CIVIL ENGINEERING

Elastic Curve

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Elastic Curve

EDITIONS

First Edition: 2015
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Fourth Edition: 2018
Fifth Edition: 2019
Sixth Edition: 2020
Seventh Edition: 2021
Eighth Edition: 2022

Ninth Edition: 2023

Elastic Curve

CONTENTS

Why Elastic Curve?

- To get better understanding about the behavior of structures subjected to loading.
- To solve the questions directly related to elastic curve.
- To determine degree of freedom and kinematic indeterminacy.
- To calculate the deflections at different points without any mathematical calculation.
- To draw the influence line diagram using Muller Breslau's principle.
- To check the correctness of directions of support reaction, which is calculated using mathematical analysis.
- To check the correctness of bending moment diagram.

How to draw Elastic curve?

There are only three simple steps to draw the elastic curve correctly.

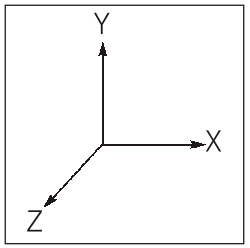
Step 1: By visual inspection

Step 2: By satisfying compatibility conditions

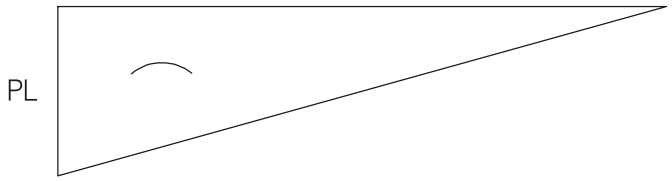
Step 3: By making elastic curve and bending moment diagram consistent

Assumptions

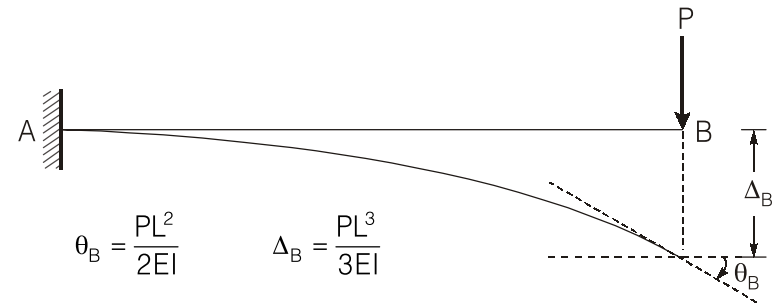
- Members of structure are linearly elastic.
- Structure is subjected to small displacement (deflection or rotation) under given loading.
- All members are axially inextensible unless mentioned.



EXAMPLE : 1



Bending Moment Diagram



Elastic Curve

DISCUSSION

Step 1: By visual inspection

Definitely, you can draw the elastic curve of the given structure by visual inspection only and that is correct. Then also, other steps are being discussed here to check the correctness of elastic curve. By visual inspection, it is evident that deflected shape will be hogging and beam is going downward.

Step 2: By satisfying compatibility conditions.

Compatibility conditions at A:

$\Delta_x = 0 \quad \Delta_y = 0 \quad \theta_A = 0$

Compatibility condition at B:

$\Delta_x = 0$ (axially inextensible) $\quad \Delta_y \neq 0 \quad \theta_B \neq 0$

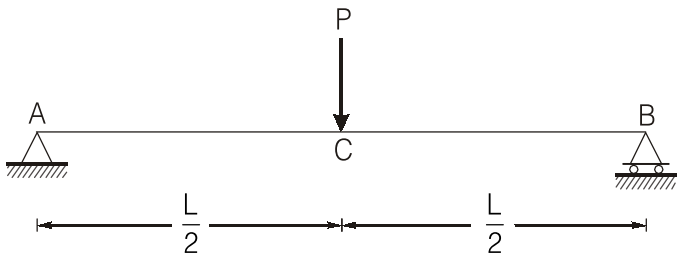
Point B will be just perpendicularly below to the member AB because structure is subjected to small displacement only.

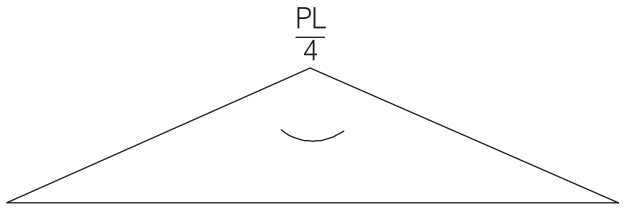
Note: Members always deflect in the perpendicular direction to its longitudinal axis.

Step 3: By making elastic curve and bending moment diagram consistent.

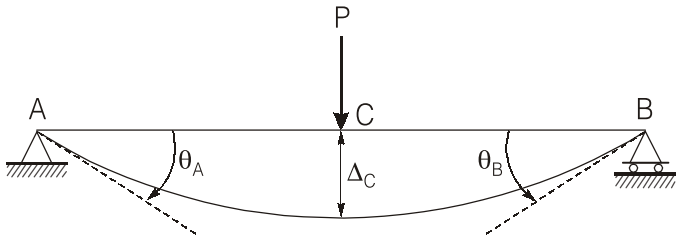
From bending moment diagram, it is clear that the entire span is under hogging shape which is clear from the deflected shape also. It means bending moment diagram and elastic curve is consistent.

EXAMPLE : 2





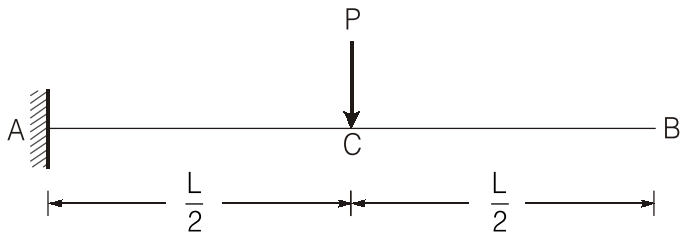
Bending Moment Diagram

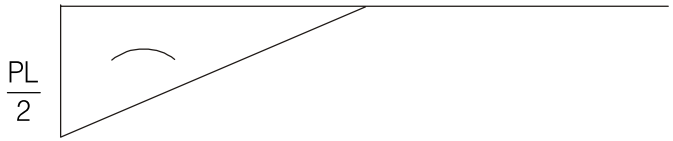


$$\theta_A = \theta_B = \frac{PL^2}{16EI} \qquad \Delta_C = \frac{PL^3}{48EI}$$

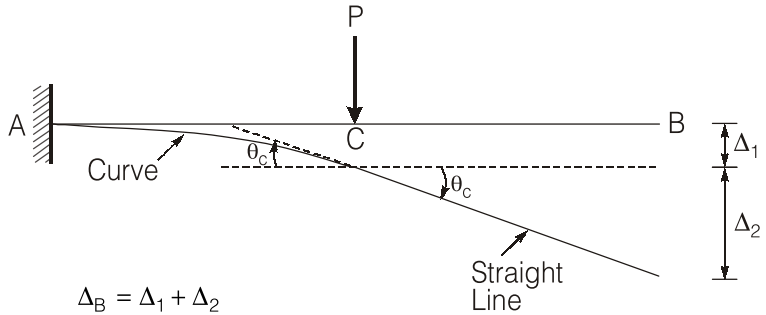
Elastic Curve

EXAMPLE : 3





Bending Moment Diagram



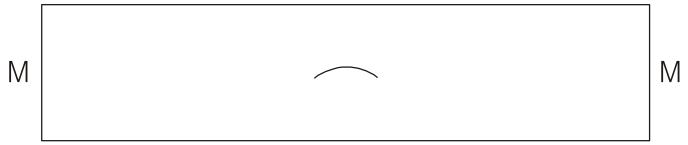
$$\Delta_B = \Delta_1 + \Delta_2$$

$$= \frac{P(L/2)^3}{3EI} + \theta_C \times \frac{L}{2} \qquad \left[\theta_C = \frac{P(L/2)^2}{2EI}\right]$$

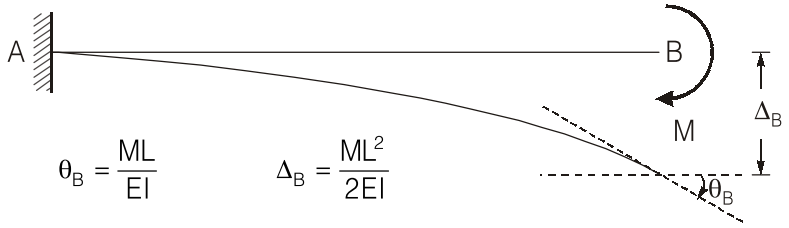
$$= \frac{5PL^3}{48EI}$$

Elastic Curve

EXAMPLE : 4



Bending Moment Diagram



$$\theta_B = \frac{ML}{EI} \qquad \Delta_B = \frac{ML^2}{2EI}$$

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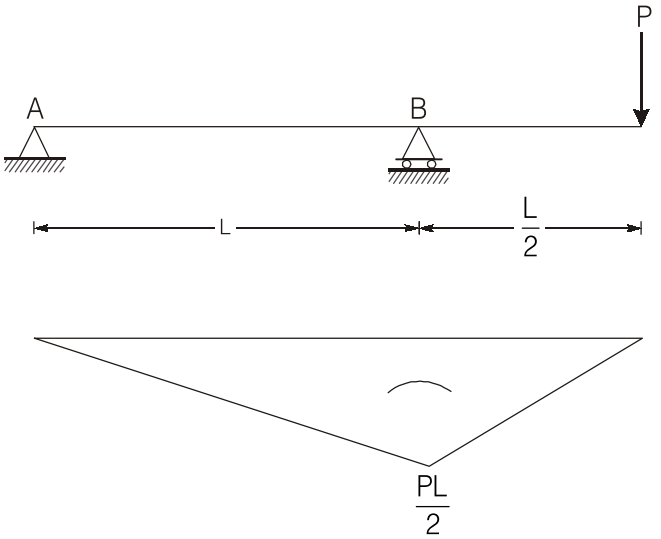
EXAMPLE : 5



