ESE GATE PSUs

State Engg. Exams

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Workbook 2025

Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

Industrial Engineering

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1 Break Even Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$\text{T.C-I} = 100 + 2x \qquad x < 800 \rightarrow s = \text{Rs } 3.5/\text{unit}$$
$$\text{T.C-II} = 200 + 1 \cdot x \qquad x < 800 \rightarrow s = \text{Rs } 3/\text{unit}$$

Now both machine are run combinally

So both combined will produce $2x$ unit

Let s = Rs. 3/unit

Then Total cost = Total sales

$$100 + 2x + 200 + x = 2x \times 3$$
$$x = 100 \text{ units}$$

It is not possible as Rs. 3 selling price is given for $x > 800$

So, now take s = Rs. 3.5/unit

$$100 + 2x + 200 + x = 2x \times 3.5$$
$$x = 75 \text{ units}$$

This is possible as $x < 800$ units for s = Rs. 3.5/units

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2 Inventory Control

Detailed Explanation of Try Yourself Questions

T1 : Solution

D = 10000 units/year

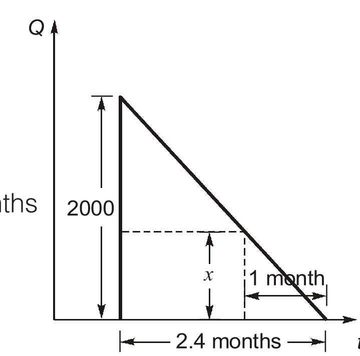
Ordering cost, (C_0) = ₹ 100/order

Carrying cost (C_c) = ₹ 0.5/unit/year

(i) $$\text{EOQ} = \sqrt{\frac{2DC_o}{C_C}} = \sqrt{\frac{20000 \times 100}{0.5}} = 2000 \text{ units}$$

(ii) $$N = \frac{D}{EOQ} = \frac{10000}{2000} = 5$$

(iii) $$T = \frac{EOQ}{D} = 0.2 \times 12 = 2.4 \text{ months}$$



(iv) $$\frac{x}{2000} = \frac{1}{2.4} \Rightarrow x = 833 \text{ units}$$

(v) Total cost = Material cost + T.I.C. = $10000 \times 2 + \sqrt{2DC_oC_c}$ = ₹ 21000

T2 : Solution

Given: A = ₹ 10000, Ordering cost, C_o = ₹ 25/order, Carrying cost, C_h = ` 12.5% of unit cost per year
Interest, i = 0.125

(*i*) $$(EOQ) = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2AC_o}{i}} = \sqrt{\frac{2 \times 10000 \times 25}{0.125}} = ₹\ 2000$$

{where, $A = D \times C_u$, $C_h = i \times C_u$}

(*ii*) Number of orders per year,

$$N = \frac{A}{EOQ} = \frac{10000}{2000} = 5$$

(*iii*) Carrying cost, C_C = Number of orders per year × carrying cost per order

= 5 × 25 = ₹ 125/year

(*iv*) Ordering cost per year,

C_h = carrying cost per yeat at EOQ

= ₹ 125/year

(*v*) Total annual cost, T.C. = $A + C_h + C_c$ = 10000 + 125 + 125 = ₹ 10250 per year

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3 Sequencing and PERT/CPM

Detailed Explanation of Try Yourself Questions

T1 : Solution

In this case, we solve the problem, by converting 3 machine problem into 2 machine problem by following method,

Max(M_2) ≤ Min(M_1 or M_3), then we will add m_i to the other machines

Since, Max(M_2) = 5 = Min(M_1)

Also, Max(M_2) = 5 = Min(M_3)

For machine $X = M_1 + M_2$, For machine $Y = M_2 + M_3$

Jobs	$X = M_1 + M_2$	$Y = M_2 + M_3$
A	18	14
B	8	10
C	10	9
D	9	8

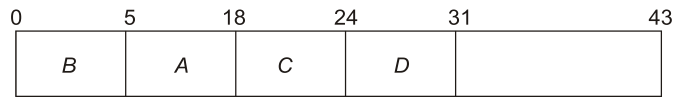
Now by solving the problem by normal procedure, we get the order as, *B* - *A* - *C* - *D*. This sequence has to be projected on the original machines.

