}{2E}\left[\frac{L^3}{8}-\frac{L^3}{24}\right]$$

$$= \frac{\rho\omega^2}{2E}\left[\frac{3L^3-L^3}{24}\right] = \frac{\rho\omega^2L^3}{12E\times 2}$$

$$\text{Total change in length} = \frac{2\times\rho\omega^2L^3}{12\times E\times 2}$$

$$\delta L = \frac{\rho\omega^2L^3}{12E}$$

Maximum stress induced the point, where F_{max} will be occur

$$F = \rho\cdot A\left(\frac{L}{2}-x\right)\overline{x}\cdot\omega^2 = \rho\cdot A\left(\frac{L}{2}-x\right)\frac{1}{2}\left(\frac{L}{2}+x\right)\cdot\omega^2$$

$$= \frac{\rho A}{2}\cdot\omega^2\left(\frac{L^2}{4}-x^2\right)$$

F_{max} at $x = 0$

$$F_{max} = \frac{F_{max}}{A} = \frac{\rho\cdot\omega^2L^2}{8}$$ **Ans.**

T4 : Solution

$$d = 1.5 \text{ m}$$

$$\sigma_{permissible} = 150 \text{ MPa}$$

$$\sigma_{induced} \le \sigma_{permissible}$$

$$E\cdot\alpha\cdot\Delta T \le 150$$

$$150\times 10^3\times 11\times 10^{-6}\times\Delta T \le 150$$

$$\Delta T \le 90.9°\text{C}$$

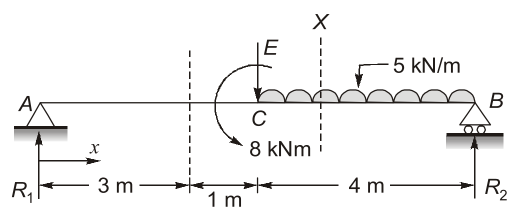
$$\text{Required least temperature} = 90.9°\text{C}$$ **Ans.**

■■■■

3 Shear Force and Bending Moment

Detailed Explanation of Try Yourself Questions

T1 : Solution



(i)

Taking moment about A

We get $8 \times 4 - 8 + 5 \times 4 \times 6 = R_2 \times 8$

$\Rightarrow \quad R_2 = 18$ kN

$\because \quad R_1 + R_2 = 8 + 5 \times 4$

$\Rightarrow \quad R_1 = 10$ kN

(ii) Let a section x - x at distance x from left support shear force at X - X

(a) if $x < 4$

$$(SF)_{X\text{-}X} = 10 \text{ kN}$$

(b) if $x > 4$

$$(SF)_{X\text{-}X} = 10 - 8 - 5(x - 4) = 22 - 5x$$

$$\text{for } (SF)_{X\text{-}X} = 0$$

$$22 - 5x = 0$$

$$x = \frac{22}{5} = 4.4 \text{ m}$$

$$(BM)_{X\text{-}X} \text{ at } x = 4.4$$

$$|M|_{x = 4.4} = 10 \times 4.4 - 8 - 8 \times 0.4 - 0.4 \times 5 \times 0.2$$

$$= 32.4 \text{ kN-m}$$

(iii) Bending moment diagram:

(a) if $x < 4$

$$(BM)_{X\text{-}X} = 10\,x \text{ kN-m}$$

(b) if $x > 4$

$$(BM)_{X\text{-}X} = 10\,x - 8 - 8\,(x-4) - 5\times(x-4)\times\left(\frac{x-4}{2}\right)$$

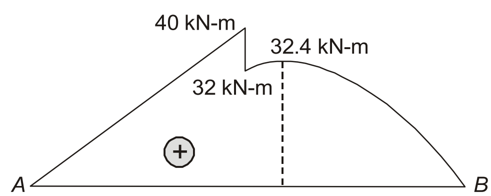
$$= -\frac{5x^2}{2} + 22x - 16$$

$$(BM)_{X=4.0} = \frac{-5\times 4^2}{2} + 22\times 4 - 16$$

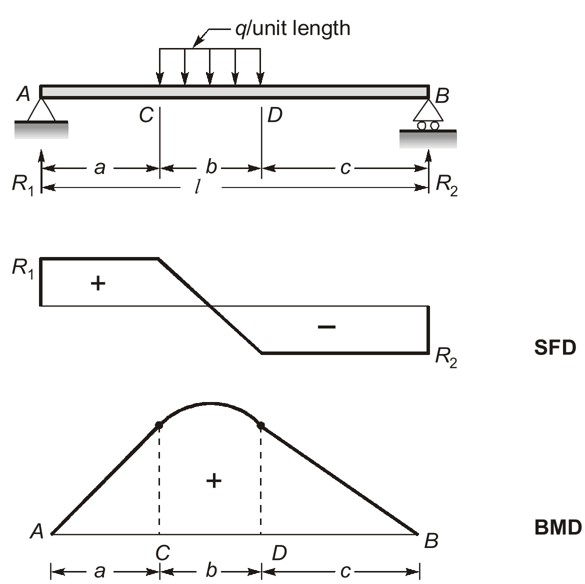
$$= 32 \text{ kNm}$$

$$(BM)_{X=4.4} = \frac{-5\times 4^2}{2} + 22\times 4.4 - 16$$

$$= 32.4 \text{ kNm}$$



T2 : Solution



$$R_1 \times l = qb\left(c + \frac{b}{2}\right)$$

$$R_1 = \frac{qb}{l}(c + 0.5b)$$

$$R_2 = qb - \frac{qb}{l}(c+0.5b) = \frac{qb}{l}[l-(c+0.5b)]$$

$$= \frac{qb}{l}\left[a+b+c-c-\frac{b}{2}\right] = \frac{qb}{l}\left(a+\frac{b}{2}\right)$$