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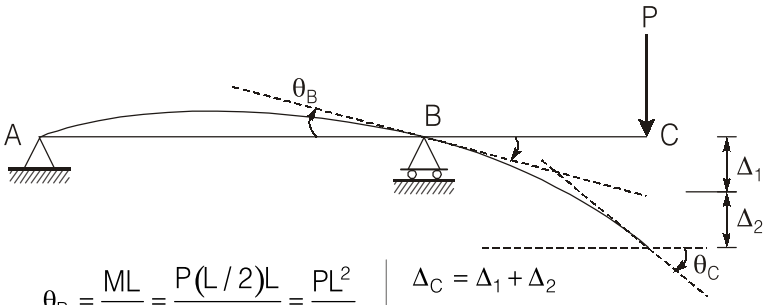


$$\theta_B = \frac{ML}{3EI} = \frac{P(L/2)L}{3EI} = \frac{PL^2}{6EI}$$

$$\theta_C = \theta_B + \frac{P(L/2)^2}{2EI} = \frac{7}{24}\frac{PL^3}{EI}$$

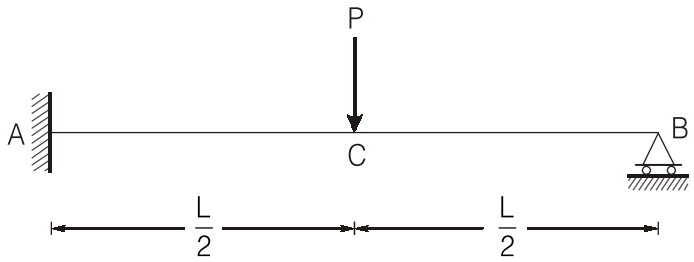
$$\Delta_C = \Delta_1 + \Delta_2$$

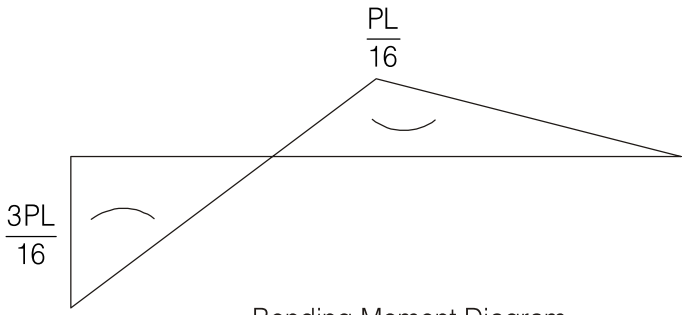
$$= \theta_B \times \frac{L}{2} + \frac{P(L/2)^3}{3EI}$$

$$= \frac{PL^3}{8EI}$$

Elastic Curve

EXAMPLE : 6





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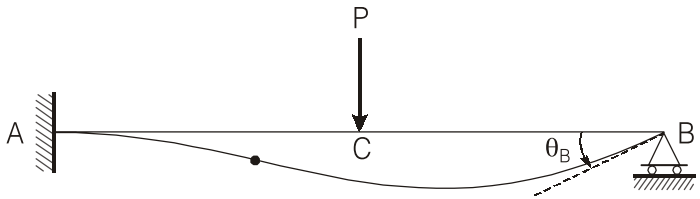

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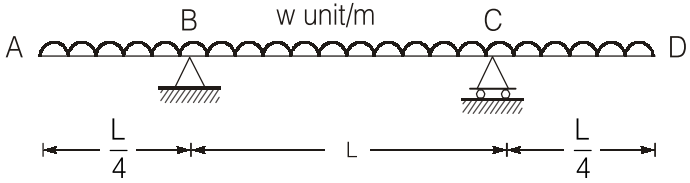
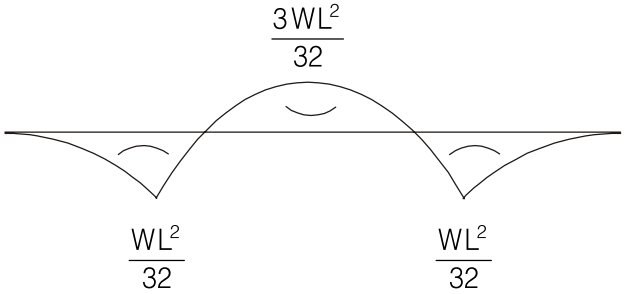
EXAMPLE : 7





Bending Moment Diagram

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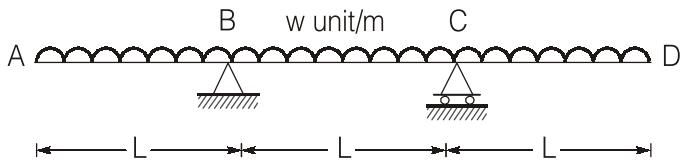
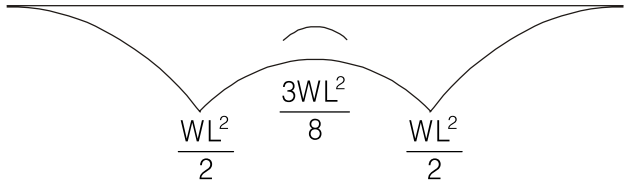
EXAMPLE : 8





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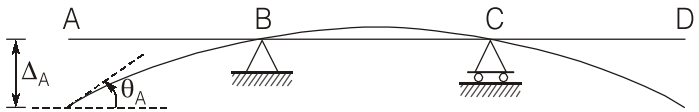

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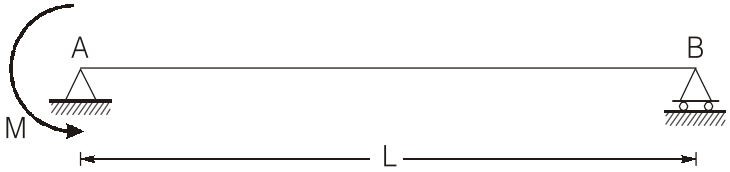
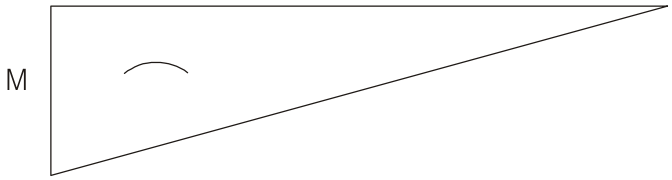
EXAMPLE : 9





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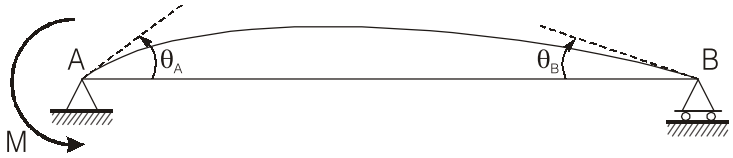

$$\theta_A = \frac{ML}{3EI} \qquad \theta_B = \frac{ML}{6EI}$$

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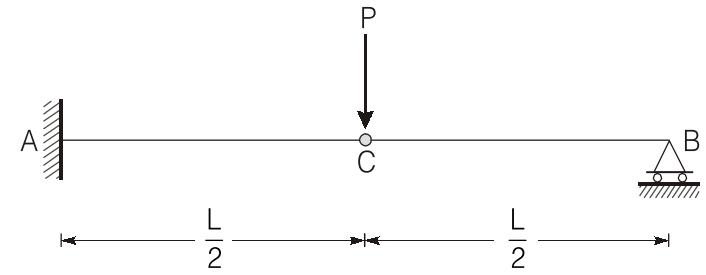
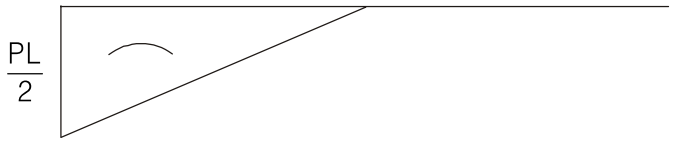
EXAMPLE : 10





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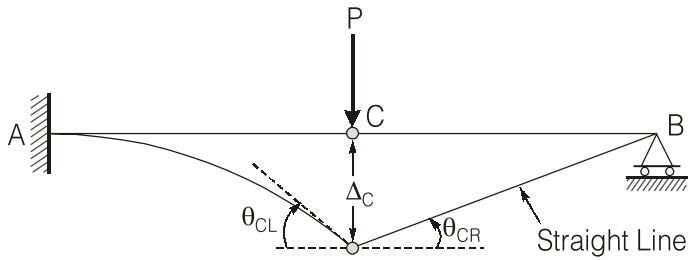

$$\theta_{CL} = \frac{P(L/2)^2}{2EI} \qquad \Delta_C = \frac{P(L/2)^3}{3EI} \qquad \theta_{CR} = \frac{\Delta_C}{L/2}$$

