ESE GATE PSUs

State Engg. Exams

MADE EASY Workbook 2025

Detailed Explanations of Try Yourself *Questions*

Mechanical Engineering

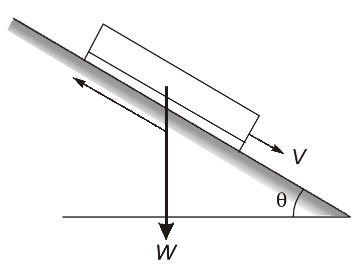
Fluid Mechanics and Machines

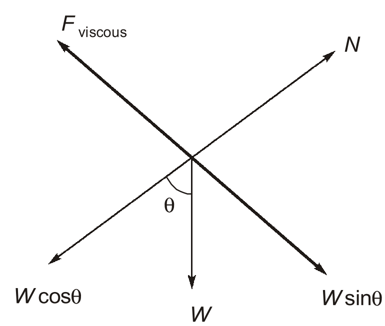
MADE EASY PUBLICATIONS

1 Fluid Properties

Detailed Explanation of Try Yourself Questions

T1 : Solution





Balancing forces along the inclined plane.

$$F_{viscous} = W\sin\theta$$

$$\Rightarrow \quad \frac{\mu AV}{y} = W\sin\theta$$

$$\Rightarrow \quad V = \frac{Wy\sin\theta}{\mu A}$$

$$= \frac{90 \times 3 \times 10^{-3} \times \sin 30}{8 \times 10^{-1} \times 0.3}$$

$$= 0.5625 \text{ m/s}$$

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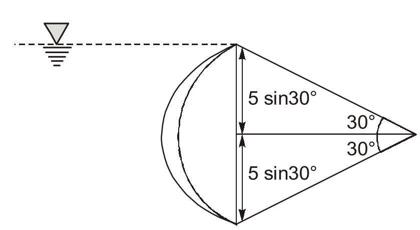
2 Fluid Statics

Detailed Explanation of Try Yourself Questions

T1 : Solution

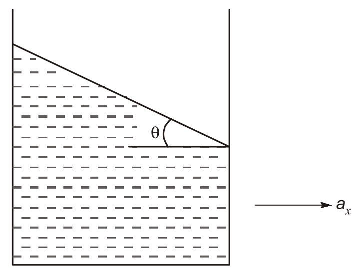
$$F_{buoyancy} = \text{Tension} + \text{Weight}$$
$$\rho_w \times \text{Volume} \times g = \text{Tension} + \text{Weight},$$
$$\text{Weight} = F_{buoyancy} - \text{Tension}$$
$$= \left[\rho_w \times \frac{4}{3} \times \pi \times r^3 \times g\right] - [5.5 \times 10^3]$$
$$= \left[1000 \times \frac{4}{3} \times \pi \times \left(\frac{1.5}{2}\right)^3 \times 9.81\right] - [5.5 \times 10^3]$$
$$= 17335.7 - 5500 = 11835.7 \text{ N} \simeq 12 \text{ kN}$$

T2 : Solution



$$F_h = \rho \times g \times A\bar{x}$$
$$= 1000 \times 10 \times 2 \times 5 \times \sin 30° \times 1 \times 5 \times \sin 30°$$
$$= 125 \text{ kN/m}$$
$$F_v = \left(\frac{\theta}{360} \times \pi R^2 - \frac{1}{2} \times b \times h\right) \times 1000 \times 10$$
$$= \left(\frac{60}{360} \times \pi \times (5)^2 - \frac{1}{2} \times 2 \times 5 \sin 30° \times 5 \cos 30°\right) \times 1000 \times 10$$
$$= 22.65 \text{ kN/m}$$
$$F_R = \sqrt{F_h^2 + F_v^2} = \sqrt{125^2 + 22.65^2} = 127.03 \text{ kN/m}$$

T3 : Solution



$$\tan\theta = \frac{-a_x}{g} = \frac{-g}{g} = -1$$

$$\theta = -45°$$

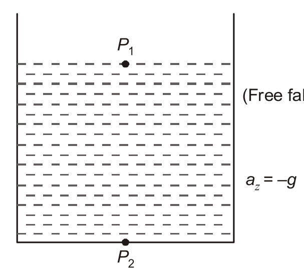
T4 : Solution

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial z}dz$$

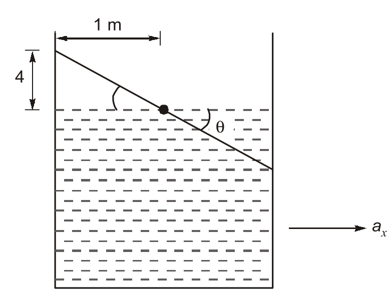
$$\frac{\partial P}{\partial z} = -\rho(-g+g) = 0$$

$$dP = 0 + 0 = 0$$

$$P_2 = P_1 = P_{atm}$$



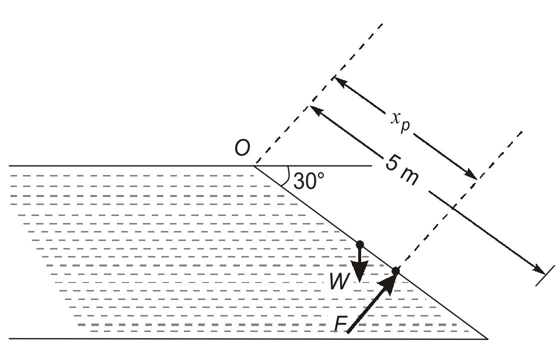
T5 : Solution



$$\tan\theta = \frac{-a_x}{g} \Rightarrow \frac{-4}{1} = \frac{-a_x}{g}$$

$$a_x = 4g$$

T6 : Solution



$$\text{Center of pressure } x_p, = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$x_p = 2.5 + \frac{1 \times 5^3 \times 2}{12 \times 1 \times 5 \times 5}$$

$$= 2.5 + \frac{5}{6} = 3.33 \text{ m}$$

$$\text{Pressure force, } F = wA\bar{h} = 1000 \times 9.81 \times 5 \times 1 \times \frac{5}{2} \times \sin 30°$$

$$= 61312.5 \text{ N}$$

Take moment about O,

$$F \times x_p = W \times \frac{5}{2} \cos 30°$$

$$61312.5 \times 3.33 = m \times 9.81 \times \frac{5}{2} \times \cos 30°$$

$$m = 9622.5 \simeq 9623 \text{ kg}$$

T7 : Solution

Let V is the volume of the body

Buoyancy force = Total weight

$$0.45 \times V \times \rho_{oil} \times g + 0.55 \times V \times \rho_{water} \times g = \rho_b \times V \times g$$

$$0.45 \times 0.7 \times 1000 + 0.55 \times 1000 = \rho_b$$

$$\rho_b = 865 \text{ kg/m}^3$$

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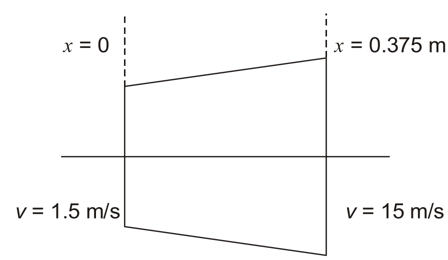
3 Fluid Kinematics

Detailed Explanation of Try Yourself Questions

T1 : Solution

Let the velocity by given by



$$u = a + bx$$

$\therefore$ At $x = 0, \quad u = 1.5$

$\therefore$ $a = 1.5$

At $x = 0.375, u = 15$

$\therefore$ $b = \dfrac{15-1.5}{0.375} = 36$

Hence $u = 1.5 + 36x$

$$a_x = \frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} + \frac{w\partial u}{\partial z}$$

$\therefore$ $\dfrac{v\partial u}{\partial y} = \dfrac{w\partial u}{\partial z} = 0$

$$a_x = (1.5+36x)\frac{\partial}{\partial x}(1.5+36x)$$

$$= (1.5 + 36x)(36)$$

$$a_x\Big|_{x = 0.375} = 36 \times \{1.5 + 36 \times 0.375\} = 540 \text{ m/s}^2$$

T2 : Solution

(i) $\psi = y^2 - x^2$

Flow to be irrotational it must satisfy the Laplace equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

checking $\psi = y^2 - x^2$

$$\therefore \quad \frac{\partial \psi}{\partial x} = -2x$$

$$\frac{\partial^2 \psi}{\partial x^2} = -2$$

$$\psi = y^2 - x^2$$

$$\therefore \quad \frac{\partial \psi}{\partial y} = 2y$$

$$\frac{\partial^2 \psi}{\partial y^2} = +2$$

Hence $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = +2 - 2 = 0$$

Hence flow is irrotational.