SFD :

in AC portion : $F_x = R_1$

in CD portion : $F_x = R_1 - q(x-a)$

For $F_x = 0$

or $x = \frac{b}{l}(c+0.5b)+a$

$F_D = R_1 - qb$

in DB portion : $F_x = R_1 - qb$

BMD :

in portion *AC* : $M_x = R_1 x$

in portion *CD* : $M_x = R_1 x - \frac{q(x-a)^2}{2}$

in portion *DB* : $M_x = R_1 x - qb(x-a-0.5b)$

$$M_{\max} = \frac{qb}{l}(c+0.5b)\left\{\frac{b}{l}(c+0.5b)+a\right\} - qb\left\{\frac{b}{l}(c+0.5b)+a-a-0.5b\right\}$$

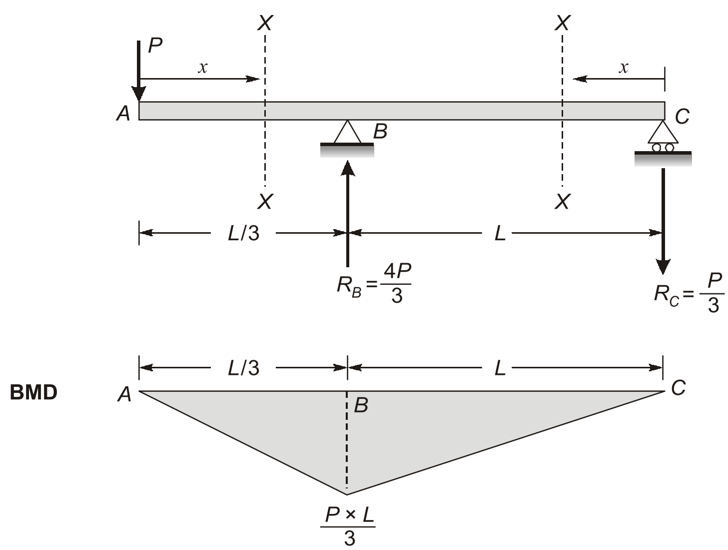
$$= \frac{qb}{l}(c+0.5b)\left\{\frac{b}{l}(c+0.5b)+a\right\} - \frac{qb^2}{l}\{(c+0.5b)-0.5l\}$$

■■■■

4 Deflection of Beams

Detailed Explanation of Try Yourself Questions

T1 : Solution



Taking moment about C

$$\Rightarrow \quad P \times \left(L + \frac{L}{3}\right) = R_B \times L$$

$$\Rightarrow \quad R_B = \frac{4P}{3}$$

$$\because \quad R_B = P + R_C$$

$$\Rightarrow \quad R_C = \frac{P}{3}$$

Strain energy for whole beam

$$U = U_{AB} + U_{BC}$$

$$= \int_0^{L/3} \frac{(P.x)^2 dx}{2EI} + \int_0^{L} \frac{\left(\frac{P}{2}\cdot x\right)^2 dx}{2EI}$$

$$= \frac{P^2L^3}{6EI \times 3^3} + \frac{P^2L^3}{9 \times 6EI}$$

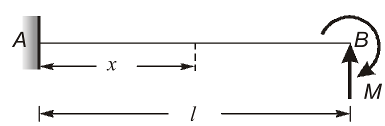
$$= \frac{P^2L^3}{3^3 \times 6 \times EI} + \frac{P^2L^3}{3^3 \times 2EI}$$

From Castigliano's thoerem,

$$\delta_A = \frac{2PL^3}{3^3 \times 6EI} + \frac{2PL^3}{3^2 \times 2EI}$$

$$= \frac{8PL^3}{27 \times 6 \times EI} = \frac{4PL^3}{81EI}$$

T2 : Solution



Net deflection at a point B is zero,

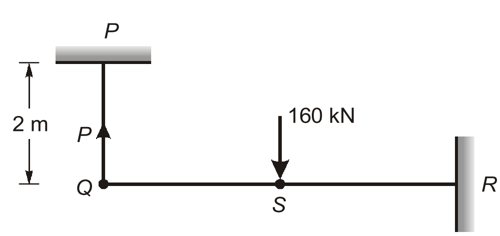
$$\frac{Ml^2}{2EI} - \frac{R_B \times l^3}{3EI} = 0$$

$$R_B = \frac{3M}{2l}$$

$$M_A = \frac{3M}{2l} \times l - M$$

$$M_A = \frac{M}{2}$$

T3 : Solution



Let the axial load in the member PQ is P

Deflection at Q due to load 160 kN at S

= Deflection at S due to 160 kN + Angle of deflection at S due to 160 kN × QS

$$= \frac{(160)(2)^3}{3EI} + \frac{160(2)^2}{2EI} \times 2 = \frac{5 \times 160 \times 2^3}{6EI} \downarrow$$

