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Detailed Solutions

ESE-2024 Mains Test Series

Mechanical Engineering Test No : 2

Section A : Heat Transfer + Refrigeration and Air Conditioning

Section B : Thermodynamics-1 + Strength of Materials & Mechanics-1

Section : A

1. (a)

Given : Q = 25 tonnes, T_E = –15 + 273 = 258 K, T_C = 40 + 273 = 313 K,

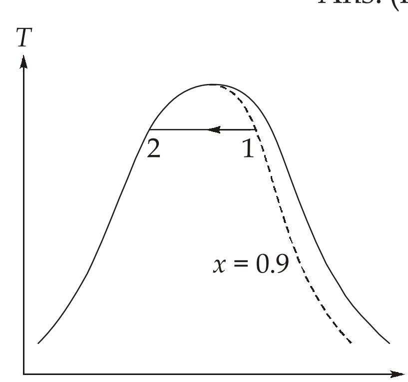
$(\text{C.O.P.})_{rel} = 0.35$

P = 3 bar, x = 0.9 and h_{fg} = 2163.2 kJ/kg

$$T_{sat} = T_G = 133.5 + 273 = 406.5 \text{ K}$$

$$\therefore \quad (\text{C.O.P.})_{max} = \frac{T_E(T_G - T_C)}{T_G(T_C - T_E)} = \frac{258(406.5-313)}{406.5(313-258)}$$

$$\therefore \quad (\text{C.O.P.})_{max} = 1.079$$ Ans. (i)



Also, $$(\text{C.O.P.})_{act} = 0.35 \times 1.079$$

$$(\text{C.O.P.})_{act} = 0.3776$$

∴ Actual heat supplied,

$$Q_{act} = \frac{R.C.}{(C.O.P.)_{act}} = \frac{25 \times 3.5}{0.3776}$$

$$\therefore \quad Q_{act} = 231.72 \text{ kW}$$

Also, $$\dot{m}_s \cdot x \cdot h_{fg} = Q_{act}$$

$$\therefore \quad \dot{m}_s = \frac{231.72}{0.9 \times 2163.2}$$

$$\dot{m}_s = 0.119 \text{ kg/sec} = 0.119 \times 3600 = 428.40 \text{ kg/hr}$$ Ans. (ii)

1. (b)

Given : $T_\infty = 12°C$, $B = 1.2$ m; $L = 1.6$ m; $T_s = 88°C$; $Q_{loss} = 3.95$ kW

$$\text{Mean temperature, } T_m = \frac{T_\infty + T_s}{2} = \frac{12+88}{2} = 50°C$$

Energy dissipated through convective heat transfer,

$$Q_{loss} = h \cdot A \cdot \Delta t$$

$$\Rightarrow \quad 3.95 \times 10^3 = h \times (1.2 \times 1.6) \times (88 - 12)$$

∴ Convective heat transfer coefficient,

$$h = 27.07 \text{ W/m}^2°C \qquad \text{Ans.}$$

$$\therefore \quad \text{Nusselt number, Nu} = \frac{hl}{k} = \frac{27.07 \times 1.6}{0.028} = 1546.86$$

Assuming laminar flow along the plate,

$$\text{Nu} = 0.664(\text{Re})^{0.5}(\text{Pr})^{0.33}$$

$$\Rightarrow \quad 1546.86 = 0.664 \times (\text{Re})^{0.5} \times (0.73)^{0.33}$$

$$\Rightarrow \quad \text{Re} = 6.68 \times 10^6$$

The Reynolds number is greater than the critical Reynolds, number (5×10^5), so assumption made of laminar flow is wrong.

As such the flow is turbulent, then

$$\text{Nu} = \frac{hl}{k} = \left[0.036\,\text{Re}^{0.8} - 836\right] \times (\text{Pr})^{0.33}$$

$$\Rightarrow \quad 1546.86 = \left[0.036\,\text{Re}^{0.8} - 836\right] \times (0.73)^{0.33}$$

$$\Rightarrow \quad \text{Re} = 1.1568 \times 10^6$$

$$\Rightarrow \quad \frac{\rho V_\infty l}{\mu} = 1.1568 \times 10^6$$

$$\Rightarrow \quad \frac{1.09 \times V_\infty \times 1.6}{2.029 \times 10^{-5}} = 1.1568 \times 10^6$$

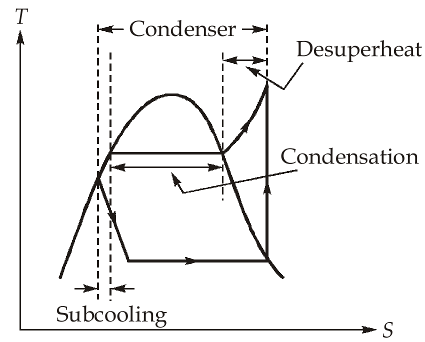
$$\Rightarrow \quad V_\infty = \frac{1.1568 \times 10^6 \times 2.029 \times 10^{-5}}{1.09 \times 1.6}$$

$$\Rightarrow \quad V_\infty = 13.458 \text{ m/s} \qquad \text{Ans.}$$

1. (c)

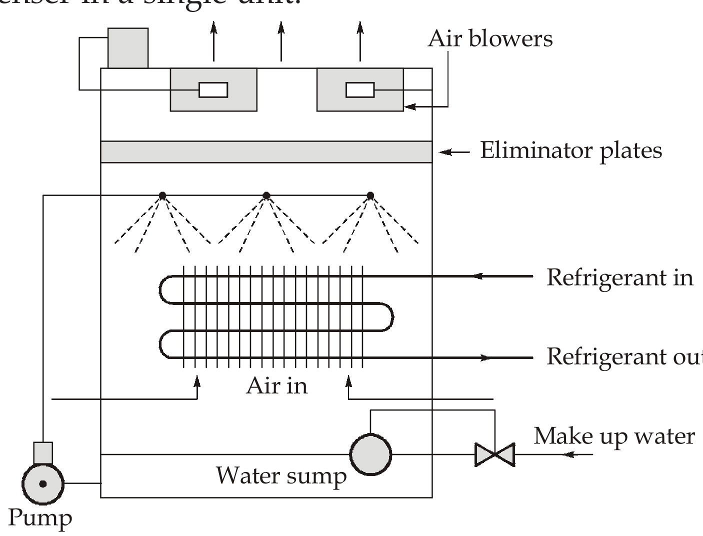
Functions of condenser in a refrigerating machine are as follows:

- To enable rejection of heat from the refrigerant in a refrigeration cycle.
- Enables desuperheat, condensation and subcooling of refrigerant to get vapour to condense to subcooled liquid stage which can then be fed to expansion device.



Different types of condensers are :

- **Air-cooled condensers :** It is used for small cooling load operation like domestic refrigerators and air conditioners. This can be further classified as:
  - Natural convection type
  - Forced convection type
- **Water-cooled condensor:** Water is used to extract heat from the refrigerant. Further classified as:
  - Double tube type
  - Shell and tube type
  - Shell and coil type
- **Evaporative condensers:** Combines the features of both a cooling tower and a water-cooled condenser in a single unit.



Consider an evaporative condenser as shown above. Water is pumped from the sump and sprayed on tubes carrying refrigerant. Water droplets on tube get evaporated by extracting latent heat of vaporization from refrigerant. Water is continuously recirculated and make up water is added as required.

Air is continuously recirculated using blowers and used to cool the refrigerant tubes which are connected to a plate to enhance area of heat exchange as shown above. The role of air is primarily to enhance evaporation of water.

Evaporative condensers are used in medium to large capacity systems. These are normally cheaper than water cooled condensers which require a separate cooling tower.

1. (d)

Given : $r_1 = \frac{d_1}{2} = \frac{0.12}{2} = 0.06$ m; $r_2 = \frac{d_2}{2} = \frac{0.14}{2} = 0.07$ m; $r_3 = r_2 + t = 0.07 + 0.03 = 0.1$ m

$h_i = 11500$ W/m^{2}K; $h_o = 20$ W/m^{2}K; $k_{Cu} = 450$ W/mK; $k_{in} = 0.20$ W/mK

- When the pipe is not insulated, the effective thermal resistance per unit length,

$$\Sigma R_{th} = R_1 + R_2 + R_3$$

$$= \frac{1}{2\pi r_1 l h_i} + \frac{\ln(r_2/r_1)}{2\pi k_{Cu} l} + \frac{1}{2\pi r_2 l h_o}$$

$$= \frac{1}{2\pi \times 0.06 \times 1 \times 11500} + \frac{\ln(0.07/0.06)}{2\pi \times 450 \times 1} + \frac{1}{2\pi \times 0.07 \times 20}$$

$$= 2.3066 \times 10^{-4} + 5.4519 \times 10^{-5} + 0.1137$$

$$= 0.1139 \text{ K/W}$$

$\therefore$ Heat loss per unit length (Q/l),

$$= \frac{\Delta t}{\Sigma R_{th}} = \frac{120 - 30}{0.1139} = 790.16 \text{ W/m}$$ **Ans.**

- When the pipe is insulated, the effective thermal resistance per unit length,

$$\Sigma R'_{th} = R_1 + R_2 + R'_3 + R'_4$$

$$= \frac{1}{2\pi r_1 l h_i} + \frac{\ln(r_2/r_1)}{2\pi k_{Cu} l} + \frac{\ln(r_3/r_2)}{2\pi k_{in} l} + \frac{1}{2\pi r_3 l h_o}$$

$$= \frac{1}{2\pi \times 0.06 \times 1 \times 11500} + \frac{\ln(0.07/0.06)}{2\pi \times 450 \times 1} + \frac{\ln(0.1/0.07)}{2\pi \times 0.2 \times 1} + \frac{1}{2\pi \times 0.1 \times 1 \times 20}$$

$$= 2.3066 \times 10^{-4} + 5.4519 \times 10^{-5} + 0.2838 + 0.0796$$

$$= 0.3637 \text{ K/W}$$

$\therefore$ Heat loss per unit length with insulation, $\left(\frac{Q'}{l}\right)$

$$= \frac{\Delta t}{\Sigma R'_{th}} = \frac{120 - 30}{0.3637} = 247.45 \text{ W/m}$$

So, with the addition of insulation reduces the heat loss from the steam by

$$= \left(\frac{790.16 - 247.45}{790.16}\right) \times 100 = 68.68\% \qquad \text{Ans.}$$

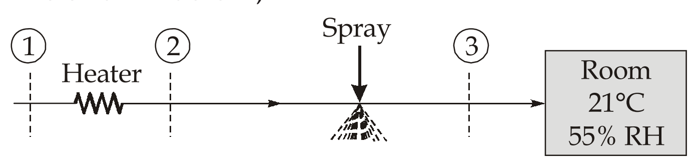
1. (e)

From steam tables,

At 5°C, $P_{vs} = 0.008726$ bar

and 21°C, $P_{vs} = 0.024882$ bar

The flow diagram is shown below,



Now, relative humidity, $\phi_3 = \dfrac{P_{v_3}}{P_{vs_3}}$

$$0.55 = \frac{P_{v_3}}{0.024882}$$

$\Rightarrow$ $P_{v_3} = 0.0137$ bar

$\therefore$ Specific humidity, $\omega_3 = \dfrac{0.622 P_{v_3}}{(P - P_{v_3})} = \dfrac{0.622 \times 0.0137}{1.0132 - 0.0137}$

$\therefore$ $\omega_3 = 8.525 \times 10^{-3}$ kg/kg.d.a.

Also, $\phi_1 = \dfrac{P_{v_1}}{P_{vs_1}}$

$\Rightarrow$ $P_{v1} = P_{vs1} = 0.008726$ bar

$\therefore$ $\omega_1 = \dfrac{0.622 \times 0.008726}{1.0132 - 0.008726}$

$\omega_1 = 5.4 \times 10^{-3}$ kg/kg.d.a.

Also, $\omega_3 - \omega_1 = (8.525 - 5.4) \times 10^{-3}$

$= 3.125 \times 10^{-3}$ kg/kg.d.a.

For air, $v_{a_3} = \dfrac{R_a T_a}{P_{a_3}} = \dfrac{0.287 \times (273 + 21)}{(1.0132 - 0.0137) \times 10^2}$

$v_{a_3} = 0.844$ m^3/kg.d.a.

