MADE EASY

India's Best Institute for IES, GATE & PSUs

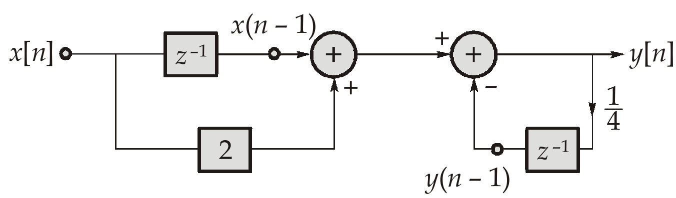
Detailed Solutions

ESE-2024 Mains Test Series

Electrical Engineering Test No : 2

Q.1 (a) Solution:

Given system,



The difference equation of the above system,

$$y(n) = 2x(n) + x(n-1) - \frac{1}{4}y(n-1)$$

Given : impulse response, i.e.,

$$x(n) = \delta(n)$$

$$\therefore \quad y(n) = 2\delta(n) + \delta(n-1) - \frac{1}{4}y(n-1)$$

By taking z-transform,

$$y(z) = 2 + z^{-1} - \frac{1}{4}y(z)\cdot z^{-1}$$

$$y(z) + \frac{1}{4}z^{-1}y(z) = 2 + z^{-1}$$

$$y(z)\left[1 + \frac{1}{4}z^{-1}\right] = 2 + z^{-1}$$

$$\therefore \qquad Y(z) = \frac{2+z^{-1}}{1+\frac{1}{4}z^{-1}}$$

$$= \frac{2}{1+\frac{1}{4}z^{-1}} + \frac{z^{-1}}{1+\frac{1}{4}z^{-1}}$$

$\because$ Using linearity property of z-transform, by taking inverse z-transform,

$$z^{-1}[Y(z)] = z^{-1}\left[\frac{2}{1+\frac{1}{4}z^{-1}}\right] + z^{-1}\left[\frac{z^{-1}}{1+\frac{1}{4}z^{-1}}\right]$$

$$y(n) = z^{-1}\left[\frac{2}{1-\left(-\frac{1}{4}\right)z^{-1}}\right] + z^{-1}\left[\frac{z^{-1}}{1-\left(-\frac{1}{4}\right)z^{-1}}\right]$$

$$y(n) = \left[2\left(\frac{-1}{4}\right)^{n} u[n] + \left(\frac{-1}{4}\right)^{n-1} u[n-1]\right]$$

Q.1 (b) Solution:

Given : $$X(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

Let $$X_1(z) = \left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) \text{ and } X_2(z) = \left(\frac{1}{1+\frac{1}{4}z^{-1}}\right)$$

Taking inverse z-transform,

$$x_1(n) = z^{-1}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) = \left(\frac{1}{2}\right)^{n} u(n)$$

$$x_2(n) = z^{-1}\left(\frac{1}{1+\frac{1}{4}z^{-1}}\right) = \left(-\frac{1}{4}\right)^{n} u(n)$$

$$x(n) = x_1(n) * x_2(n) = \sum_{K=0}^{n} x_1(n-K)x_2(K)$$

$$= \sum_{K=0}^{n}\left(\frac{1}{2}\right)^{n-K}\left(\frac{-1}{4}\right)^{K} = \left(\frac{1}{2}\right)^{n}\sum_{K=0}^{n}\left[\frac{\left(\frac{-1}{4}\right)}{\left(\frac{1}{2}\right)}\right]^{K}$$

$$= \left(\frac{1}{2}\right)^{n}\sum_{K=0}^{n}\left(\frac{-1}{2}\right)^{K} = \left(\frac{1}{2}\right)^{n}\frac{1-\left(\frac{-1}{2}\right)^{(n+1)}}{1-\left(-\frac{1}{2}\right)}$$

Sum of $(n + 1)$ terms of G.P. (from 0 to n) is

$$S = \frac{a(1-r^{n+1})}{1-r} = \left(\frac{1}{2}\right)^{n}\cdot\frac{2}{3}\left[1-\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^{n}\right]$$

$$x(n) = \left[\frac{2}{3}\left(\frac{1}{2}\right)^{n} + \frac{1}{3}\left(\frac{-1}{4}\right)^{n}\right]u(n)$$

Q.1 (c) Solution:

We know that,

Inverse DFT, $$y(n) = \frac{1}{N}\sum_{K=0}^{N-1} Y(K)e^{j\frac{2\pi nK}{N}}$$

where, the given discrete time sequence,

$$Y(K) = \{1, 0, 1, 0\}$$

i.e., $$N = 4$$

$$y(n) = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\frac{2\pi nK}{4}}$$

$$y(n) = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\frac{\pi nK}{2}}$$

$\therefore$ $$y(0) = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j0} = \frac{1}{4}\sum_{K=0}^{3} Y(K)$$

$$= \frac{1}{4}\{1+0+1+0\} = \frac{1}{2} = 0.5$$

At $n = 1$,
$$y(1) = \frac{1}{4}\sum_{K=0}^{3} y(K)e^{j\frac{\pi K}{2}}$$
$$= \frac{1}{4}\left\{y(0)e^{j0} + y(1)e^{j\frac{\pi}{2}} + y(2)e^{j\pi} + y(3)e^{j\frac{3\pi}{2}}\right\}$$
$$= \frac{1}{4}\{1+0+1(\cos\pi + j\sin\pi)+0\} = \frac{1}{4}\{1-1\} = 0$$

At $n = 2$,
$$y(2) = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\frac{2\pi\times 2K}{4}} = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\pi K}$$
$$= \frac{1}{4}\left[Y(0)e^{j0} + Y(1)e^{j\pi} + Y(2)e^{j2\pi} + Y(3)e^{j3\pi}\right]$$
$$= \frac{1}{4}\left[1+0+1(\cos 2\pi + j\sin 2\pi)+0\right] = \frac{1}{4}[1+1] = 0.5$$

At $n = 3$,
$$y(3) = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\frac{2\pi\times 3K}{4}} = \frac{1}{4}\sum_{K=0}^{3} Y(K)e^{j\frac{3\pi K}{2}}$$
$$= \frac{1}{4}\left[Y(0)e^{j0} + Y(1)e^{j\frac{3\pi}{2}} + Y(2)e^{j3\pi} + Y(3)e^{j\frac{9\pi}{2}}\right]$$
$$= \frac{1}{4}\left[1+0+1(\cos 3\pi + j\sin 3\pi)+0\right]$$
$$y(3) = 0$$

$\therefore$
$$y(n) = \{y(0), y(1), y(2), y(3)\}$$
$$y(n) = \{0.5, 0, 0.5, 0\}$$

Q.1 (d) Solution:

Given : Analog filter transfer function,

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$ by using bilinear transfer function

$$= \frac{2}{(s+1)(s+2)}\Bigg|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

Given : $T = 1$ sec

$$\therefore \quad H(z) = \frac{2}{\left\{2\left(\frac{z-1}{z+1}\right)+1\right\}\left\{2\left(\frac{z-1}{z+1}\right)+2\right\}}$$

$$= \frac{2}{\frac{2(z-1)+z+1}{z+1}\cdot\frac{2(z-1)+2(z+1)}{z+1}}$$

$$= \frac{2}{\frac{2z-2+z+1}{z+1}\cdot\frac{2z-2+2z+2}{z+1}} = \frac{2}{\frac{3z-1}{z+1}\cdot\frac{4z}{z+1}}$$

$$= \frac{2(z+1)^2}{(3z-1)(4z)} = \frac{2(z+1)^2}{12z^2-4z} = \frac{2z^2(1+z^{-1})^2}{z^2(12-4z^{-1})}$$

$$H(z) = \frac{0.166(1+z^{-1})^2}{1-0.33z^{-1}}$$

Q.1 (e) Solution:

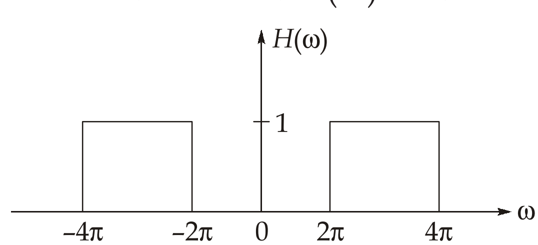
LXI H,	3025H;	Address of 1st number in HL pair
MOV	A,M;	1st number in accumulator
INX	H;	Address of 2nd number in HL pair
CMP	M;	Compare 2nd number with 1st
JC	LOOP;	Yes, smaller number is in accumulator
MOV	A,M;	No, get 2nd number in accumulator
LOOP :	STA 3027H;	Store smaller number in 3027H
	HLT;	Stop

Q.2 (a) Solution:

Given, an ideal band pass filter with pass band $1 < |f| < 2$ Hz

i.e., $1 < f < 2$ (or) $-2 < f < -1$

or $2\pi < \omega < 4\pi$ (or) $-4\pi < \omega < -2\pi$



Given input to bandpass filter,

$$V(t) = 4e^{-3t}u(t) \text{ V}$$

By taking Fourier transform,

$$F_v(\omega) = \int_{-\infty}^{\infty} V(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty}\left[4e^{-3t}u(t)\right]e^{-j\omega t}dt$$

$$= \int_{0}^{\infty} 4e^{-3t}e^{-j\omega t}dt = 4\int_{0}^{\infty} e^{-(3+j\omega)t}dt$$

$$= \frac{4}{-(3+j\omega)}\left[e^{-(3+j\omega)t}\right]_0^{\infty}$$

$$F_v(\omega) = \frac{4}{3+j\omega}$$

The energy of input signal,

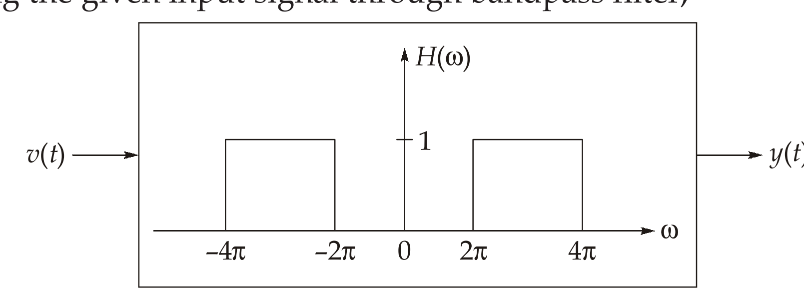
$$W = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F_v(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{4}{3+j\omega}\right|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{4}{\sqrt{9+\omega^2}}\right]^2 d\omega = \frac{16}{2\pi}\int_{-\infty}^{\infty}\frac{1}{9+\omega^2}d\omega$$

or $$W = \frac{16}{\pi}\int_{0}^{\infty}\frac{d\omega}{9+\omega^2} = \frac{16}{\pi}\left[\frac{1}{3}\tan^{-1}\left(\frac{\omega}{3}\right)\right]_0^{+\infty} = \frac{16}{3\pi}\left[\frac{\pi}{2}-0\right]$$

$$W_v = \frac{8}{3}\text{J}$$

After passing the given input signal through bandpass filter,



The output energy of bandpass filter :