(ii) $$\psi = Ax^2y^2$$

For flow to be irrotational stream function should satisfy the Laplace equation.

$$\therefore \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Checking $$\psi = Ax^2y^2$$

$$\frac{\partial \psi}{\partial x} = 2Ay^2x$$

$$\frac{\partial^2 \psi}{\partial x^2} = 2Ay^2$$

Checking $$\psi = Ax^2y^2$$

$$\frac{\partial \psi}{\partial y} = Ax^2 2y$$

$$\therefore \quad \frac{\partial^2 \psi}{\partial y^2} = 2Ax^2$$

$$\therefore \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 2A(x^2 + y^2)$$

Flow is not irrotational.

(iii) $$\psi = Ax - By^2$$

For flow to be irrotational stream function should satisfy the Laplace equation.

∴ Checking $$\psi = Ax - By^2$$

$$\frac{\partial \psi}{\partial x} = A$$

$$\frac{\partial^2 \psi}{\partial x^2} = 0$$

Checking $$\psi = Ax - By^2$$

$$\frac{\partial \psi}{\partial y} = -2By$$

$$\frac{\partial^2 \psi}{\partial y^2} = -2B$$

Hence $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 - 2B \neq 0$$

Hence flow is not irrotational.

T3 : Solution

At stagnation point, $u = 0$, $v = 0$, and $\omega = 0$

$0.5 + 0.8x = 0$, $1.5 - 0.8y = 0$

$x = \frac{5}{8}$, $y = \frac{15}{8}$

So, there will be only one stagnation point.

Therefore, option (c) is correct.

T4 : Solution

$$\text{Radial distance} = 120 \text{ m}$$

$$V_r = \frac{-60 \times 10^3}{2\pi r}$$

$$V_\theta = \frac{300 \times 1000}{2\pi r}$$

Given: Angle turned by leaf, $\theta = \frac{1}{2}$ Revolution $= \pi$ radian

$$V_r = \frac{dr}{dt}$$

$$V_\theta = r\omega = \frac{rd\theta}{dt}$$

and $$\frac{V_\theta}{V_r} = -5$$

$$\frac{rd\theta / dt}{dr / dt} = -5$$

$$d\theta = -5\frac{dr}{r}$$

Integrating above equation,

$$\int_0^\pi d\theta = -5\int_{120}^{r} \frac{dr}{r}$$

$$\pi = -5\ln\left(\frac{r}{120}\right)$$

$$r = 120 \times e^{-\pi/5} = 64.01 \text{ m}$$

Therefore, option (b) is correct.

T5 : Solution

$$u = x^2,\ V = -2xy$$

$$\text{Circulation} = \iint_c \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dxdy = \iint (-2y - 0)\, dxdy$$

$$= \int_1^2 dx \int_1^2 (-2y)dx = -[x]_1^2 \times \left[\frac{2y^2}{2}\right]_1^2$$

$$= -1 \times (4 - 1) = -3$$

T6 : Solution

(i) $\phi = 2x + 5y$

For valid potential function, continuity equation should be satisfied and continuity equation in terms of potential function is laplace equation, i.e.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

LHS $\Rightarrow$ $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2}{\partial x^2}(2x + 5y) + \frac{\partial^2}{\partial y^2}(2x + 5y) = 0 + 0$$

$\therefore$ $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Hence, It is a valid potential function.

(ii) $\phi = 4x^2 - 5y^2$

$$\frac{\partial^2 \phi}{\partial x^2} = 8 \text{ and } \frac{\partial^2 \phi}{\partial y^2} = -10$$

So, $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 8 - 10 = -2 \neq 0$$

$\therefore$ It is not a valid potential function.

T7 : Solution

$$u = \frac{y^3}{3} + 2x - x^2 y$$

$$v = xy^2 - 2y - \frac{x^3}{3}$$

$$\frac{\partial u}{\partial x} = 0 + 2 - 2xy = 2 - 2xy$$

$$\frac{\partial v}{\partial y} = 2xy - 2$$

So, $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$$

Hence, it is possible case of flow as it satisifes 2D continuity equation

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

$$\psi = \int u dy + F(x) = \int \left(\frac{y^3}{3} + 2x - x^2 y \right) dy + F(x)$$

$$\psi = \frac{y^4}{12} + 2xy - \frac{x^2 y^2}{2} + F(x)$$

$$\frac{\partial \psi}{\partial x} = 2y - \frac{2xy^2}{2} + F'(x) = -\left(xy^2 - 2y - \frac{x^3}{3} \right)$$

$$F'(x) = -xy^2 + 2y + \frac{x^3}{3} - 2y + xy^2 = \frac{x^3}{3}$$

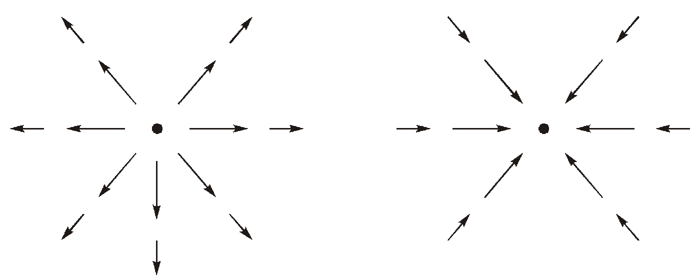
Integrating, we get

$$F(x) = \frac{x^4}{12} + C$$

$\therefore$

$$\psi = \frac{y^4}{12} + 2xy - \frac{x^2 y^2}{2} + \frac{x^4}{12} + C$$

T8 : Solution



Source Sink

T9 : Solution

Euler's equation for vector form (incompressible and frictionless),

$$\vec{F}_P + \vec{F}_g = \vec{F}_i$$

$$\vec{g} = g_x \hat{i} + g_y \hat{j} + g_z \hat{k}$$

$$= -g(\hat{k})$$

$$-\nabla P + \rho \vec{g} = \frac{\rho \Delta \vec{V}}{\Delta t}$$

$\Rightarrow$

$$-\nabla P + \rho \vec{g} = \rho \left[\frac{u \partial \vec{V}}{\partial x} \hat{i} + \frac{u \partial \vec{V}}{\partial y} \hat{j} + \frac{w \partial \vec{V}}{\partial z} \hat{k} + \frac{\partial \vec{V}}{\partial t} \right]$$

$$\Rightarrow \quad -\nabla P + \rho(0\hat{i} + 0\hat{j}) = \rho\left[x\hat{i} + (-y)(-\hat{j}) + 0 + 0\right]$$