$\therefore$ Spray water $= \dfrac{3.125 \times 10^{-3}}{0.844} = 3.703 \times 10^{-3}$ kg moisture/m^3

2. (a)

Refer to figure,

$$P_B = 10 \text{ bar}, t_{wf} = 6°\text{C}, P_C = 0.06 \text{ bar}, \eta_{nozzle} = 0.85$$
$$\eta_{ent} = 0.65, \eta_{comp} = 0.8, x_5 = 0.9$$

From steam tables, corresponding to 10 bar (dry saturated), we have

$$h_1 = 2777.1, s_1 = 6.585 \text{ kJ/kgK}, t_1 = 179.878°\text{C}$$

Corresponding to a temperature of 6°C, from steam tables we have

$$P_E = 0.0009354 \text{ MPa}; h_{f2} = 25.22 \text{ kJ/kg}, h_{fg2} = 2486.7 \text{ kJ/kg}$$
$$s_f = 0.09134 \text{ kJ/kgK}, s_{fg2} = 8.908 \text{ kJ/kgK}$$

For isentropic expansion 1-2, we have

$$s_1 = s_2 = s_{f2} + x_2 s_{fg2}$$
$$6.585 = 0.09134 + x_2 \times 8.908$$

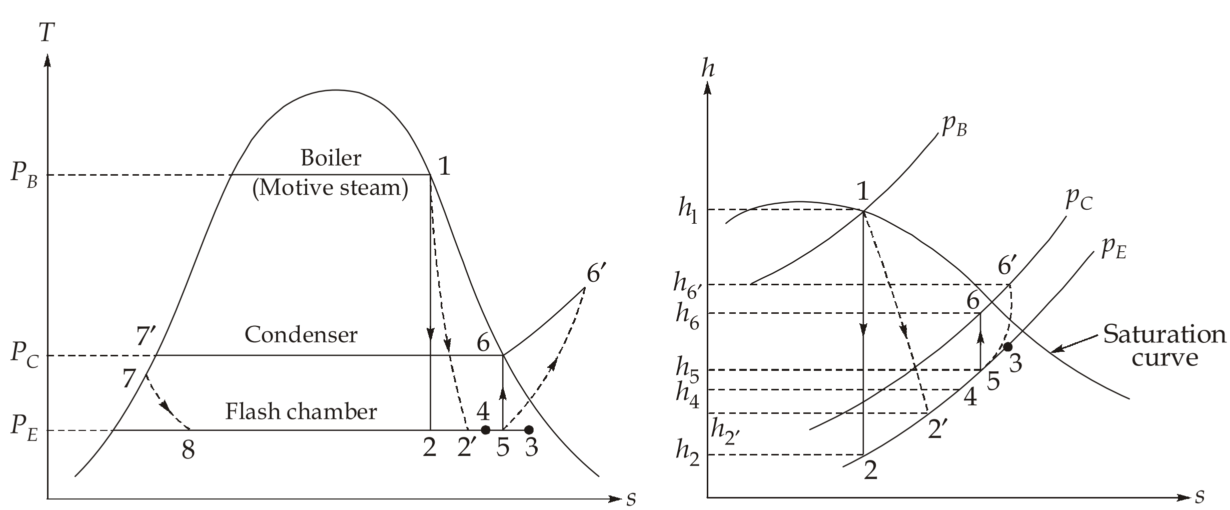
$\therefore$ $$x_2 = 0.729$$

$\therefore$ $$h_2 = h_{f2} + x_2 h_{fg2} = 25.22 + 0.729 \times 2486.7$$
$$h_2 = 1838.02 \text{ kJ/kg}$$

Now, $$\eta_{nozzle} = \frac{h_1 - h_2'}{h_1 - h_2}$$

$$0.85 = \frac{2777.1 - h_2'}{2777.1 - 1838.02}$$

$\therefore$ $$h_2' = 1978.88 \text{ kJ/kg}$$



(a) **T-s diagram** (b) **h-s diagram**

Since the points 2, 2′, 4 and 5 lie on the same pressure line, therefore,

$$h_{f2} = h_{f2'} = h_{f4} = h_{f5} = 25.22 \text{ kJ/kg}$$

$$h_{fg_2} = h_{fg2'} = h_{fg4} = h_{fg5} = 2486.7 \text{ kJ/kg}$$

Also, $$\eta_{ent} = \frac{h_1 - h_4}{h_1 - h_{2'}}$$

$$0.65 = \frac{2777.1 - h_4}{2777.1 - 1978.88}$$

∴ $$h_4 = 2258.26 \text{ kJ/kg}$$

Also $$h_4 = h_{f4} + x_4 \times h_{fg4}$$

∴ $$2258.26 = 25.22 + x_4 \times 2486.7$$

∴ $$x_4 = 0.898$$

Enthalpy at point 5,

$$h_5 = h_{f5} + x_5 \times h_{fg5}$$

$$h_5 = 25.22 + 0.9 \times 2486.7$$

$$h_5 = 2263.25 \text{ kJ/kg}$$

Also, $$s_5 = s_{f5} + x_5 \times s_{fg5}$$

$$s_5 = 0.09134 + 0.9 \times 8.908$$

$$s_5 = 8.108 \text{ kJ/kgK}$$

From steam tables, at $P_c = 0.06$ bar we have

$$h_{f6} = 151.48 \text{ kJ/kg}; \ h_{fg6} = 2415.2 \text{ kJ/kg}$$

$$s_{f6} = 0.52082 \text{ kJ/kgK}; \ s_{fg6} = 7.8082 \text{ kJ/kgK}$$

Since the compression of the mixture is isentropic, therefore,

$$s_5 = s_6 = s_{f6} + x_6 s_{fg6}$$

$$8.108 = 0.52082 + x_6 \times 7.8082$$

∴ $$x_6 = 0.97$$

∴ $$h_6 = h_{f6} + x_6 h_{fg6}$$

$$h_6 = 151.48 + 0.97 \times 2415.2 = 2494.22 \text{ kJ/kg}$$

Also, compression efficiency

$$\eta_{comp} = \frac{h_6 - h_5}{h_{6'} - h_5}$$

$$0.8 = \frac{2494.22 - 2263.25}{h_{6'} - 2263.25}$$

∴ $$h_{6'} = 2551.96 \text{ kJ/kg}$$

(i) Mass of motive steam required.

According to the law of conservation of energy, the energy available for compression must be equal to energy required for compression.

$$\therefore \quad m_s(h_1 - h_4) = (m_s + m_v)(h_{6'} - h_5)$$

$$\text{or} \quad m_s(2777.1 - 2258.26) = (m_s + 1)(2551.96 - 2263.25)$$

$$\therefore \quad 518.84\, m_s = 288.71 m_s + 288.71$$

$$\therefore \quad m_s = 1.254 \text{ kg/kg of flash vapour}$$

Ans. (i)

(ii) Refrigerating effect per kg of flash vapour.

We know that

$$m_v h_3 + m_s h_4 = (m_s + m_v) h_5$$

$$\therefore \quad h_3 + 1.254 \times 2258.26 = (1.254 + 1) \times 2263.25$$

$$\therefore \quad h_3 = 2269.51 \text{ kJ/kg}$$

$$\therefore \quad \text{R.E.} = h_3 - h_{f7}$$

$$= 2269.51 - 4.18 \times 22 = 2177.55 \text{ kJ/kg}$$

Ans. (ii)

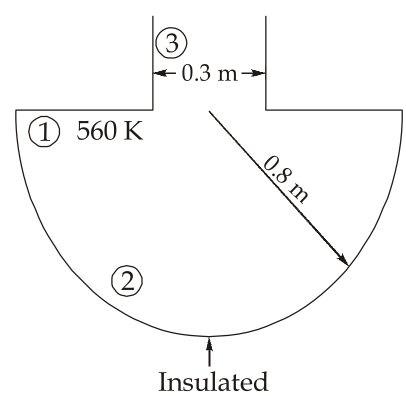
(iii) COP of the system

$$\text{COP} = \frac{m_v(h_3 - h_{f7})}{m_s(h_1 - h_{f7'})} = \frac{1 \times (2269.51 - 4.18 \times 22)}{1.254(2777.1 - 151.48)}$$

$$\therefore \quad \text{COP} = 0.66$$

Ans. (iii)

2. (b)



Let the inner surface of the plate be 1, the surface of the hemisphere be 2, and the projected surface of the hole be 3. Since the surface 1 is completely surrounded, we have

$$F_{11} + F_{12} + F_{13} = 1$$

Since, the surface 1 can neither see itself nor surface 3, so

$$F_{11} = F_{13} = 0$$

$$\Rightarrow \quad F_{12} = 1$$

By reciprocity theorem, $A_1F_{12} = A_2F_{21}$

$$\Rightarrow \quad F_{21} = \frac{A_1}{A_2} \times F_{12} = 1 \times \frac{\pi \times \left[(0.8)^2 - \left(\frac{0.3}{2}\right)^2\right]}{2\pi \times 0.8^2}$$

$$\Rightarrow \quad F_{21} = 0.48242$$

Again for surface 3,

$$F_{31} + F_{32} + F_{33} = 1$$

$\Rightarrow$ $F_{32} = 1$ $[\because \quad F_{33} = F_{31} = 0]$

Again, $A_2F_{23} = A_3F_{32}$

$\therefore$ $$F_{23} = \frac{A_3}{A_2} \times F_{32} = 1 \times \frac{\pi \times 0.15^2}{2 \times \pi \times 0.8^2} = 0.01758$$

$\therefore$ Rate of energy incident on surface of the hemisphere

$$= A_1F_{12} \cdot \sigma \cdot T_1^4 = A_1 \cdot \sigma \cdot T_1^4$$

Note : The rate of energy entering through the hole from outside being negligible since the surrounding are very large and at normal temperature.

Rate of energy emitted by surface 2 would be

$$= A_2F_{21}\sigma T_2^4 + A_2F_{23}\sigma T_2^4 = A_2\sigma T_2^4 \times (0.01758 + 0.48242)$$

$$= 0.5 \times A_2\sigma T_2^4$$

Under steady state conditions,

$$A_1\sigma T_1^4 = 0.5 \times A_2\sigma T_2^4$$

$\Rightarrow$ $$\left(\frac{T_2}{T_1}\right)^4 = 2 \times \frac{A_1}{A_2} = 2 \times \left[\frac{\pi \times (0.8^2 - 0.15^2)}{2\pi \times 0.8^2}\right] = 2 \times 0.48242$$

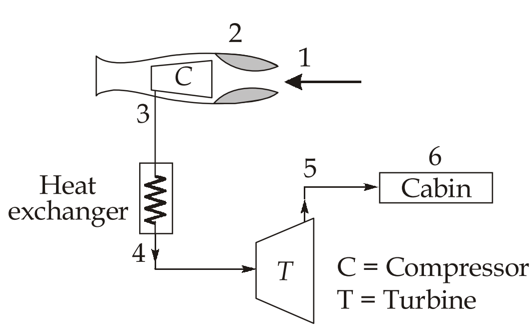
$$T_2 = (0.96484)^{1/4} \times 560 = 555.01 \text{ K}$$ **Ans.**

Heat input to the heater, $Q_1 = A_1F_{12}\sigma_1\left(T_1^4 - T_2^4\right)$

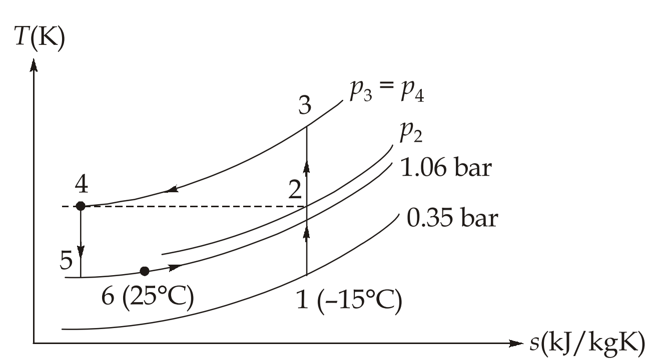
$$= \pi \times [(0.8)^2 - (0.15)^2] \times 1 \times 5.67 \times 10^{-8} \times (560^4 - 555.01^4)$$

$$= 380.44 \text{ W}$$ **Ans.**

2. (c)



(a) Schematic arrangement



(b) T-s diagram

Given : $C = 1200 \text{ km/h} = 1200 \times \frac{5}{18} = 333.33 \text{ m/s}$

$P_1 = 0.35$ bar, $T_1 = -15 + 273 = 258$ K

$T_6 = 25 + 273 = 298$ K, $P_5 = P_6 = 1.06$ bar

$P_3 = 5P_2$, $Q = 30$ tonnes

Now, $$T_2 = T_1 + \frac{C^2}{2000c_p} = 258 + \frac{333.33^2}{2000 \times 1.005}$$

$$T_2 = 313.278 \text{ K} \quad \text{Ans.}$$

and $$P_2 = P_1 \times \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 0.35 \times \left(\frac{313.278}{258}\right)^{\frac{1.4}{1.4-1}} = 0.69 \text{ bar} \quad \text{Ans.}$$

also, $$P_3 = 5 \times P_2 = 5 \times 0.69 = 3.45 \text{ bar} \quad \text{Ans.}$$