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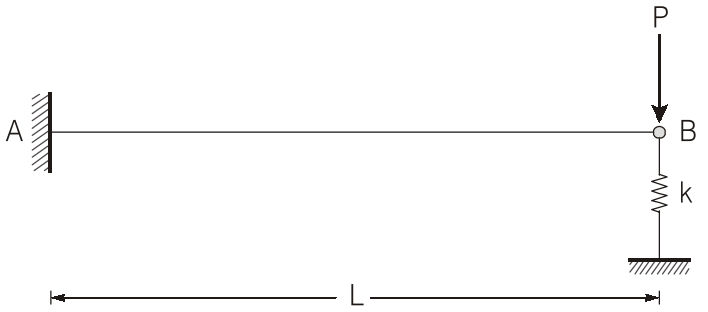
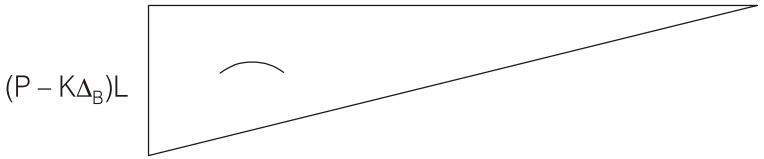
EXAMPLE : 11





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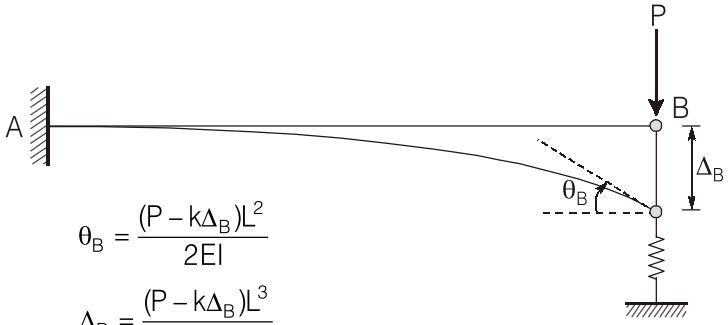
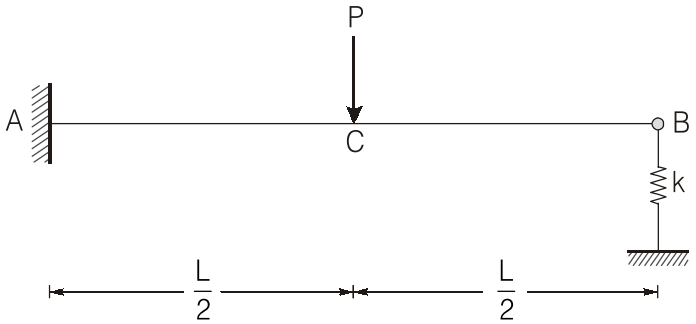


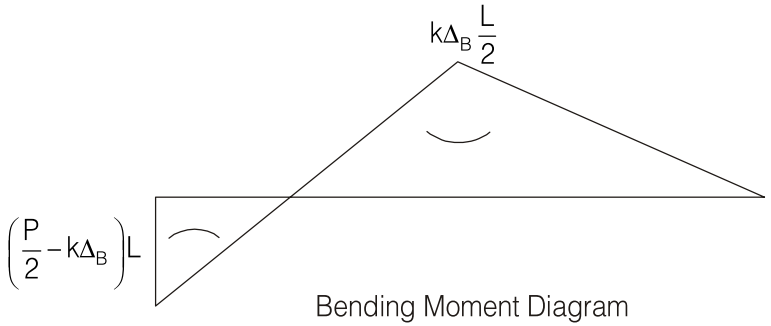
$$\theta_B = \frac{(P - k\Delta_B)L^2}{2EI}$$

$$\Delta_B = \frac{(P - k\Delta_B)L^3}{3EI}$$

Elastic Curve

EXAMPLE : 12





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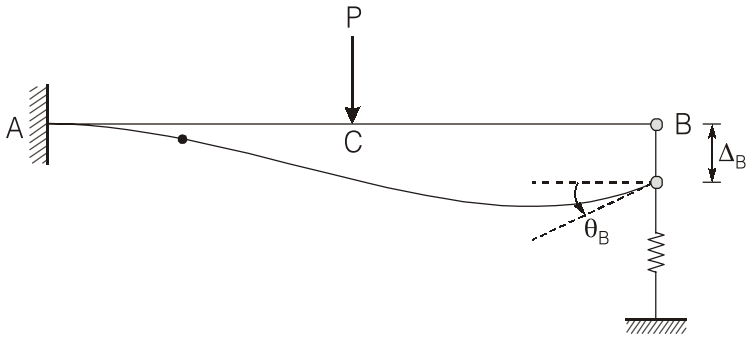

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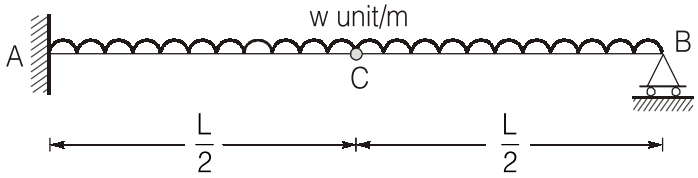
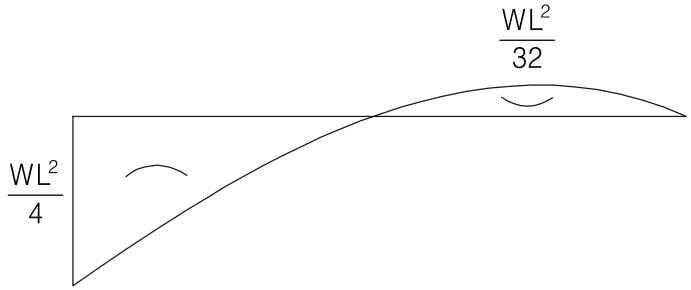
EXAMPLE : 13





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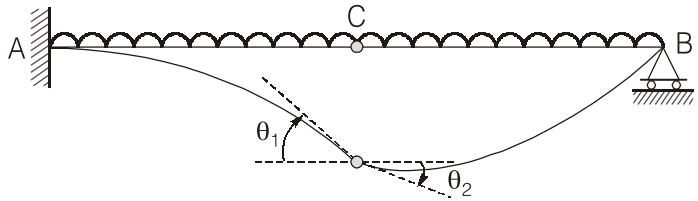
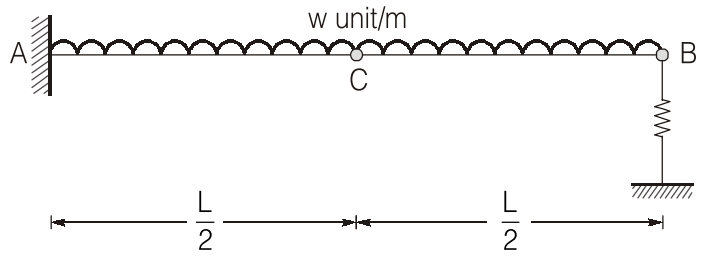
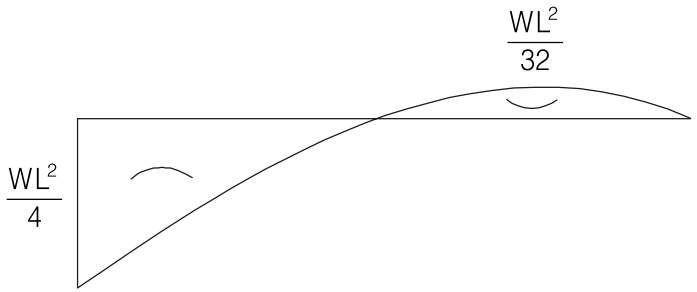


$\theta_1 \neq \theta_2$

Elastic Curve

EXAMPLE : 14





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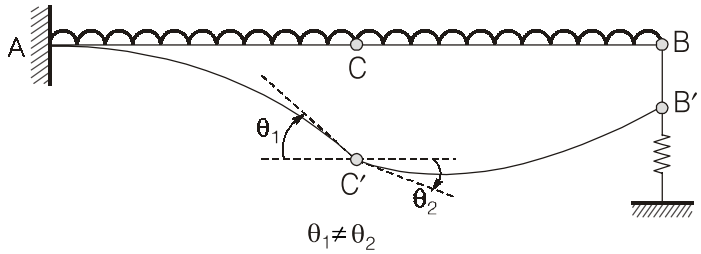