	M_1		M_2		M_3	
	IN	OUT	IN	OUT	IN	OUT
B	0	5	5	8	8	15
A	5	18	18	23	23	32
C	18	24	24	28	32	37
D	24	31	31	33	37	43 (Cycle time)

Gantt Charts:

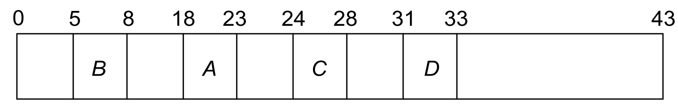
% Idle time

For machine M_1



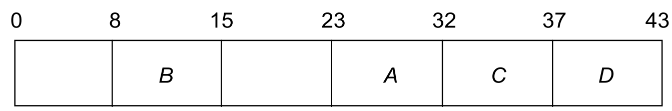
$\Rightarrow \frac{12}{43} \times 100 = 27.9\%$

For machine M_2



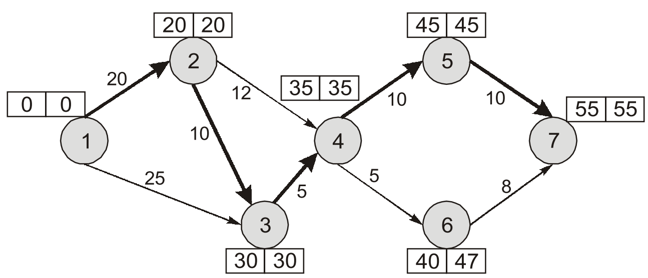
$\Rightarrow \frac{29}{43} \times 100 = 67.44\%$

For machine M_3



$\Rightarrow \frac{16}{43} \times 100 = 37.21\%$

T2 : Solution



Activity	Normal Time	Crash Time	Normal Cost	Crash Cost	Cost Slope	EST	EFT	LST	LFT	TF
1-2	20	17	600	720	40	0	20	0	20	0
1-3	25	25	200	200	0	0	25	5	30	5
2-3	10	8	300	440	70	20	30	20	30	0
2-4	12	6	400	700	50	20	32	23	35	3
3-4	5	2	300	420	40	30	35	30	35	0
4-5	10	5	300	600	60	35	45	35	45	0
4-6	5	3	600	900	150	35	40	42	47	7
5-7	10	5	500	800	60	45	55	45	55	0
6-7	8	3	400	700	60	40	48	47	55	7
			3600							

For activity 1-2:

$$\text{Cost slope} = \frac{720-600}{20-17} = 40$$

EST = 0, LFT = 20

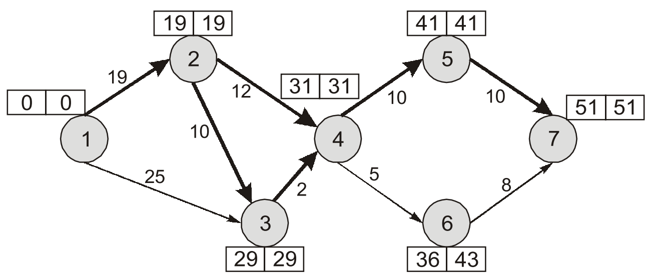
EFT = LFT + t_{1-2} = 0 + 20 = 20

LST = LFT – t_{1-2} = 20 – 20=0

TF = LST – EST = LFT – EFT= 0

Therefore 1–2 is critical activity, similarly 2–3, 3–4, 4–5, 5–7 have 0 total float. Total float of 1–3 is 5 days and total float of 2–4 is 3 days. So we can crash 3 days on activity 3–4 and 1 day on activity 1–2. Therefore project cost = Total normal cost + 4(40) = 3760