Further, $$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 313.278(5)^{\frac{1.4-1}{1.4}} = 496.17 \text{ K} \quad \text{Ans.}$$

Also, $$T_4 = T_2 = 313.278 \text{ K} \quad \text{Ans.}$$

$$P_4 = P_3 = 3.45 \text{ bar} \quad \text{Ans.}$$

Now, $$T_5 = T_4\left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 313.278\left(\frac{1.06}{3.45}\right)^{\frac{1.4-1}{1.4}} = 223.61 \text{ K} \quad \text{Ans}$$

Now, Refrigeration capacity,

$$\text{R.C.} = \dot{m}c_p(T_6 - T_5)$$

$\therefore$ $$30 \times 3.5 = \dot{m} \times 1.005(298 - 223.61)$$

$\therefore$ $$\dot{m}_a = 1.4 \text{ kg/s} \quad \text{Ans.}$$

$\therefore$ At compressor inlet, $$\dot{V}_2 = \frac{\dot{m}_a R T_2}{P_2} = \frac{1.4 \times 0.287 \times 223.61}{0.69 \times 10^2}$$

$$\dot{V}_2 = 1.82 \text{ m}^3/\text{s} \quad \text{Ans.}$$

At turbine outlet, $$\dot{V}_5 = \frac{\dot{m}_a R T_5}{P_5} = \frac{1.4 \times 0.287 \times 313.278}{1.06 \times 10^2}$$

$\therefore$ $$\dot{V}_5 = 0.8476 \text{ m}^3/\text{s} \quad \text{Ans.}$$

Work requirement, $W = W_C - W_E$

$$W = \dot{m}c_p\left[(T_3 - T_2) - (T_4 - T_5)\right]$$

$$W = 1.4 \times 1.005[(496.17 - 313.278) - (313.278 - 223.61)]$$

$\therefore$ $W = 131.16$ kW **Ans.**

Now, $$\text{COP} = \frac{R.E.}{W} = \frac{30 \times 3.5}{131.16} = 0.8$$ **Ans.**

3. (a)

(i)

Analysis of heat flow form the finned surface is made with the following assumptions:

- Thickness of the fin is small compared with the length and width, temperature gradients over the cross-section are neglected and heat conduction treated one dimensional.
- Homogeneous and isotropic fin material; the thermal conductivity *(k)* of the fin material is constant.
- Uniform heat transfer coefficient *(h)* over the entire fin surface.
- No heat generation within the fin itself.
- Joint between the fin the heated wall offers no bond resistance; temperature at root or base of the fin is uniform and equal to temperature (t_0) of the wall.
- Negligible radiation exchange with the surroundings; radiation effects, if any, are considered as included in the convection coefficient *(h)*.
- Steady state heat dissipation.

(ii)

Given : $L = 0.06$ m; $A = 4.8 \times 10^{-4}$ m^2; $P = 0.12$ m; $k = 110$ kJ/m-hr-deg; $T_\infty = 880$°C; $T_w = 520$°C; $h = 1200$ kJ/m^2-hr-deg

For a fin losing heat at the tip, the temperature distribution is given by,

$$\frac{\theta_x}{\theta_0} = \frac{T_x - T_\infty}{T_w - T_\infty} = \frac{\cos h\, m(l-x) + \frac{h}{mk}\sin h\, m(l-x)}{\cos h\, ml + \frac{h}{mk}\sin h\, ml}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{1200 \times 0.12}{110 \times 4.8 \times 10^{-4}}} = 52.223 \text{ m}^{-1}$$

$$ml = 52.33 \times 0.06 = 3.133$$

$$\frac{h}{mk} = \frac{1200}{110 \times 52.223} = 0.2089$$

So, at $x = \frac{l}{2} = \frac{0.06}{2} = 0.03$ m

$$\frac{t_x - 880}{520 - 880} = \frac{\cos h\, 52.223(0.06-0.03) + 0.2089 \sin h 52.223(0.06-0.03)}{\cos h 3.133 + 0.2089 \sin h 3.133}$$

$$t_x = 802.78° \qquad \text{Ans.}$$

The heat flow through the blade is given by,

$$Q = k \cdot A_c \cdot m(t_w - t_\infty) \cdot \frac{\tan h(ml) + \frac{h}{mk}}{1 + \frac{h}{mk} \cdot \tan h(ml)}$$

$$= 110 \times 4.8 \times 10^{-4} \times 52.223 \times (520 - 880) \cdot \frac{\tan h(3.133) + 0.2089}{1 + 0.2089 \tan h(3.133)}$$

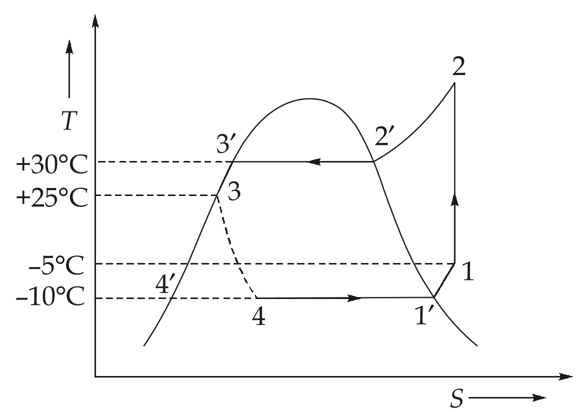
$$= -990.19 \text{ kJ/hr}$$

The negative sign indicates that the heat flows from the combustion gases to the blade.

3. (b)

Refer to figure,

At point '1'



$$h_1 = h_1' + c_{p_v}(T_1 - T_1')$$

$$= 347.96 + 0.615 \times 5$$

$$= 351.035 \text{ kJ/kg}$$

Also, $$s_1 = s_1' + c_{p_v} \ln\left(\frac{T_1}{T_1'}\right)$$

$$s_1 = 1.5632 + 0.615 \ln \frac{268}{263}$$

$$s_1 = 1.5747 \text{ kJ/kgK}$$

Process 1-2 is isentropic compression, therefore,

$$s_1 = s_2 = s_2' + c_{pv} \ln \frac{T_2}{T_2'}$$

$$1.5747 = 1.5481 + 0.615 \ln \frac{T_2}{303}$$

or $$\ln \frac{T_2}{303} = 0.04325$$

∴ $$T_2 = 316.4 \text{ K}$$

$\therefore$ $$h_2 = h_2' + c_{p_v}\left(T_2 - T_2'\right)$$

$$h_2 = 364.96 + 0.615(316.4 - 303)$$

$$h_2 = 373.2 \text{ K}$$

Again enthalpy at point '3'

$$h_3 = h_3' + c_{pl}\left(T_3' - T_3\right)$$

$$= 229.11 - 0.963(30 - 25)$$

$$h_3 = 224.295 \text{ kJ/kg} = h_4$$

Also, $$\frac{v_1}{T_1} = \frac{v_1'}{T_1'}$$

$\therefore$ $$v_1 = \frac{T_1}{T_1'} \times v_1' = \frac{268}{263} \times 0.07702$$

$$= 0.07848 \text{ m}^3\text{/kg}$$

Now, $$\text{R.E.} = h_1 - h_4 = 351.035 - 224.295$$

$$= 126.74 \text{ kJ/kg}$$ Ans. (i)

Also, $$w_{\text{in}} = h_2 - h_1 = 373.2 - 351.035$$

$$= 22.165 \text{ kJ/kg}$$

Now, $$\dot{m} \times R \cdot E = 20 \times 3.5$$

or $$\dot{m} = \frac{20 \times 3.5}{126.74} = 0.552 \text{ kg/sec}$$

$\therefore$ Theoretical power, $$\dot{p}_{th} = \dot{m} \times w_{in} = 0.552 \times 22.165$$

$$\dot{p}_{th} = 12.235 \text{ kW}$$ Ans. (ii)

$$\text{COP} = \frac{R.E.}{w_{in}} = \frac{126.74}{22.165} = 5.7$$ Ans. (iii)

Theoretical piston displacement, [Assuming 100% volumetric efficiency]

$$\dot{m}v_1 = \frac{\pi}{4} D^2 L \cdot \frac{N}{60} \cdot K$$

or $$0.552 \times 0.07848 = \frac{\pi}{4} D^3 \times 1.5 \times 2 \times \frac{1200}{60}$$

$\therefore$ $$D^3 = \frac{0.552 \times 0.07848 \times 4 \times 60}{\pi \times 1.5 \times 2 \times 1200}$$

$\therefore$ $D = 0.0972$ m

or $D = 9.72$ cm Ans. (iv)

and stroke, $L = 1.5 \times 9.72$

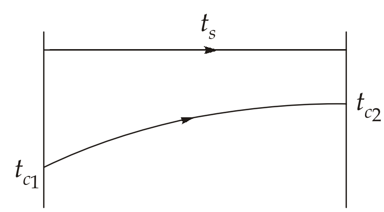
$\therefore$ $L = 14.58$ cm Ans. (iv)

3. (c)

Given : $A = 24$ m^2; $t_{c_1} = 42°C$; $U = 130$ W/m^{2}K; $\dot{m}_w = \dot{m}_c = 0.94$ kg/s;

$h_{fg} = 2257$ kJ/kg at 100°C

From energy balance, $Q = U \cdot A \cdot \Delta T_m = \dot{m}_c \cdot c_{pc} \cdot \left(t_{c_2} - t_{c_1}\right)$



Also,
$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)} = \frac{\left(t_s - t_{c_1}\right) - \left(t_s - t_{c_2}\right)}{\ln\left(\frac{t_s - t_{c_1}}{t_s - t_{c_2}}\right)}$$

$$= \frac{(100-42)-(100-t_{c_2})}{\ln\left(\frac{100-42}{100-t_{c_2}}\right)} = \frac{t_{c_2} - 42}{\ln\left(\frac{58}{100-t_{c_2}}\right)}$$

$$\Rightarrow \quad 130 \times 24 \times \left[\frac{t_{c_2} - 42}{\ln\left(\frac{58}{100-t_{c_2}}\right)}\right] = 0.94 \times 4.187 \times 10^3 \times \left(t_{c_2} - 42\right) \quad ...(i)$$

On solving, above equation, we get

$\Rightarrow$ $t_{c_2} = 73.75°C$ Ans.

Again, applying the energy balance between the condensing steam and the feed water gives:

$$\dot{m}_s \times h_{fg} = m_c \cdot c_{pc} \cdot \left(t_{c_2} - t_{c_1}\right)$$

$$\Rightarrow \quad \dot{m}_s = \frac{0.94 \times 4.187 \times (73.75 - 42)}{2257}$$

$\therefore$ Steam condensation rate, $\dot{m}_s = 0.05536$ kg/s Ans.

With overall heat transfer coefficient,

$$U' = 2 \times 130 = 260 \text{ W/m}^2\text{K}$$

From equation (i), we get

$$260 \times 24 \times \left[\frac{t'_{c_2} - 42}{\ln\left(\frac{58}{100 - t'_{c_2}}\right)}\right] = 0.94 \times 4.187 \times 10^3 \times \left(t'_{c_2} - 42\right)$$

On solving, above equation, we get

$$\Rightarrow \qquad t'_{c_2} = 88.12°$$

$\therefore$ Then, the steam condensation rate is,

$$\dot{m}_s = \frac{0.94 \times 4.187 \times (88.12 - 42)}{2257}$$

$$= 0.08042 \text{ kg/s}$$

$$\text{Increase in steam condensation} = \left(\frac{0.08042 - 0.05536}{0.05536}\right) \times 100 = 45.27\%$$

Thus, a 100% increase in the overall heat transfer coefficient produces only 45.27% increase in the rate of steam condensation.

4. (a)

Given : $\dot{m} = 900 \text{ kg/hr} = \frac{900}{3600} = 0.25$ kg/s; $T_1 = 16°\text{C}$; $L = 1.6$ m; $D_i = 0.09$ m; $T_s = 98°\text{C}$;

$\rho = 1150$ kg/m^3; $\mu = 22.5$ kg/m-s; $c_p = 2750$ J/kgK; $k = 0.42$ W/mK

$$\therefore \qquad \text{Reynolds number, Re} = \frac{4\dot{m}}{\pi\mu D_i} = \frac{4 \times 0.25}{\pi \times 22.5 \times 0.09}$$

$$\text{Re} = 0.1572$$

Hence, the flow is laminar and the given correlation is valid.