$$\nabla P = -\rho\left[x\hat{i} + y\hat{j}\right]$$

T10 : Solution

$$\vec{V} = 2x\hat{i} - 2y\hat{j}$$

$$u = 2x, \; v = -2y$$

For velocity potential function

$$\frac{\partial\phi}{\partial x} = -u$$

$$\Rightarrow \quad \frac{\partial\phi}{\partial x} = -2x$$

$$\phi = -x^2 + f(y)$$

Differentiating ϕ with respect to y

$$\frac{\partial\phi}{\partial y} = f'(y)$$

$$\frac{\partial\phi}{\partial y} = -v = 2y$$

$$f'(y) = 2y$$

Integrating, $$f(y) = y^2 + c$$

$$\phi = -x^2 + y^2 + c$$

T11 : Solution

$$\vec{V} = 2x\hat{i} - 2y\hat{j}$$

$$u = 2x, \; v = -2y$$

Streamline equation

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \quad \frac{dx}{2x} = \frac{dy}{-2y}$$

$$\Rightarrow \quad \ln x = -\ln y + \ln c$$

$$\Rightarrow \quad \ln x + \ln y = \ln c$$

$$xy = c \quad \text{(hyperbola)}$$

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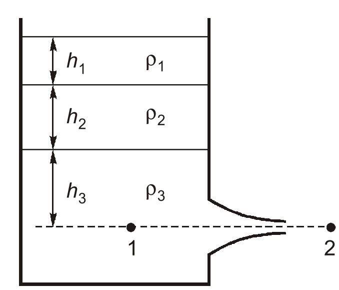
4 Fluid Dynamics & Flow Measurement

Detailed Explanation of Try Yourself Questions

T1 : Solution

Applying Bernoullis between points 1 and 2



$$\therefore \quad \frac{P_1}{\rho_3 g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho_3 g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$$P_1 = (\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)g$$

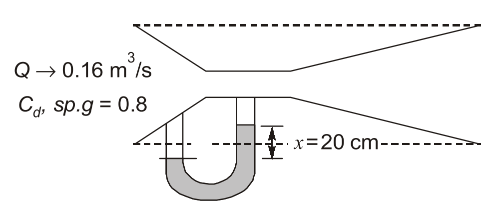
$$P_2 = 0 \qquad \text{(gauge pressure)}$$

$$V_1 = 0$$

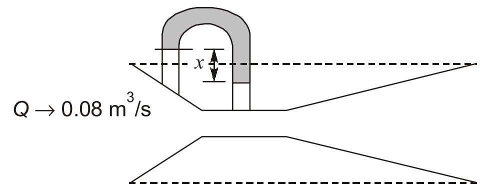
$$\therefore \quad \frac{V_2^2}{2g} = \frac{(\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3)g}{\rho_3 g}$$

$$V_2 = \sqrt{2gh_3\left\{\frac{\rho_1 h_1}{\rho_3 h_3} + \frac{\rho_2 h_2}{\rho_3 h_3} + 1\right\}}$$

T2 : Solution



$$\Delta h_1 = \left[\frac{S_m}{S_p} - 1\right]x = \left[\frac{13.6}{0.8} - 1\right]20 = 320 \text{ cm}$$



$$\Delta h_2 = \left[1 - \frac{s_m}{s_P}\right] x = \left[1 - \frac{\frac{\rho_{air}}{1000}}{0.8}\right] x$$

$$\rho_{air} = \frac{1.013 \times 10^5}{287 \times 298} = 1.184 \text{ kg/m}^3$$

$$\Delta h_2 = \left[1 - \frac{1.184 \times 10^{-3}}{0.8}\right] x = 0.99852x \text{ m}$$

$$\frac{Q_1}{Q_2} = \sqrt{\frac{\Delta h_1}{\Delta h_2}}$$

$$\Rightarrow \quad \frac{0.16}{0.08} = \sqrt{\frac{320}{0.99852x}}$$

$$\Rightarrow \quad 2 = \sqrt{\frac{320}{0.99852x}}$$

$$\Rightarrow \quad 4 = \frac{320}{0.99852x}$$

$$\therefore \quad x = \frac{320}{4 \times 0.99852} = 80.12 \text{ cm}$$

T3 : Solution

$$P_1A_1 - F_x = \dot{m}\left(V_{2_x} - V_{1_x}\right) \qquad (\dot{m} = 0)$$

$$P_1A_1 = F_x$$

$$\text{or} \quad F_x = \left(140 \times 10^3\right)\left[\frac{\pi}{4}\left(\frac{0.27}{\sqrt{\pi}}\right)^2\right]$$

$$= 2551.5 \text{ N}$$

$$-P_2A_2 + F_y = \dot{m}\left(V_{2_y} - V_{1_y}\right) \qquad (\dot{m} = 0)$$

$$F_y = P_2A_2$$

$$= \left(1.013 \times 10^5\right)\left[\frac{\pi}{4}\left(\frac{0.14}{\sqrt{\pi}}\right)^2\right]$$

$$= 496.37 \text{ N}$$

$$\text{Resultant force} = \sqrt{F_x^2 + F_y^2} = \sqrt{(2551.5)^2 + (496.37)^2}$$

$$= 2599.33 \text{ N} = 2.60 \text{ kN}$$

T4 : Solution

Given data :

D = 50 cm = 0.50 m

Specific gravity, S = 0.8

ω = 2.0 rad/s

For the sphere, the circular line of maximum pressure is a horizontal circle, at a distance $h = g/\omega^2$ below the centre of sphere.

$$h = \frac{g}{\omega^2} = \frac{9.81}{(20)^2} = 0.0245 \text{ m} = \mathbf{2.45 \text{ cm}}$$

T5 : Solution

At bottom due weight of liquid above and rotation of cylinder,

$$\text{Pressure, } P = \frac{\rho\omega^2 r^2}{2}$$

$$\text{Thrust force at bottom, } F = \int_0^R p dA = \int_0^R \rho g h 2\pi r dr + \int_0^R \frac{\rho\omega^2 r^3}{2} \times 2\pi dr$$

$$= \rho g h \pi R^2 + \frac{\pi\rho\omega^2 R^4}{4}$$

T6 : Solution

Both statements are correct.

The pressure difference at inlet and throat of venturimeter is due to the change in cross-sectional area.

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5 Dimensional Analysis

Detailed Explanation of Try Yourself Questions

T1 : Solution

As per Reynold's model law

$$\frac{\rho_r V_r l_r}{\mu_r} = 1$$

$$\Rightarrow \quad \frac{V_r l_r}{\nu_r} = 1$$

Viscosity scale ratio, $V_r = \dfrac{\nu_r}{l_r}$

Discharge scale ratio, $Q_r = V_r \times A_r$

and $A_r = l_r^2$

So, $Q_r = \dfrac{\nu_r}{l_r} \times l_r^2 = \nu_r \times l_r$

T2 : Solution

$$\left[\frac{\rho VL}{\mu}\right]_{model} = \left[\frac{\rho VL}{\mu}\right]_P$$