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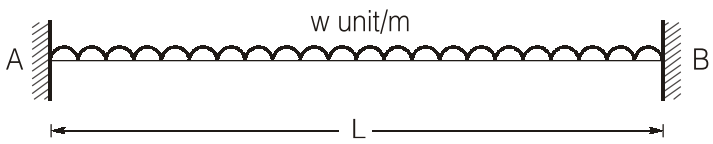
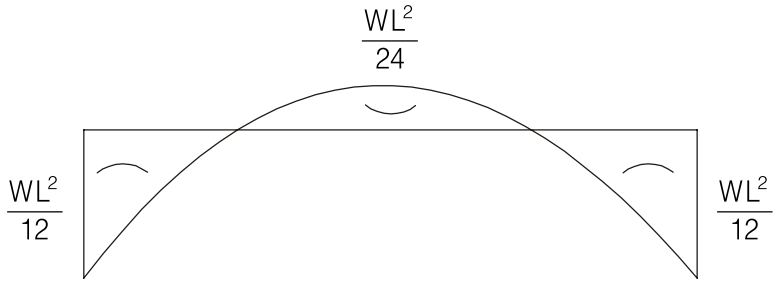
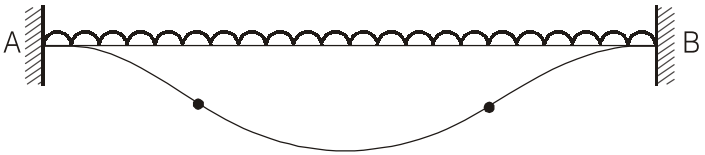
EXAMPLE : 15





Bending Moment Diagram



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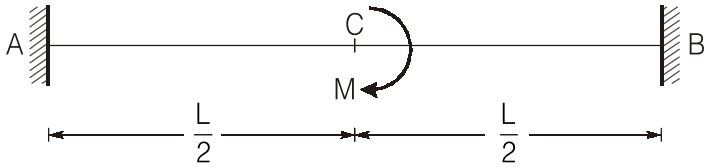
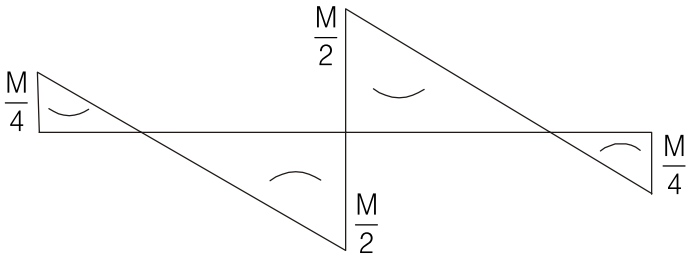
EXAMPLE : 16





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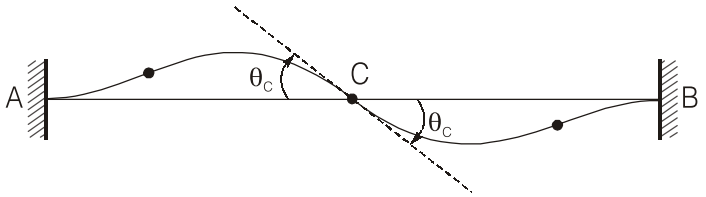

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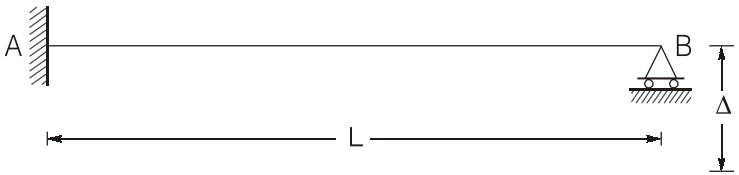
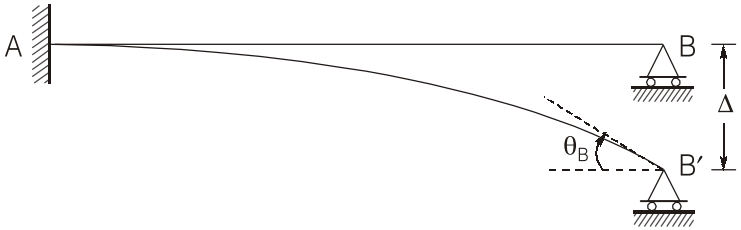
EXAMPLE : 17

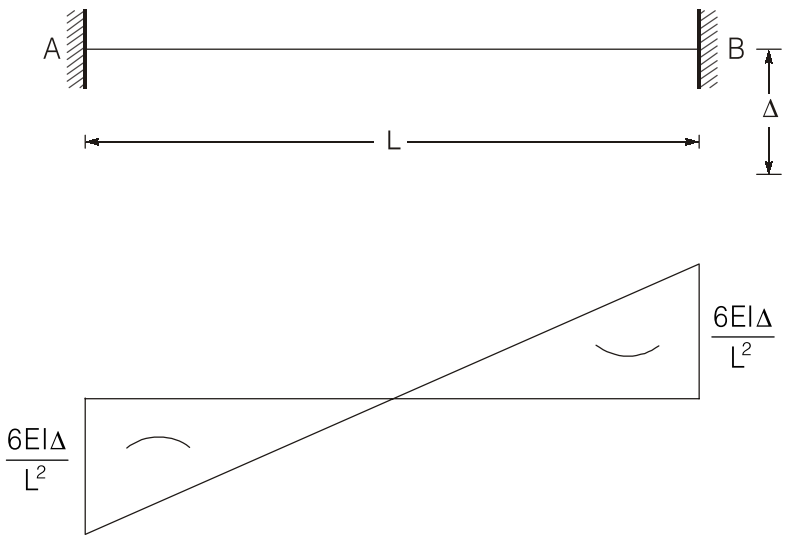


Bending Moment Diagram



Elastic Curve

EXAMPLE : 18



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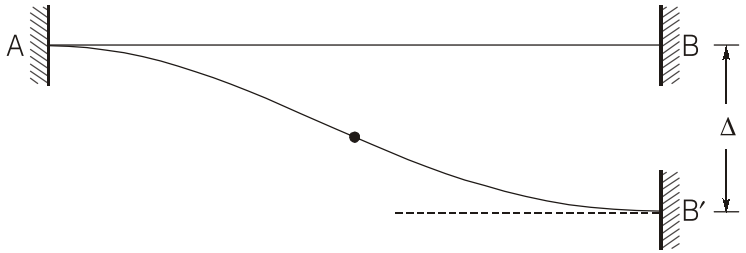

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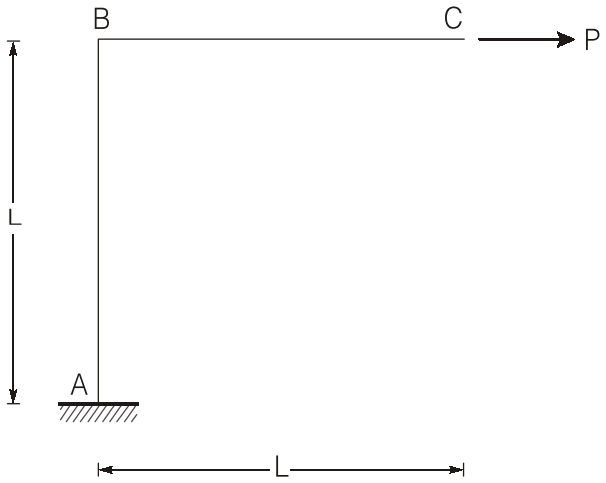
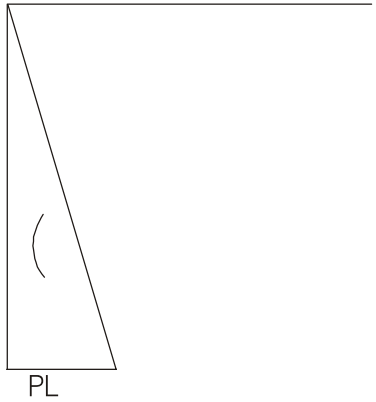
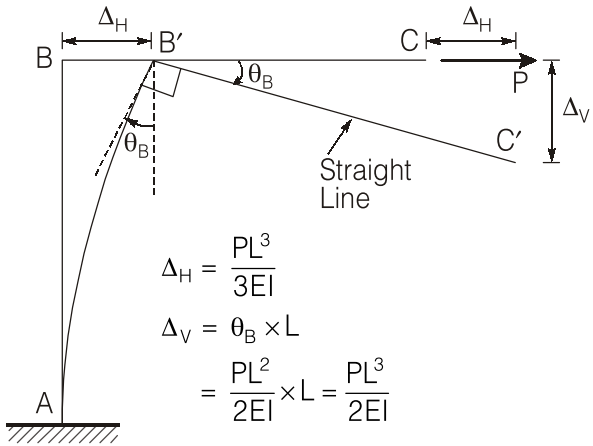
EXAMPLE : 19