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{22.5 \times 2750}{0.42} = 147321.43$$

$$\frac{d}{l}\text{Re Pr} = \frac{0.09}{1.6} \times 0.1572 \times 147321.43 = 1302.69$$

$$\therefore \qquad \text{Nu} = \frac{h \cdot d}{k} = 3.65 + \frac{0.067 \frac{d}{l}\text{Re Pr}}{1 + 0.04\left(\frac{d}{l}\text{Re Pr}\right)^{0.67}}$$

$$\text{Nu} = 3.65 + \frac{0.067 \times 1302.69}{1 + 0.04 \times (1302.69)^{0.67}}$$

$$\text{Nu} = 18.477$$

$$\Rightarrow \quad \frac{h \cdot D_i}{k} = 18.477$$

$$\Rightarrow \quad h = \frac{18.477 \times 0.42}{0.09} = 86.23 \text{ W/m}^2\text{K}$$

Let T_1 and T_2 denote the temperature of cheese at entrance and exit of the heated section. Then, mean bulk temperature of cheese is,

$$T_b = \frac{T_2 + T_1}{2} = \frac{T_2 + 16}{2}$$

The heat gained by cheese equals the convective heat flow from the tube to cheese.

$$\dot{m} c_p (T_2 - T_1) = h \cdot A (T_s - T_b)$$

$$\Rightarrow \quad 0.25 \times 2750 \times (T_2 - 16) = 86.23 \times \pi \times 0.09 \times 1.6 \times \left(98 - \frac{T_2 + 16}{2}\right)$$

$$\Rightarrow \quad T_2 = 20.52°\text{C}$$ Ans.

$\therefore$ Rise in temperature of cheese = 20.52 – 16 = 4.52°C

$\therefore$ Heat transfer from tube to cheese

$$= h \cdot A \times (T_s - T_b) = \dot{m} c_p (T_2 - T_1)$$

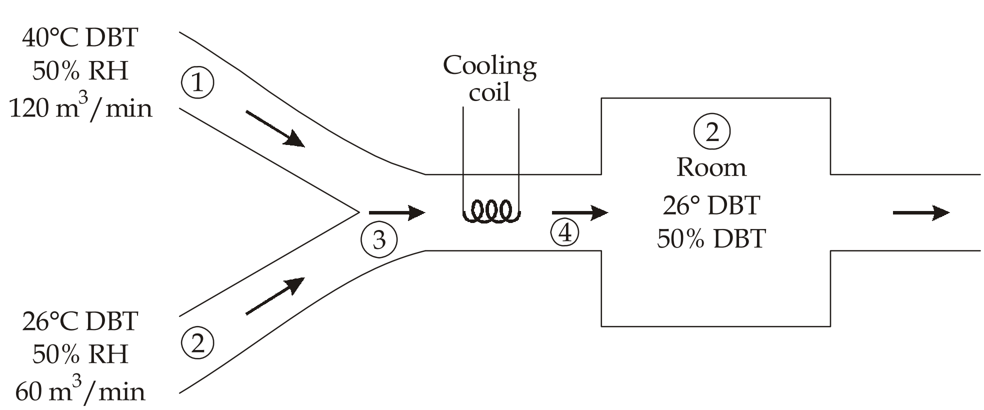
$$= 0.25 \times 2750 \times (20.52 - 16)$$

$$= 3107.5 \text{ W}$$ Ans.

4. (b)

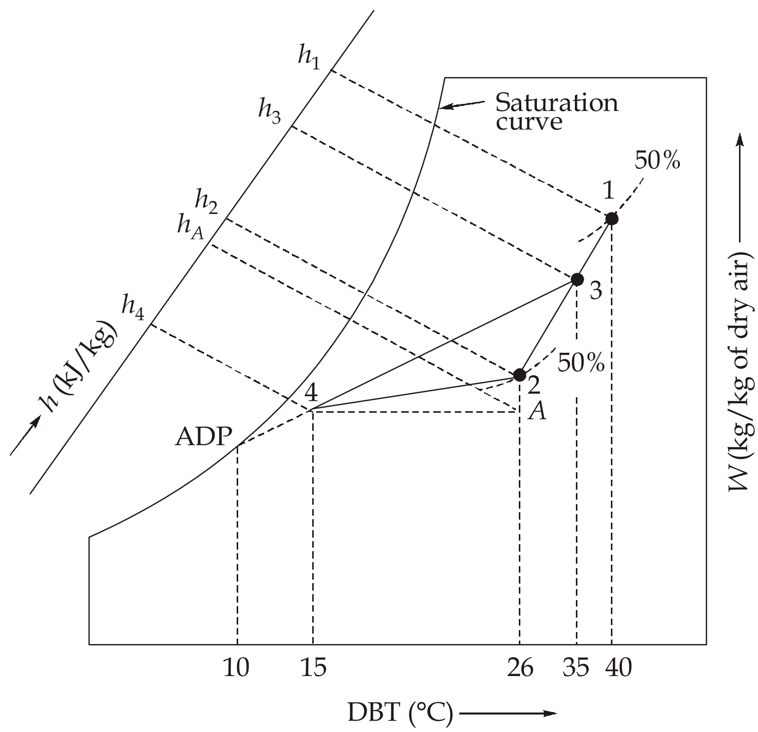
Given : t_{db1} = 40°C; ϕ_1 = 50%; V_1 = 120 m^3/min; t_{db2} = 26°C; ϕ_2 = 50%; V_2 = 60 m^3/min; ADP = 10°C; BF = 0.2

The flow diagram is shown below:



The various process shown on the Psychrometric chart are discussed below:

- Locate point 1 at the intersection of 40°C DBT and 50% RH lines.
- Locate point 2 at the intersection of 26°C DBT and 50% RH lines.
- Join point 1 and 2.



From Psychrometric chart we find,

$$h_1 = 99.6 \text{ kJ/kg.d.a.};\ h_2 = 54 \text{ kJ/kg.d.a.}$$

$$v_{s_1} = 0.92 \text{ m}^3\text{/kg.d.a.};\ v_{s_2} = 0.862 \text{ m}^3\text{/kg.d.a.}$$

Now,
$$\dot{m}_1 = \frac{V}{v_{s_1}} = \frac{120}{0.92} = 130.43 \text{ kg/min}$$

Similarly,
$$\dot{m}_2 = \frac{V}{v_{s_2}} = \frac{60}{0.862} = 69.6 \text{ kg/min}$$

Also,
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$\therefore$
$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_1 + \dot{m}_2} = \frac{130.43 \times 99.6 + 69.6 \times 54}{130.43 + 69.6}$$

$\therefore$
$$h_3 = 83.73 \text{ kJ/kg.d.a.}$$

Locate point 3 on the line 1-2 such that

$$h_3 = 83.73 \text{ kJ/kg.d.a.}$$

From Psychrometric chart, we find

$$t_{db_3} = 35°C$$

Now, locate ADP = 10°C, on the saturation curve

Also,
$$BF = 0.2 = \frac{t_{d_4} - ADP}{t_{db_3} - ADP} = \frac{t_{db_4} - 10}{35 - 10}$$

or
$$t_{db_4} = 10 + 0.2(35 - 10) = 15°C$$

Locate point 4 (t_{db4} = 15°C) on the line joining point 3 and ADP = 10°C

∴ From the Psychrometric chart we find

$$\phi_4 = 91\%$$

∴ State of air leaving the coil : 15°DBT, 91% RH **Ans.**

From the Psychrometric chart, we find:

$$h_2 = 54 \text{ kJ/kg.d.a.};\ h_4 = 40 \text{ kJ/kg.d.a.};\ h_A = 53 \text{ kJ/kg.d.a.}$$

∴
$$RSHF = \frac{h_A - h_4}{h_2 - h_4} = \frac{53 - 40}{54 - 40} = 0.928$$ **Ans.**

Cooling load of the coil:

$$Q_c = \frac{\dot{m}_3(h_3 - h_4)}{60 \times 3.5} = \frac{200.03(83.73 - 40)}{60 \times 3.5} \quad (\because \dot{m}_3 = \dot{m}_1 + \dot{m}_2)$$

$$Q_c = 41.65 \text{ tonnes}$$ **Ans.**

4. (c)

(i)

Given : D = 0.026 m; H = 0.032 m; T_i = 30°C; T_∞ = 760°C; ρ = 7850 kg/m^3; c_p = 480 J/kgK; k = 42 W/mK; h = 84 W/m^{2}K

For a cylindrical surface, the characteristic linear dimension is,

$$s = \frac{\text{Volume}}{\text{Surface area}} = \frac{\pi R^2 H}{2\pi R(R+H)} = \frac{RH}{2(R+H)}$$

$$= \frac{0.013 \times 0.032}{2 \times (0.013 + 0.032)} = 4.6222 \times 10^{-3} \text{ m}$$

$$\text{Biot number, Bi} = \frac{hs}{k} = \frac{84 \times 4.6222 \times 10^{-3}}{42} = 0.00924 < 0.1$$

∵ Bi < 0.1, the lumped parameter model can be adopted. Therefore,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(\frac{-hA}{\rho VC}\tau\right)$$

$$\frac{hA}{\rho VC} = \frac{84}{7850 \times 4.6222 \times 10^{-3} \times 480} = 4.823 \times 10^{-3}$$

$$\frac{620 - 760}{30 - 760} = \exp(-4.823 \times 10^{-3} \times \tau)$$

$$\tau = 342.39 \text{ seconds} \qquad \text{Ans.}$$

Let T' be the temperature attained when the pieces are taken out from the furnace after 270 seconds.

Then,
$$\frac{T' - 760}{30 - 760} = \exp(-4.823 \times 10^{-3} \times 270)$$

$$T' = 561.49°\text{C}$$

$\therefore$ Shortfall in temperature = 620 – 561.49

= 58.51°C

(ii)

The process of boiling depends upon the nature of the surface, thermo-physical properties of the fluid and vapour bubble dynamics.

There are three definite regimes of boiling (Interface evaporation, nucleate boiling and film boiling) associated with progressively increasing heat flux, as shown in figure below. This specific curve has been obtained from an electrically heated platinum wire submerged in a pool of water (at saturation temperature) by varying its surface temperature and measuring the surface heat flux q_s.

1. **Interface evaporation :** Interface evaporation (evaporation process with no bubble formation) exists in region *I*, called the free convection zone. Here the excess temperature, Δt_e, is very small and ≈ 5°C. In this region the liquid near the surface is superheated slightly, the convection currents circulate the liquid and evaporation takes place at the liquid surface.

2. **Nucleate boiling :** This type of boiling exists in regions II and III. With the increase in Δt_e (excess temperature) the formation of bubbles on the surface of the wire at certain localised spots commences. The bubbles condense in the liquid without reaching the liquid surface. In fact, it is the region II where nucleate boiling starts. With further increase in Δt_e the bubbles are formed more rapidly and rise to the surface of the liquid resulting in rapid evaporation, as indicated in the region III. The nucleate boiling is thus characterised by formation of bubbles at the nucleation sites and the resulting liquid agitation. The bubble agitation induces considerable

fluid mixing and that promotes substantial increase in the heat flux and the boiling heat transfer heat transfer coefficient (The equipment used for boiling should be designed to operate in this region only).

Nucleate boiling exists upto $\Delta t_e \approx 50°C$. The maximum heat flux, known as the critical heat flux, occurs at point A and is of the order of 1 MW/m^2.