Thus the precedence diagram becomes



Now, there are two critical paths 1–2–3–4–5–7 and 1–2–4–5–7

4 Forecasting

Detailed Explanation of Try Yourself Questions

T1 : Solution

1. (a) In simple moving average method 3 point moving forecast for 13th year

$$= \frac{\text{Sum of last three years demand}}{3}$$

$$= \frac{34.3 + 35.2 + 36.0}{3} = 35.167$$

1. (b) In weighted moving average method 3 point moving forecast for 13th year

$$= \frac{(12^{th}\text{ demand}) \times (12^{th}\text{ weightage}) + (11^{th}\text{ demand}) \times (11^{th}\text{ weightage}) + (10^{th}\text{ demand}) \times (10^{th}\text{ weightage})}{12^{th}\text{ weightage} + 11^{th}\text{ weightage} + 10^{th}\text{ weightage}}$$

$$= \frac{36 \times 0.5 + 35.2 \times 0.3 + 34.3 \times 0.2}{0.5 + 0.3 + 0.2} = 35.42$$

Year	Demand		Simple Moving Average	Waited Moving Average
1	28.0		f_i	F_i
2	29.0		-	-
3	28.5		-	-
4	31.0		28.50	28.55
5	34.2		29.50	29.85
6	32.7		31.23	32.10
7	33.5		32.62	32.81
8	31.8		33.47	33.40
9	31.9		32.67	32.49
10	34.3		32.40	32.19
11	35.2	3 Point average	32.67	33.08
12	36.0		33.80	34.27
13			35.17	35.42

2. (a)

$$\text{MAD} = \frac{\sum_{i=1}^{n} |D_i - f_i|}{n}$$

$$= \frac{2.5 + 4.7 + 1.47 + 0.87 + 1.67 + 0.77 + 1.9 + 2.53 + 2.2}{9}$$

$$= 2.067$$

2. (b)

$$\text{MAD} = \frac{\sum_{i=1}^{n} |(D_i - f_i)|}{n}$$

$$= \frac{2.45 + 4.35 + 0.6 + 0.69 + 1.6 + 0.59 + 2.11 + 2.12 + 1.73}{9}$$

$$= 1.8044$$

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5 Line Balancing

Detailed Explanation of Try Yourself Questions

T1 : Solution

By largest candidate rule:
Step I: List all the element in the decreasing order of their task time.
Step II: Assigning element in workstation.

Element	Time (minutes)	Precedance
3	0.8	1
10	0.8	6, 9
6	0.6	3
1	0.5	-
8	0.5	3, 5
7	0.4	4, 5
2	0.3	-
9	0.3	7, 8
4	0.2	1
5	0.1	2

Workstation	Element	T_s	Idle time $(T_C - T_s)$
I	1 2 4	1	0
II	3 5	0.9	0.1
III	6 7	1	0
IV	8 9	0.8	0.2
V	10	0.8	0.2

where T_s = Station time, T_c = Cycle time

Total work content, (TWC) = 4.5 minutes
Cycle time, (T_C) = 1 minutes

Theoretically minimum number of workstation required $(n) = \frac{TWC}{T_C} = \frac{4.5}{1} = 4.5 \approx 5$

Actual number of workstations, $n = 5$

$$\text{Balance delay} = \left(\frac{nT_C - TWC}{nT_C}\right) \times 100 = \frac{0.5}{5} \times 100 = 10\%$$

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6 Queuing Theory

Detailed Explanation of Try Yourself Questions

T1 : Solution

$$\lambda(\text{Arrival rate}) = 1.5 \text{ per minutes}$$

$$\mu(\text{Service rate}) = 2 \text{ per minutes}$$

$$\text{Utilization factor } (\rho) = \frac{1.5}{2} = 0.75$$

(i) Average queue length, $L_q = \dfrac{\rho^2}{1-\rho} = \dfrac{(0.75)^2}{1-0.75} = 2.25$

(ii) Average length of non-empty queue $= \dfrac{1}{1-\rho} = \dfrac{1}{1-0.75} = 4$

(iii) Average number of worker in the system, $L_s = \dfrac{\rho}{1-\rho} = \dfrac{0.75}{0.25} = 3$

(iv) Mean waiting time of an arrival $= \dfrac{L_q}{\lambda} = \dfrac{\rho^2}{(1-\rho)\lambda} = \dfrac{(0.75)^2}{(1-0.75)\times 1.5} = 1.5$ minutes