Bending Moment Diagram



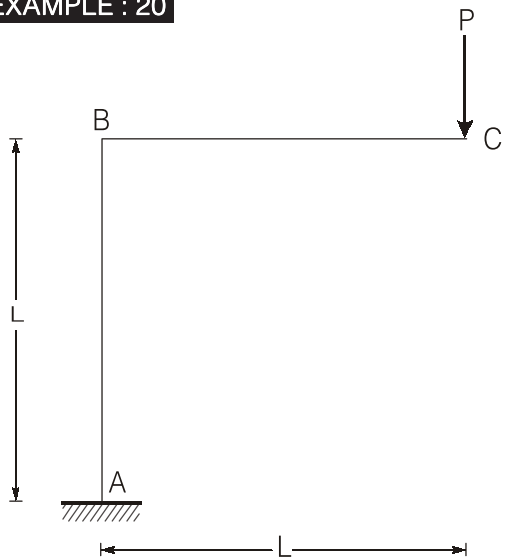
$$\Delta_H = \frac{PL^3}{3EI}$$

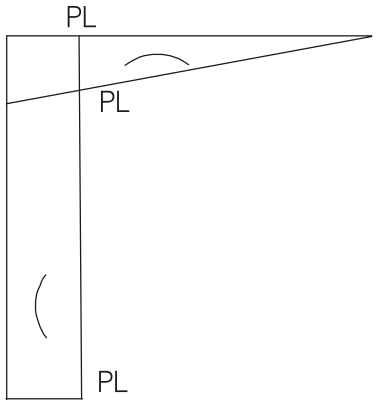
$$\Delta_V = \theta_B \times L$$

$$= \frac{PL^2}{2EI} \times L = \frac{PL^3}{2EI}$$

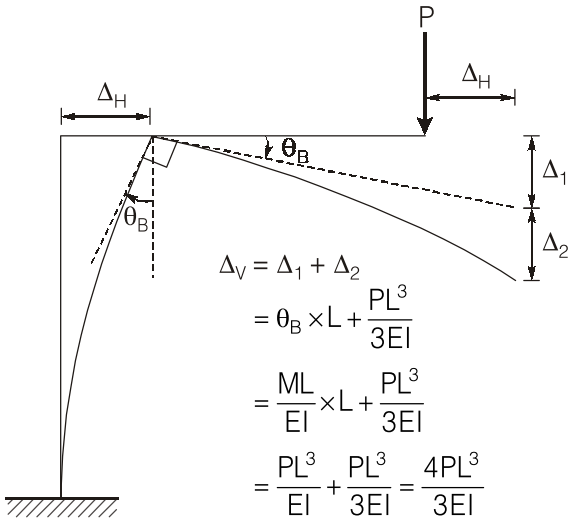
Elastic Curve

EXAMPLE : 20





Bending Moment Diagram



$$\Delta_V = \Delta_1 + \Delta_2$$

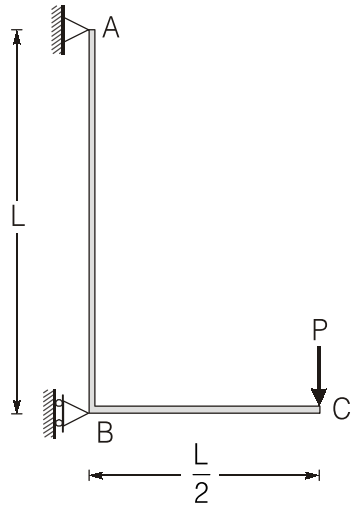
$$= \theta_B \times L + \frac{PL^3}{3EI}$$

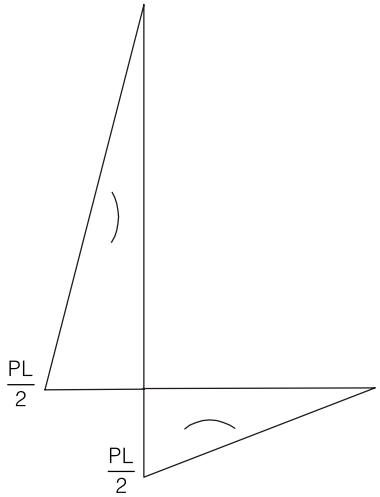
$$= \frac{ML}{EI} \times L + \frac{PL^3}{3EI}$$