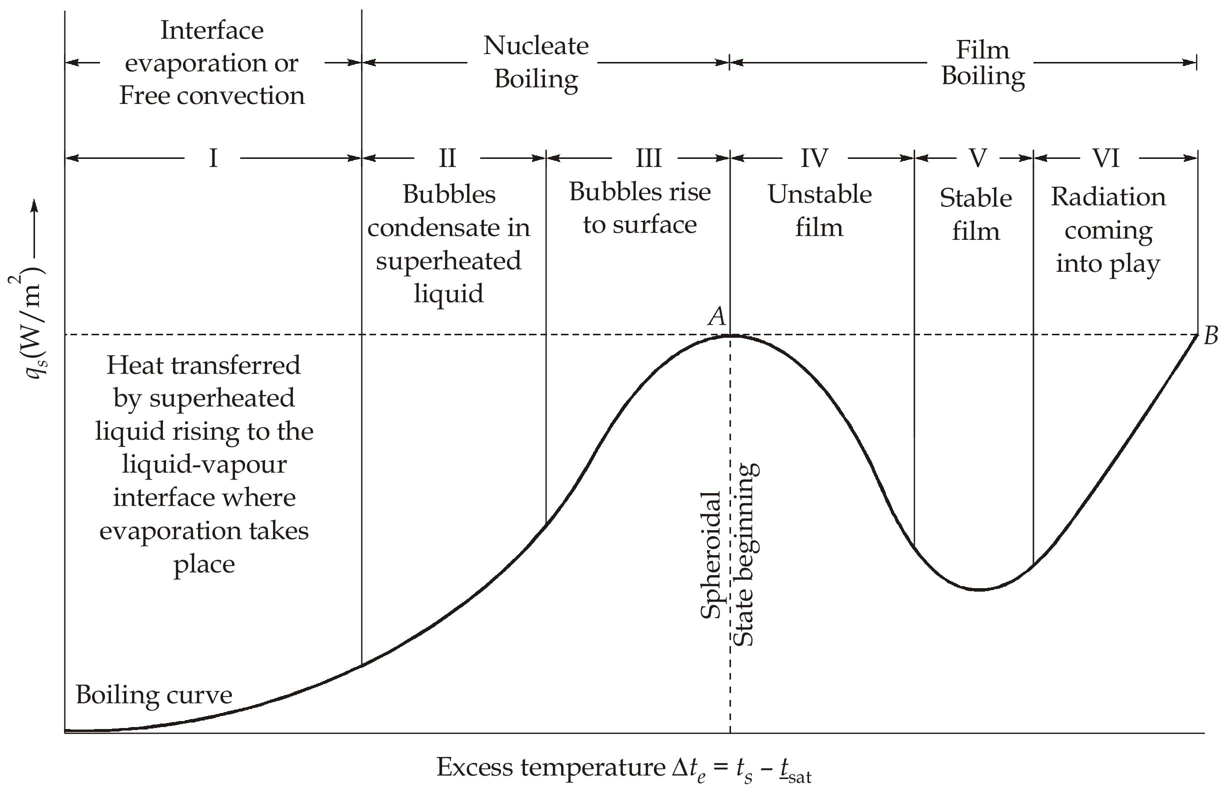


Figure : The boiling curve for water

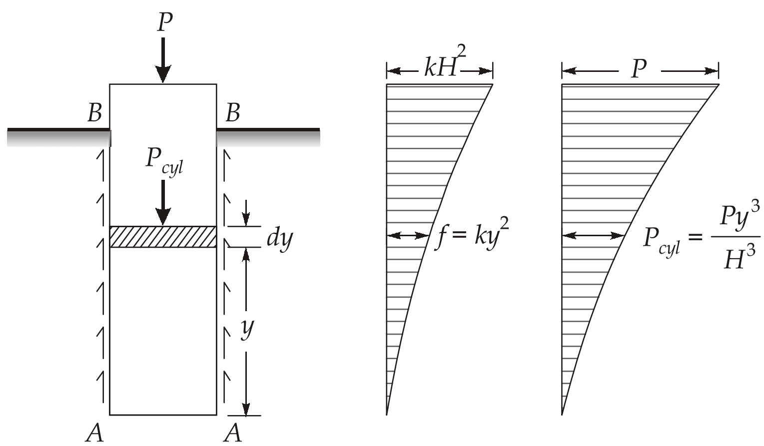
3. **Film boiling :** Film boiling comprises of regions IV, V and VI. The trend of increase of heat flux with increase in excess temperature observed upto region III is reversed in region IV (called film boiling region). This is due to the fact that the bubble formation is very rapid and the bubbles blanket the heating surface and prevent the incoming fresh liquid from taking their place. Eventually the bubbles coalesce and form a vapour film which covers the surface completely. Since the thermal conductivity of vapour film is much less than that of the liquid the heat flux drops with growth in Δt_e. Within the temperature range $50°C < \Delta t_e < 150°C$, conditions oscillate between nucleate and film boiling and the phase is referred to as transition boiling, unstable film boiling or partial film boiling (region IV). With further increase in Δt_e the vapour film is stabilised and the heating surface is completely covered by a vapour blanket and the heat flux is the lowest as shown in region V.

The surface temperatures required to maintain a stable film are high and under these conditions a sizeable amount of heat is lost by the surface due to radiation, as indicated in the region VI.

Section : B

5. (a)

Refer to figure,



(a) **(b)** **(c)**

The variation of frictional resistance (f) along the depth is shown in above figure (b).

$$\text{Total frictional resistance, } F = \int_0^H f \cdot dy = \int_0^H ky^2 dy = \left[\frac{ky^3}{3}\right]_0^H$$

$$\Rightarrow \quad F = P = \frac{k \cdot H^3}{3} \quad \text{...(i)}$$

$$\Rightarrow \quad k = \frac{3P}{H^3} \quad \text{...(ii)}$$

The compressive force on pile varies with the depth. At any height 'y' above the bottom of the pile, the total compressive force will be equal to the frictional resistance (F_y) of clay for the bottom height 'y'.

$$\therefore \quad \text{Total compression, } P_{cyl} = \int_0^y f \cdot dy = \int_0^y ky^2\, dy = \frac{ky^3}{3}$$

$$\Rightarrow \quad P_{cyl} = \frac{ky^3}{3} = \frac{3P}{H^3} \times \frac{y^3}{3} \quad \text{[From equation (ii)]}$$

$$\Rightarrow \quad P_{cyl} = \frac{Py^3}{H^3}$$

The variation of P_{cyl} along the depth of the pile is shown above in figure (c).

Now, shortening of small length dy of the pile,

$$= \frac{P_{cyl'}\, dy}{AE}$$

$\therefore$ Total shortening of the pile $= \int_0^H \frac{P_{cyl'}\, dy}{AE} = \int_0^H \frac{\frac{Py^3}{H^3} dy}{AE} = \frac{P}{AEH^3} \int_0^H y^3 dy$

$$= \frac{P}{AEH^3 \times 4}\left[y^4\right]_0^H = \frac{PH^4}{4AEH^3} = \frac{PH}{4AE}$$

5. (b)

Given : m_s = 15 kg; T_s = 800°C; T_w = 30°C; m_w = 10 kg; c_{ps} = 0.5 kJ/kgK; c_{pw} = 4.27 kJ/kgK; $h_{fg@100°C}$ = 2257 kJ/kg

If the lump of steel reaches a final temperature of 100°C.

The energy lost by steel $= m_s c_{ps}(T_s - 100)$

$= 15 \times 0.5\,(800 - 100)$

$E_1 = 5250$ kJ

If the water temperature changes from 30°C to 100°C.

The energy gained by water,

$$E_2 = m_s c_{pw}(100 - T_{w1})$$
$$= 10 \times 4.27 \times (100 - 30) = 2989 \text{ kJ}$$

$\because$ $E_1 > E_2$, so some water vaporizes, the energy available for vaporization.

$$\Delta E = E_1 - E_2 = 5250 - 2989 = 2261 \text{ kJ}$$

Mass of water vaporized $= \frac{2261}{h_{fg}} = \frac{2261}{2257} = 1.0018$ kg

Therefore in the final state, the system is at 100°C and has 1.0018 kg of steam and 8.9982 kg of liquid water.

$\therefore$

$$\Delta s_{steel} = m_s c_{p_s} \ln\frac{T_F}{T_1}$$

$$= 15 \times 0.5 \ln\frac{373}{1073} = -7.92 \text{ kJ/K}$$

$$\Delta s_{water} = m_v \times \frac{h_{fg}}{T_{sat}} + m_w c_{p_w} \ln\frac{T_F}{T_1}$$

$$= 1.0018 \times \frac{2257}{373} + 10 \times 4.27 \ln \frac{373}{303}$$

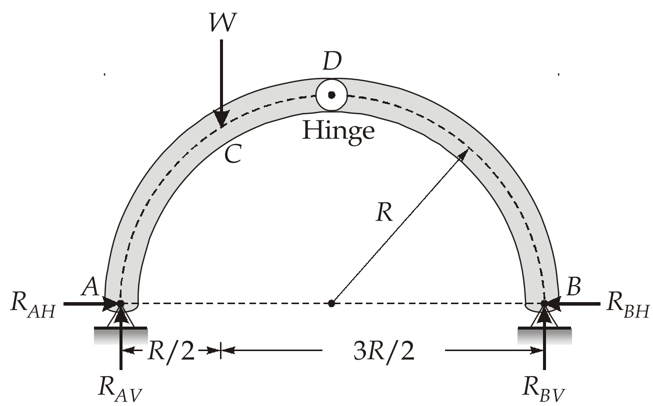
$$= 14.9368 \text{ kJ/K}$$

$\therefore$ $$\Delta s_{univ} = \Delta s_{steel} + \Delta s_{water}$$

$$= -7.92 + 14.9368$$

$$= 7.017 \text{ kJ/K}$$ Ans.

5. (c)



$\Sigma F_V = 0$

$\Rightarrow$ $$R_{AV} + R_{BV} = 0 \quad \text{...(i)}$$

$\Sigma F_H = 0$

$\Rightarrow$ $$R_{AH} - R_{BH} = 0$$

$\Rightarrow$ $$R_{AH} = R_{BH} \quad \text{...(ii)}$$

Taking moment at end A, we get

$\Sigma M_A = 0$

$\Rightarrow$ $$W \times \frac{R}{2} - R_{BV} \times 2R = 0$$

$\Rightarrow$ $$R_{BV} = \frac{W}{4}$$

From equation (i), we get

$\Rightarrow$ $$R_{AV} = W - R_{BV} = W - \frac{W}{4} = \frac{3W}{4}$$

For equilibrium, considering portion ACD only, taking moment about D,

$\Sigma M_D = 0$

$\Rightarrow$ $$R_{AH} \times R + W \times \frac{R}{2} - R_{AV} \times R = 0$$

$$\Rightarrow \qquad R_{AH} = R_{AV} - \frac{W}{2} = \frac{3W}{4} - \frac{W}{2}$$

$$\Rightarrow \qquad R_{AH} = \frac{W}{4}$$

From equation (ii), we get

$$R_{BH} = \frac{W}{4}$$

Resultant reaction forces at A and B are given by

$$R_{A,\text{resultant}} = \sqrt{R_{AV}^2 + R_{AH}^2} = \sqrt{\left(\frac{3W}{4}\right)^2 + \left(\frac{W}{4}\right)^2}$$

$$= \frac{\sqrt{10}}{4}W = 0.79\,\text{W} \qquad \text{Ans.}$$

$$R_{B,\text{resultant}} = \sqrt{R_{BV}^2 + R_{BH}^2} = \sqrt{\left(\frac{W}{4}\right)^2 + \left(\frac{W}{4}\right)^2}$$

$$= \frac{\sqrt{2}}{4}\text{W} = 0.35\,\text{W} \qquad \text{Ans.}$$

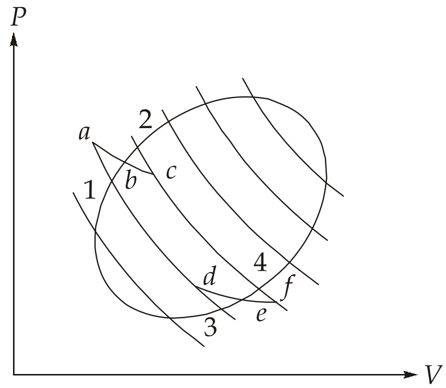
5. (d)

The Clausius inequality states that whenever a system undergoes a cycle, the algebraic sum of the ratios of heat interactions to the temperature at which the heat interaction occurs, over the complete cycle is less than or equal to zero. Stated mathematically $\oint \frac{dQ}{T} \leq 0.$ The equality sign holds good if the cycle consists of only reversible processes.

Consider a system which undergoes the cyclic change as shown in figure below. Now, draw a family of closely spaced adiabatic lines similar to a-3 and 2-f covering the entire cycle. Then join the adjacent adiabatic lines a-3 and 2-f by the isotherms a-b-c and d-e-f as shown in figure, such that the area of 1-a-b = area of b-c-2 and area 3-d-e = area of e-f-4.

Thus the segments 1-b-2 and 3-e-4 of the reversible cycle are replaced by 1-a-b-c-2 and 3-d-e-f-4, respectively. Now the differential cycle a-b-c-4-f-e-d-a is a Carnot cycle. Thus the original cycle can be transformed into a combination of several differential Carnot cycles. We know that for a Carnot cycles $\Sigma \frac{Q}{T} = 0$ or for a differential Carnot cycle $\oint \frac{dQ}{T} = 0.$ Therefore, for the given reversible cycle, we get

$$\oint \frac{dQ}{T} = 0 \qquad \text{For a reversible cycle} \qquad \text{...(i)}$$



Now consider an irreversible cycle. That is, some processes of the cycle may be irreversible. We can follow the above argument and find that the given cycle splits into a large number of differential cycles of which some are irreversible.