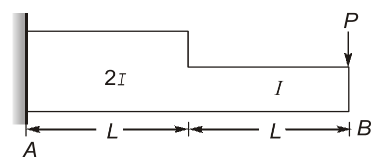
Deflection at Q due to load $P = \dfrac{P \times (4)^3}{3EI} \uparrow$

$\because$ Point Q is hinged, so there will be no net deflection point Q.

$$\therefore \qquad \frac{P \times (4)^3}{3EI} = \frac{5 \times 160 \times 2^3}{6EI}$$

$$\Rightarrow \qquad P = 50 \text{ kN}$$

T4 : Solution



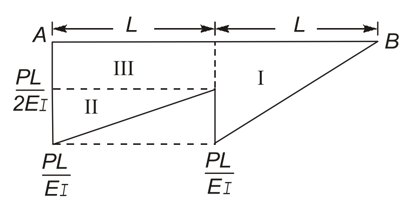
$$I = 375 \times 10^{-6} \text{ m}^4$$

$$L = 0.5 \text{ m}$$

$$E = 200 \text{ GPa}$$

$$\text{Stiffness of beam, } k_b = \frac{P}{y_{max}}$$

$\frac{M}{EI}$ diagram



Using moment area method:

$$y_B - y_A = (A\bar{x})_I + (A\bar{x})_{II} + (A\bar{x})_{III}$$

$$y_B = \left(\frac{L}{2} \times \frac{PL}{EI} \times \frac{2L}{3}\right) + \left(\frac{PL}{2EI} + L \times \frac{3L}{2}\right) + \left(\frac{1}{2} \times \frac{PL}{2EI} \times L \times \frac{5L}{3}\right)$$

$$y_B = \frac{PL^3}{3EI} + \frac{3PL^3}{4EI} + \frac{5PL^3}{12EI}$$

$$= \frac{(4+9+5)PL^3}{12EI}$$

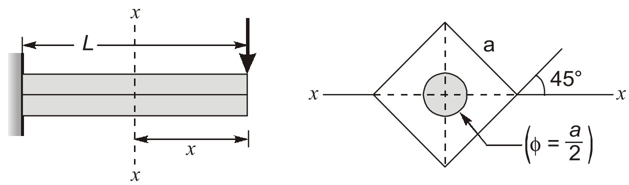
$$= \frac{18PL^3}{12EI}$$

$$\text{Stiffness, } k_b = \left(\frac{P \times 12EI}{18L^3 P}\right) = \frac{12 \times EI}{18L^3}$$

$$= \frac{12 \times 200 \times 10^9 \times 375 \times 10^{-6}}{18 \times 0.5^3}$$

$$k_b = 400 \text{ MN/m}$$

T5 : Solution



$$I_{xx} = \left(\frac{a^3}{12} - \frac{\pi\left(\frac{a}{2}\right)^4}{64}\right)$$

$$I_{xx} = \left(\frac{a^4}{12} - \frac{\pi a^4}{16 \times 64}\right)$$

$$\delta_{\text{deflection}} = \frac{PL^3}{3EI} = \frac{PL^3}{3E\left(\frac{a^4}{12} - \frac{\pi a^4}{16 \times 64}\right)}$$

$$= \frac{PL^3 \times 1024}{3E\left(\frac{256a^4}{3} - \pi a^4\right)} = \frac{1024PL^3}{(256 - 3\pi)E \cdot a^4}$$

$$\delta_{\text{deflection}} = \frac{1024P}{(256 - 3\pi)E} \frac{L^3}{a^4}$$

Hence, option (b) is correct.

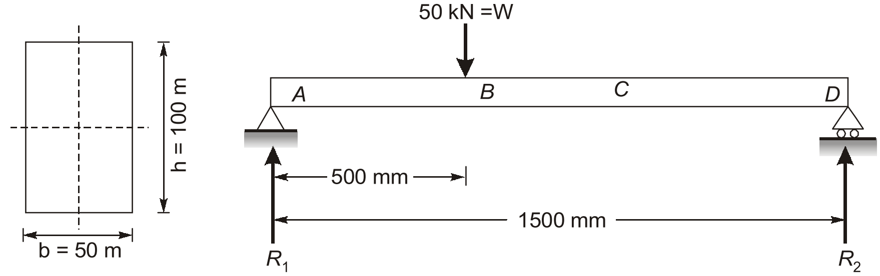
■■■■

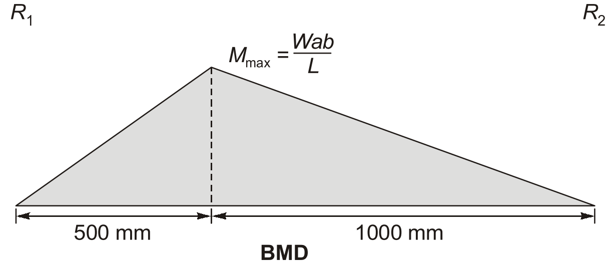
5 Bending and Shear Stresses in Beams

Detailed Explanation of Try Yourself Questions

T1 : Solution

Reactions,
$$R_1 = \frac{2}{3} \times 50 = \frac{100}{3} \text{ kN}$$
$$R_2 = \frac{50}{3} \text{ kN}$$





Bending moment is maximum when shear force is zero or changes sign i.e. at 500 mm from left support.