Given $\dfrac{L_m}{L_P} = \dfrac{1}{6}$

$$[VL]_m = [VL]_P$$

$$V_m = 60 \times \frac{L_P}{L_m} = 60 \times 6 = 360 \text{ km/hr}$$

$$F_D = C_D \frac{1}{2} \rho A V^2$$

or $F_D \propto (LV)^2$

$\therefore$ $(F_D)_P = k[L_P v_P]^2$

$$(F_D)_m = k[L_m V_m]^2$$

$$\frac{(F_D)_P}{(F_D)_m} = \frac{L_P^2\, V_P^2}{L_m^2\, V_m^2}$$

$$= 6^2 \times \left(\frac{60}{360}\right)^2$$

$$\frac{(F_D)_P}{250} = 1$$

$\therefore$ $$(F_D)_P = 250\text{ N}$$

Power required to overcome the drag in prototype

$$= (F_D)_P \times V_P$$

$$= 250 \times \frac{60 \times 1000}{3600}$$

$$= 4167.67\text{ W} = 4.167\text{ kW}$$

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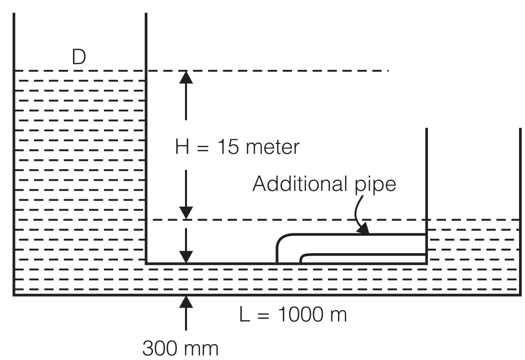
6 Flow Through Pipes

Detailed Explanation of Try Yourself Questions

T1 : Solution



All the losses are negligible except friction.

$$\therefore \qquad H = \frac{fL}{d}\cdot\frac{V^2}{2g}$$

$$15 = \frac{0.02\times1000\times V^2}{0.3\times2\times9.81}$$

$$V^2 = \frac{15\times0.3\times2\times9.81}{0.02\times1000}$$

$$V = 2.101 \text{ m/sec}$$

$$\therefore \qquad \text{Flow rate, } \dot{Q} = AV = \frac{\pi}{4}(0.3)^2\times2.101$$

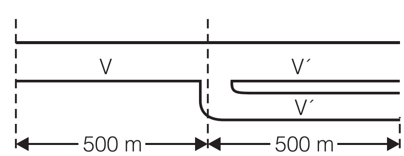
$$\dot{Q} = 0.1485 \text{ m}^3/\text{sec}$$

Now addition same pipe of length is added in later half of pipe as

$$\therefore \qquad Q_1 = Q_2 + Q_3$$

$$AV = AV' + AV'$$

$$V' = \frac{V}{2}$$



$\therefore$ Friction head loss is same

$$h_f = 15 = \frac{fL'}{d}\cdot\frac{V^2}{2g} + \frac{fL'}{d}\cdot\frac{V'^2}{2g}$$

$$15 = \frac{0.02\times500}{0.3}\frac{V^2}{2g} + \frac{0.02\times500}{0.3}\times\frac{1}{4}\cdot\frac{V^2}{2g}$$

$$15 = 2.124\,V^2$$

$$V = 2.657 \text{ m/sec}$$

$$V' = \frac{V}{2} = 1.329 \text{ m/sec}$$

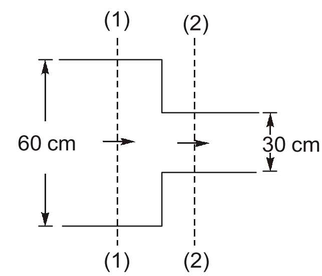
Discharge rate $\quad Q' = A.V = \frac{\pi}{4}.(0.3)^2 \times 2.657 = 0.18781 \text{ m}^3\text{/sec}$

$$\text{Increase in discharge} = \frac{Q' - Q}{Q} = 26.47\%.$$

T2 : Solution

Using the Bernaulli's equation, at points 1 and 2

∴ Let p_1, V_1, Z_1 be the pressure, velocity and head at point 1, and p_2 V_2, Z_2, be the corresponding values at point 2.



$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$h_L = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

$$\therefore \quad h_L = \left(\frac{1}{0.65} - 1\right)^2 \frac{V_2^2}{2g}$$

$$\therefore \quad h_L = 0.2899 \frac{V_2^2}{2g}$$

Also, $\quad Q = A_1 V_1 = A_2 V_2$

$$\Rightarrow \quad \frac{\pi}{4} \times (60)^2 V_1 = \frac{\pi}{4}(30)^2 \times V_2$$

$$\therefore \quad V_1 = \frac{V_2}{4}$$

Using the Bernaulli's equation

$$\therefore \quad \frac{100 \times 10^3}{1000 \times 9.81} + \frac{1}{2g}\left(\frac{V_2}{4}\right)^2 + Z_1 = \frac{80 \times 10^3}{1000 \times 9.81} + \frac{V_2^2}{2g} + Z_2 + 0.2899\frac{V_2^2}{2g}$$

$$\therefore \quad 10.1936 + \frac{V_2^2}{32g} = 8.1549 + 1.2899\frac{V_2^2}{2g} \qquad [\because Z_1 = Z_2]$$

$$\therefore \quad 10.1936 - 8.1549 = 1.2899\frac{V_2^2}{2g} - \frac{V_2^2}{32g}$$

$$2.0387 = 0.06255\, V_2^2$$

$$\Rightarrow \quad V_2^2 = 32.5886$$

$$\therefore \quad V_2 = 5.7086 \text{ m/s}$$

∴ Flow rate, $\quad Q = A_2 V_2 = \frac{\pi}{4} \times (0.3)^2 \times 5.7086$

$$Q = 0.4035 \text{ m}^3\text{/s}$$

Also,
$$h_L = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$
$$h_L = \left(\frac{1}{0.65} - 1\right)^2 \times \frac{(5.7086)^2}{2 \times 9.81}$$
$$h_L = 0.482 \text{ m}$$

T3 : Solution

As per given,
Discharge, uniformly along its length
$$q_x = \frac{Q}{L} \cdot x$$
$$\int_0^Q dQ = \int_0^L \frac{f \cdot (q_x)^2 dx}{12.1 d^5} = \int_0^L \frac{f \cdot \left(\frac{Q}{L} \cdot x\right)^2 dx}{12.1 d^5}$$
$$Q_1 = \frac{f \cdot Q^2}{12.1 d^5 L^2}\left(\frac{L^3}{3}\right) = \frac{f \cdot Q^2 L}{12.1 \cdot d^5 \times 3}$$
$$Q_2 = \frac{fLQ^2}{12.1 d^5}$$
$$Q_1 = \frac{Q_2}{3}$$
Therefore, option (b) is correct.

T4 : Solution

$$c = 1414 \text{ m/s}$$
$$c = \sqrt{\frac{k}{\rho}}$$
$$c' = \frac{\sqrt{\frac{k}{\rho}}}{\sqrt{1 + \frac{DK}{EL_P}}} = \frac{1414}{\sqrt{1 + \frac{40 \times 10^{-2} \times 2.1 \times 10^9}{2.1 \times 10^{11} \times 4 \times 10^{-3}}}}$$
$$= \frac{1414}{\sqrt{2}} = \frac{1414}{1.414}$$
$$= 1000 \text{ m/s}$$

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7 Laminar and Turbulent Flow

Detailed Explanation of Try Yourself Questions

T1 : Solution

Reynolds number, $$Re = \frac{\rho VD}{\mu} = \frac{1260 \times 5.0 \times 0.10}{1.50} = 420$$

(i) As this value is less than 2000, the flow is laminar. In laminar flow in a conduit

$$\tau_0 = \frac{8\mu V}{D} = \frac{8 \times 1.50 \times 5.0}{0.10} = 600 \text{ Pa}$$

(ii) In laminar flow the head loss

$$h_f = \frac{32\mu VL}{\omega D^2} = \frac{32 \times 1.50 \times 5.0 \times 12}{(1260 \times 9.81)(0.1)^2} = 23.3 \text{ m}$$

(iii) Power expended $P = \omega Q h_f$

Discharge $$Q = AV = \frac{\pi \times (0.1)^2}{4} \times 5.0 = 0.03927 \text{ m}^3\text{/s}$$