For the reversible cycles we have $\oint \frac{dQ}{T} = 0$ and for the irreversible cycles we have $\oint \frac{dQ}{T} < 0$. Then for the original irreversible cycle, we get

$$\oint \frac{dQ}{T} < 0 \qquad \text{For an irreversible cycle} \qquad ...(ii)$$

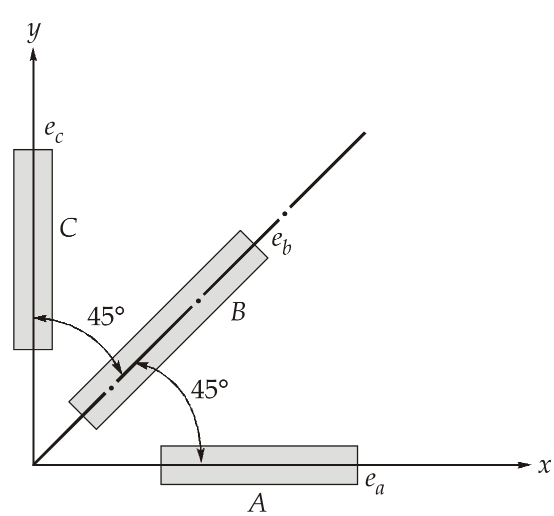
The equations (i) and (ii) can be generalized as

$$\oint \frac{dQ}{T} \leq 0$$

which is the Clausius inequality.

5. (e)

Given : $e_x = 520 \times 10^{-6}$, $e_y = -140 \times 10^{-6}$ and $e_{45} = 270 \times 10^{-6}$



Normal strain at angle θ is given by

$$(e_n)_\theta = \frac{1}{2}(e_x + e_y) + \frac{1}{2}(e_x - e_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

At θ = 0°

$$e_a = (e_n)_{\theta = 0^\circ} = 10^{-6} \times \left[\frac{1}{2}\{520 + (-140)\} + \frac{1}{2}\{520 - (-140)\} \times \cos(2 \times 0^\circ) + \frac{\gamma_{xy}}{2} \times \sin(2 \times 0^\circ)\right]$$

$\Rightarrow$ $$e_a = (e_n)_{\theta = 0^\circ} = 520 \times 10^{-6}$$

At θ = 90°

$$\in_c = (e_n)_{\theta = 90^\circ} = 10^{-6} \times \left[\frac{1}{2}\{520 + (-140)\} + \frac{1}{2}\{520 - (-140)\} \times \cos(2 \times 90^\circ) + \frac{\gamma_{xy}}{2} \times \sin(2 \times 90^\circ)\right]$$

$\Rightarrow$ $$e_c = -140 \times 10^{-6} = (e_n)_{\theta = 90^\circ}$$

At θ = 45°

$$e_{45^\circ} = 270 \times 10^{-6}$$

$$\therefore\ 270 \times 10^{-6} = 10^{-6} \times \left[\frac{1}{2}\{520 + (-140)\} + \frac{1}{2}\{520 - (-140)\}\cos(2 \times 45^\circ) + \frac{\gamma_{xy}}{2}\sin(2 \times 45^\circ)\right]$$

$\Rightarrow$ $$\gamma_{xy} = 160 \times 10^{-6}$$

$$\in_{1,2} = \frac{1}{2}\left[\in_x + \in_y \pm \sqrt{(\in_x - \in_y)^2 + 4 \times \left(\frac{\gamma_{xy}}{2}\right)^2}\right]$$

$$= \frac{10^{-6}}{2} \times \left[\{520 + (-140)\} \pm \sqrt{(520 - (-140))^2 + 4 \times \left(\frac{160}{2}\right)^2}\right]$$

$$= \frac{10^{-6}}{2} \times (380 \pm 679.117) = 529.55 \times 10^{-6}; -149.55 \times 10^{-6}$$

The principal strains are :

$$\in_1 = 529.55 \times 10^{-6} \text{ and } \in_2 = -149.55 \times 10^{-6}$$ **Ans.**

The principal stresses are given by,

$$\sigma_1 = \frac{E(\in_1 + \mu \in_2)}{1 - \mu^2}$$

$$= \frac{205 \times 10^3 (529.55 + 0.32 \times (-149.55)) \times 10^{-6}}{1 - 0.32^2}$$

$$\sigma_1 = 110.01 \text{ MPa}$$ **Ans.**

$$\sigma_2 = \frac{E(\epsilon_2 + \mu \epsilon_1)}{1-\mu^2}$$

$$\sigma_2 = \frac{205\times10^3[(-149.55)+0.32\times529.55]\times10^{-6}}{1-0.32^2}$$

$$\sigma_2 = 4.54 \text{ MPa}$$ Ans.

6. (a)

Given : P_{Ri} = 60 MPa; D_i = 120 mm; R_i = 60 mm; D_j = 180 mm; R_j = 90 mm; D_o = 220 mm; R_o = 110 mm; p_j = 10 MPa

Let i, j and o are the suffix used for inner, junction and outer radius, respectively.

For inner tube : $$p_x = -A + \frac{B}{x^2} \quad ...(1)$$

$$\sigma_{h_x} = A + \frac{B}{x^2} \quad ...(2)$$

For outer tube : $$p_x = -A' + \frac{B'}{x^2} \quad ...(3)$$

$$\sigma_{h_x} = A' + \frac{B'}{x^2} \quad ...(4)$$

(a) Before the fluid is admitted : Inner tube

At $x = R_i$ = 60 mm, $$p_{x_{Ri}} = 0 = -A + \frac{B}{3600} \Rightarrow A = \frac{B}{3600}$$

At $x = R_j$ = 90 mm, $$\sigma_{x_{Rj}} = 10 = \frac{-B}{3600} + \frac{B}{8100}$$

$\Rightarrow$ $$B = -64800$$

$\Rightarrow$ $$A = -\frac{64800}{3600} = -18$$

Hence, $$\sigma_{h,x} = -18 - \frac{64800}{x^2}$$

The minus sign indicates that σ_{hx} will be compressive throughout.

At $x = R_i$ = 60 mm, $$\sigma_{h_{Ri}} = -18 - \frac{64800}{60^2} = -36 \text{ MPa or } 36 \text{ MPa (compressive)}$$

At $x = R_j$ = 90 mm, $$\sigma_{h_{Rj}} = -18 - \frac{64800}{90^2} = -26 \text{ MPa or } 26 \text{ MPa (compressive)}$$

(b) Before the fluid is admitted: Outer tube

At $x = R_o = 110$ mm, $\quad p_{xRo} = 0 \Rightarrow -A' + \frac{B}{110^2} = 0 \Rightarrow A' = \frac{B'}{110^2}$

At $x = R_j = 90$ mm, $\quad p_j = p_{xRj} = 10 = -A' + \frac{B'}{90^2}$

$$\Rightarrow \quad 10 = -\frac{B'}{110^2} + \frac{B'}{90^2}$$

$$\Rightarrow \quad B' = 245025$$

$$\Rightarrow \quad A' = \frac{B'}{110^2} = \frac{245025}{110^2} = 20.25$$

Hence, $\quad \sigma_{h_x} = 20.25 + \frac{245025}{x^2}$

The plus sign indicates that σ_{h_x} will be tensile throughout.

At $x = R_j = 90$ mm $\Rightarrow \sigma_{hRj} = 20.25 + \frac{245025}{90^2} = 50.5$ MPa (Tensile)

At $x = R_o = 110$ mm $\Rightarrow \sigma_{hRo} = 20.25 + \frac{245025}{110^2} = 40.5$ MPa (Tensile)

(c) After the fluid is admitted : Compound tube

From Lame's equation: $\quad P_x = -a + \frac{b}{x^2}$ ...(5)

$$\sigma_{h_x} = a + \frac{b}{x^2} \quad ...(6)$$

At $x = R_o = 110$ mm, $p_x = 0 \Rightarrow -a + \frac{b}{110^2} \Rightarrow a = \frac{b}{110^2}$

At $x = R_i = 60$ mm $p_x = p_{Ri} = 60 \Rightarrow -a + \frac{b}{60^2}$

$$\Rightarrow \quad 60 = \frac{-b}{110^2} + \frac{b}{60^2}$$

$$\Rightarrow \quad b = 307482.35$$

$$\Rightarrow \quad a = \frac{307482.35}{110^2} = 25.41$$

Hence, $\quad \sigma_{h_x} = 25.41 + \frac{307482.35}{x^2}$

At $x = R_i = 60$ mm, $\sigma_{hRi} = 25.41 + \frac{307482.35}{60^2} = 110.82$ MPa (Tensile)

At $x = R_j = 90$ mm, $\sigma_{hRj} = 25.41 + \frac{307482.35}{90^2} = 63.37$ MPa (Tensile)

At $x = R_o = 110$ mm, $\sigma_{hRo} = 25.41 + \frac{307482.35}{110^2} = 50.82$ MPa (Tensile)

For inner tube : Final hoop stress

$$(\sigma_h)_{R_{i,f}} = -36 + 110.82 = 74.82 \text{ MPa}$$ Ans.

$$(\sigma_h)_{R_{0,f}} = -26 + 63.37 = 37.37 \text{ MPa}$$

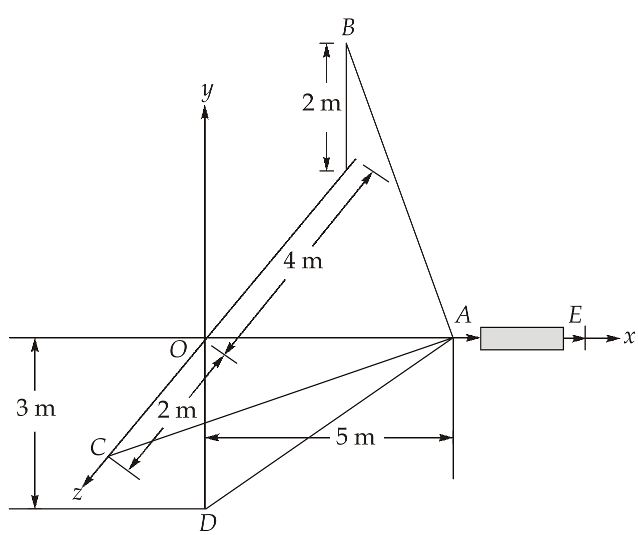
For Outer tube : Final hoop stress

$$(\sigma_h)_{R_{i,f}} = 50.5 + 63.37 = 113.87 \text{ MPa}$$

$$(\sigma_h)_{R_{o,f}} = 40.5 + 50.82 = 91.32 \text{ MPa}$$ Ans.

6. (b)

The coordinates of points A, B, C and D are $A(5, 0, 0)$, $B(0, 2, -4)$, $C(0, 0, 2)$ and $D(0, -3, 0)$



$$\vec{r}_{AB} = -5\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{r}_{AC} = -5\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\vec{r}_{AD} = -5\hat{i} - 3\hat{j} + 0\hat{k}$$

$\therefore$ $$\vec{r}_{AB} = \sqrt{(-5)^2 + (2)^2 + (-4)^2} = 3\sqrt{5} \text{ m} = 6.708 \text{ m}$$

∴ Unit vector in the direction AB,

$$n_1 = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{-5\hat{i} + 2\hat{j} - 4\hat{k}}{6.708}$$

$$n_1 = -0.745\hat{i} + 0.298\hat{j} - 0.596\hat{k}$$

∴ $$F_1 = F_1 \cdot n_1 = -0.745F_1\hat{i} + 0.298F_1\hat{j} + (-0.596)F_1\hat{k}$$

Similarly, $$r_{AC} = \sqrt{(-5)^2 + (0)^2 + (2)^2} = \sqrt{29}\text{ m} = 5.385\text{ m}$$

$$n_2 = \frac{\vec{r}_{AC}}{r_{AC}} = \frac{-5\hat{i} + 2\hat{k}}{5.385}$$

$$n_2 = -0.929\,\hat{i} + 0\,\hat{j} + 0.371\,\hat{k}$$

∴ $$F_2 = -0.929F_2\,\hat{i} + 0.371F_2\,\hat{k}$$

and $$r_{AD} = \sqrt{(-5)^2 + (-3)^2 + (0)^2} = \sqrt{34}\text{ m} = 5.831\text{ m}$$

$$n_3 = \frac{\vec{r}_{AD}}{r_{AD}} = \frac{-5\hat{i} - 3\hat{j}}{5.831}$$

∴ $$n_3 = -0.857\,\hat{i} - 0.514\,\hat{j}$$

and $$F_3 = -0.857F_3\,\hat{i} - 0.514\,F_3\,\hat{j}$$

Let the force applied on the turn buckle AE be F_4.