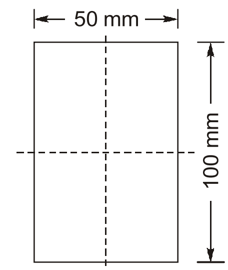
$$\therefore \quad M_{max} = R_1 \times 0.5 = \frac{100 \times 10^3}{3} \times 0.5 = 16666.67 \text{ N-m}$$

Moment of area
$$I = \frac{1}{12} bh^3 = \frac{1}{12} \times 50 \times (100)^3$$
$$I = 4.1667 \times 10^6 \text{ mm}^4$$

$$I = 4.1667 \times 10^{-6}\ m^4$$

$$\text{Maximum stress } (\sigma_{max}) = \frac{My}{I} = \frac{16666.67 \times 0.05}{4.1667 \times 10^{-6}}$$

$$\sigma_{max} = 200\ N/mm^2$$



Cross-Section

$$y_c = \frac{Wa^2b^2}{3EIL}$$

$$a = 0.5\ m,\ b = 1\ m,\ L = 1.5\ m$$

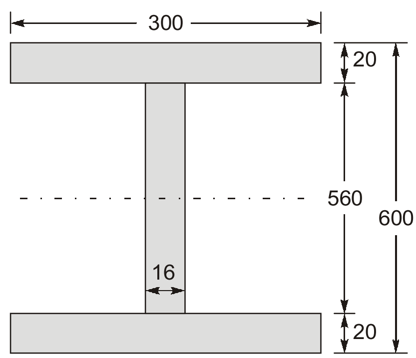
$$EI = \frac{2 \times 10^{11} \times 4.1667 \times 10^{-6}}{10^3}\ kN.m^2$$

$$= 833.34\ kNm^2$$

$$y_c = \frac{50 \times 0.5^2 \times 1^2}{3 \times 833.34 \times 1.5} = 3.33 \times 10^{-3}\ m = 3.33\ mm$$

T2 : Solution

(i) I-section



$$A_I = 2 \times 30 \times 2 + 56 \times 1.6 = 209.6\ cm^2$$

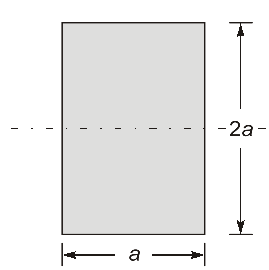
$$I_I = \frac{1}{12} \times (56)^3 \times (1.6) + 2 \times \left[\frac{1}{12} \times 30 \times 2^3 + 30 \times 2 \times 29^2\right]$$

$$= 124375.47\ cm^4$$

$$Z_I = \frac{I_I}{Y} = \frac{124375.47}{30}$$

$$= 4145.85\ cm^3 = 41.4585 \times 10^5\ mm^3$$

(ii) Rectangular section



$$A_{\text{rect c/s}} = 2a^2 = 209.6$$

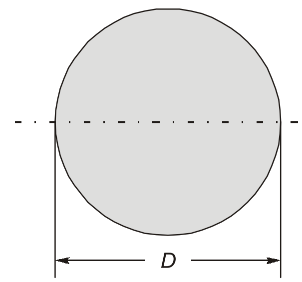
$\Rightarrow$ $$a = 10.237 \text{ cm}$$

$$I = \frac{1}{12}(a)(2a)^3 = \frac{1}{12}(10.237)^4 \times 8$$

$$= 7321.491 \text{ cm}^4$$

$$Z = \frac{I}{Y} = \frac{7321.491}{10.237} = 715.1989 \text{ cm}^3 = 7.152 \times 10^5 \text{ mm}^3$$

(iii) Circular section



$$A = \frac{\pi}{4}D^2 = 209.6$$

$\Rightarrow$ $$D = 16.336 \text{ cm}$$

$$I = \frac{\pi}{64}D^4 = 3496.01 \text{ cm}^4$$

$$Z = \frac{I}{Y} = \frac{3496.01}{(16.336/2)} = 428.013 \text{ cm}^3$$

$$= 4.28 \times 10^5 \text{ mm}^3$$

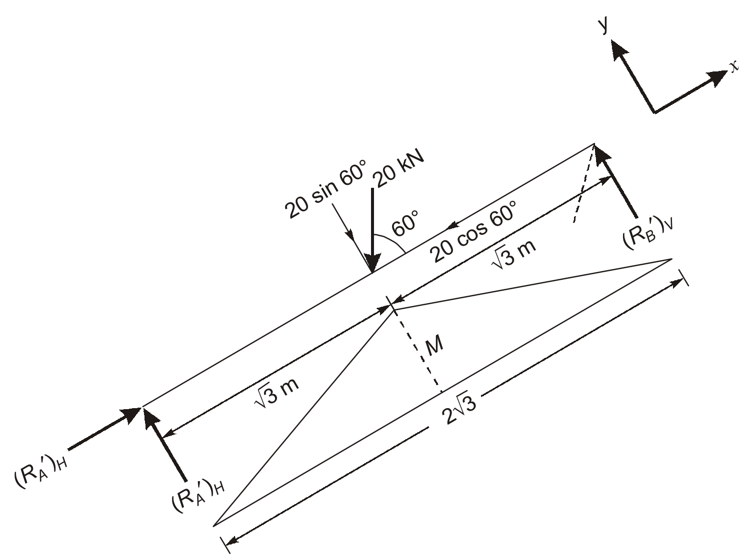
Comparison:

Ratio of flexural strength $= Z_{\text{I-section}} : Z_{\text{rect. C/s}} : Z_{\text{circular c/s}}$