Power, $$P = (1260 \times 9.81) \times 0.03927 \times 23.3$$
$$= 11309.8 \text{ W} = 11.31 \text{ kW}$$

T2 : Solution

(i) For two-dimensional laminar flow between two parallel plates

$$u_m = \text{Maximum velocity} = \frac{3}{2}V$$
$$= \frac{3}{2} \times 1.40 = 2.10 \text{ m/s}$$

(ii) Since Mean velocity, $$V_m = \left(-\frac{dp}{dx}\right)\frac{B^2}{12\mu}$$

$$\left(-\frac{dp}{dx}\right) = \frac{12\mu V}{B^2} = \frac{12 \times 0.105 \times 1.40}{(0.012)^2} = 12250$$

Boundary shear stress $\tau_0 = \left(-\frac{dp}{dx}\right)\frac{B}{2} = 12250 \times \frac{0.012}{2} = 73.5 \text{ Pa}$

(iii) Shear stress τ at any y from the boundary

$$\tau = \left(-\frac{dp}{dx}\right)\left(\frac{B}{2} - y\right)$$

At $y = 0.002$ m

$$\tau = (12250)\left(\frac{0.012}{2} - 0.002\right) = 49 \text{ Pa}$$

Velocity

$$V = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)(By - y^2)$$

$$= \frac{1}{2 \times 0.105} \times 12250\left[0.012 \times 0.002 - (0.002)^2\right]$$

$$V = 1.167 \text{ m/s}$$

T3 : Solution

Given:

$$u_m = 1.50 \text{ m/s}, \quad y = 0.1 \text{ m} = r_0$$

$$u = 1.35 \text{ m/s}, \quad y = 0.05 \text{ m}$$

$$\frac{u_m - u}{u^*} = 5.75\log_{10}\left(\frac{r_0}{y}\right)$$

$$\frac{1.5 - 1.35}{u^*} = 5.75\log_{10}\left(\frac{0.1}{0.05}\right)$$

$$u^* = 0.086658 \text{ m/s}$$

$$\frac{u_m - V}{u^*} = 3.75$$

$$\frac{1.5 - V}{0.086658} = 3.75$$

$$V = 1.175 \text{ m/s}$$

Discharge, $Q = \frac{\pi D^2}{4}V = \frac{\pi \times 0.2^2}{4} \times 1.175$

$$= 0.037 \text{ m}^3/\text{s}$$

Shear velocity, $u^* = V\sqrt{\frac{f}{8}}$

$$0.086658 = 1.175\sqrt{\frac{f}{8}}$$

$\Rightarrow$ $f = 0.0436$

Height of roughness, $\frac{u_1}{u^*} = 5.75\log_e\left(\frac{y_1}{\varepsilon}\right) + 8.5$

$$\frac{1.35}{0.086658} = 5.75\log_e\left(\frac{0.05}{\varepsilon}\right) + 8.5$$

$$\varepsilon = 14.599 \text{ mm}$$

T4 : Solution

(i) As per given,

$$\text{Average roughness of height, } k = 1.165 \text{ mm}$$

$$\text{Laminar sublayer thickness, } \delta' = \frac{11.6\nu}{u^*}$$

$$u^* = \sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{3.6}{1000}} = 0.06 \text{ m/s}$$

$$\nu = \frac{\mu}{\rho} = 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Water dynamic viscosity} = 10^{-3} \text{ N.s/m}^2$$

$$\delta' = \frac{11.6 \times 10^{-6} \text{ m}^2/\text{s}}{0.06 \text{ m/s}}$$

$$= 0.19333 \text{ mm}$$

From Nikuradse's conditions

$$\frac{k}{\delta'} = \frac{1.165}{0.19333} = 6.025$$

$$\frac{k}{\delta'} > 6 \Rightarrow \text{Rough-Nature of the pipe surface}$$

(ii) The flow is rough-turbulent

$$\frac{u}{u^*} = 5.75\log_e\left(\frac{y}{\varepsilon}\right) + 8.5$$

$$\frac{u_2 - u_1}{u^*} = 5.75\log_e\left(\frac{y_2}{y_1}\right)$$

$$u_2 = 1.3u_1$$

$$u_2 = 2.0 \text{ cm}$$

$$y_1 = 1.0 \text{ cm}$$

$$\frac{0.3u_1}{u^*} = 5.75\log_e(2)$$

$$\frac{u_1}{u^*} = 13.285$$

$$\frac{u_1}{u^*} = 5.75\log_e\left(\frac{y_1}{\varepsilon}\right) + 8.5$$

$$13.285 = 5.75\log_e\left(\frac{10^{-2}}{\varepsilon}\right) + 8.5$$

$$\varepsilon = 0.435 \times 10^{-3}$$

$$\text{Roughness} = 4.35 \times 10^{-3} \text{ m}$$

$$= 4.35 \text{ mm}$$

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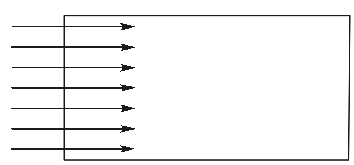
8 Boundary Layer Theory, Drag and Lift

Detailed Explanation of Try Yourself Questions

T1 : Solution



$$F_{D1} = C_{fx}\rho\frac{1}{2}AV_\infty^2 \qquad \text{[For first half]}$$

$$C_{fx} = \frac{k}{\sqrt{\text{Re}_x}}$$

$$= \frac{k}{\sqrt{\text{Re}_x}} \times \rho \times \frac{1}{2} \times b \times \frac{L}{2} \times U_\infty^2$$

$$= \frac{k\sqrt{2\mu}}{\sqrt{\rho VL}} \times \frac{\rho \times bU_\infty^2 \times L}{4} \qquad \text{....(1)}$$

$$F_{D2} = C_{fx}\rho\frac{1}{2}AV_\infty^2 \qquad \text{[for full plate]}$$

$$C_{fx} = \frac{k}{\sqrt{\text{Re}_\text{L}}}$$

$$= \frac{k \times \rho \times b \times L \times U_\infty^2\sqrt{\mu}}{\sqrt{\rho VL} \times 2}$$

$$\frac{F_{D_1}}{F_{D_2}} = \frac{\sqrt{2}/4}{1/2}$$

$$= \frac{\sqrt{2}}{4} \times 2 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

T2 : Solution

Given:

(i)

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

or
$$u = \frac{3U_\infty}{2}\left(\frac{y}{\delta}\right)-\frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3$$

Differentiating w.r.t y, the above equation becomes,

$$\frac{\partial u}{\partial y} = \frac{3U_\infty}{2}\times\frac{1}{\delta}-\frac{U_\infty}{2}\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

At $y = 0$,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{3U_\infty}{2\delta}-\frac{3U_\infty}{2}\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta}=\frac{3U_\infty}{2\delta}$$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is positive. Hence flow will not separate or flow will remain attached with the surface.

(ii)

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right)^2-\left(\frac{y}{\delta}\right)^3$$

∴
$$u = 2U\left(\frac{y}{\delta}\right)^2-U_\infty\left(\frac{y}{\delta}\right)^3$$

∴
$$\frac{\partial u}{\partial y} = 2U\times 2\left(\frac{y}{\delta}\right)\times\frac{1}{\delta}-U_\infty\times 3\left(\frac{y}{\delta}\right)^2\times\frac{1}{\delta}$$

at $y = 0$,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 2U_\infty\times 2\left(\frac{0}{\delta}\right)\times\frac{1}{\delta}-U_\infty\times 3\left(\frac{0}{\delta}\right)^2\times\frac{1}{\delta}=0$$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}=0$, the flow is on the verge of separage.