Let $$W_y = \frac{1}{2\pi}\left[\int_{-4\pi}^{-2\pi}|F_v(\omega)|^2 d\omega + \int_{2\pi}^{4\pi}|F_v(\omega)|^2 d\omega\right]$$

$$= \frac{1}{2\pi}\int_{-4\pi}^{-2\pi}\frac{16d\omega}{9+\omega^2} + \frac{1}{2\pi}\int_{2\pi}^{4\pi}\frac{16d\omega}{9+\omega^2}$$

$$= \frac{16}{2\pi}\left[\frac{1}{3}\tan^{-1}\left(\frac{\omega}{3}\right)\right]_{-4\pi}^{-2\pi} + \frac{16}{2\pi}\left[\frac{1}{3}\tan^{-1}\left(\frac{\omega}{3}\right)\right]_{2\pi}^{4\pi}$$

$$= \frac{16}{6\pi}\left\{\left[\tan^{-1}\left(\frac{-2\pi}{3}\right) - \tan^{-1}\left(\frac{-4\pi}{3}\right)\right] + \left[\tan^{-1}\left(\frac{4\pi}{3}\right) - \tan^{-1}\left(\frac{2\pi}{3}\right)\right]\right\}$$

$$= \frac{16}{6\pi}\times 2\left[\tan^{-1}\frac{4\pi}{3} - \tan^{-1}\frac{2\pi}{3}\right]$$

$$W_y = 358\text{ mJ} \quad \text{(or)} \quad 0.358\text{ J}$$

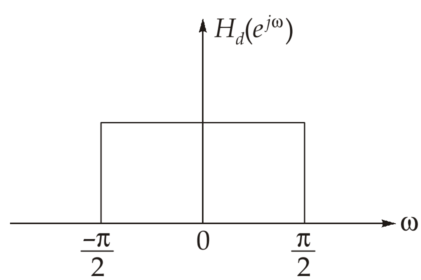
where, $W_v = \frac{8}{3}\text{J} = 2.67\text{ J}$

$\therefore \quad W_y = 13.44\%\ W_v$

Q.2 (b) Solution:

(i) Given : $H(e^{j\omega}) = 1 \quad \text{for } \frac{-\pi}{2} \le \omega \le \frac{\pi}{2}$

$\qquad = 0 \quad \text{for } \frac{\pi}{2} \le |\omega| \le \pi$



From the above frequency response, we get a non-causal filter coefficients symmetrical about $n = 0$,

i.e., $h_d(n) = h_d(-n)$

From the Inverse Discrete Time Fourier Transform,

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi/2}^{\pi/2} = \frac{1}{2j\pi n}\left[e^{j\omega n}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi jn}\left[e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}\right] = \frac{1}{2\pi jn} \times \frac{2j}{2j}\left[e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}\right]$$

$$h_d(n) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{2}\right)$$

$$h(n) = \begin{cases} \dfrac{\sin\frac{\pi}{2}n}{\pi n}; & \text{for } |n| \le 5 \\ 0; & \text{otherwise} \end{cases}$$

For $n = 0$,
$$h(0) = \lim_{n\to 0}\frac{\sin\frac{n\pi}{2}}{\pi n} = \frac{1}{2}\lim_{n\to 0}\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{2}}$$

$\therefore$
$$h(0) = \frac{1}{2} \qquad \left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right)$$

For $n = 1$,
$$h(1) = h(-1) = \frac{\sin\frac{\pi}{2}}{\pi} = \frac{1}{\pi} = 0.3183$$

For $n = 2$,
$$h(2) = h(-2) = \frac{\sin\pi}{2\pi} = 0$$

For $n = 3$,
$$h(3) = h(-3) = \frac{\sin\frac{3\pi}{2}}{3\pi} = \frac{-1}{3\pi} = -0.106$$

For $n = 4$,
$$h(4) = h(-4) = \frac{\sin\frac{4\pi}{2}}{4\pi} = 0$$

For $n = 5$,
$$h(5) = h(-5) = \frac{\sin\frac{5\pi}{2}}{5\pi} = \frac{1}{5\pi} = 0.0637$$

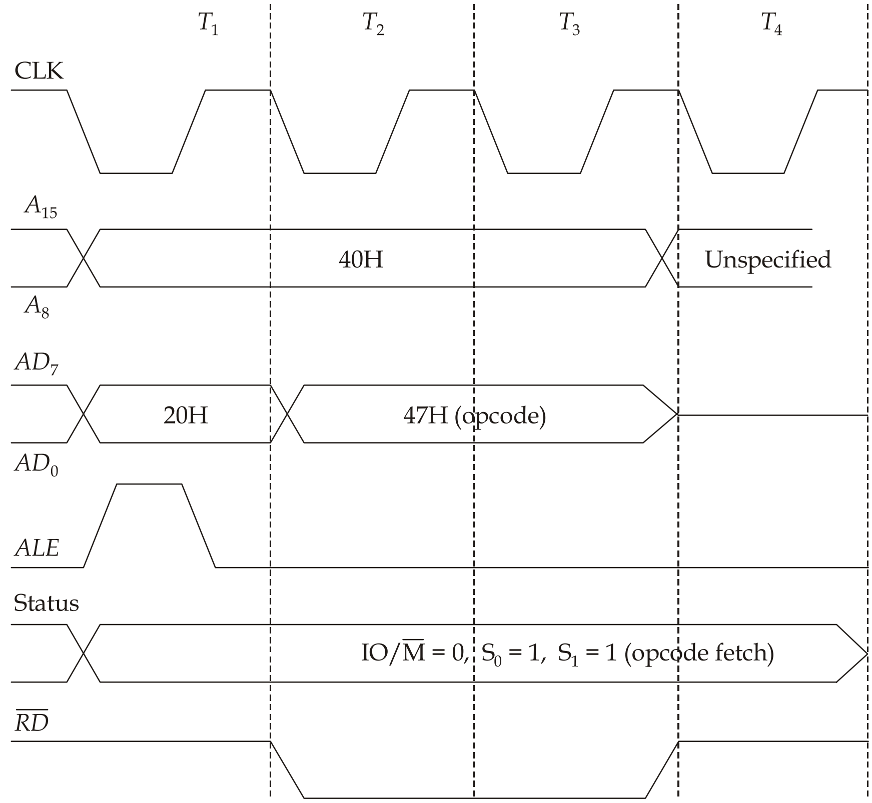
(ii) The transfer function of the filter,

$$H(z) = h(0) + \sum_{K=1}^{\frac{N-1}{2}}\{h(n)[z^n + z^{-n}]\} = 0.5 + \sum_{K=1}^{5} h(n)[z^n + z^{-n}]$$

$$= 0.5 + h(1)(z + z^{-1}) + h(2)(z^2 + z^{-2}) + h(3)(z^3 + z^{-3}) + h(4)(z^4 + z^{-4}) + h(5)(z^5 + z^{-5})$$

$\therefore$
$$H(z) = 0.5 + 0.3183(z + z^{-1}) - 0.106(z^{+3} + z^{-3}) + 0.637(z^5 + z^{-5})$$

Q.2 (c) (i) Solution:



While the system executes the instruction, the following takes place one after another :

1. The microprocessor places the address of 4020H (residing in program counter) on the address bus = 40H on the high order bus $A_{15} - A_8$ and 20H on the low order bus $AD_7 - AD_0$.
2. The CPU raises the ALE signal to go high. The high to low transition of ALE at the end of the first T-state specifies the lower order bus as the data bus.
3. The CPU identifies the nature of the machine cycle by means of the three status signal $IO/\bar{M}$, S_0 and S_1

$$IO/\bar{M} = 0; S_1 = 1 \text{ and } S_0 = 1$$

4. In T_2, memory is enabled by the $\bar{M}$ signal. The content at memory location 4020H, i.e., opcode 47H is placed on the data bus. The program counter is incremented to 4021H.
5. In T_3, CPU reads 47H H and places it in the instruction register.
6. In T_4, CPU decodes the instruction and places the accumulator content, 05H into register B.

Q.2 (c) (ii) Solution:

Every instruction of program has to operate on data. The method of specifying the data to be operated with an instruction is called addressing.

8085 has following addressing modes :

1. **Immediate Addressing Mode :** In an immediate addressing mode 8 or 16 bit data can be specified as a part of the instruction.

   Example : MVI B, 30H

2. **Register Addressing Mode :** The register addressing mode specialities the source operand, destination operand or both to be contained in 8085 register.

   Example : MOV B, C

3. **Direct Addressing Mode :** The direct addressing mode specify 16-bit address of the operand within the instruction itself. Since, this address is a part of instruction, the second and third byte of this instruction contain this 16-bit address.

   Example : LDA, 3050H

4. **Indirect Addressing Mode :** In indirect addressing mode, the memory address where two operands are located in specified by the contents of register pair.

   Example: LDAX D

5. **Implicit Addressing Mode :** In this addressing mode, opcode specifies the address of the operand.

   Example : RAR, CMA.

Q.3 (a) (i) Solution:

(a) Given : $y(t) = x(\cos 3t)$

For linearity, $ax_1(t) \leftrightarrow ay_1(t) = ax_1(\cos 3t)$

$bx_2(t) \leftrightarrow by_2(t) = bx_2(\cos 3t)$

$\therefore \quad ax_1(t) + bx_2(t) \leftrightarrow ay_1(t) + by_2(t)$

$\therefore$ The above given system is linear.

For time invariance,

delay the input, $x(t - t_0) \leftrightarrow y_1(t) = x(\cos 3t - t_0)$

delayed output, $y(t - t_0) = x[\cos 3(t - t_0)]$

$\therefore \quad y_1(t) \neq y(t - t_0)$

$\therefore$ Given system is time variant.

For static, output must be independent of past inputs.

$$y(t) = x(\cos 3t)$$

At $t = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = x\left(\cos\frac{3\pi}{2}\right) = x(0)$

∴ System is dynamic.

(b) Given : $y(t) = (t^2 - 1)x(t)$

For linearity, $ax_1(t) \leftrightarrow a(t^2 - 1)x_1(t) = ay_1(t)$

$bx_2(t) \leftrightarrow b(t^2 - 1)x_2(t) = by_2(t)$

$ax_1(t) + bx_2(t) \leftrightarrow a(t^2 - 1)x_1(t) + b(t^2 - 1)x_2(t)$

$= ay_1(t) + by_2(t)$

∴ System is linear.

For time invariance,

For delayed input, $x_1(t - t_0) \leftrightarrow (t^2 - 1)x(t - t_0)$

delayed output, $y(t - t_0) = ((t - t_0)^2 - 1)x(t - t_0)$

∴ $y_1(t) \neq y(t - t_0)$

Hence, system is time variant.

For static, $y(1) = 0.x(1) = 0$

$y(0) = -x(0)$

Output depends on present input only.

∴ System is static.

Q.3 (a) (ii) Solution:

Given : $x_1(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$

∴ $x_1(n) = \{\underset{\uparrow}{1}, 1, -1, -1\}$

∴ $x_2(n) = \delta(n) - \delta(n-2) + \delta(n-4)$

∴ $x_2(n) = \{\underset{\uparrow}{1}, 0, -1, 0, 1\}$

Add one zero to sequence $x_1(n)$ to bring its length to 5.