(v) Average waiting time of an arrival who waits $= \dfrac{L_s}{\lambda} = \dfrac{3}{1.5} = 2$ minutes

(vi) When one attendent

$$L_s = 3 \qquad [\rho = 0.75]$$

$$\text{Total cost} = 3 \times \frac{\text{Rs.}15}{\text{hr}} \times 8\text{hr} + \frac{\text{Rs.}4}{\text{hr}} \times 8\text{hr}$$

$$= \text{Rs. } 392$$

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7 LPP, Transportation and Assignment

Detailed Explanation of Try Yourself Questions

T1 : Solution

By introducing slack variable s_1, s_2, s_3 and s_4 the problem may be written as follows

Maximum $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

$$x_1 + x_2 + s_1 = 6$$
$$2x_1 + x_2 + s_2 = 8$$
$$-x_1 + x_2 + s_3 = 1$$
$$x_2 + s_4 = 2$$
$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

The above information is tabulated

Table - 1

Basis \ C_j	3 x_1	2 x_2	0 s_1	0 s_2	0 s_3	0 s_4	b	$\theta = \frac{b}{C_i}$
0 s_1	1	1	1	0	0	0	6	6
← 0 s_2	[2]*	1	0	1	0	0	8	4 ←
0 s_3	–1	1	0	0	1	0	1	–1
0 s_4	0	1	0	0	0	1	2	∞
E_j	0	0	0	0	0	0		
$E_j - C_j$	–3 ↑	–2	0	0	0	0		

E_j and $E_j - C_j$ are calculated

The entering column is identified and the θ values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

s_2 leaves the basis, x_1 enters the basis

The following row operations are performed

$$R_{2\text{new}} = R_{2\text{old}}/2$$
$$R_{1\text{new}} = R_{1\text{old}} - R_{2\text{new}}$$
$$R_{3\text{new}} = R_{3\text{old}} + R_{2\text{new}}$$

Then the table 1 becomes

E_j and $E_j - C_j$ are calculated

The entering column is identified and the θ values are calculated, then the leaving row are identified

The pivotal element/value is identified from the table

s_4 leaves the basis, x_2 enters the basis

Table - 2

Basis \ C_j	3 x_1	2 x_2	0 s_1	0 s_2	0 s_3	0 s_4	b	$\theta = \frac{b}{C_i}$
0 s_1	1	1/2	1	–1/2	0	0	2	4
← 0 s_2	1	1/2	0	1/2	0	0	4	8
0 s_3	0	3/2	0	1/2	1	0	5	10/3
0 s_4	0	[1]*	0	1	0	1	2	2
E_j	3	3/2	0	3/2	0	0		
$E_j - C_j$	0	–1/2 ↑	0	3/2	0	0		

The following row operations are performed

$$R_{1\text{new}} = R_{1\text{old}} - (R_{4\text{new}})/2$$
$$R_{2\text{new}} = R_{2\text{old}} - (R_{4\text{new}})/2$$
$$R_{3\text{new}} = 2R_{3\text{old}} - 3(R_{4\text{new}})/2$$

Then the table 2 becomes table 3

E_j and $E_j - C_j$ are calculated

Table - 3

Basis \ C_j	3 X_1	2 X_2	0 S_1	0 S_2	0 S_3	0 S_4	b
0 S_1	0	0	1	–1/2	0	–1/2	1
← 0 X_1	1	0	0	1/2	0	–1/2	3
0 S_3	0	0	0	1/2	1	–3/2	2
0 X_2	0	1	0	0	0	1	2
E_j	3	2	0	3/2	0	1/2	
$E_j - C_j$	0	0	0	3/2	0	1/2	

It is an optimum matrix since all the values obtained in '$E_j - C_j$' row are positive

Thus the optimum solution will change as

$$\Rightarrow \left.\begin{array}{l} x_1 = 3 \\ x_2 = 2 \\ S_1 = 1 \\ S_3 = 2 \end{array}\right\} \text{Basic variables} \quad \left.\begin{array}{l} S_2 = 0 \\ S_4 = 0 \end{array}\right\} \text{Non basic variables}$$