$$= \frac{PL^3}{EI} + \frac{PL^3}{3EI} = \frac{4PL^3}{3EI}$$

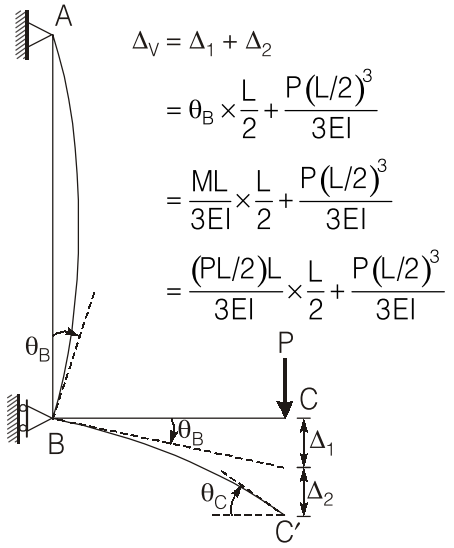
Elastic Curve

EXAMPLE : 21



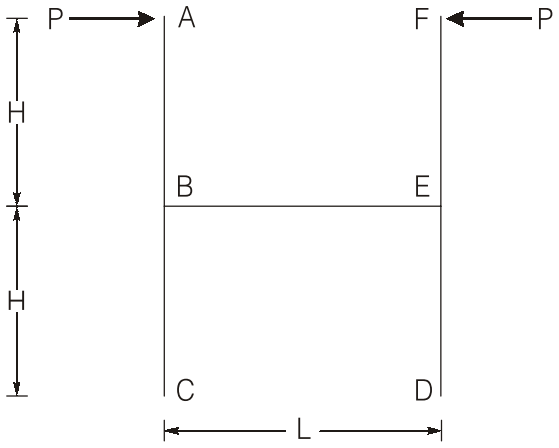


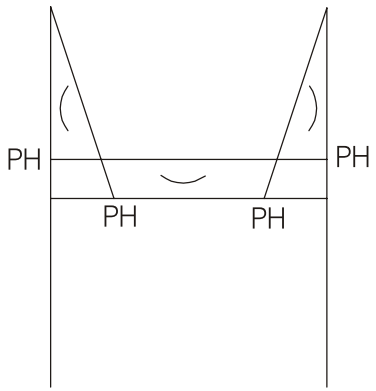
Bending Moment Diagram



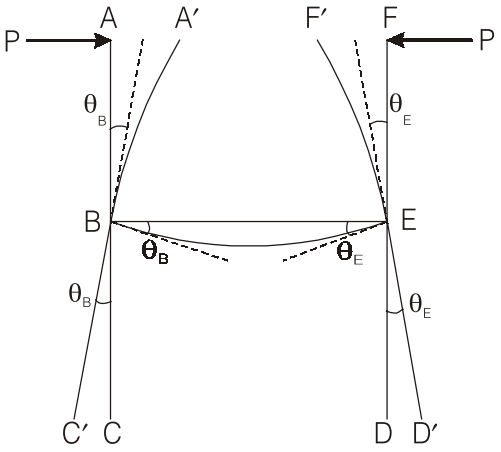
Elastic Curve

EXAMPLE : 22



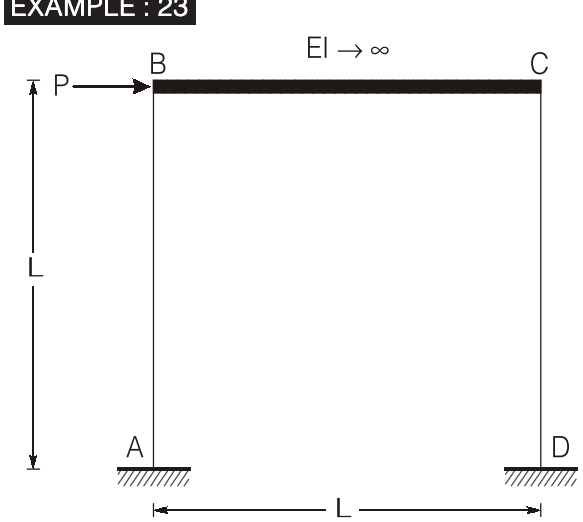


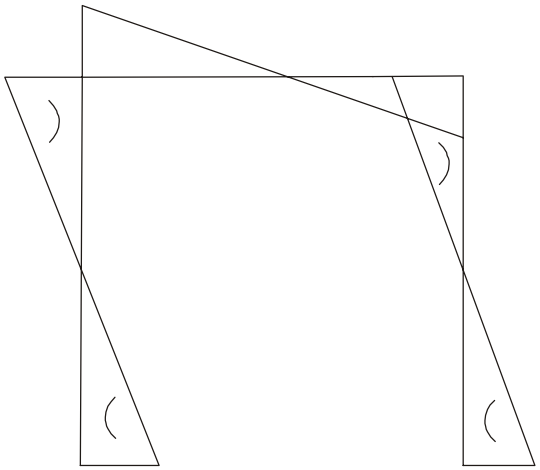
Bending Moment Diagram



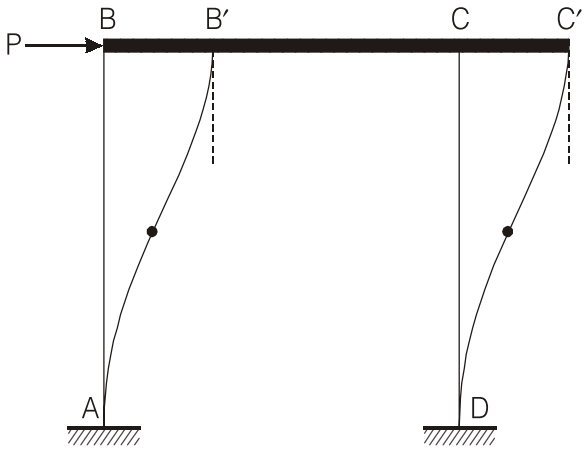
Elastic Curve

EXAMPLE : 23





Bending Moment Diagram

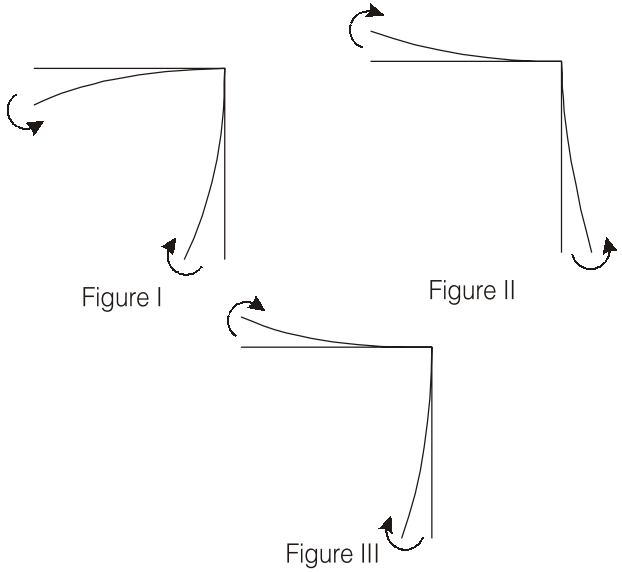


Elastic Curve

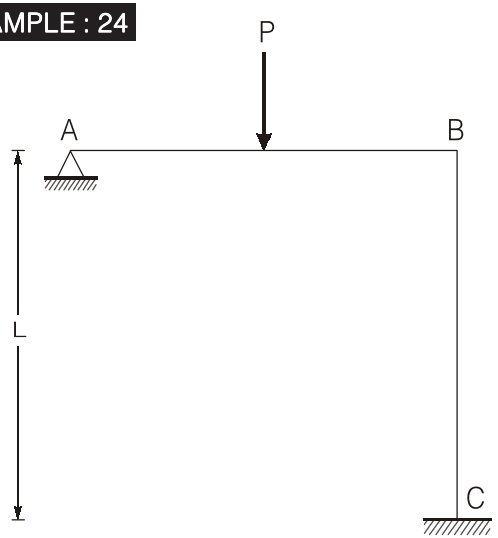
CONCEPT

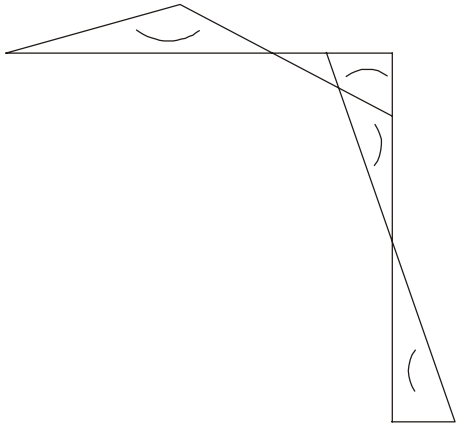
"If two members of a frame is meeting at a rigid joint and subjected to no external point moment at that joint then elastic curve at the joint will be like figure I or figure II. Figure III is not possible."

Figure I and Figure II is under rotational equilibrium because one moment is clockwise and other one is anticlockwise. But in case of Figure III, both the moment is clockwise so joint is not in rotational equilibrium.