$x_1(n) = \{1, 1, -1, -1, 0\}$

Using matrix approach,

$$\begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \dots & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & \dots & x_2(2) \\ \vdots & \vdots & \vdots & & \vdots \\ x_2(N-2) & x_2(N-3) & x_2(N-4) & \dots & x_2(N-1) \\ x_2(N-1) & x_2(N-2) & x_2(N-3) & \dots & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ \vdots \\ x_1(N-2) \\ x_1(N-1) \end{bmatrix} = \begin{bmatrix} x_3(0) \\ x_3(1) \\ \vdots \\ x_3(N-2) \\ x_3(N-1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix}_{5\times5} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}_{5\times1} = \begin{bmatrix} 3 \\ 0 \\ -3 \\ -2 \\ 2 \end{bmatrix}$$

$$\therefore \quad x_3(n) = \{3, 0, -3, -2, 2\}$$

Q.3 (b) (i) Solution:

Given :
$$f(t) = \begin{cases} \sin \omega t; & 0 < t < \dfrac{\pi}{\omega} \\ 0; & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases}$$

$$f\left(t + \frac{2\pi}{\omega}\right) = f(t)$$

i.e., the above given signal is periodic with period,

$$T = \frac{2\pi}{\omega}$$

For periodic signal, Laplace transform is defined as,

$$L[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

where, period, $T = \dfrac{2\pi}{\omega}$

$$\therefore \quad L[f(t)] = \frac{1}{1 - e^{-s\left(\frac{2\pi}{\omega}\right)}} \int_0^{2\pi/\omega} e^{-st} (\sin \omega t) dt = \frac{1}{1 - e^{-\frac{2\pi s}{\omega}}} \left[\int_0^{\pi/\omega} e^{-st} \sin \omega t dt + \int_{\pi/\omega}^{2\pi/\omega} e^{-st} (0) dt \right]$$

$$= \frac{1}{1 - e^{-\frac{2\pi s}{\omega}}} \left[\int_0^{\pi/\omega} e^{-st} \sin \omega t \, dt \right] = \frac{1}{1 - e^{-\frac{2\pi s}{\omega}}} \left[\frac{e^{-st}}{(-s)^2 + \omega^2} (-s \sin \omega t - \omega \cos \omega t) \right]_0^{\pi/\omega}$$

$$= \frac{1}{1 - e^{\frac{-2\pi s}{\omega}}} \left[\frac{e^{\frac{-\pi s}{\omega}} \omega}{s^2 + \omega^2} + \frac{\omega}{s^2 + \omega^2} \right] = \frac{1}{(1)^2 - \left(e^{\frac{-\pi s}{\omega}}\right)^2} \left[\frac{\omega \left(e^{\frac{-s\pi}{\omega}} + 1\right)}{s^2 + \omega^2} \right]$$

$$= \frac{1}{\left(1 - e^{\frac{-\pi s}{\omega}}\right)\left(1 + e^{\frac{-\pi s}{\omega}}\right)} \left[\frac{\omega\left(e^{\frac{-\pi s}{\omega}} + 1\right)}{s^2 + \omega^2}\right]$$

$$F(s) = \frac{\omega}{\left(1 - e^{-\frac{\pi s}{\omega}}\right)(s^2 + \omega^2)}$$

Q.3 (b) (ii) Solution:

Given, $$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

Convolution Property of *z* Transform :

$$y(n) = x(n) * h(n) = \sum_{K=-\infty}^{\infty} x(K)h(n-K) \quad \Rightarrow \quad X(z)H(z)$$

where ROC of $X(z)$ is R_1 and ROC of $H(z)$ is R_2.

ROC : The ROC of $Y(z)$ is at least the intersection of the ROCs of $x(n)$ and $h(n)$, i.e., ROC of $Y(z)$ is $R = (R_1 \cap R_2)$.

We can write, $$X(z) = X_1(z) \cdot X_2(z)$$

where, $$X_1(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)}$$

and $$X_2(z) = \frac{1}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

Now consider, $X_1(z)$,

Taking inverse z-transform, we get

$$x_1(n) = IZT\left[\frac{1}{1 - \frac{1}{2}z^{-1}}\right]$$

$$x_1(n) = \left(\frac{1}{2}\right)^n u(n) \qquad \text{...(1)}$$

Consider $X_2(z)$,

Taking inverse z-transform, we get

$$x_2(n) = IZT\left[\frac{1}{1+\frac{1}{4}z^{-1}}\right]$$

$$x_2(n) = \left(\frac{-1}{4}\right)^n u(n) \quad ...(2)$$

Using convolution property of z-transform, $x(n)$ is the convolution of $x_1(n)$ and $x_2(n)$

$$\therefore \quad x(n) = x_1(n) * x_2(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x_1(n-k)\cdot x_2(k) = \sum_{k=-\infty}^{\infty}\left(\frac{1}{2}\right)^{(n-k)}\cdot\left(\frac{-1}{4}\right)^k u(k)u(n-k)$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n}\left(\frac{1}{2}\right)^{-k}\cdot\left(\frac{-1}{4}\right)^k = \left(\frac{1}{2}\right)^n \sum_{k=0}^{n}\left[\frac{(-1/4)}{(1/2)}\right]^k$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n}\left(\frac{-1}{2}\right)^k = \left(\frac{1}{2}\right)^n\left[\frac{1-\left(\frac{-1}{2}\right)^{n+1}}{1-\left(-\frac{1}{2}\right)}\right]$$

$$= \left(\frac{1}{2}\right)^n\left[\frac{1-\left(\frac{-1}{2}\right)^{n+1}}{\frac{3}{2}}\right] = \left(\frac{1}{2}\right)^n\cdot\frac{2}{3}\cdot\left[1-\left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right)^n\right] = \frac{2}{3}\left(\frac{1}{2}\right)^n + \frac{1}{2}\times\frac{2}{3}\left(\frac{1}{2}\right)^n\cdot\left(\frac{-1}{2}\right)^n$$

$$x(n) = \left[\frac{2}{3}\left(\frac{1}{2}\right)^n + \frac{1}{3}\left(\frac{-1}{4}\right)^n\right]u(n)$$

Q.3 (c) (i) Solution:

The delay loop includes two instruction, i.e., DCR and JNZ with 14 T-states.

Time delay at loop DELAY,

$$T_L = 14 \text{ T-states} \times T_{CLK} \times \text{Count}$$

$$= 14 \times 0.5 \times 10^{-6} \times 17$$

(count is stored in register C as 11H = 17_{10})

$$T_L = 119 \text{ µsec}$$

The delay outside the loop includes following instructions :

MVI B,	00H	(7T states)
DCR	B	(4T states)
MVI C,	11H	(7T states)
MOV	A, B	(4T states)
OUT	PORT	(10T states)
	HLT	(5T states)

$$\text{Total T-states} = 7 + 4 + 7 + 4 + 10 + 5 = 37 \text{ T-states}$$

Delay outside the loop

$$T_0 = 37 \times T = 37 \times 0.5 \times 10^{-6}$$
$$= 18.5 \ \mu s$$
$$\text{Total delay} = T_D = T_0 + T_L$$
$$= 119 \times 10^{-6} + 18.5 \times 10^{-6}$$
$$T_D = 137.5 \ \mu\text{sec}$$

Q.3 (c) (ii) Solution:

Consider the difference equation relating $w(n)$ and $x(n)$ for system S_1.

$$w(n) = \frac{1}{2}w(n-1) + x(n)$$

Taking z-transform,

$$W(z) = \frac{1}{2}z^{-1}W(z) + X(z)$$

$$W(z) = \frac{X(z)}{\left(1 - \frac{1}{2}z^{-1}\right)}$$

$$\frac{W(z)}{X(z)} = H_1(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)} \quad \text{...(1)}$$

Let $H_1(z)$ is the transfer function of system S_1.

Now, consider the difference equation relating $y(n)$ and $w(n)$ for system S_2.

$$y(n) = \alpha y(n-1) + \beta w(n)$$

Taking z-transform

$$Y(z) = \alpha z^{-1}Y(z) + \beta W(z)$$
$$Y(z)[1 - \alpha z^{-1}] = \beta W(z)$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{\beta}{(1-\alpha z^{-1})} \quad \text{...(2)}$$

where, $H_2(z)$ is the transfer function of system S_2.

Now, consider $H(z)$ as the transfer function of whole system. Hence,

$$H(z) = \frac{Y(z)}{X(z)} = H_1(z)\cdot H_2(z) \quad \text{...(3)}$$

$$H(z) = H_1(z)\cdot H_2(z)$$

$\therefore$

$$H(z) = \frac{\beta}{(1-\alpha z^{-1})\left(1-\frac{1}{2}z^{-1}\right)} \quad \text{(from eqn (1) and (2))}$$

$$H(z) = \frac{\beta}{1+\frac{\alpha z^{-2}}{2}-\frac{1}{2}z^{-1}-\alpha z^{-1}}$$

$$H(z) = \frac{\beta}{1-\left(\alpha+\frac{1}{2}\right)z^{-1}+\frac{\alpha}{2}z^{-2}} \quad \text{...(4)}$$

Now consider the difference equation relating $x(n)$ and $y(n)$,

$$y(n) = \frac{-1}{8}y(n-2)+\frac{3}{4}y(n-1)+x(n)$$

Taking z-transform,

$$Y(z) = \frac{-1}{8}z^{-2}Y(z)+\frac{3}{4}z^{-1}Y(z)+X(z)$$

$$Y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right] = X(z)$$

$\therefore$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right]} \quad \text{...(5)}$$

Now comparing equation (4) and (5), we get

$$\alpha+\frac{1}{2} = \frac{3}{4} \Rightarrow \alpha = \frac{1}{4}$$

and $\beta = 1$

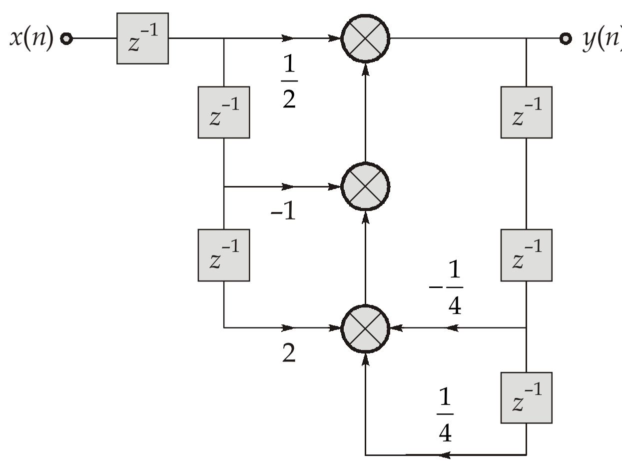
Q.4 (a) Solution:

(i) Given transfer function,

$$H(z) = \frac{z^2 - 2z + 4}{\left(z - \frac{1}{2}\right)(2z^2 + z + 1)} = \frac{z^2 - 2z + 4}{2z^3 + z^2 + z - z^2 - \frac{1}{2}z - \frac{1}{2}} = \frac{z^2 - 2z + 4}{\left(2z^3 + \frac{1}{2}z - \frac{1}{2}\right)}$$

$$= \frac{\frac{1}{2}z^{-1} - z^{-2} + 2z^{-3}}{1 + \frac{1}{4}z^{-2} - \frac{1}{4}z^{-3}} = \frac{\left[\frac{1}{2} - z^{-1} + 2z^{-2}\right]z^{-1}}{1 - \left[-\frac{1}{4}z^{-2} + \frac{1}{4}z^{-3}\right]} \quad \text{...(i)}$$

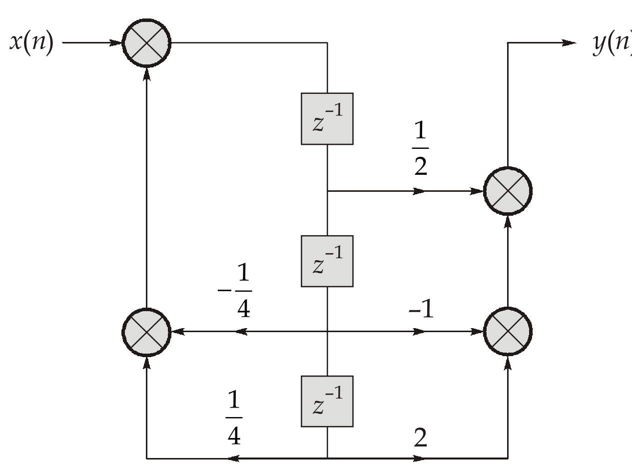
Now Direct form-I →



Direct form II →

From equation (i),

$$H(z) = \frac{\left[\frac{1}{2} - z^{-1} + 2z^{-2}\right]z^{-1}}{1 - \left[-\frac{1}{4}z^{-2} + \frac{1}{4}z^{-3}\right]}$$



(ii) Given transfer function,

$$H(z) = \frac{(z-1)}{(4z^3+2z^2+2z+3)} = \frac{(z-1)\times(0.25)}{\left(z^3+\frac{1}{2}z^2+\frac{1}{2}z+\frac{3}{4}\right)}$$

By making the factor of the denominator,

We get, $$H(z) = \frac{(z-1)\times(0.25)}{(z+0.888)(z^2-0.388z+0.844)}$$

Let, $$H(z) = H_1(z)\cdot H_2(z) \quad \text{...(ii)}$$

Where, $$H_1(z) = \frac{(z-1)}{(z+0.888)}$$

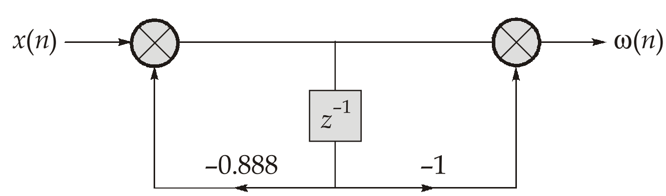
and $$H_2(z) = \frac{0.25}{(z^2-0.388z+0.844)}$$

Now consider $H_1(z)$,

$$H_1(z) = \frac{(z-1)}{(z+0.888)}$$

$$H_1(z) = \frac{(1-z^{-1})}{(1+0.888z^{-1})} = \frac{(1-z^{-1})}{1-(-0.888z^{-1})}$$

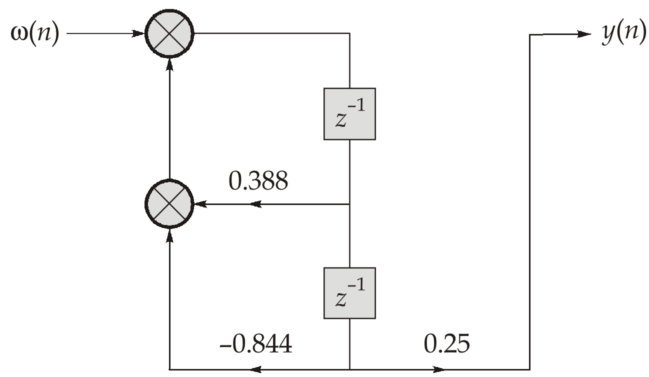
Direct form-II structure uses minimum delay elements, Hence drawing the direct form-II of $H_1(z)$,



Now consider $H_2(z)$

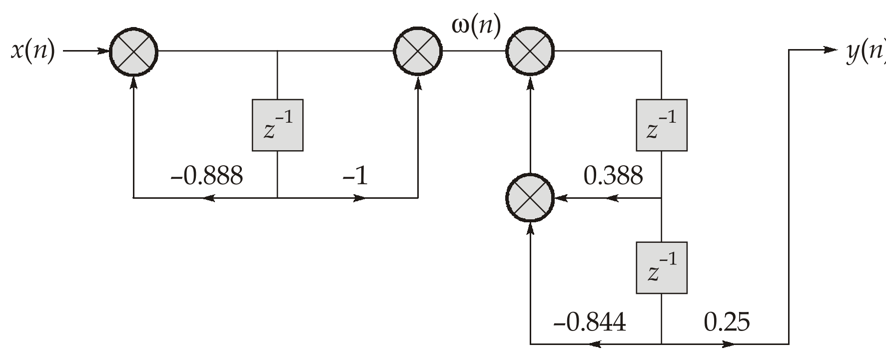
$$H_2(z) = \frac{0.25}{(z^2-0.388z+0.844)}$$

$$= \frac{0.25z^{-2}}{1-(0.388z^{-1}-0.844z^{-2})}$$



Now from equation (ii),

The cascade structure is obtained by cascading $H_1(z)$ and $H_2(z)$ direct form-II structures as below:



Q.4 (b) (i) Solution:

LXI H, 4480 H	:	Initialize the memory pointer
MOV E, M	:	Get multiplicand
MVI D, 00H	:	Extends to 16-bit
INX H	:	Increment memory pointer
MOV A, M	:	Get multiplier
LXIH, 0000H	:	Product = 0
MVI B, 08H	:	Initialize counter with count = 8
MULT : DAD H	:	Product = Product x 2
RAL	:	
JNC SKIP		is carry from multiplier 1?
DAD D	:	Yes, product = product + multiplicant
SKIP : DCR B	:	is counter = Zero
JNZ MULT	:	no, repeat
SHLD 5500H	:	Store the result
HLT	:	End of program

Q.4 (b) (ii) Solution:

1. Machine cycles :

Instruction	Machine Cycle M₁	M₂	M₃
IN 24H	opcode fetch	memory read	IO read
JMP START	opcode fetch	memory read	memory read

2. Given : Clock frequency = 6 MHz

Time period of clock

$$T_{CLK} = \frac{1}{f_{CLK}} = \frac{1}{6 \times 10^6} = 0.1667 \times 10^{-6} \text{ sec}$$

Time required to execute the program

$$= (\text{Total T-states}) \times T_{CLK}$$

$$= (10 + 10) \times 0.167 \times 10^{-6}$$

$$= 3.33 \ \mu\text{sec}$$

Q.4 (c) (i)

LXI H 2050 H : Load H-L pair with 2050H

20 → H = 20, 50H → L = 50

MVI A, 22 H : Load 22H data to accumulator

A = 22H ← 22H

INR A : Increment content of accumulator with (01) H

$A \leftarrow A + (01)H$

$A \leftarrow 23H$

$A \rightarrow$ 23 H

STA 2050 H : Store the content of accumulator to (2050) H memory address

[2050] H → 23 H

INR A : Increment the content of accumulator by (01) H

$A \leftarrow A + (01)H$

$A \leftarrow 23 + 01$

$A \rightarrow$ 24 H

XRA M : Bit by bit EX-OR operation between accumulator and content of memory point

$$[A] \oplus [H - L]$$

$$24H \oplus 23H \begin{cases} \rightarrow C_y = 0 \\ \rightarrow A_c = 0 \end{cases}$$

$$A \longrightarrow 24H \longrightarrow 0010\ 0100$$

$$[H - L] \longrightarrow 23H \longrightarrow \underline{0010\ 0011} \longleftarrow \text{Ex-OR}$$

$$0000\ 0111$$

$$A \longleftarrow (07)H$$

$$A \longrightarrow \boxed{07}\ H$$

Status of flag :

S	Z	X	AC	X	P	X	C_y
0	0	0	0	0	0	0	0

$$F \longrightarrow \boxed{00}$$

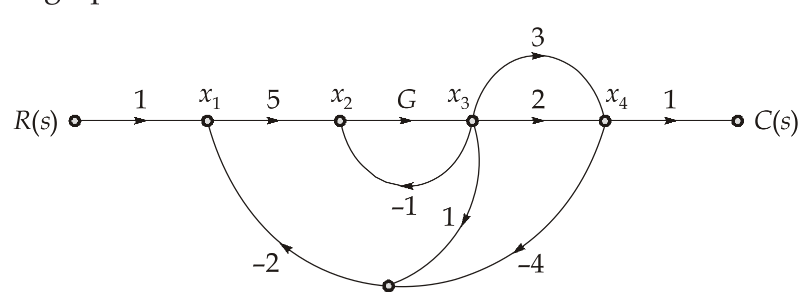
$$PSW \longrightarrow \boxed{A \mid F} = \boxed{07 \mid 00}\,H$$

Q.4 (c) (ii) Solution:

1. **Cycle Stealing DMA :** This is word by word transfer based on CPU cycle stealing. When DMA steals a cycle, CPU is stopped completely for one cycle. In this CPU pause, for just one cycle. It is not an interrupt.
2. **Interleaved DMA :** In this DMA controller takes the control of system bus only when CPU is not using it. In this mode, CPU will not be blocked due to DMA at all. This is the slowest mode of DMA transfer since DMAC has to wait (might be) for so long time to just even get the access of system buses from the CPU itself.
3. **Block Transfer DMA :** In block transfer DMA, DMA controller takes the bus control by CPU. Here, CPU has no access to bus untill the transfer is complete. During this time, CPU can perform internal operations which do not need buses. This is the quickest mode of DMA transfer since at once a huge amount of data being transferred.

Q.5 (a) Solution:

The signal flow graph can be redrawn as



Here, the forward paths are

$P_1 \Rightarrow \quad R - x_1 - x_2 - x_3 - x_4 - C = 10G$

and $P_2 \Rightarrow R - x_1 - x_2 - x_3 - x_4 - C = 15G$

Also, $\Delta_1 = \Delta_2 = 1$

Loops : $L_1 = 5 \times G \times 2 \times 4 \times 2 = 80G$

$L_2 = -5 \times G \times 2 = -10G$

$L_3 = -G$

and $L_4 = 5 \times G \times 3 \times 4 \times 2 = 120G$

$\therefore$ Overall gain

$$\frac{C(s)}{R(s)} = \frac{10G + 15G}{1 + 10G - 120G - 80G + G}$$

$$\frac{13}{17} = \frac{25G}{1 - 189G}$$

$$13 - 2457G = 425G$$

$$13 = 2882G$$

$$G = \frac{13}{2882} = 4.51 \times 10^{-3}$$

Q.5 (b) Solution:

The Routh table is formulated as follows :

s^4	1	6	8
s^3	2	8	
s^2	$\frac{2 \times 6 - 1 \times 8}{2} = 2$	$\frac{2 \times 8 - 1 \times 0}{2} = 8$	
s^1	$\frac{2 \times 8 - 2 \times 8}{2} = 0$	0	
s^0			

All the elements in the s^1 row are zeros. That means there are symmetrically located roots of the characteristic equation with respect to the origin of the s-plane. So, the system is unstable.