(iii)

$$\frac{u}{U_\infty} = -2\left(\frac{y}{\delta}\right)+\left(\frac{y}{\delta}\right)^2$$

∴
$$u = -2U_\infty\left(\frac{y}{\delta}\right)+U_\infty\left(\frac{y}{\delta}\right)^2$$

∴
$$\frac{\partial u}{\partial y} = -2U_\infty\left(\frac{1}{\delta}\right)+2U_\infty\left(\frac{y}{\delta}\right)\times\frac{1}{\delta}$$

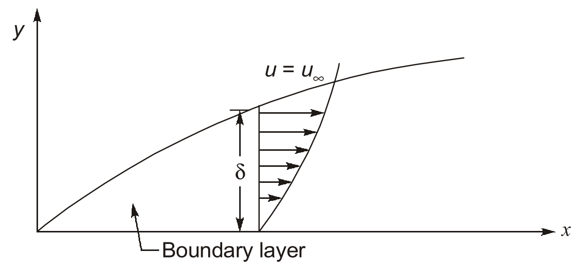
At $y = 0$,
$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{2U_\infty}{\delta}+2U_\infty\left(\frac{0}{\delta}\right)\times\frac{1}{\delta}=-\frac{2U_\infty}{\delta}$$

As $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative the flow has separated.

T3 : Solution

For laminar boundary layer:

$$\frac{u}{u_\infty} = c_0 + c_1\left(\frac{y}{\delta}\right) + c_2\left(\frac{y}{\delta}\right)^2 + c_3\left(\frac{y}{\delta}\right)^3$$



At
$$y = 0$$
$$u = 0$$
$$0 = c_0 + 0 + 0 + 0$$
$$c_0 = 0 \qquad \text{Ans.}$$

At
$$y = \delta$$
$$u = u_\infty$$
$$1 = 0 + c_1\left(\frac{\delta}{\delta}\right) + c_2\left(\frac{\delta}{\delta}\right)^2 + c_3\left(\frac{\delta}{\delta}\right)^3$$
$$c_1 + c_2 + c_3 = 1$$
$$y = \delta, \frac{du}{dy} = 0$$
$$0 = u_\infty\left[0 + \frac{c_1}{\delta} + 2c_2\left(\frac{y}{\delta}\right)\frac{1}{\delta} + 3c_3\left(\frac{y}{\delta}\right)^2\right]\frac{1}{\delta}$$
$$\frac{c_1}{\delta} + \frac{2c_2}{\delta} + \frac{3c_3}{\delta} = 0$$
$$c_1 + 2c_2 + 3c_3 = 0$$

At
$$y = 0$$
$$\frac{d^2y}{dy^2} = 0$$
$$0 = \frac{2c_2}{\delta^2} + 6c_3\left(\frac{y}{\delta}\right)\frac{1}{\delta^2}$$
$$c_2 = 0$$

Therefore,
$$c_1 + c_3 = 1 \qquad \text{...(i)}$$
$$c_1 + 3c_3 = 0 \qquad \text{...(ii)}$$

Subtracting equation (i) and (ii), we get
$$-2c_3 = 1$$
$$c_3 = -\frac{1}{2}$$
$$c_1 = \frac{3}{2}$$

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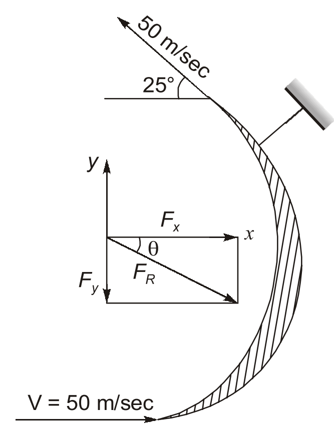
9 Hydraulic Machines

Detailed Explanation of Try Yourself Questions

T1 : Solution

Given: **(a)** Velocity of jet, V = 50 m/s

Angle at outlet = 25°



For the stationary vane, the force in the direction of jet is given as

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where, V_{1x} = 50 m/s

$$V_{2x} = -50\cos 25° = -45.315$$

∴ Force in direction of jet per unit weight of water

$$= \frac{\text{Mass/sec}\left[50-(-45.315)\right]}{\text{Weight of water/sec}}$$

or

$$F_x = \frac{(Mass/\text{sec})[50+45.315]}{(\text{Mass/sec})\times g}$$

$$= \frac{1}{g}[50+45.315]\ \text{N/N} = \frac{95.315}{9.81} = 9.716\ \text{N/N/s}$$

Force exerted by jet in the direction perpendicular to the direction of the jet per unit weight of the flow,

$$F_y = \frac{(\text{Mass per sec})\left[V_{1y}-V_{2y}\right]}{g\times \text{ Mas per sec}}$$

$$= \frac{1}{g}\left[V_{1y}-V_{2y}\right] = \frac{1}{g}[O-50\sin 25°] \qquad (\because V_{1y} = 0,\ V_{2y} = 50\sin 25°)$$

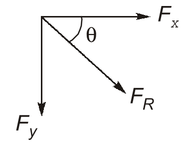
$$= \frac{-50\sin 25°}{9.81} = -2.154\ \text{N/N/s}$$

–ve sign means the force F_y is acting in the downward direction.

$\therefore$ Resultant force per unit weight of water = $\sqrt{F_x^2 + F_y^2}$

or $$F_R = \sqrt{(9.716)^2 + (2.154)^2} = 9.952 \text{ N/N/s}$$

The angle made by the resultant with the x-axis.

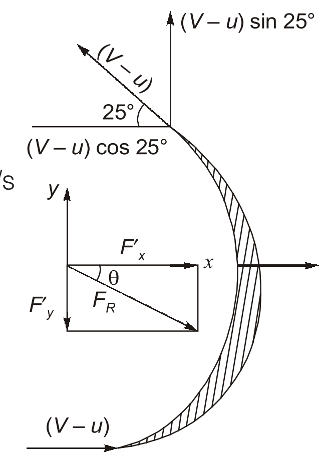


$$\tan\theta = \frac{F_y}{F_x} = \frac{2.154}{9.716} = 0.2217$$

$$\therefore \quad \theta = \tan^{-1} 0.2217 = 12.50°$$

(b) Velocity of the vane = 20 m/s

When the vane is moving in the direction of the jet, the force exerted by the jet on the plate in the direction of jet,



$$F'_x = [\text{Mass of water striking/sec}] \times [V_{1x} - V_{2x}]$$

where, V_{1x} = Initial velocity of the striking water

$= (V - u) = 50 - 20 = 30$ m/s

V_{2x} = Final velocity in the direction of x

$= -(V - u) \cos 25° = 30 \times \cos 25° = -27.189$ m/s

$\therefore$ F_x = Mass per sec [30 + 27.189]

Force in the direction of jet per unit weight,

$$F'_x = \frac{\text{Mass per sec } [30 + 27.189]}{\text{Mass per sec} \times g}$$

$$= \frac{(30 + 27.189)}{9.81} = 5.829 \text{ N/N/s}$$

Force exerted by the jet in the direction perpendicular to direction of jet, per unit weight

$$F'_y = \frac{1}{g}\left[V_{1y} - V_{2y}\right]$$

where, $V_{1y} = 0;\ V_{2y} = (V - u) \sin 25° = (50 - 20) \sin 25° = 30 \sin 25°$

$$F'_y = \frac{1}{9.81}\left[0 - 30 \sin 25°\right] = -1.292 \text{ N/N/s}$$

$$\therefore \quad \text{Resultant force} = \sqrt{(5.829)^2 + (1.292)^2} = 5.917 \text{ N/N/s}$$