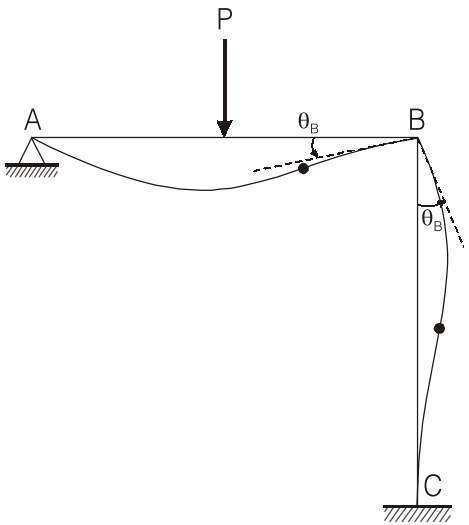


EXAMPLE : 24



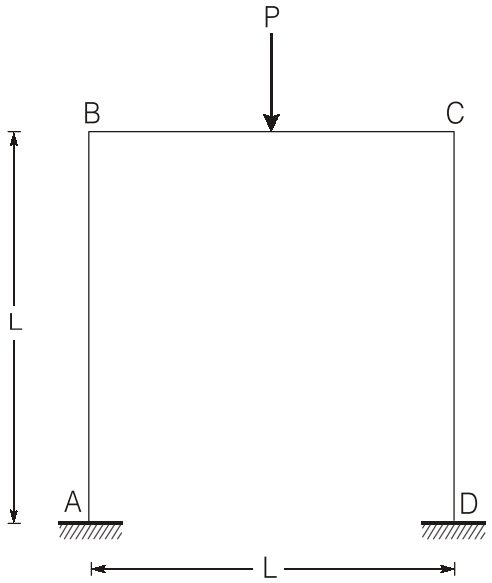


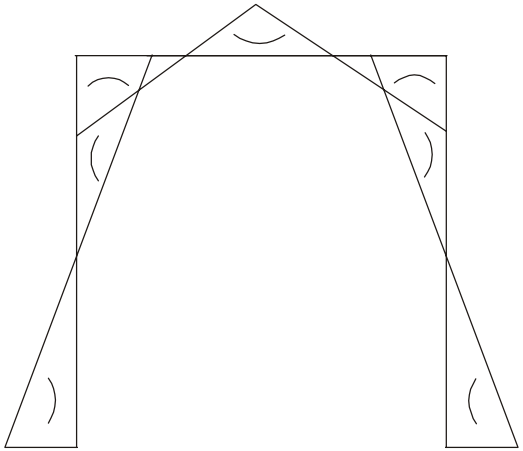
Bending Moment Diagram



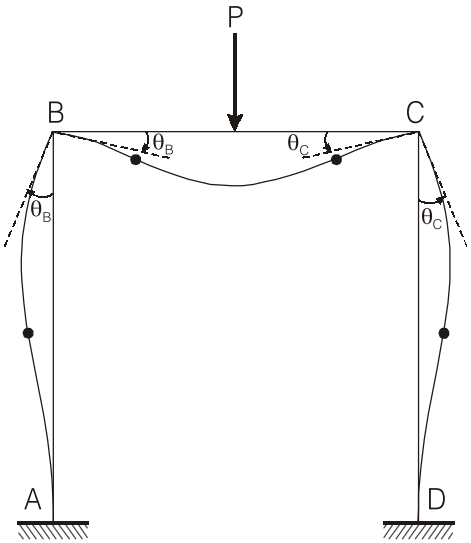
Elastic Curve

EXAMPLE : 25



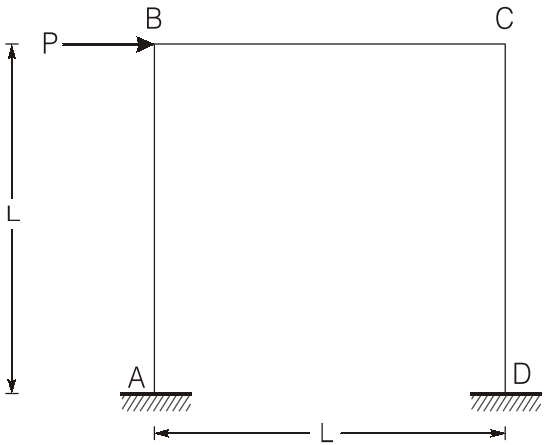


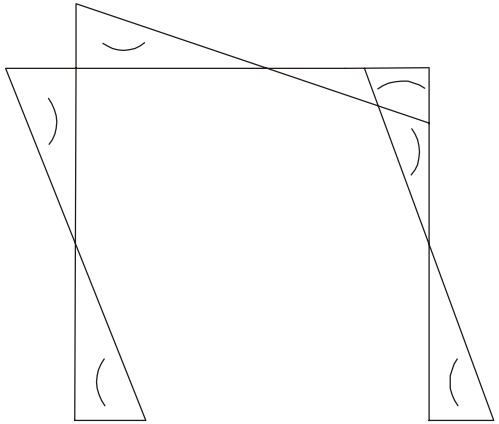
Bending Moment Diagram



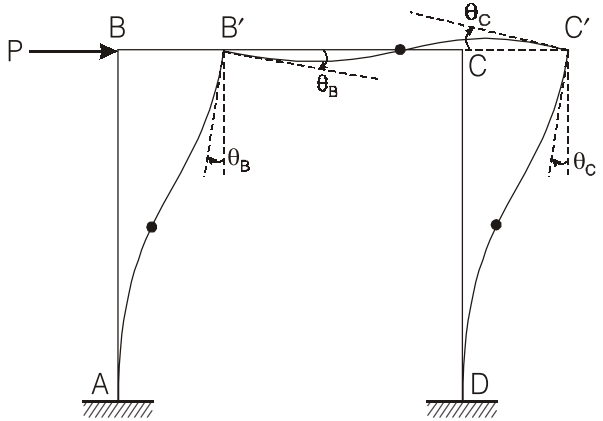
Elastic Curve

EXAMPLE : 26



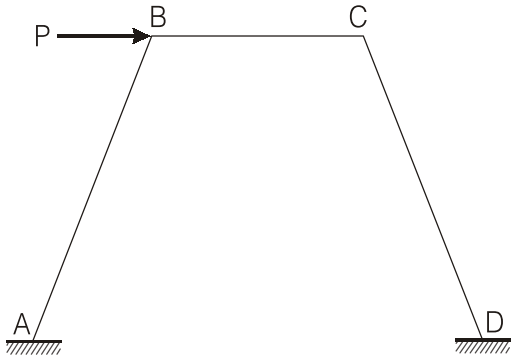


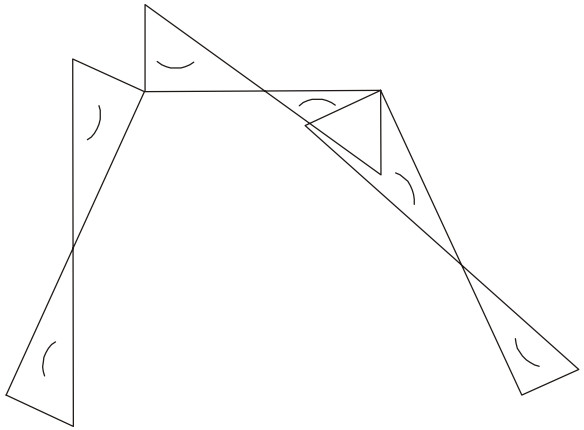
Bending Moment Diagram



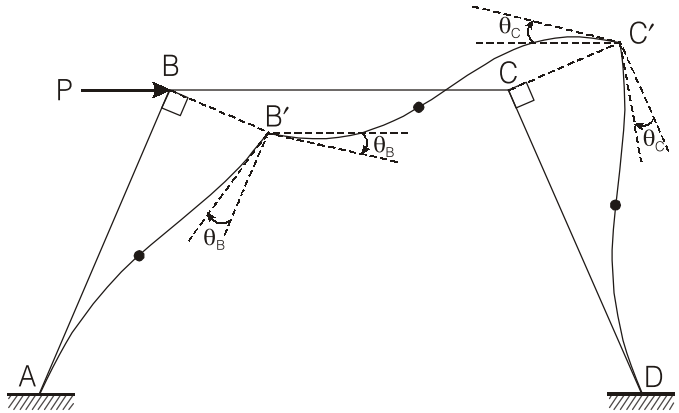
Elastic Curve

EXAMPLE : 27



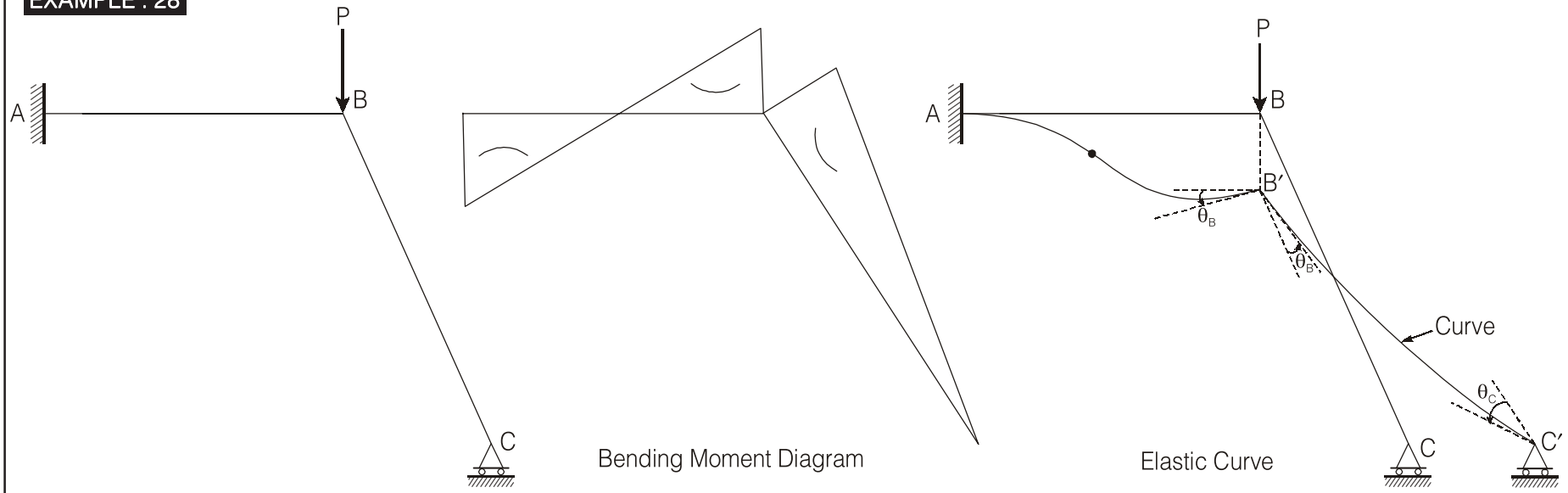


Bending Moment Diagram

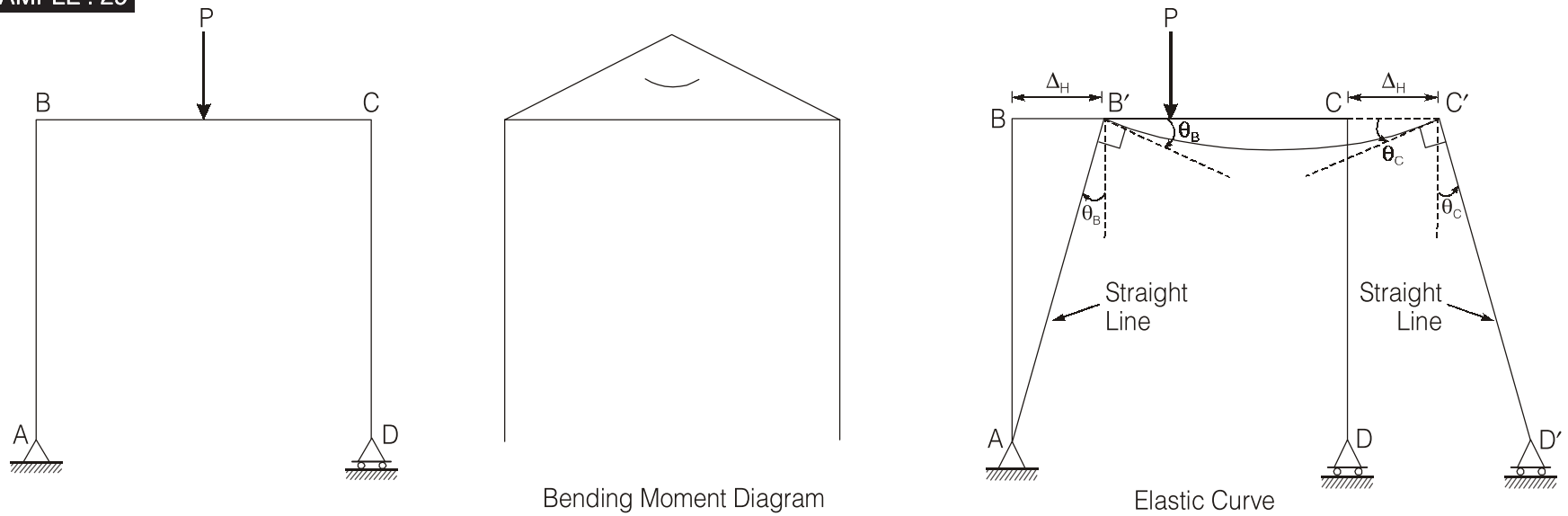


Elastic Curve

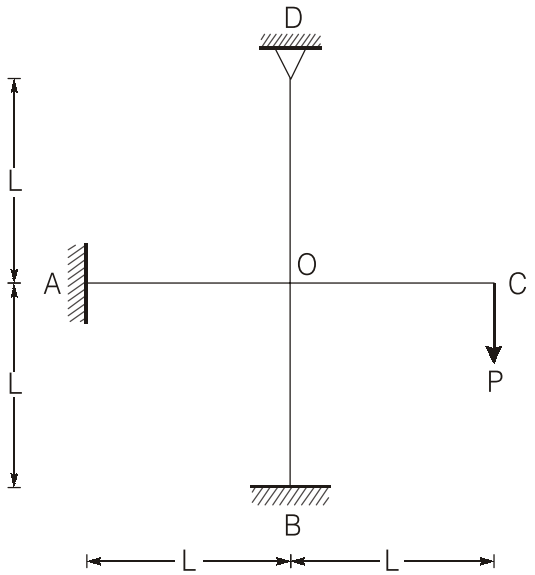
EXAMPLE : 28

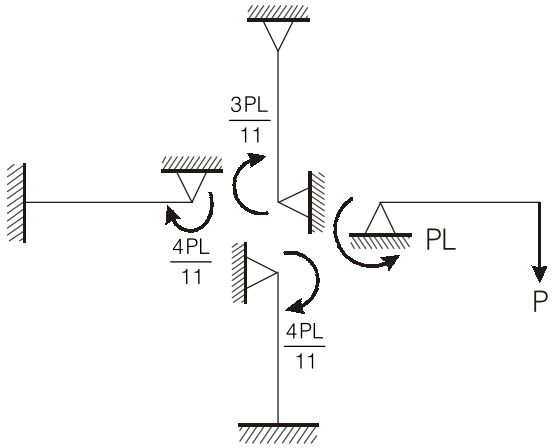


EXAMPLE : 29

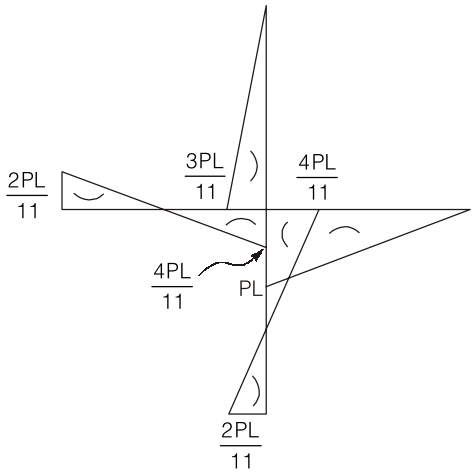


EXAMPLE : 30

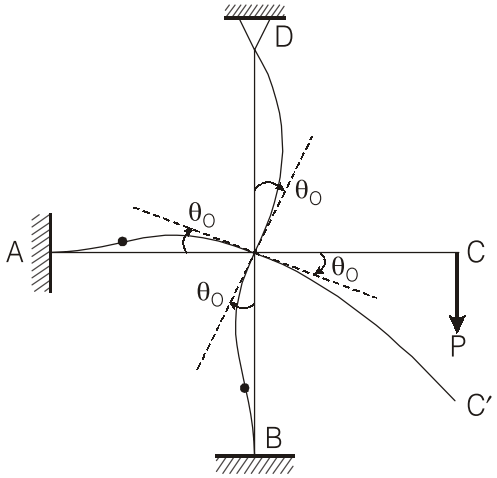




Free Body Diagram



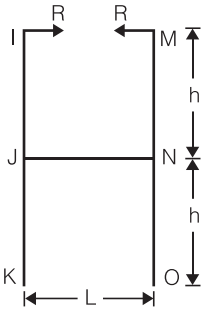
Bending Moment Diagram



Elastic Curve

EXERCISE : 1

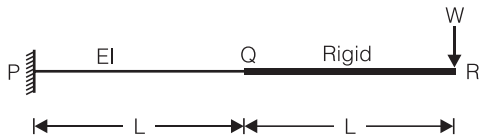
A "H" shaped frame of uniform flexural rigidity EI is loaded as shown in the figure. The relative outward displacement between points K and O is



(a) $\frac{RLh^2}{EI}$ (b) $\frac{RL^2h}{EI}$ (c) $\frac{RLh^2}{3EI}$ (d) $\frac{RL^2h}{3EI}$

EXERCISE : 2

In the cantilever beam PQR shown in figure below, the segment PQ has flexural rigidity EI and the segment QR has infinite flexural rigidity



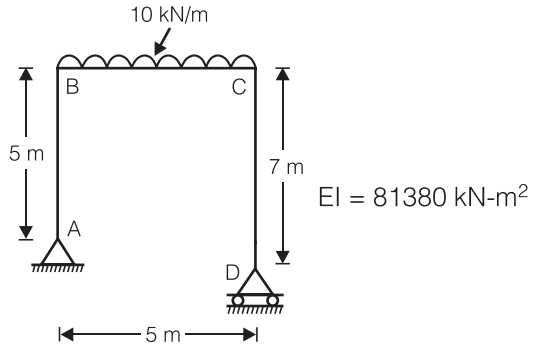
The deflection and slope of the beam at Q are respectively

(a) $\frac{5WL^3}{6EI}$ and $\frac{3WL^2}{2EI}$ (b) $\frac{WL^3}{3EI}$ and $\frac{WL^2}{2EI}$

(c) $\frac{WL^3}{2EI}$ and $\frac{WL^2}{EI}$ (d) $\frac{WL^3}{3EI}$ and $\frac{3WL^2}{2EI}$

EXERCISE : 3

The plane frame below is analyzed by neglecting axial deformations. Following statements are made with respect to the analysis:



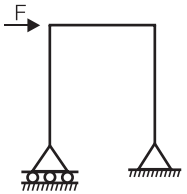
1. Column AB carries axial force only.
2. Vertical deflection at the center of beam BC is 1 mm.

With reference to the above statements, which of the following applies?

(a) Both the statements are true
(b) Statement 1 is true but 2 is false
(c) Statement 2 is true but 1 is false
(d) Both the statements are false

EXERCISE : 4

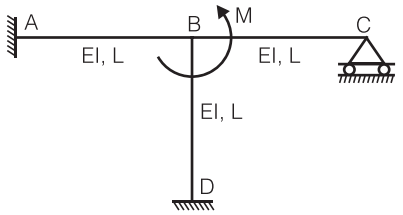
Considering beam as axially rigid, the degree of freedom of a plane frame shown below is



(a) 9 (b) 8 (c) 7 (d) 6

EXERCISE : 5

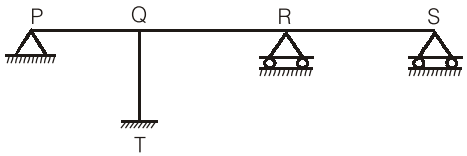