To determine the location of the roots, from the auxiliary equation $A(s)$ by using the coefficients of the row just above the row of zeros, i.e.,

$$A(s) = 2s^2 + 8 = 0$$

Take the first derivative of the auxiliary equation, i.e.,

$$\frac{dA(s)}{ds} = 4s + 0 = 0$$

Replace the row of zeros with the coefficients of the first derivative of the auxiliary equation and complete the formation of the Routh table :

s^4	1	6	8
s^3	2	8	
s^2	2	8	
s^1	4	0	
s^0	8		

There are no sign changes in the elements of the first column of the Routh array and hence, there are no roots of the characteristic equation in the right-half of the s-plane. Since, the system is marginally stable, there must be roots on the imaginary axis of the s-plane which can be determined by solving the auxiliary equation.

$$2s^2 + 8 = 0$$

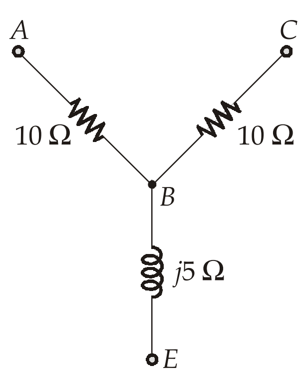
$$s^2 + 4 = 0$$

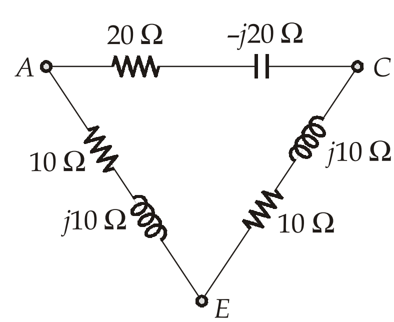
$$s = \pm j2$$

This shows that there is a pair of roots at $s = \pm j2$, and so the system oscillates and the frequency of sustained oscillations is $\omega = 2$ rad/sec.

Q.5 (c) Solution:

Consider the star connected at node B as shown in figure.





Define $Z_s = (10)(10) + 10(j5) + (j5)(10)$

$= 100 + j100$

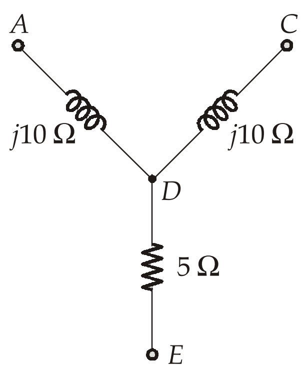
The equivalent impedance in delta connection are

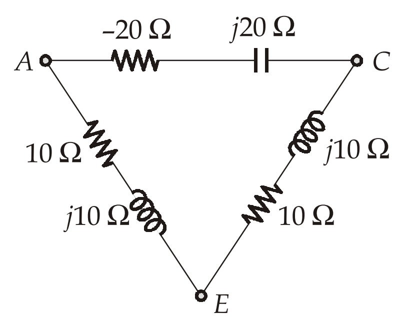
$$Z_{AC} = \frac{100 + j100}{j5} = (20 - j20)\ \Omega$$

$$Z_{AE} = \frac{100 + j100}{10} = 10 + j10\ \Omega$$

$$Z_{CE} = \frac{100 + j100}{10} = 10 + j10\ \Omega$$

The impedance are shown in figure. Now consider the star connection at node D as shown in figure.





Define $$Z'_s = (j10)(j10) + 5(j10) + (j10)(5) = -100 + j100$$

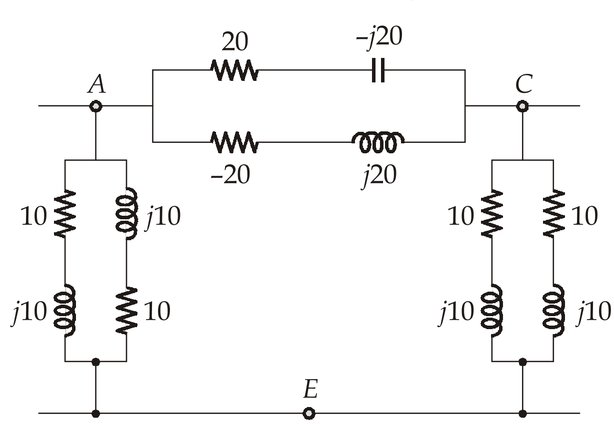
The equivalent impedance in delta connections are

$$Z'_{AC} = \frac{-100 + j100}{5} = -20 + j20$$

$$Z'_{AE} = \frac{-100 + j100}{j10} = 10 + j10$$

$$Z'_{CE} = \frac{-100 + j100}{j10} = 10 + j10$$

These impedances have been indicated on the figure :



Equivalent impedance between nodes A and C

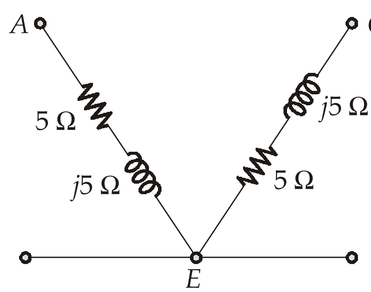
$$Z_{AC} = \frac{(20 - j20)(-20 + j20)}{(20 - j20) + (-20 + j20)} = \infty \text{ (open circuit)}$$

Similarly, equivalent impedance between A and E

$$Z_{AE} = \frac{(10 + j10)(10 + j10)}{2(10 + j10)} = (5 + j5)\ \Omega$$

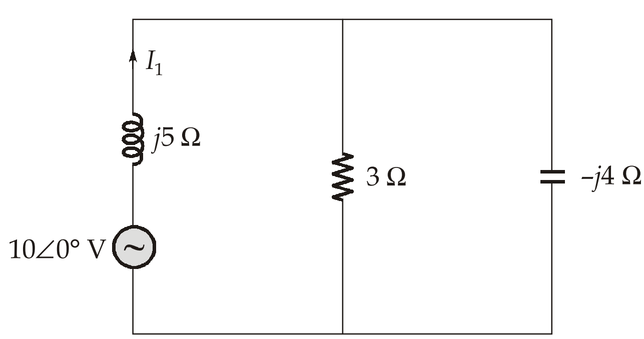
and equivalent impedance between node C and E

$$Z_{CE} = \frac{(10 + j10)(10 + j10)}{2(10 + j10)} = (5 + j5)\ \Omega$$



Q.5 (d) Solution:

Let us consider $10\angle 0°$ V source alone in the circuit as shown in the circuit as shown in figure below :

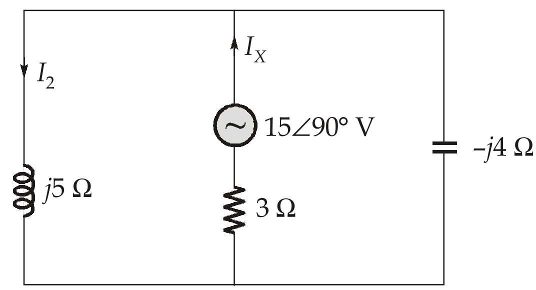


The current,

$$I_1 = \frac{10\angle 0°}{j5 + \dfrac{(3)(-j4)}{3 + (-j4)}}$$

$$= 2.47\angle -61.66° \text{ A}$$

Next we take 15∠90° volt source in the circuit alone.



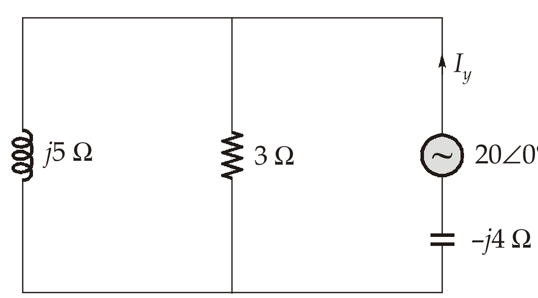
The current given out by the 15∠90° V voltage source

$$I_X = \frac{15\angle 90^\circ}{\frac{(j5)(-j4)}{(j5)+(-j4)}+3} = \frac{j15}{(3-j20)}\text{A}$$

The current through the inductor

$$I_2 = \left(\frac{j15}{3-j20}\right)\left(\frac{-j4}{j5+(-j4)}\right)$$
$$= 2.96\angle{-8.54^\circ}\ \text{A}$$

Finally, considering 20∠0° V source alone



$$I_y = \frac{20\angle 0^\circ}{\frac{(j5)(3)}{(j5+3)}+(-j4)} = \frac{20(3+j5)}{20+j3}$$

Current through the inductor

$$I_3 = \frac{20(3+j5)}{20+j3}\cdot\frac{3}{3+j5} = \frac{60}{20+j3}$$
$$= 2.96\angle{-8.54^\circ}$$

By superposition, the resultant current

$$I = I_1 - I_2 - I_3 = 2.47\angle{-61.66^\circ} - 2.96\angle{-8.54^\circ} - 2.96\angle{-8.54^\circ}$$
$$= 4.84\angle{-164.50^\circ}\ \text{A}$$

Q.5 (e) Solution:

The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(1+sT)}}{1+\frac{K}{s(1+sT)}} = \frac{\frac{K}{T}}{s^2+\frac{s}{T}+\frac{K}{T}}$$

Comparing the characteristic

$$s^2+\frac{s}{T}+\frac{K}{T} = 0$$

with the standard from of the characteristic equation

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \text{ of a second-order system}$$

$$2\xi\omega_n = \frac{1}{T}$$

and $$\omega_n^2 = \frac{K}{T}$$

i.e., $$\omega_n = \sqrt{\frac{K}{T}}$$

$\therefore$ $$2\xi\sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$\xi = \frac{1}{2\sqrt{KT}}$$

(i) When $\xi = \xi_1 = 0.2$, let $K = K_1$

When $\xi = \xi_2 = 0.8$, let $K = K_2$

$\therefore$ $$\frac{\xi_1}{\xi_2} = \frac{0.2}{0.8} = \frac{1}{4} = \frac{1}{2\sqrt{K_1T}} \times 2\sqrt{K_2T} = \sqrt{\frac{K_2}{K_1}}$$

i.e., $$\frac{K_2}{K_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 = \frac{1}{16}$$

or $$K_2 = \frac{1}{16}K_1$$

Hence, the gain K_1 at which $\xi = 0.2$ should be multiplied by $\frac{1}{16}$ to increase the damping ratio from 0.2 to 0.8.