Then, $$F_4 = 800\,\hat{i}$$

From equilibrium condition, we have

$$F_1 + F_2 + F_3 + F_4 = 0$$

$$\Rightarrow (-0.745F_1 - 0.929F_2 - 0.857F_3 + 800)\,\hat{i} + (0.298F_1 - 0.514F_3)\,\hat{j} + (-0.596F_1 - 0.371F_2)\,\hat{k} = 0$$

$$\Rightarrow -0.745F_1 - 0.929F_2 - 0.857F_3 + 800 = 0 \quad \text{...(i)}$$

$$\Rightarrow 0.298F_1 - 0.514F_3 = 0$$

$$\Rightarrow F_3 = 0.5797 \times F_1 \quad \text{...(ii)}$$

$$\Rightarrow -0.596 \times F_1 + 0.371F_2 = 0$$

$$\Rightarrow F_2 = 1.6064 \times F_1 \quad \text{...(iii)}$$

From equation (i), (ii) and (iii), we get

$$-0.745F_1 - 0.929 \times 1.6064 \times F_1 - 0.857 \times 0.5797 \times F_1 + 800 = 0$$

$\Rightarrow$ $F_1 = 292.59$ kN **Ans.**

$F_2 = 1.6064 \times 292.59$

$\Rightarrow$ $F_2 = 470.01$ kN **Ans.**

$\Rightarrow$ $F_3 = 0.5797 \times 292.59$

$F_3 = 169.61$ kN **Ans.**

6. (c)(i)

Suppose a system follows the path 1-a-2 in reaching the final state 2 starting from the initial state 1. Then the system can be restored to the initial state 1 either by the path 2-b-1 or 2-c-1 as shown in figure below.

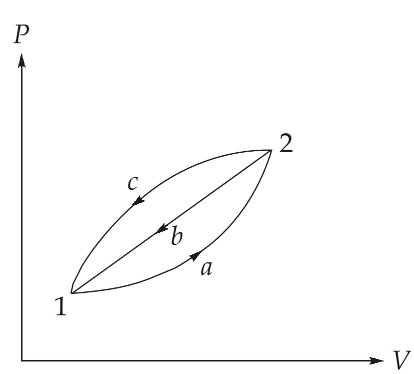
Then the combination of the processes leads to two different cycles, namely 1-a-2-b-1 and 1-a-2-c-1. Applying the first law of thermodynamics to the cycles 1-a-2-b-1 and 1-a-2-c-1, we get

$$\int_{1a2} dQ + \int_{2b1} dQ - \int_{1a2} dW - \int_{2b1} dW = 0 \quad \text{...(A)}$$

$$\int_{1a2} dQ + \int_{2c1} dQ - \int_{1a2} dW - \int_{2c1} dW = 0 \quad \text{...(B)}$$

Subtracting equation (B) from equation (A), we get

$$\int_{2b1} dQ - \int_{2c1} dQ - \int_{2b1} dW - \int_{2c1} dW = 0 \quad \text{...(C)}$$



We know that work interaction is a path function $\int dW$ represents the area under the P-V curve. Hence

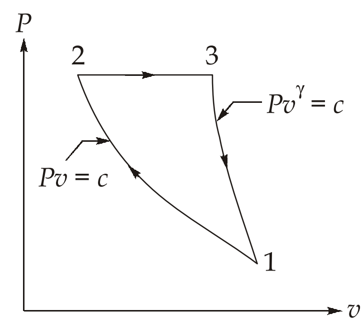
$$\int_{2b1} dW - \int_{2c1} dW \neq 0 \quad ...(D)$$

From equation (C) and (D), we get

$$\int_{2b1} dQ - \int_{2c1} dQ \neq 0 \text{ or } \int_{2b1} dQ \neq \int_{2c1} dQ$$

That is, the heat interaction along the path 2-b-1 is different from the heat interaction along the path 2-c-1. Thus, heat interaction depends on the path followed by a system and heat interaction is a path function. It is not a point function and hence its differential is not exact. Heat interaction is not a property of a thermodynamic system.

6. (c)(ii)



Given :

State 1 : $P_1 = 100$ kPa; $T_1 = 300$ K

State 2 : $T_2 = T_1$, $v_2 = \frac{1}{5}v_1$

State 3 : $P_3 = P_2$, $s_3 = s_1$

$$R = 0.5 \text{ kJ/kgK}; c_p = 1.25 \text{ kJ/kgK}$$

$\therefore$
$$c_v = c_p - R = 1.25 - 0.5 = 0.75 \text{ kJ/kgK}$$

and
$$\gamma = \frac{c_p}{c_v} = \frac{1.25}{0.75} = 1.667$$

Now,
$$w_{1\text{-}2} = p_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = RT_1 \ln\left(\frac{v_2}{v_1}\right)$$

$$w_{1\text{-}2} = 0.5 \times 300 \ln\left(\frac{1}{5}\right)$$

$$w_{1\text{-}2} = -241.415 \text{ kJ/kg}$$

Now, maximum pressure in the cycle,

$$p_2 = p_3 = \frac{p_1 v_1}{v_2} = 1 \times 5 = 5 \text{ bar} \quad \text{Ans.}$$

Now considering process 3 - 1

$$T_3 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \because P_3 = P_2$$

$$\therefore \quad T_3 = 300(5)^{\frac{1.667-1}{1.667}} = 571.2 \text{ K} \quad \text{Ans.}$$

For constant pressure process,

$$w_{2\text{-}3} = p_2(v_3 - v_2) = R(T_3 - T_1) \quad \because \quad T_2 = T_1$$

$$w_{2\text{-}3} = 0.5(571.2 - 300) = 135.6 \text{ kJ/kg}$$

and

$$w_{3\text{-}1} = \frac{R(T_1 - T_3)}{1-\gamma} = \frac{0.5(300 - 571.2)}{1 - 1.667}$$

$$w_{3\text{-}1} = 203.3 \text{ kJ/kg}$$

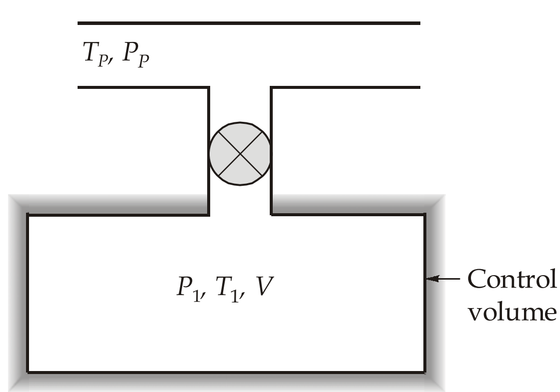
∴ Net work transfer in the cycle

$$w_{net} = -241.415 + 135.6 + 203.3$$

$$= 97.485 \text{ kJ/kg} \quad \text{Ans.}$$

7. (a)

Assuming the tank as the control volume and apply the Ist law for the charging of a tank to obtain,



$$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_i \quad \text{...(a)}$$

where, m_2 = Mass of gas in the tank at the end of filling operation, m_1 = Initial mass of gas in the tank

Equation (a) can be rewritten as,

$$\frac{P_2V}{RT_2}\times c_v \times T_2 - \frac{P_1V}{RT_1}\times c_v\times T_1 = \left(\frac{P_2V}{RT_2}-\frac{P_1V}{RT_1}\right)\times c_p \times T_p$$

or $$(P_2 - P_1)c_v = \left(\frac{P_2}{T_2}-\frac{P_1}{T_1}\right)\times c_p\times T_p$$

or $$(P_2 - P_1) = \gamma\times T_p\left(\frac{P_2}{T_2}-\frac{P_1}{T_1}\right)$$

where P_1 and P_2 are the initial and final pressure of gas in the tank, respectively. V is the volume of the tank and T_p is the temperature of the gas in the supply line.

Substituting the values of P_1, P_2, T_1 and T_p, we get

$$1.4\times 550\left(\frac{25}{T_2}-\frac{1}{300}\right) = 25 - 1$$

or $$\frac{25}{T_2}-\frac{1}{300} = \frac{24}{1.4\times 550}$$

or $$\frac{25}{T_2} = \frac{24}{1.4\times 550}+\frac{1}{300}$$

$\therefore$ $$T_2 = 724.6 \text{ K}$$ **Ans.**

$\therefore$ Amount of gas that entered the tank,

$$\Delta m = m_2 - m_1$$

$$= \left(\frac{P_2}{T_2}-\frac{P_1}{T_1}\right)\times\frac{V}{R} = \left(\frac{25}{724.6}-\frac{1}{320}\right)\times 100\times\frac{2.5}{0.287}$$

$\therefore$ $$\Delta m = 27.33 \text{ kg}$$ **Ans.**

7. (b) (i)

The given load on bracket CDE can be taken as an equivalent, point load of $2P$ at point C and clockwise moment of $\frac{PL}{2}$ at point C.

$\Sigma F_V = 0$

$\Rightarrow$ $$R_A + R_B = 2P \quad \text{...(i)}$$

Taking moment about B,

$\Sigma M_B = 0$

$\Rightarrow$ $$R_A\times L - 2P\times\frac{3L}{4}+\frac{PL}{2} = 0$$

$\Rightarrow \qquad R_A = P(\uparrow)$

Hence, from equation (i), $R_B = P(\uparrow)$

S.F.D. for AC,

$$F_x = P \text{ (constant)}$$

$$F_A = F_C\text{(left)} = P$$

For CB,

$$F_x = P - 2P = -P \text{ (constant)}$$

$$F_C\text{(right)} = F_C = -P$$

B.M.D. : For AC,

$$M_x = R_A \times x = P.x \text{ (linear variation)}$$

At $x = 0$, $M_A = 0$

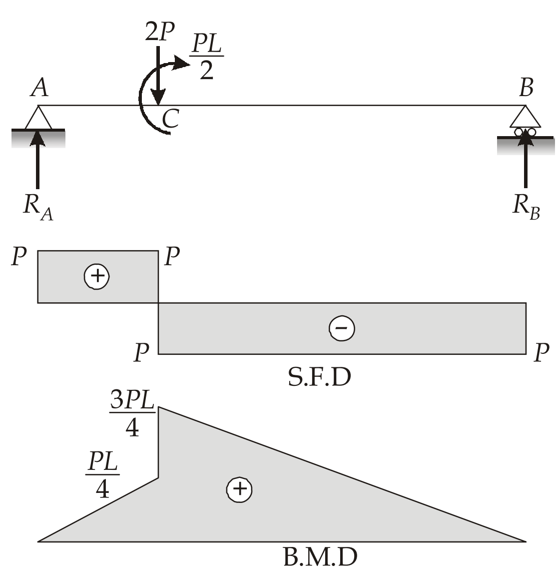
At $x = \frac{L}{4}$, $\qquad M_C\text{(left)} = \frac{PL}{4}$

For CB, $\qquad M_x = P.x + \frac{PL}{2} - 2P \times \left(x - \frac{L}{4}\right) \qquad$ (Linear variation)

At $x = \frac{L}{4}$, $\qquad M_C\text{(right)} = \frac{PL}{4} + \frac{PL}{2} = \frac{3PL}{4}$

At $x = 0$, $\qquad M_B = PL + \frac{PL}{2} - 2P \times \left(L - \frac{L}{4}\right) = 0$

S.F.D. and B.M.D. of the beam are shown below:



7. (b) (ii)

Let suffix 1 is used for steel rod and suffix 2 for brass rod.