The angle made by the resultant with x-axis,

$$\tan\theta = \frac{1.292}{5.829} = 0.2217$$

$$\therefore \quad \theta = \tan^{-1} 0.2217 = 12.30°$$

$\therefore$ Work done per second per unit weight of flow

$= F'_x \times u = 5.829 \times 20 = 116.58$ N m/s

$$\therefore \quad \text{Power developed} = \frac{\text{Work done per second}}{1000} = \frac{116.58}{1000} = 0.116 \text{ kW}$$

T2 : Solution

Given:

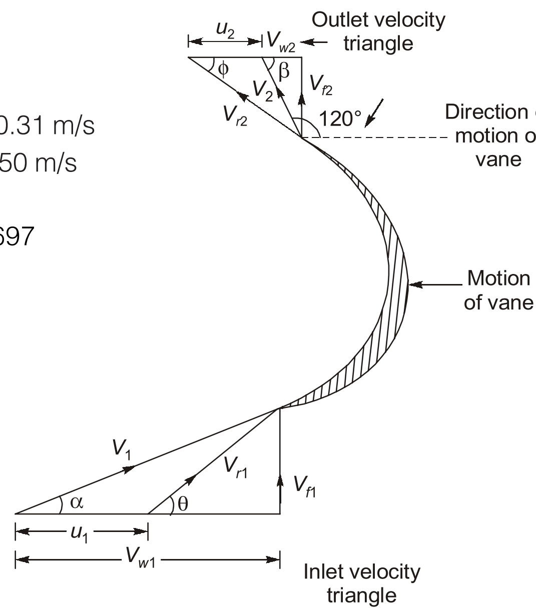
Velocity of jet, $V_1 = 35$ m/s

Velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes = 120°

∴ Angle $\beta = 180° - 120° = 60°$



(a) Angle of vanes tips.

From inlet velocity triangle,

$$V_{w1} = V_1 \cos\alpha = 35 \cos 30° = 30.31 \text{ m/s}$$

$$V_{f1} = V_1 \sin\alpha = 35 \sin 30° = 17.50 \text{ m/s}$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

∴ $\theta = \tan^{-1} 1.697 = 59.49°$

By sine rule, $$\frac{V_{r1}}{\sin 90°} = \frac{V_{f1}}{\sin\theta}$$

or $$\frac{V_{r1}}{1} = \frac{17.50}{\sin 59.49°}$$

∴ $V_{r1} = 20.31$ m/s

Now, $V_{r2} = V_{r1} = 20.31$ m/s

From outlet velocity triangle, by sine rule

$$\frac{V_{r2}}{\sin 120°} = \frac{u_2}{\sin(60° - \phi)}$$

or $$\frac{20.31}{0.866} = \frac{20}{\sin(60° - \phi)}$$

∴ $$\sin(60° - \phi) = \frac{20 \times 0.866}{20.31} = 0.85278 \Rightarrow 60° - \phi = \sin^{-1}(0.85278) = 58.515°$$

∴ $\phi = 60° - 58.515° = 1.485°$

(b) Work done per unit weight of water entering $= \frac{1}{g}(V_{w1} + V_{w2}) \times u_1$ ...(i)

$$V_{w1} = 30.31 \text{ m/s and } u_1 = 20 \text{ m/s}$$

The value of V_{w2} is obtained from outlet velocity triangle

$$V_{w2} = V_{r2} \cos\phi - u_2 = 20.31 \cos 1.485° - 20.0 = 0.24 \text{ m/s}$$

∴ Work done/unit weight $= \frac{1}{9.81}[30.31 + 0.24] \times 20 = 62.28$ Nm/N

(c)

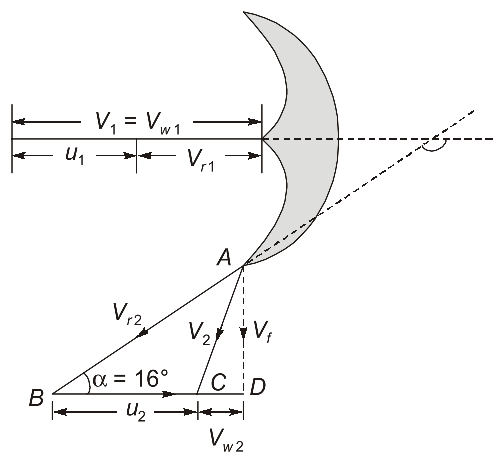
$$\text{Efficiency} = \frac{\text{Work done per kg}}{\text{Energy supplied per kg}}$$

$$= \frac{62.28}{\frac{V_1^2}{2g}} = \frac{62.28 \times 2 \times 9.81}{35 \times 35} = 99.74\%$$

T3 : Solution

Gross head, $H_g = 220$ m, Net head, $H = 200$ m, $C_V = 0.98$, $N = 200$ rpm, power = 3.7 MW, $u_1 = u_2 = u$

Given: $\frac{u}{V_1} = 0.46$, $D = ?$



Speed of jet at vena contracta i.e. max. speed of jet

$$V_1 = C_V\sqrt{2gH}$$
$$= 0.98\sqrt{2 \times 9.81 \times 200}$$
$$= 61.4 \text{ m/sec}$$

Speed of wheel

$$u = 0.46 \times V_1$$
$$= 0.46 \times 61.4 = 28.24 \text{ m/sec}$$

$$u = \frac{\pi DN}{60} = 28.24 \qquad [u = u_1 = u_2]$$

$$D = \frac{28.24 \times 60}{\pi \times 200}$$

$$D = 2.697 \text{ m}$$

$$\therefore \quad V_{r2} = V_{r1} = V_1 - u$$
$$= 61.4 - 28.24$$
$$= 33.16 \text{ m/s}$$
$$V_{w2} = V_{r2}\cos 16° - u$$
$$= 33.16 \times \cos 16° - 28.24$$
$$V_{w2} = 3.635 \text{ m/s}$$

Blade efficiency,

$$\eta_b = \frac{2u(V_{w1} + V_{w2})}{V_1^2} = \frac{2 \times 28.24\,(61.4 + 3.635)}{61.4^2}$$

$$\eta_b = 97.5\%$$

Hydraulic efficiency

$$= \frac{u(V_{w1} + V_{w2})}{gH} = \frac{28.24(61.4 + 3.635)}{9.81 \times 200} = 0.936 = 93.6\%$$

T4 : Solution

Given: Gross head, $H_g = 500$ m

Head lost in friction, $h_f = \frac{H_g}{3} = \frac{500}{3} = 166.7$ m

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.3$ m

Discharge, $Q = 2.0\ m^3/s$

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio, $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

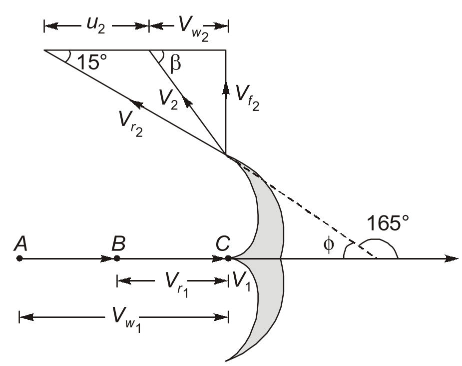
or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

∴ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $V_{w_1} = V_1 = 80.86$ m/s

From outlet velocity tringle, we have



$V_{r_2} = V_{r_1} = 44.473$ m/s

$V_{r_2}\cos\phi = u_2 + V_{w_2}$

or $44.473 \cos 15° = 36.387 + V_{w_2}$

or $V_{w_2} = 44.473 \cos 15° - 36.387$

$= 6.57$ m/s

Work done by the jet on the runner per second is given by equation as

$$\rho a V_1\left[V_{w_1} + V_{w_2}\right] \times u = \rho Q\left[V_{w_1} + V_{w_2}\right] \times u \quad (\because aV_1 = Q)$$