$= 41.4585 \times 10^5 : 7.152 \times 10^5 : 4.28 \times 10^5$

$= 9.6865 : 1.671 : 1$

T3 : Solution



$$M = \frac{WL}{4} = \frac{20 \times \frac{\sqrt{3}}{2} \times 2\sqrt{3}}{4} = \frac{20 \times \frac{\sqrt{3}}{2} \times 2\sqrt{3}}{4} = 15 \times 10^3 \text{ N.m}$$

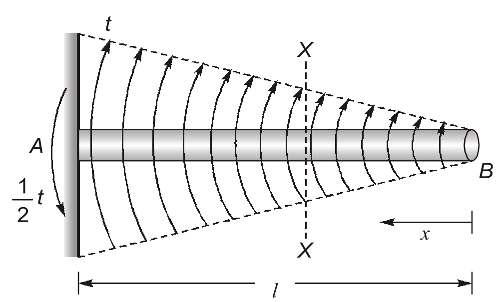
Maximum compressive stress $= \dfrac{M}{Z} = \dfrac{15 \times 10^3}{24 \times 10^{-3}} = 0.625 \text{ MPa}$

■■■■

6 Torsion of Shafts

Detailed Explanation of Try Yourself Questions

T1 : Solution



(i) $$\tau_{max} = \frac{T_{max}}{Z_p} = \frac{tl/2}{\frac{\pi}{16}d^3} = \frac{8tl}{\pi d^3}$$

(ii) $$T_x = \frac{1}{2}\left(\frac{tx}{l}\right) \times x = \frac{tx^2}{2l}$$

$$d\theta_x = \frac{T_x dx}{GJ}$$

$$\theta_B - \theta_A = \int_0^l \frac{\frac{tx^2}{2l} \times dx}{GJ}$$

$\Rightarrow$ $$\theta_B = \left[\frac{t}{2lGJ}\left(\frac{x^3}{3}\right)\right]_0^l = \frac{tl^2}{6GJ} = \frac{8tL^2}{48GJ}$$

(iii) Let point C at $x = l/2$ from free end

$$\theta_C - \theta_A = \int_{l/2}^{l} \frac{\frac{tx^2}{2l} \times dx}{GJ}$$

$$\theta_C = \frac{t}{6LGJ}[x^3]_{l/2}^{l} = \frac{t}{6lGJ}\left(l^3 - \left(\frac{l}{2}\right)^3\right) = \frac{7tl^2}{48GJ}$$

(iv) $$U = \int_0^l \frac{T_x^2 dx}{2GJ_x} = \int_0^l \frac{\left(\frac{tx^2}{2l}\right)^2 dx}{2GJ} = \frac{t^2}{8l^2GJ}\left[\frac{x^5}{5}\right]_0^l = \frac{t^2l^3}{40GJ}$$

■■■■

7 Principal Stresses and Strains

Detailed Explanation of Try Yourself Questions

T1 : Solution

Moment of area

$$J = \frac{\pi}{32}(110^4 - 100^4)$$

$$J = 4.5563 \times 10^6 \text{ mm}^4$$

Shear stress

$$\tau = \frac{T \times 55}{4.5563 \times 10^6} = \frac{1000 \times 10^3 \times 55}{4.5563 \times 10^6}$$

$$\tau_{xy} = 12.07 \text{ N/mm}^2$$

Principal stress

$$\therefore \quad \sigma = \frac{1}{2}\left[(\sigma_x + \sigma_y) + (\sigma_x - \sigma_y)\cos 2\theta\right] + \tau_{xy}\sin 2\theta \qquad \sigma_x = \sigma_y = 0$$

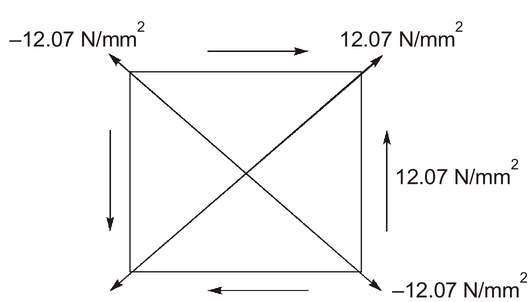
$$\Rightarrow \quad \sigma = \tau_{xy} \sin 2\theta$$

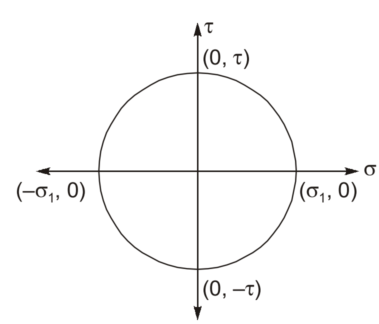
$$\sigma_{1,2} = 12.07 \sin 2\theta$$

$$\sigma_1 = 12.07 \text{ N/mm}^2, \theta = 45°$$

$$\sigma_2 = -12.07 \text{ N/mm}^2, \theta = 135°$$

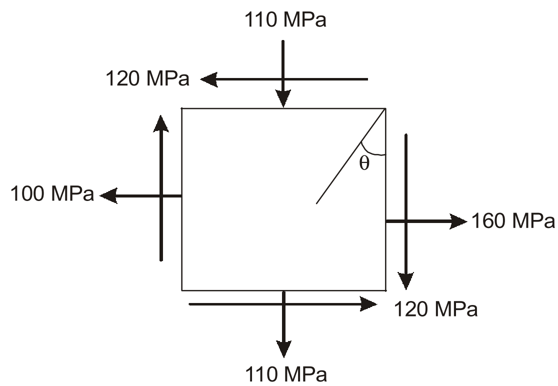
θ's are with direction of torsion moment.