(ii) When $\xi = \xi_1 = 0.9$, let $T = T_1$

When $\xi = \xi_2 = 0.3$, let $T = T_2$

$$\frac{\xi_1}{\xi_2} = \frac{0.9}{0.3} = 3$$

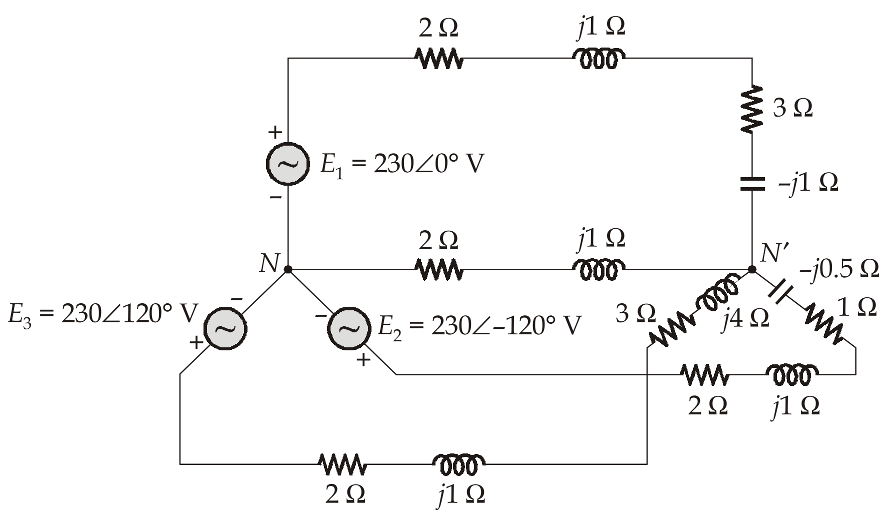
$$\frac{1}{2\sqrt{KT_1}} \times 2\sqrt{KT_2} = \sqrt{\frac{T_2}{T_1}}$$

$\therefore$
$$\frac{T_2}{T_1} = \left(\frac{\xi_1}{\xi_2}\right)^2 = 9$$

or
$$T_2 = 9T_1$$

Hence, the original time constant T_1 should be multiplied by 9 to reduce the damping ratio from 0.9 to 0.3.

Q.6 (a) (i) Solution:



$E_1 = 230\angle 0°$, $\quad Z_1 = (2 + j1) + (3 - j1) = 5 + j0$

$E_2 = 230\angle -120°$, $\quad Z_2 = (2 + j1) + (1 - j0.5) = 3 + j0.5$

$E_3 = 230\angle 120°$, $\quad Z_3 = (2 + j1) + (3 + j4) = 5 + j5$

$E_{N'N} = 10$ V, $\quad Z_{N'N} = (2 + j1)$

The admittance,
$$Y_1 = \frac{1}{Z_1} = \frac{1}{5 + j0} = 0.2 - j0$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{3 + j0.5} = 0.324 - j0.054$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{5 + j5} = 0.10 - j0.10$$

$$Y_{N'N} = \frac{1}{Z_{NN'}} = \frac{1}{2+j1} = 0.40 - j0.20$$

$$\Sigma Y = Y_1 + Y_2 + Y_3 + Y_{NN'}$$

$$= (0.2 - j0) + (0.324 - j0.054) + (0.10 - j0.10) + (0.40 - j0.20)$$

$$= 1.024 - j0.354$$

The potential difference between points N and N'

$$\therefore \quad V_{NN'} = \frac{\Sigma EY}{\Sigma Y} = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3 + E_{N'N}Y_{N'N}}{\Sigma Y}$$

$$= \frac{(230\angle 0°)(0.2 - j0) + (230\angle -120°)(0.324 - j0.054) + (230\angle 120°)(0.10 - j0.10)}{1.024 - j0.354}$$

$$= 13.73 - j21.50 = 25.51\angle -57.43° \text{ V}$$

Q.6 (b) Solution:

The overall transfer function is determined as :

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s(s+1)}}{1 + \frac{1}{s(s+1)} \times (s\alpha + 1)}$$

The characteristic equation is :

$$s^2 + (\alpha + 1)s + 1 = 0$$

which can be rearranged as :

$$s^2 + s + 1 + \alpha s = 0$$

or

$$1 + \frac{\alpha s}{s^2 + s + 1} = 0$$

Therefore, the open loop transfer function for sketching the root contour is given by :

$$G_1(s)H_1(s) = \frac{\alpha s}{s^2 + s + 1}$$

Open-loop zero : $s = 0$

Open-loop poles : $s = \frac{-1 \pm \sqrt{1^2 - 4 \times 1}}{2} = -0.5 \pm j0.866$

The number of root contour branches = 2

The starting point ($\alpha = 0$) of root contours is at $s = -0.5 \pm j0.866$

The terminating point ($\alpha \to \infty$) of root contours is at $s = 0$ and $s \to \infty$.

The angle of asymptotes :

$$\angle\text{Asymptotes} = \frac{(2k+1)}{P-Z} \times 180°\,;\ K = 0$$

$$= \frac{(2\times 0+1)}{2-1} \times 180° = 180°$$

The root contour is present on entire negative real axis. The characteristic equation is

$$1 + \frac{\alpha s}{s^2 + s + 1} = 0$$

or
$$\alpha = \frac{-(s^2+s+1)}{s}$$

$\therefore$
$$\frac{d\alpha}{ds} = -\left\{\frac{s(2s+1) - (s^2+s+1)\times 1}{s^2}\right\}$$

$$= -\left\{\frac{s^2-1}{s^2}\right\}$$

The breakaway point is determined using $\frac{d\alpha}{ds} = 0$.

$\therefore$
$$s^2 - 1 = 0$$

$$s = \pm 1$$

The breakaway point is identified at $s = -1$ as it lies on root contour branch.

The angles of departure from complex poles $s = -0.5 \pm j0.866$ are determined below :

$$\theta_{d\,(-0.5+j0.866)} = 180° - \{(\theta_{P2} - \phi_z)\}$$

$$= 180° - \left\{90° - \left(90° + \tan^{-1}\frac{0.866}{0.5}\right)\right\}$$

$$= 180° - \{90° - (90° + 30°\} = 210°$$

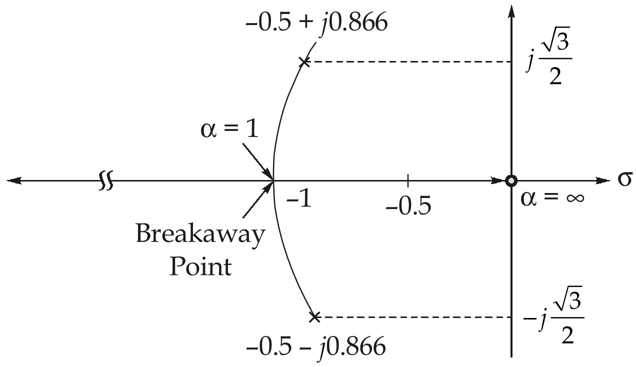
$$\theta_{d\,(-0.5-j0.866)} = 180° - \{(\theta_{P1} - \phi_z)\}$$

$$= 180° - \left\{-90° + \left(90° + \tan^{-1}\frac{0.866}{0.5}\right)\right\}$$

$$= 180° - \{-90° + (90° + 30°\}$$

$$= 150°$$

As per data calculated above the root contour plot is drawn and shown in figure below :



The critical damping occurs at breakaway point on real axis, i.e., $s = -1$. As the point, $s = -1$ lies on the root contour.

Therefore, $$|G_1(-1)H_1(-1)| = 1$$

$$\therefore \quad 1 = \left|\frac{\alpha(-1)}{(-1)^2 + (-1) + 1}\right|$$

$$\therefore \quad \alpha = 1$$

Q.6 (c) Solution:

The rise time, percentage overshoot, peak time and settling time remain the same for unit-step input and step input of any amplitude. Only the peak overshoot varies. The peak overshoot for a step input of 10 units is 10 times the peak overshoot for a unit-step input.

The closed-loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\dfrac{5}{s(s+1)}}{1+\dfrac{5}{s(s+1)}}$$

$$= \frac{5}{s^2 + s + 5}$$

Comparing this transfer function with the standard form of the transfer function of a second-order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{5}{s^2 + s + 5}$$

$$\therefore \quad \omega_n^2 = 5$$

or $\omega_n = \sqrt{5} = 2.236 \text{ rad/sec}$

$$2\xi\omega_n = 1$$

$\therefore$ $\xi = \dfrac{1}{2\omega_n} = \dfrac{1}{2 \times 2.236} = 0.223$

$$\omega_d = \omega_n = \sqrt{1-\xi^2} = 2.236\sqrt{1-(0.223)^2} = 2.179 \text{ rad/sec}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.223)^2}}{0.223} = 1.346 \text{ rad/sec}$$

The rise time, $t_r = \dfrac{\pi - \theta}{\omega_d} = \dfrac{3.141 - 1.346}{2.179}$

$$= 0.824 \text{ sec}$$

The percentage overshoot

$$\%M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100\%$$

$$= e^{\left(-\pi \times 0.223/\sqrt{1-0.223^2}\right)} \times 100\%$$

$$= 0.487 \times 100 = 48.7\%$$

The peak overshoot for a unit-step input is

$$\frac{48.7}{100} = 0.487$$

For an input of 10 units, the peak overshoot is

$$0.487 \times 10 = 4.87$$

The peak time, $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{2.179} = 1.442 \text{ sec}$

The time constant, $T = \dfrac{1}{\xi\omega_n} = \dfrac{1}{0.223 \times 2.236} = 2 \text{ sec}$

For 5% error, the settling time

$$t_s = 3T = 3 \times 2 = 6 \text{ sec}$$

For 2% error, the settling time

$$t_s = 4T = 4 \times 2 = 8 \text{ sec}$$

Q.7 (a) (i) Solution:

The shape of the magnitude curve suggests a zero at the corner frequency 0.1 rad/sec and a pole at the corner frequency 0.5 rad/sec. Hence, let the transfer function is given by

$$G(s)H(s) = \frac{K\left(1+\frac{s}{0.1}\right)}{1+\frac{s}{0.5}} = \frac{K'(0.1+s)}{0.5+s}$$

The phase factor of this function in the frequency domain is given by :

$$\phi = \tan^{-1}\frac{\omega}{0.1} - \tan^{-1}\frac{\omega}{0.5} = \tan^{-1} 10\omega - \tan^{-1} 2\omega$$

To maximize phase, it is necessary that

$$\frac{d\phi}{d\omega} = \frac{d}{d\omega}(\tan^{-1} 10\omega) - \frac{d}{d\omega}(\tan^{-1} 2\omega) = 0$$

$$\frac{10}{1+100\omega^2} - \frac{2}{1+4\omega^2} = 0$$

$$\frac{5}{1+100\omega^2} = \frac{1}{1+4\omega^2}$$

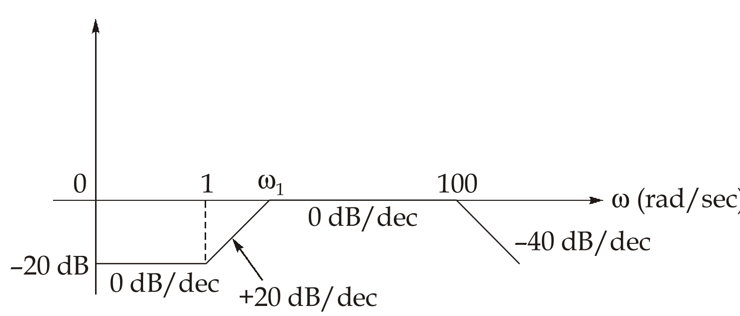
$$1 + 100\omega^2 = 5 + 20\omega^2$$

$$80\omega^2 = 4$$

$$\omega = \frac{1}{\sqrt{20}}$$

Q.7 (a) (ii) Solution:

The given bode plot can be redrawn as :



It can be seen that the initial slope is 0 dB/dec and intercept is –20 dB. This is only possible due to factor K. Since the intercept is minus, the value of K will be less than 1.