$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630$ Nm/s

∴ Power given by the water to the runner in MW

$$= \frac{\text{Work done per second}}{10^6} = \frac{6362630}{10^6} = 6.36 \text{ MW} = 6.36 \text{ MW}$$

Hydraulic efficiency of the turbine is given by equation as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$= 0.9731$ or 97.31%

T5 : Solution

Given: Head, $H = 60$ m

Speed, $N = 200$ rpm

Shaft power, SP = 95.6475 kW

Velocity of bucket, $u = 0.45 \times$ Velocity of jet

Overall efficiency, $\eta_0 = 0.85$

Co-efficient of velocity, $C_v = 0.98$

Design of Pelton wheel means to find diameter of jet (d), diameter of wheel (D), Width and depth of buckets and number of buckets on the wheel

(i) Velocity of jet,

$$V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62 \text{ m/s}$$

$\therefore$ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $$u = \frac{\pi DN}{60}$$ where D = Diameter of wheel

$$\therefore \quad 15.13 = \frac{\pi \times D \times 200}{60}$$

or $$D = \frac{60 \times 15.13}{\pi \times 200} = 1.44 \text{ m}$$

(ii) Diameter of the jet (d)

Overall efficiency $\eta_0 = 0.85$

But $$\eta_0 = \frac{SP}{WP} = \frac{95.6475}{\left(\frac{WP}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H}$$ ($\because$ WP = ρgQH)

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$$\therefore \quad Q = \frac{95.6475 \times 1000}{\eta_0 \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3\text{/s}$$

But the discharge, Q = Area of jet $\times$ Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = 85 \text{ mm}$$

(iii) Size of buckets

Width of bucket $= 5 \times d = 5 \times 85 = 425$ mm

Depth of bucket $= 1.2 \times d = 1.2 \times 85 = 102$ mm

(iv) Number of buckets on the wheel is given by eq. as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times 0.085}$$

$$= 15 + 8.5 = 23.5 \text{ Say } 24$$

T6 : Solution

Inlet diameter, $D_1 = 1.0$ m

Rotational speed, $N = 400$ rpm

Area of flow, $A = 0.25$ m^2

Net available head, $H = 65$ m

Velocity of flow at inlet, $V_{f1} = 8.0$ m/s

Velocity of whirl at inlet, $V_{w1} = 25.0$ m/s

Flow is radial at outlet i.e. velocity of whirl at outlet, $V_{w2} = 0$

Let the peripheral velocity at inlet and outlet be u_1 and u_2 respectively

$$\therefore \quad u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1 \times 400}{60} = 20.94 \text{ m/s}$$

Discharge, $Q = A \times V_{f1} = 0.25 \times 8 = 2$ m^3/s

Power developed by the wheel is expressed as

$$P = \rho Q(u_1 V_{w1} - u_2 V_{w2})$$

$$= 1000 \times 2 \times (20.94 \times 25 - u_2 \times 0) \times 10^{-3} = 1047 \text{ kW}$$

Hydraulic efficiency,
$$\eta_h = \left[\frac{u_1 V_{w1} - u_2 V_{w2}}{gH}\right] \times 100$$

$$= \left[\frac{20.94 \times 25 - u_2 \times 0}{9.81 \times 65}\right] \times 100 = 82.1\%$$

T7 : Solution

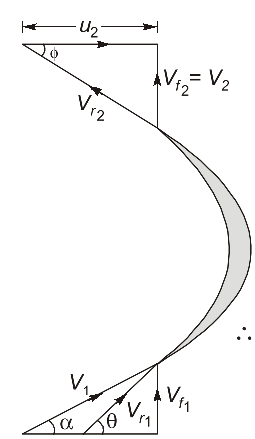
Given:

Head, $H = 12$ m

Hub diameter, $D_b = 0.35 \times D_0$ Where D_0 = Dia. of runner

Speed, $N = 100$ rpm

Vane angle at outlet, $\phi = 15°$



Flow ratio
$$= \frac{V_{f_1}}{\sqrt{2gH}} = 0.6$$

$$\therefore \quad V_{f_1} = 0.6 \times \sqrt{2gH} = 0.6 \times \sqrt{2 \times 9.81 \times 12} = 9.2 \text{ m/s}$$

From the outlet velocity triangle, $V_{w_2} = 0$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{V_{f_1}}{u_2} \qquad \left(\because V_{f_2} = V_{f_1} = 9.2 \text{ m/s}\right)$$

$$\tan 15° = \frac{9.2}{u_2}$$

$$\therefore \quad u_2 = \frac{9.2}{\tan 15°} = 34.33 \text{ m/s}$$

But for Kaplan turbine, $u_1 = u_2 = 34.33$ m/s

Now, using the relation, $u_1 = \frac{\pi D_0 \times N}{60}$ or $34.33 = \frac{\pi \times D_0 \times 100}{60}$

$$D_0 = \frac{60 \times 34.33}{\pi \times 100} = 6.56 \text{ m}$$

$\therefore$ $D_b = 0.35 \times D_0 = 0.35 \times 6.56 = 2.30$ m

Discharge through turbine is given by eq. as

$$Q = \frac{\pi}{4}\left[D_0^2 - D_b^2\right] \times V_{f_1} = \frac{\pi}{4}\left[6.56^2 - 2.3^2\right] \times 9.2$$

$$= \frac{\pi}{4}(43.033 - 5.29) \times 9.2 = 272.71 \text{ m}^3/\text{s}$$

T8 : Solution

Given:

Head, $H = 25$ m

Speed, $N = 200$ rpm

Discharge, $Q = 9$ cumec $= 9$ m^3/s

Efficiency, $\eta_0 = 90\% = 0.90$ (Take the efficiency as overall η)

Now using relation, $\eta_0 = \frac{\text{Work developed}}{\text{Water power}} = \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$

$$\therefore \quad P = \eta_0 \times \frac{\rho \times g \times Q \times H}{1000} = \frac{0.90 \times 9.81 \times 1000 \times 9 \times 25}{1000} = 1986.5 \text{ kW}$$

(i) Specific speed of the machine (N_s)

Using equation $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{200 \times \sqrt{1986.5}}{25^{5/4}} = 159.46$ units

(ii) Power generated $P = 1986.5$ kW

(iii) As the specific speed lies between 51 and 255, the turbine is a Francis turbine.

T9 : Solution

Given: $Q = 0.04$ m^3/s

$H_g = 20$ m

$$\eta_0 = \frac{\rho g Q H}{P}$$

$f = 0.015$

$l = 100$ m

$D = 0.15$ m

$\eta_0 = 70\%$, $\eta_0 = 0.7$

$$h_f = \frac{4flQ^2}{12D^5} = \frac{4 \times 0.015 \times 100 \times (0.04)^2}{12 \times (0.15)^5} = 10.534 \text{ m}$$

$\therefore$ $H_{net} = H_g + h_f = 20 + 10.534$

$$\Rightarrow \qquad H_{net} = 30.534 \text{ m}$$

$$\eta_0 = \frac{\frac{\rho g Q H_{net}}{1000}}{P}$$

$$0.70 = \frac{\frac{1000 \times 9.81 \times 0.04 \times 30.534 \text{ kW}}{1000}}{P}$$

$$\therefore \qquad P = \frac{9.81 \times 0.04 \times 30.534}{0.7} \text{ kW}$$

$$P = 17.116 \text{ kW}$$

Hence power required to derive the pump is 17.116 kW.

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