Given : A_1 = 2500 mm^2; L_1 = 500 mm; E_1 = 200 GPa; A_2 = 1500 mm^2; L_2 = 400 mm; E_2 = 100 GPa; P = 800 kN

From the equilibrium of the system,

$$P_1 + 2P_2 = P$$

$$\Rightarrow \quad \sigma_1 A_1 + 2\sigma_2 A_2 = P$$

$$\Rightarrow \quad \sigma_1 \times 2500 + 2 \times 1500 \times \sigma_2 = 800000$$

$$\Rightarrow \quad 2500\sigma_1 + 3000\sigma_2 = 800000$$

$$\Rightarrow \quad \sigma_1 + 1.2\sigma_2 = 320 \qquad ...(1)$$

Also, the deformation in rods to be same,

$$\therefore \quad \Delta_1 = \Delta_2$$

$$\Rightarrow \quad \frac{\sigma_1 L_1}{E_1} = \frac{\sigma_2 L_2}{E_2}$$

$$\Rightarrow \quad \frac{\sigma_1 \times 500}{200 \times 10^3} = \frac{\sigma_2 \times 400}{100 \times 10^3}$$

$$\Rightarrow \quad \sigma_1 = 1.6\sigma_2 \qquad ...(2)$$

From equation (1) and (2), we get

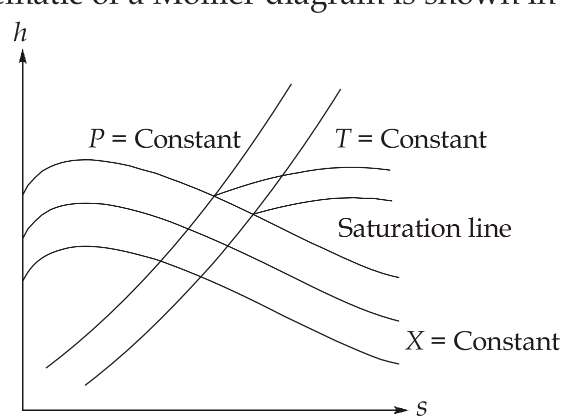
$$1.6\sigma_2 + 1.2\sigma_2 = 320$$

$$\Rightarrow \quad \sigma_2 = \frac{320}{2.8} = 114.28 \text{ MPa} \qquad \text{Ans.}$$

$$\therefore \quad \sigma_1 = 1.6 \times \sigma_2 = 1.6 \times 114.28 = 182.86 \text{ MPa} \qquad \text{Ans.}$$

7. (c) (i)

The Mollier diagram shows the two phase region and the vapor region. It contains constant pressure lines, constant temperature lines and also the constant quality lines in the two phase region. A schematic of a Mollier diagram is shown in the figure below.



Schematic of a Mollier diagram

It may be noted that

1. The constant temperature lines and constant pressure lines coincide in the two phase region.
2. The constant temperature and constant pressure lines are straight in the two phase region.
3. The constant temperature lines are usually curved in the superheat region but at increased temperatures they tend to become horizontal as the gas behaviour approaches that of an ideal gas [$h = f(T)$ only].
4. The constant pressure lines are usually curved.
5. In general or in most of the diagrams, the constant volume lines may not be shown.
6. The saturation line corresponds to quality (X) is equal to one.
7. The constant quality lines are usually marked as constant dryness fraction lines and they are approximately parallel to the saturation line.

7. (c) (ii)

From steam tables, we have

At 230°C; $P_1 = 2.7971$ MPa; $v_{f_1} = 0.00120902\ \text{m}^3/\text{kg}$

$v_{g_1} = 0.071503\ \text{m}^3/\text{kg}$; $u_1 = u_{g_1} = 2602.9\ \text{m}^3/\text{kg}$

At 190°C; $P_2 = 1.2552$ MPa; $v_{f_2} = 0.00114145\ \text{m}^3/\text{kg}$

$v_{g_2} = 0.15636\ \text{m}^3/\text{kg}$; $u_{f_2} = 806$ kJ/kg; $u_{g_2} = 2589$ kJ/kg

Since the tank is rigid and sealed, the mass of steam and the total volume remains constant.

Therefore, the specific volume $\left(v = \dfrac{V}{m}\right)$ also remains constant. Hence,

$$v_2 = v_g = v_1 = 0.071503$$

or $$v_{f2} + x_2(v_{g2} - v_{f2}) = 0.071503$$

$$0.00114145 + x_2(0.15636 - 0.00114145) = 0.071503$$

$\therefore$ $$x_2 = \frac{0.071503 - 0.00114145}{0.15636 - 0.00114145} = 0.4533 \quad \text{Ans.}$$

$\therefore$ Internal energy at final state,

$$u_2 = u_{f2} + xu_{fg2}$$

$$u_2 = 806 + 0.4533(2589 - 806)$$

$$= 1614.234\ \text{kJ/kg} \quad \text{Ans.}$$

Mass of steam in the tank, $m = \frac{V}{\nu} = \frac{1}{0.071503} = 13.985$ kg

For a constant volume process, energy transferred as heat is

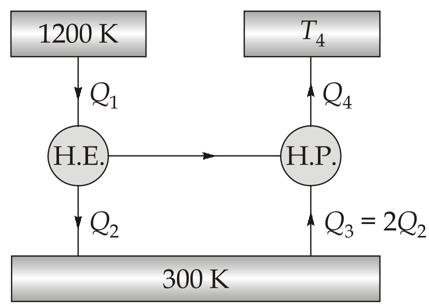
$$Q = m(u_2 - u_1)$$

$$= 13.985(1614.234 - 2602.9) = -13826.49 \text{ kJ}$$

or $Q = 13826.49$ kJ (Rejected)

8. (a)

Given data :



Now, $\eta_E = 0.4 \times \eta_{Carnot}$

and $(COP)_{HP} = 0.6 \times (COP)_{rev}$

$$\eta_E = \left(1 - \frac{300}{1200}\right) \times 0.4 = 0.3$$

or, $W = 0.3Q_1$ ...(i)

Similarly,

$$(COP)_{HP} = \left(\frac{T_4}{T_4 - T_3}\right) \times 0.6 = \frac{0.6T_4}{T_4 - 300}$$

or $$\frac{Q_4}{W} = \frac{W + Q_3}{W} = 1 + \frac{Q_3}{W} = \frac{0.6T_4}{T_4 - 300}$$

$\therefore$ $$W = \frac{Q_3(T_4 - 300)}{(300 - 0.4T_4)}$$ ...(ii)

$\because$ $Q_3 = 2Q_2$ (Given)

and from heat engine balance.

$$Q_1 = W + Q_2$$

$\therefore$ $Q_2 = Q_1 - W = Q_1 - 0.3Q_1 = 0.7Q_1$

$\therefore$ $Q_3 = 2 \times 0.7Q_1 = 1.4Q_1$ ...(iii)

∴ From equation (i), (ii) and (iii), we get

$$0.3Q_1 = \frac{1.4Q_1(T_4 - 300)}{(300 - 0.4T_4)}$$

or $$90 - 0.12T_4 = 1.4T_4 - 420$$

or $$T_4 = 335.52 \text{ K}$$ Ans. (i)

Further, $$(\text{COP})_{\text{H.P.}} = \frac{Q_4}{W} = \frac{T_4}{T_4 - 300} \times 0.6$$

or $$Q_4 = \frac{335.52}{335.52 - 300} \times 0.6 \times 0.3 \times 100$$

[∵ $W = 0.3 \times Q_1 = 0.3 \times 100$ from equation (i)]

∴ $$Q_4 = 170 \text{ kW}$$ Ans. (ii)

8. (b)

Given : σ_x = 135 MPa; σ_y = 70 MPa; τ_{xy} = 90 MPa

(i) Angle of principal plane from vertical plane,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 90}{135 - 70}$$

⇒ $$\theta_p = 35.07°, 125.07°$$

$$(\sigma_n)_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y) \cdot \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$(\sigma_n)_{\theta = 35.07°} = \frac{1}{2}(135 + 70) + \frac{1}{2}(135 - 70) \cdot \cos(2 \times 35.07) + 90 \times \sin(2 \times 35.07)$$

$$= 198.19 \text{ MPa}$$

$$(\sigma_n)_{\theta = 125.07°} = \frac{1}{2}(135 + 70) + \frac{1}{2}(135 - 70) \cdot \cos(2 \times 125.07) + 90 \times \sin(2 \times 125.07)$$

$$= 6.81 \text{ MPa}$$ Ans.

The magnitude and direction of major principal stress is 198.19 MPa at 35.07°, and that of minor principal stress is 6.81 MPa at 125.07°.

(ii) Angle of maximum shear stress,

$$\tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = \frac{70 - 135}{2 \times 90}$$

$$\theta = -9.93°, 80.07°$$ Ans.

∴ Maximum shear stress, $$(\tau_s)_\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$(\tau_s)_{\theta = -9.93°} = -\frac{1}{2}(135-70)\sin(2\times(-9.93)) + 90\times\cos(2\times(-9.93))$$
$$= 95.69 \text{ MPa} \qquad \text{Ans.}$$

$$(\tau_s)_{\theta = 80.07°} = -\frac{1}{2}(135-70)\sin(2\times 80.07) + 90\times\cos(2\times 80.07)$$
$$= -95.69 \text{ MPa} \qquad \text{Ans.}$$

(iii)

Angle of inclination from x-phase is $\theta = 60°$.

$$(\sigma_n)_{\theta = 60°} = \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

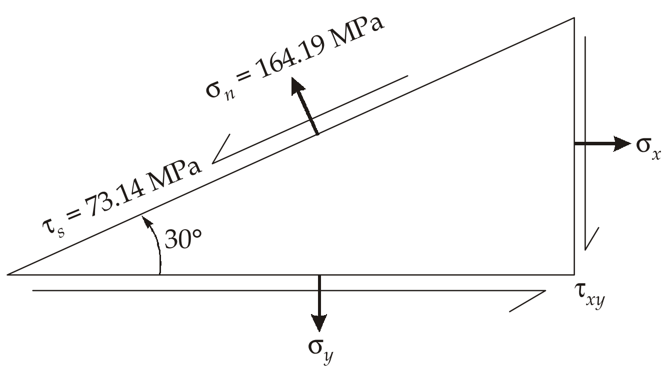
$$(\sigma_n)_{\theta = 60°} = \frac{1}{2}(135+70) + \frac{1}{2}(135-70)\cos(2\times 60) + 90\times\sin(2\times 60)$$

$$(\sigma_n)_{\theta = 60°} = 164.19 \text{ MPa} \qquad \text{Ans.}$$

$$(\tau_s)_{\theta = 60°} = -\frac{1}{2}\left(\sigma_x-\sigma_y\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$(\tau_s)_{\theta = 60°} = -\frac{1}{2}(135-70)\sin(2\times 60) + 90\cos(2\times 60°)$$

$$(\tau_s)_{\theta = 60°} = -73.14 \text{ MPa} \qquad \text{Ans.}$$



8. (c)

Vessel 'A'

$P_1 = 0.4$ MPa; $T_1 = 85°\text{C} = 358$ K; $V_1 = 2\ \text{m}^3$

Vessel 'B'

$P_1 = 0.2$ MPa; $T_2 = 220°\text{C} = 493$ K; $V_2 = 3.5\ \text{m}^3$

Now for vessel 'A'

$$m_A = \frac{P_1V_1}{RT_1} = \frac{0.4\times 10^3\times 2}{0.287\times 358} = 7.786 \text{ kg}$$

Similarly for vessel 'B'

$$m_B = \frac{P_2V_2}{RT_2} = \frac{0.2\times10^3\times3.5}{0.287\times493} = 4.947 \text{ kg}$$

Let T_f and P_f are the final temperature and pressure after mixing the air of two vessel.
Now, from 1st law, we have

$$dU = 0 \qquad (\because \quad Q = 0; W = 0)$$

or $\quad m_Ac_vT_1 + m_Bc_vT_2 = (m_A + m_B)c_vT_f$

$$\therefore \quad T_f = \frac{m_AT_1 + m_BT_2}{m_A + m_B} = \frac{7.786\times358 + 4.947\times493}{7.786 + 4.947}$$

$$T_f = 410.45 \text{ K}$$

Total volume after mixing,

$$V_f = V_1 + V_2 = 2 + 3.5 = 5.5 \text{ m}^3$$

Now applying, $\quad P_f V_f = mRT_f$

$$\therefore \quad P_f = \frac{mRT_f}{V_f} = \frac{12.733\times0.287\times410.45}{5.5}$$

$$P_f = 272.71 \text{ kPa}$$

Now, change in entropy of vessel A,

$$\Delta S_A = m_A\left[c_p\ln\left(\frac{T_f}{T_1}\right) - R\ln\left(\frac{P_f}{P_1}\right)\right]$$

$$= 7.786\left[1.005\ln\left(\frac{410.45}{358}\right) - 0.287\ln\left(\frac{272.71}{0.4\times10^3}\right)\right]$$

$$= 1.9258 \text{ kJ/K}$$

Similarly change in entropy of air of vessel 'B'

$$\Delta S_B = m_B\left[c_p\ln\left(\frac{T_f}{T_2}\right) - R\ln\left(\frac{P_f}{P_2}\right)\right]$$

$$= 4.947\left[1.005\ln\left(\frac{410.45}{493}\right) - 0.287\ln\left(\frac{272.71}{0.2\times10^3}\right)\right]$$

$$= -1.3513 \text{ kJ/K}$$

Hence, total change in entropy,

$$\Delta S_T = \Delta S_A + \Delta S_B$$

$$= 1.9258 + (-1.3513)$$

[ΔS_{surr} = 0, since the system is insulated]

$$\Delta S_T = 0.5745 \text{ kJ/K}$$ Ans.

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