$\therefore$ $$20 \log K = -20$$

$$K = 0.1$$

Find the value of ω_1 :

From the slope between ω_1 and ω = 1 rad/sec

$$-\frac{0+20}{\log\omega - \log\omega_1} = 20 \text{ dB/dec}$$

$$-\frac{20}{\log\left(\frac{1}{\omega_1}\right)} = 20$$

$$-\log\frac{1}{\omega_1} = 1$$

$$\omega_1 = 10 \text{ rad/sec}$$

Here, the slope of the line changes at ω = 1 rad/sec by +20 dB/dec. Thus, a zero is located at ω = 1.

- At ω = 10 rad/sec the slope again changes to 0 dB/dec that is obtained by adding a pole in the system.
- Again at ω = 100 rad/sec, the slope changes to –40 dB/dec which results in addition of two poles in the system.

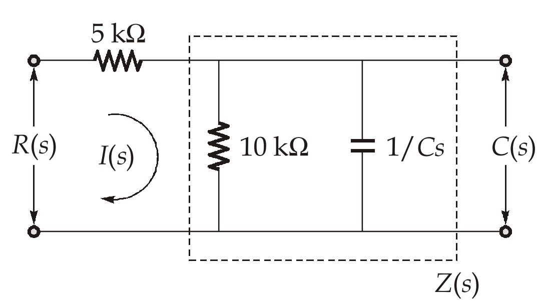
∴ The resultant transfer function is

$$T(s) = \frac{K(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)^2} = \frac{0.1(1+s)\times 10\times 100^2}{(s+10)(s+100)^2}$$

$$T(s) = \frac{10^4(s+1)}{(s+10)(s+100)^2}$$

Q.7 (b) (i) Solution:

Given circuit :



Here,
$$Z(s) = \frac{10\times 10^3 \times \frac{1}{0.1\times 10^{-6}s}}{10\times 10^3 + \frac{1}{0.1\times 10^{-6}s}} = \frac{10^7}{s+1000}$$

$$C(s) = I(s).Z(s) = \frac{R(s)}{\left(5\times10^3 + \frac{10^7}{s+1000}\right)} \times \frac{10^7}{s+1000}$$

Transfer function : $$H(s) = \frac{C(s)}{R(s)} = \frac{2\times10^3}{s+3\times10^3}$$

$$H(s) = \frac{\frac{2}{3}}{\left(1+\frac{s}{3\times10^3}\right)} = \frac{k}{\left(1+\frac{s}{\omega_c}\right)} \quad ...(1)$$

$$H(j\omega) = \frac{k}{\left(1+\frac{j\omega}{\omega_c}\right)}$$

$$|H(j\omega)| = \frac{k}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}} \quad ...(2)$$

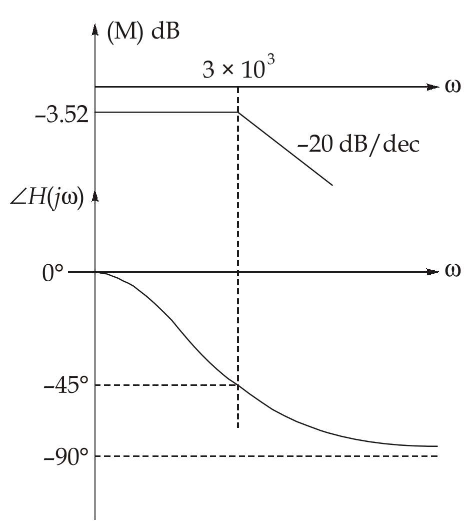
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right) \quad ...(3)$$

Now, at $\omega = 0$, $$|H(j\omega)| = 20\log_{10} K = 20\log_{10}\frac{2}{3} = -3.52 \text{ dB}$$

and

	$\omega = 0$	$\omega = \omega_c$	$\omega = \infty$
$\angle H(j\omega)$	0°	–45°	–90°

Magnitude Plot :



Q.7 (b) (ii) Solution:

Applying KVL and KCL to the circuit, we get

$$i_1 = \frac{V_i - V_1}{R_1}, \qquad V_1 = \frac{\int (i_1 - i_2)dt}{C_1}$$

$$i_2 = \frac{V_1 - V_0}{R_2}, \qquad V_0 = \frac{\int i_2 dt}{C_2}$$

Their laplace transfer, under relaxed conditions, yield

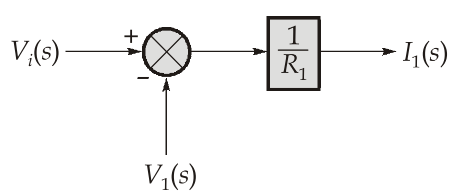
$$I_1(s) = \frac{1}{R_1}[V_i(s) - V_1(s)] \quad \text{...(i)}$$

$$V_1(s) = \frac{1}{sC_1}[I_1(s) - I_2(s)] \quad \text{...(ii)}$$

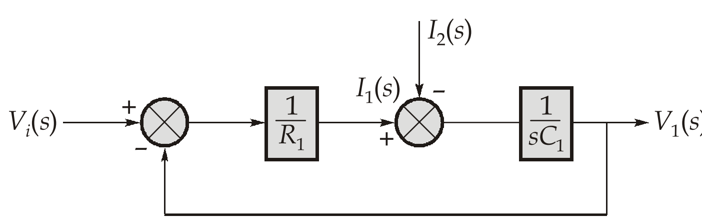
$$I_2(s) = \frac{1}{R_2}[V_1(s) - V_0(s)] \quad \text{...(iii)}$$

$$V_0(s) = \frac{1}{sC_2} I_2(s) \quad \text{...(iv)}$$

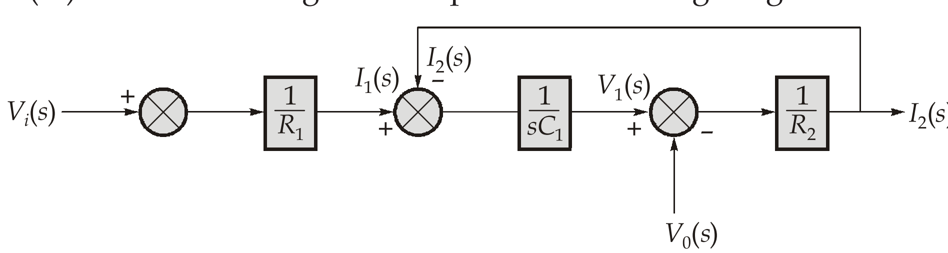
Equation (i) can be converted to a block diagram as follows, we note that $V_i(s)$ is the input corresponding to $V_i(t)$ of the given circuit. So, putting this in the input of a summing point, we draw figure to represent eqn. (i).



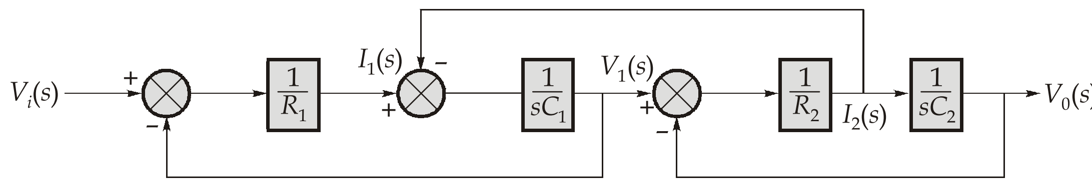
Equation (ii) modifies the diagram to its form given in below figure.



Equation (iii) extends the diagram to representation in figure given below :

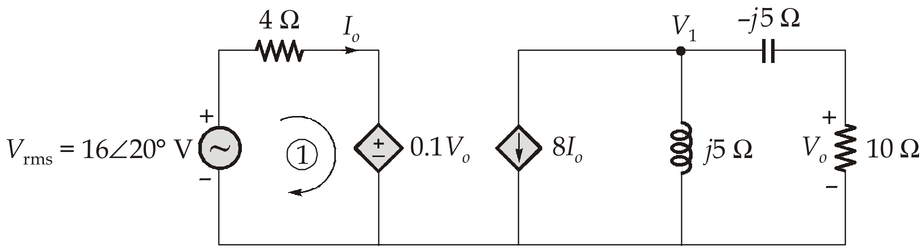


The last equation, eqn. (iv) completes the diagram :



Q.7 (c) (i) Solution:

Redrawing the circuit, we get



Applying KVL in loop (1), we get

$$4I_0 + 0.1V_0 = 16\angle 20° \text{ V} \quad \text{...(i)}$$

Applying KCL to the right side of the circuit, we get

$$8I_0 + \frac{V_1}{j5} + \frac{V_1}{10-j5} = 0 \quad \text{...(ii)}$$

$$V_0 = \frac{10}{10-j5}V_1$$

$$V_1 = \frac{10-j5}{10}V_0 \quad \text{...(iii)}$$

Using equation (ii) and (iii), we get

$$8I_0 + \frac{10-j5}{j50}V_0 + \frac{V_0}{10} = 0$$

By solving the above equation

$$I_0 = j0.025V_0 \quad \text{...(iv)}$$

Substituting (iv) into (i), we get

$$16\angle 20° = 0.1V_0(1+j)$$

$$V_0 = \frac{160\angle 20°}{1+j} = \frac{160}{\sqrt{2}}\angle -25°$$

The power absorbed by the 10 Ω resistor is,

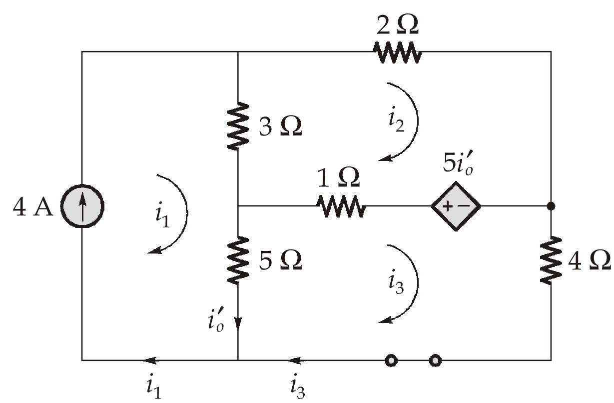
$$P_{10\,\Omega} = \frac{\left(\frac{160}{\sqrt{2}}\right)^2}{10} = \frac{160 \times 160}{2 \times 10}$$

$$= 1280 \text{ W}$$

Q.7 (c) (ii) Solution:

We turn off the 20 V source so that we have the circuit in figure.