$$\sigma_1 = -\sigma_2 = \tau = 12.07 \text{ MPa}$$

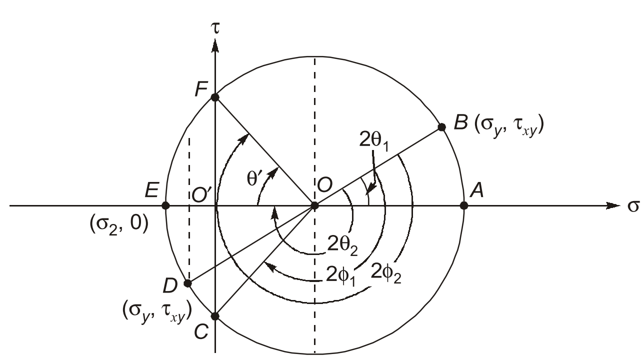
T2 : Solution



Given:
$$\sigma_h = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

For principal plane,
$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$
$$\tan 2\theta = \frac{2 \times 120}{(160 + 110)}$$
$$\theta_1 = 20.816°$$
$$\theta_2 = 110.816°$$
$$(\sigma_n)_{\theta = 20.816°} = \left(\frac{160 - 110}{2}\right) + \left(\frac{160 + 110}{2}\right)\cos(2 \times 20.816) + 120\sin(2 \times 20.816)$$
$$= 205.623 \text{ N/mm}^2 \text{ (tensile)} = \sigma_1 \text{ (Major principal stress)}$$
$$(\sigma_n)_{\theta = 100.816°} = \left(\frac{160 - 110}{2}\right) + \left(\frac{160 + 110}{2}\right)\cos(2 \times 110.816) + 120\sin(2 \times 110.816)$$
$$\sigma_2 = -155.623 \text{ N/mm}^2 \text{ (compresive)} = \sigma_2 \text{ (Minor principal stress)}$$

Plane of zero normal stress.



Radius of Mohr's circle,

$$OF = R = \frac{\sigma_1 - \sigma_2}{2} = \frac{205.623 + 155.623}{2} = 180.623 \text{ MPa}$$

$$OO' = 205.623 - 180.623 = 25 \text{ MPa}$$

$$\theta' = \cos^{-1}\left(\frac{OO'}{OF}\right) = \cos^{-1}\left(\frac{25}{180.623}\right) = 82.044°$$

$$2\phi_1 = 180 - 82.044 + 2 \times 20.816$$

$$\phi_1 = 69.794°$$ From x-face in CW direction

$$\phi_2 = \frac{2\phi_1 + 2\theta'}{2}$$

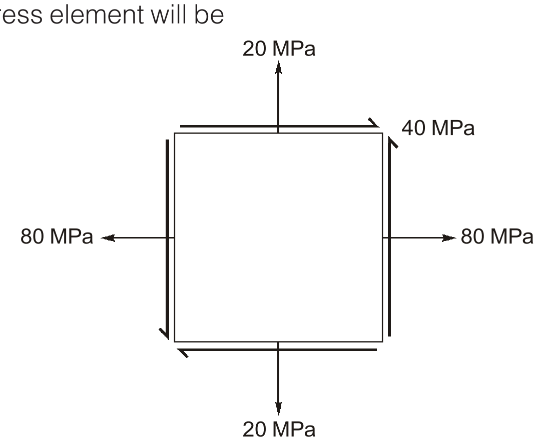
$$\phi_2 = \phi_1 + \theta'$$

$$\phi_2 = 69.794 + 82.044$$

$$\phi_2 = 151.838°$$ From x-face in CW direction

T3 : Solution

According to problem, stress element will be



$$\text{Principal stress, } (\sigma_1, \sigma_2) = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{80 + 20}{2} \pm \sqrt{\left(\frac{80 - 20}{2}\right)^2 + 40^2}$$

$$= 50 \pm 50$$

$$\Rightarrow \quad \sigma_1 = 100 \text{ MPa}; \sigma_2 = 0$$

$$\text{Major principal strain, } \varepsilon_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} = \frac{100}{200000} - \frac{0.25 \times 0}{200000} = 5 \times 10^{-4}$$

$$\text{Length of major axis, } l_1 = d + \varepsilon_1 d$$

$$= 100 + 5 \times 10^{-4} \times 100 = 100.05 \text{ mm}$$

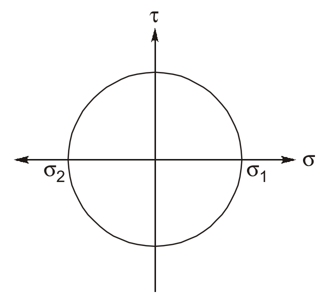
$$\text{Minor principal strain, } \varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} = 0 - \frac{0.25 \times 100}{200000} = 1.25 \times 10^{-4}$$