Hence, $Z = 3 \times 3 + 2 \times 2 = 13$

T2 : Solution

The simplex problem can be expressed in the form of matrices as follows

$$\text{Minimum } Z = \overset{A}{(5\ \ 2\ \ 2)} \overset{X}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$

$$\overset{B}{\begin{pmatrix} 2 & 3 & 1 \\ 6 & 8 & 5 \\ 7 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}} \overset{X}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} \geq \overset{C}{\begin{pmatrix} 20 \\ 30 \\ 40 \\ 50 \end{pmatrix}}$$

Note: If primal problem is $\begin{pmatrix} \min Z = AX \\ BX \geq C \end{pmatrix}$ then Dual problem is $\begin{pmatrix} \min Z = C^T Y \\ B^T Y \leq A^T \end{pmatrix}$

So the dual for the given problem becomes

$$\text{Maximum } Z = (20\ 30\ 40\ 50) \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 6 & 7 & 1 \\ 3 & 8 & 1 & 2 \\ 1 & 5 & 3 & 4 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \leq \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$$

T3 : Solution

Let s_1, s_2, s_3 be slack variables then the constraints become

$$x + y + s_1 = 90$$
$$x + 2y + s_2 = 80$$
$$x + y + s_3 = 50$$
$$x, y, s_1, s_2, s_3 \geq 90$$

$\Rightarrow$ $$Z = 45x + 40y + 0s_1 + 0s_2 + 0s_3$$

The above information is tabulated, E_j and $E_j - C_j$ are calculated.

The entering column is identified and the θ values are calculated, then the leaving row are identified

Table - 1

Basis ↓ C_j →	45 x	40 y	0 s_1	0 s_2	0 s_3	b	$\theta = \frac{b}{C_i}$
0 s_1	1	1	1	0	0	90	90
0 s_2	1	2	0	1	0	80	80
0 s_3	[1]*	1	0	0	1	50	50 ←
E_j	0	0	0	0	0		
$E_j - C_j$	–45 ↑	–40	0	0	0		

s_3 leaves the basis and x enters into the basis, then the table 1 becomes table 2, E_j and $E_j - C_j$ are calculated.

Table - 2

Basis ↓ C_j →	45 x	40 y	0 s_1	0 s_2	0 s_3	b	$\theta = \frac{b}{C_i}$
0 s_1	0	0	1	0	–1	40	
0 s_2	0	1	0	1	–1	30	
45 x	1	1	0	0	1	50	
E_j	45	45	0	0	45		
$E_j - C_j$	0	5	0	0	46		

As '$E_j - C_j$' values present in the matrix are positive. So it is an optimum matrix.
So solution can be read out from matrix i.e.,
$x = 50$, $y = 0$,
$\therefore$ by putting $(x, y) = (50, 0)$ in the objective function the maximum value of $Z = 45x + 40y = 2250$.

T4 : Solution

6	5	8	11	16
1	13	16	1	10
16	11	8	8	8
9	14	12	10	16
10	13	11	8	16

Steps I - If the number of lines is equal to the order of matrix then it is the optimal table. If the number of lines is less than the order of matrix, then select the minimum number which is not covered by the lines. Subtract this value from each and every element which is uncovered and add this value at the intersection point of lines. Here adding "3" minimum uncovered value at intersection and subtracting at non covered values.

1	[0]	3	6	11
~~0~~	12	15	~~0~~	9
8	3	[0]	~~0~~	~~0~~
[0]	5	3	1	7
2	5	3	[0]	8

We will continue to repeat step I, till number of lines are equal to the order of the matrix. Now number of lines are equal to the order of the matrix, so select zeros such that every column and row consists of one zero i.e., one allocation for one machine.

1	[0]	~~0~~	6	8
~~0~~	12	12	[0]	6
11	6	~~0~~	3	[0]
[0]	5	~~0~~	1	4
2	5	[0]	~~0~~	5

A–II, B–IV, C–V, D–I, E–III

The total processing time is 34.