We apply mesh analysis in order to obtain i'_o.



$$i_o = i'_o + i''_o \quad \text{...(i)}$$

For loop 1,

$$i_1 = 4 \text{ A} \quad \text{...(ii)}$$

For loop 2,

$$-3i_1 + 6i_2 - i_3 - 5i_0 = 0 \quad \text{...(iii)}$$

For loop 3,

$$-5i_1 - i_2 + 10i_3 + 5i'_0 = 0 \quad \text{...(iv)}$$

But at node 0,

$$i_3 = i_1 - i'_0 = 4 - i'_0 \quad \text{...(v)}$$

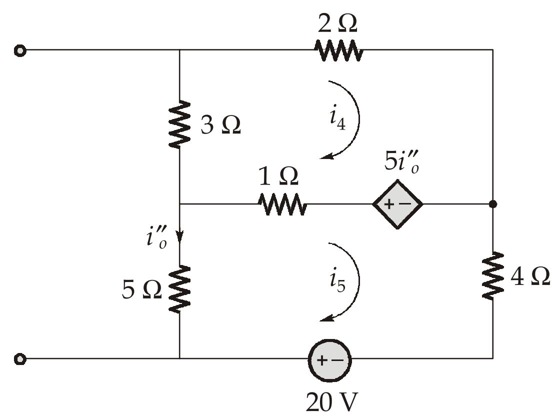
Substituting equations (ii) and (v) into equations (iii) and (iv) gives two simultaneous equations

$$3i_2 - 2i'_0 = 8 \quad \text{...(vi)}$$

$$i_2 + 5i'_0 = 20 \quad \text{...(vii)}$$

which can be solved to get $i'_0 = \frac{52}{17}$ A

To obtain i''_0, we turn off the 4 A current source, so that the circuit becomes that shown in figure :



For loop 4, KVL gives,

$$6i_4 - i_5 - 5i''_0 = 0 \quad ...(ix)$$

and for loop 5,

$$-i_4 + 10i_5 - 20 + 5i''_0 = 0 \quad ...(x)$$

But $i_5 = -i'_0$, substituting this in equations (ix) and (x) gives

$$6i_4 - 4i''_0 = 0 \quad ...(xi)$$

$$i_4 + 5i''_0 = -20 \quad ...(xii)$$

which we solve to get

$$i''_0 = \frac{-60}{17} \text{ A} \quad ...(xiii)$$

Now, $$i_0 = i'_0 + i''_0 = \frac{-8}{17} = -0.4706 \text{ A}$$

Q.8 (a) Solution:

(i) Apply KVL in each mesh

$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$

$$3i_1 - i_2 - 2i_3 = 1 \quad ...(1)$$

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$-i_1 + 6i_2 - 3i_3 = 0 \quad ...(2)$$

$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

$$-2i_1 - 3i_2 + 6i_3 = 6 \quad ...(3)$$

Using Crammer's rule

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$[R]\,[I] = [V]$$

$$R = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}$$

Determinent of Resistance matrix

$$\Delta = 3[36 - 9] + 1[-6 - 6] -2[3 + 12]$$

$$\Delta = 39$$

$$i_1 = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}} = \frac{117}{39} = 3A$$

$$i_2 = \frac{\begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}} = \frac{78}{39} = 2A$$

$$i_3 = \frac{\begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}} = \frac{117}{39} = 2A$$

$$i_1 = 3A; \quad i_2 = 2A; \quad i_3 = 3A$$

(ii) Apply KVL in mesh 2 :

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

$$-i_1 + 6i_2 - 3i_3 = 0 \qquad \text{...(1)}$$

The current sources appear in meshes 1 and 3. Since the 15A source is located on the perimeter of the circuit.

$$\therefore \qquad i_1 = 15A$$

and
$$\frac{1}{9}V_x = \frac{1}{9}\times 3(i_3 - i_2) = i_3 - i_1$$

$$-i_1 + \frac{1}{3}i_2 + \frac{2}{3}i_3 = 0 \qquad ...(2)$$

Substitute $i_1 = 15A$ in eqn. (1) and eqn. (2)

$$6i_2 - 3i_3 = 15$$

$$\frac{1}{3}i_2 + \frac{2}{3}i_3 = 15$$

Solve above equations to get

$$i_2 = 11A;\ i_3 = 17A$$

$$i_1 = 15A;\quad i_2 = 11A;\quad i_3 = 17A$$

Q.8 (b) (i) Solution:

Any point on the root locus must satisfy the phase angle condition.

$$\angle G(s)H(s)|_{s=\sigma+j\omega} = (2n+1)180°;\ n = 0, 1, 2, ...$$

$$G(s)|_{s=\sigma+j\omega} = \frac{K(\sigma+j\omega+1)^2}{(\sigma+j\omega+2)^2} = \frac{K(1+\sigma+j\omega)^2}{(2+\sigma+j\omega)^2}$$

$$\angle G(\sigma+j\omega) = 2\tan^{-1}\left(\frac{\omega}{\sigma+1}\right) - 2\tan^{-1}\left(\frac{\omega}{\sigma+2}\right)$$

As per phase angle condition,

$$\tan^{-1}\left(\frac{\omega}{\sigma+1}\right) - \tan^{-1}\left(\frac{\omega}{\sigma+2}\right) = 90°$$

$$\frac{\left(\frac{\omega}{\sigma+1}\right) - \left(\frac{\omega}{\sigma+2}\right)}{1+\frac{\omega^2}{(\sigma+1)(\sigma+2)}} = \tan(90°) = \infty$$

$$1+\frac{\omega^2}{(\sigma+1)(\sigma+2)} = 0$$

So,
$$1+\frac{\omega^2}{(\sigma+1)(\sigma+2)} = 0$$

$$(\sigma+1)(\sigma+2)+\omega^2 = 0$$

$$\sigma^2 + 3\sigma + 2 + \omega^2 = 0$$

$$\sigma^2 + 3\sigma + 2.25 - 2.25 + 2 + \omega^2 = 0$$

$$(\sigma + 1.5)^2 + (\omega - 0)^2 = (0.50)^2$$

The above equation is independent of the value of K and it follows the equation of a circle with centre at (–1.5, 0) and radius of 0.50.

So, the root locus lies on a circle for any value of $K(K > 0)$.

(ii) (a) Given : $r(t) = u(t)$ and $C(t) = 1 + 0.2e^{-60t} - 1.2e^{-10t}$

Transfer function of closed loop system is given as

$$T(s) = \frac{C(s)}{R(s)}$$

Now,
$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$= \frac{1}{s} + \frac{0.2(s+10) - 1.2(s+60)}{s^2 + 70s + 600}$$

$$= \frac{1}{s} + \frac{(-s-70)}{s^2 + 70s + 600}$$

$$C(s) = \frac{600}{s(s^2 + 70s + 600)} \quad \text{...(1)}$$

Also,
$$R(s) = \frac{1}{s} \quad \text{...(2)}$$

From equation (1) and (2)

$$T(s) = \frac{C(s)}{R(s)} = \frac{600}{s^2 + 70s + 600} \quad \text{...(3)}$$

(b) This is a second order system. For this type standard form of

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{...(4)}$$

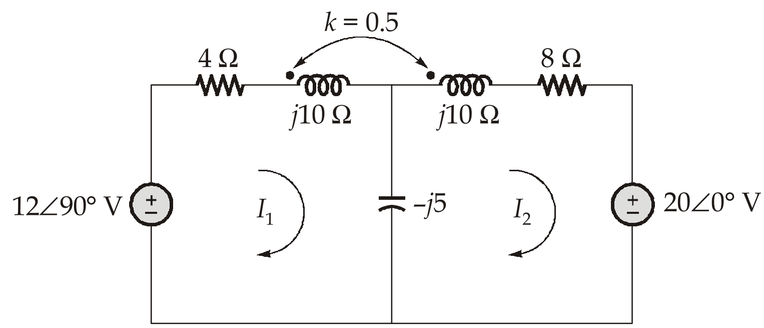
On comparing eqn. (3) and (4)

$$\omega_n^2 = 600 \Rightarrow \omega_n = \sqrt{600} = 24.5 \text{ rad/sec}$$

$$\xi = \frac{70}{2\omega_n} = \frac{70}{2 \times 24.5} = 1.43$$

Q.8 (c) Solution:

Transfer the current source to a voltage source as shown below :



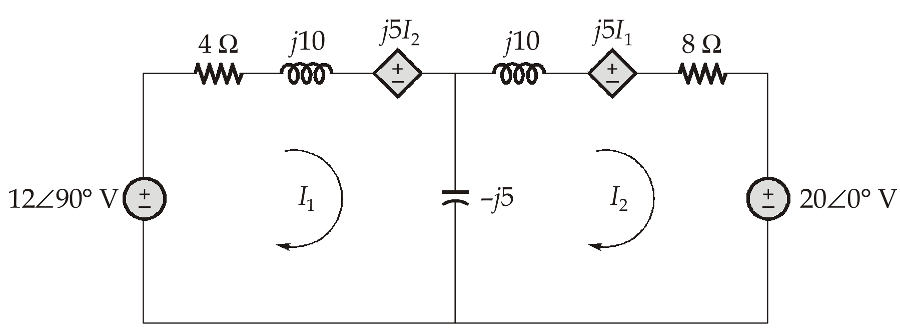
$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = K\sqrt{L_1 L_2}$$

$$\omega M = K\sqrt{\omega L_1 \omega L_2}$$

$$\omega M = 0.5 \times \sqrt{10 \times 10} = 5\ \Omega$$

Now above circuit can be converted as,



For Mesh 1,

$$j12 = (4 + j10)I_1 + j5I_2 - j5(I_1 - I_2)$$

$$j12 = (4 + j5)I_1 + j10I_2 \quad \text{...(1)}$$

For Mesh 2,

$$-j5(I_2 - I_1) + j10I_2 + 8I_2 + j5I_1 = -20$$

$$j10I_1 + (8 + j5)I_2 = -20 \quad \text{...(2)}$$

From eqn. (1) and (2)

$$\begin{bmatrix} 4 + j5 & j10 \\ j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j12 \\ -20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 4+j5 & j10 \\ j10 & 8+j5 \end{vmatrix} = 122.67\angle 29.28°$$

Using Cramer's rule

$$I_1 = \frac{\Delta_1}{\Delta}$$

where, $$\Delta_1 = \begin{vmatrix} j12 & j10 \\ -20 & 8+j5 \end{vmatrix} = 302\angle 101.45°$$

Therefore, $$I_1 = \frac{302\angle 101.45°}{122.67\angle 29.28} = 2.4620\angle 72.17°$$

And $$I_2 = \frac{\Delta_2}{\Delta}$$

where, $$\Delta_2 = \begin{vmatrix} 4+j5 & j12 \\ j10 & -20 \end{vmatrix} = 107.70\angle -68.19°$$

Therefore, $$I_2 = \frac{107.70\angle -68.18°}{122.67\angle 29.28°} = 0.878\angle -97.48° \text{ A}$$

and $$\vec{I}_3 = \vec{I}_1 - \vec{I}_2 = 2.4620\angle 72.17° - 0.878\angle -97.48°$$

$$\vec{I}_3 = 3.329\angle 74.89° \text{ A}$$

Therefore,

$$I_1 = 2.4620\angle 72.19° \text{ A}$$

$$I_2 = 0.878\angle -97.48° \text{ A}$$

$$I_3 = 3.329\angle 74.89° \text{ A}$$

❍❍❍❍