$$\text{Length of minor axis, } l_2 = d + \varepsilon_2 d$$

$$= 100 - 1.25 \times 10^{-4} \times 100 = 99.9875 \text{ mm}$$

T4 : Solution

Shaft is subjected to only torsion. So, Mohr circle will be



Let τ is maximum shear stress in shaft then,

$$\sigma_1 = -\sigma_2 = \tau$$

$$\text{Strain, } \varepsilon = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$3.98 \times 10^{-4} = \frac{\tau}{105000}(1+0.3)$$

$$\Rightarrow \quad \tau = 32.15 \text{ MPa}$$

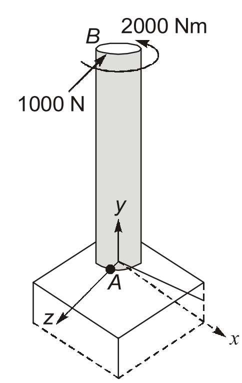
$$\text{Torque, } T = \frac{\pi}{16}\tau d^3 = \frac{\pi}{16} \times 32.15 \times 60^3$$

$$= 1363366.63 \text{ Nmm}$$

$$\text{Power, } P = T\omega = 1363.37 \times 2 \times \pi \times \frac{800}{60}$$

$$= 114217.1 \text{ Nm}$$

T5 : Solution (b)



$$\text{Bending stress at A, } \sigma_b = \frac{My}{I} = \frac{1000 \times 10^3 \times 32}{\pi \times 60^3} = 47.16 \text{ MPa}$$

$$\text{Shear stress at A, } \tau = \frac{Tr}{J} = \frac{2000 \times 10^3 \times 16}{\pi \times 60^3} = 47.16 \text{ MPa}$$

$$\text{Principal stress at A, } (\sigma_1, \sigma_2) = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{47.16}{2} \pm \sqrt{\left(\frac{47.16}{2}\right)^2 + 47.16^2} = 23.58 \pm 52.73$$

$$\Rightarrow \quad \sigma_1 = 76.31 \text{ MPa}; \ \sigma_2 = -29.15 \text{ MPa}$$

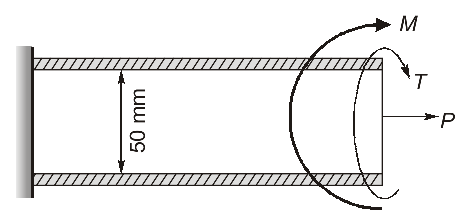
$$\text{Maximum shear stress at A} = \frac{\sigma_1 - \sigma_2}{2} = \frac{76.31-(-29.15)}{2} = 52.73 \text{ MPa}$$

■■■■

8 Theories of Failure

Detailed Explanation *of* Try Yourself Questions

T1 : Solution



$L = 120 \text{ mm}$

Axial load, $P = 90 \text{ kN}$

Torsion, $T = 72.0 \text{ N.m}$

Bending load, $F = 1.75 \text{ kN at free end}$

$\sigma_{yt} = 267 \text{ MPa}$

$d_i = 50 \text{ mm}$

$\text{FOS} = 4$

As per give condition critical point on the surface that ‘A’

$$(\sigma_{max})_{bending} = \frac{F\times L}{\dfrac{\;\dfrac{\pi d_i t_i \cdot d_i^2}{2}\;}{\dfrac{d_i}{2}}} = \frac{F\times L}{\pi\times t\times d_i^2}$$

$$= \frac{1.75\times 10^3\times 120}{\pi\times t\times 50^2} = \frac{20.738}{t}$$

$$(\sigma_{axial}) = \frac{P}{\pi d_i t} = \frac{9000}{\pi\times 50\times t} = \frac{57.295}{t}$$

$$\tau_{xy} = \frac{T}{\dfrac{\pi d_i t\times d_i^2}{d_i/2}} = \frac{2T}{\pi d_i^2 t}$$

$$= \frac{2\times 72\times 10^3}{\pi\times 30^2\times t} = \frac{18.334}{t}$$

$$\sigma_x = (\sigma_{max})_{bending} + \sigma_{axial}$$

$$\sigma_x = \frac{26.738}{t} + \frac{57.295}{t} = \frac{84.033}{t}$$

Major principal stress,

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{84.033}{t} + \sqrt{\left(\frac{84.033}{t}\right)^2 + \left(\frac{18.334}{t}\right)^2}$$

$$\sigma_1 = \frac{84.033}{t} + \frac{86.009}{t}$$

$$\sigma_1 = \frac{170.043}{t}$$

$\Rightarrow$ FromMPST

$$\sigma_1 \le \frac{\sigma_{yt}}{FOS}$$

$$\Rightarrow \quad \frac{170.043}{t} \le \frac{267}{4}$$

$$\Rightarrow \quad t \ge 2.547 \text{ mm}$$

T2 : Solution

Principal stress = p, 0.5p, 0

ν = 0.30

(i) **Maximum shear stress theory:** The maximum shear is equal to half the difference between the maximum and minimum principal stress and since the maximum shear in simple tension is equal to the half of the tensile stress, so we have

$$\tau_{max} = \left(\frac{\sigma_1 - \sigma_3}{2}\right) = \tau_{per} = \frac{\sigma_y}{2} \qquad [\sigma_y = \text{failure stress in tension}]$$

$$\frac{p-0}{2} = \frac{\sigma}{2}$$

$$p = \sigma$$

(ii) **Strain energy theory:** Here one of principal stress has zero value

$$\therefore \quad \sigma_1^2 + \sigma_2^2 - \frac{2}{m}\sigma_1\sigma_2 = \sigma_y^2$$

$$p^2 + (0.5p)^2 - (2 \times p \times 0.5p \times 0.30) = \sigma_y^2$$

$$p^2 + 0.25p^2 - 0.3p^2 = \sigma_y^2$$

$$0.95p^2 = \sigma_y^2$$

$$0.97p = \sigma_y$$

$$p = 1.03\,\sigma$$

(iii) **Distortion energy theory:**

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

$$p^2 + (0.5p)^2 - p \times 0.5p = \sigma_y^2$$

$$0.75\,p^2 = \sigma_y^2$$

$$0.866p = \sigma_y$$

$$p = 1.15\,\sigma_y$$

■■■■

9 Thick and Thin Cylinders and Spheres

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given:

$$P = 500 \text{ atm} = 49035 \text{ kPa} = 49.035 \text{ MPa}$$
$$D = 100 \text{ mm}$$
$$\sigma_y = 500 \text{ MPa, FOS} = 2$$

For safety;

Using maximum principal stress theory

$$(\sigma_{max})_{induced} \leq \sigma_{per}$$
$$\frac{PD}{2t} \leq \frac{\sigma_y}{N}$$
$$\Rightarrow \quad t \geq 9.807 \text{ mm}$$
$$\Rightarrow \quad t = 10 \text{ mm}$$

Alternate Solution:

Using maximum shear stress theory

$$\tau_{allowable} = \frac{\sigma_y}{2\,\text{FOS}} = \frac{500}{2 \times 2} = 125 \text{ MPa}$$

$$\tau_{max} = \frac{PD}{4t} \text{ (for this cylindrical pressure vessel)}$$

$$\frac{PD}{4t} = 125$$

or
$$t = \frac{PD}{500} = \frac{49.035 \times 100}{500} = 9.807 \text{ mm say } 10 \text{ mm}$$

$\therefore$ Required thickness, $t = 10$ mm **Ans.**

■■■■

10 Theory of Columns

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$\text{For both ends hinged } P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\text{For both ends fixed } P_{cr} = \frac{4\pi^2 EI}{L^2}$$

$$\frac{d_0}{d_i} = 1.25 \text{ and } \frac{d_i}{d_0} = 0.8$$

Now if P is load for both ends hinged and P + 300 is load for both ends fixed

$$P = \frac{\pi^2 EI}{L^2} \text{ and } P + 300 = \frac{4\pi^2 EI}{I^2}$$

we get,
$$\frac{\pi^2 EI}{L^2} + (300 \times 10^3) = \frac{4\pi^2 EI}{L^2}$$

$$\frac{3\pi^2 EI}{L^2} = 300 \times 10^3$$

$$I = \frac{300 \times 10^3 \times 9}{3 \times \pi^2 \times 100 \times 10^9} = 9.1189 \times 10^{-7} \qquad \left(\therefore \text{Where } k = \frac{d_i}{d_0} = 0.8\right)$$

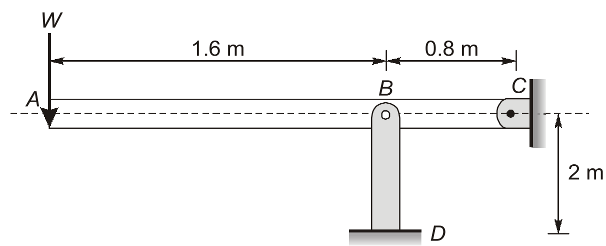
$$I = 9.12 \times 10^{-7} \text{ m}^4$$

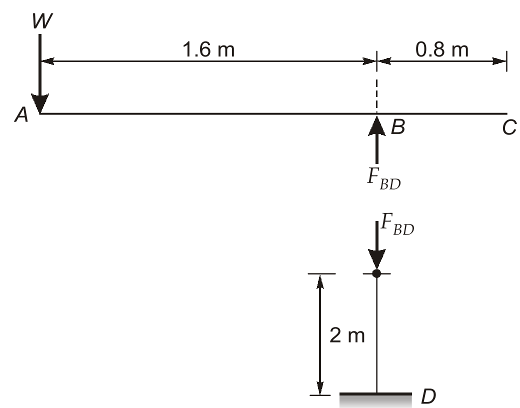
Now
$$I = \frac{\pi}{64} d_0^4 (1 - k^4) \text{ for hollow column.}$$

$$\frac{\pi}{64} d_0^4 (1 - k^4) = 9.12 \times 10^{-7}$$

$$d_0 = 74.9 \text{ mm}$$

T2 : Solution





From euler equation,

$$\sigma_{induced} \leq \frac{P_{cr}}{A \cdot FOS}$$

$$\frac{F_{BD}}{A} \leq \frac{P_{cr}}{A \cdot FOS}$$

$$F_{BD} \leq \frac{\pi^2 EI_{min}}{(L_c^2) \cdot FOS}$$

$$F_{BD} \leq \frac{\pi^2 \times 210 \times 10^3 \times \frac{60 \times 40^3}{12}}{\left(\frac{2000}{\sqrt{2}}\right)^2 \times 2}$$

$$F_{BD} \leq 165809.3539 \text{ N}$$

By taking moment about '*c*'

$$W \times 2.4 = F_{BD} \times 0.8$$

$$W = \frac{165809.3539 \times 0.8}{2.4}$$

$$= 55.269 \text{ kN}$$

Ans.

■■■■