All members of the frame shown below have the same flexural rigidity EI and length L. If a moment M is applied at joint B, the rotation of the joint is



(a) $\frac{ML}{12EI}$ (b) $\frac{ML}{11EI}$ (c) $\frac{ML}{8EI}$ (d) $\frac{ML}{7EI}$

EXERCISE : 6

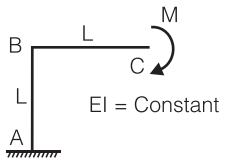
The span(s) to be loaded uniformly for maximum positive (upward) reaction at support P, as shown in the figure below, is(are)



(a) PQ only
(b) PQ and QR
(c) QR and RS
(d) PQ and RS

EXERCISE : 7

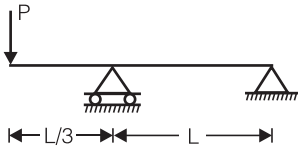
What is the horizontal deflection at free end C of the frame shown in the given figure?



(a) $\frac{ML^2}{2EI}$ (b) $\frac{ML^2}{EI}$ (c) $\frac{3ML^2}{2EI}$ (d) $\frac{2ML^2}{EI}$

EXERCISE : 8

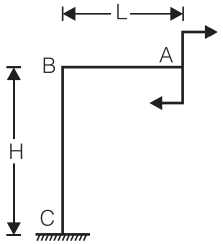
An overhang beam of uniform EI is loaded as shown in the figure below. The deflection at the free end will be



(a) $\frac{PL^3}{81EI}$ (b) $\frac{PL^3}{27EI}$ (c) $\frac{4PL^3}{81EI}$ (d) $\frac{2PL^3}{27EI}$

EXERCISE : 9

A rigid cantilever frame ABC is fixed at C and carries a couple μ at the free end A as shown in the given figure below. Neglecting axial deformation and assuming the flexural rigidity EI to be constant throughout the frame, the vertical deflection of A is

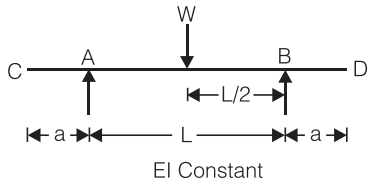


(a) $\frac{\mu L}{EI}\left(H+\frac{L}{2}\right)$ (b) $\frac{\mu L^2}{EI}\left(H+\frac{L}{2}\right)$

(c) $\frac{\mu H^2}{2EI}\left(\frac{H}{2}+L\right)$ (d) $\frac{\mu H}{EI}\left(\frac{H}{2}+L\right)$

EXERCISE : 10

Consider the loaded beam shown in the given figure:



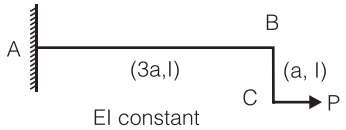
Assertion (A): The deflection at the free end C is 'a' times the slope at A.

Reason (R): The elastic curve for the overhang portion AC or BD is a straight line tangential to the elastic curve at A and B.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

EXERCISE : 11

What is the vertical displacement at the point C of the structure shown in the figure given below?



(a) $\frac{9Pa^3}{2EI}$ (b) $\frac{27Pa^3}{2EI}$

(c) $\frac{27Pa^3}{8EI}$ (d) $\frac{3Pa^3}{8EI}$

EXERCISE : 12

What is the moment at A for a frame as shown in figure below ?

Each member indicated in dark lines has very large moment of inertia.

(a) $\frac{PL}{2}$ (b) $\frac{PL}{4}$ (c) $\frac{PL}{8}$ (d) $\frac{PL}{16}$

EXERCISE : 13

For the rigid frame shown in the figure below, the force required for moving the girder AB through a horizontal displacement Δ is given by

(a) $\frac{6EI\Delta}{L^3}$ (b) $\frac{8EI\Delta}{L^3}$

(c) $\frac{9EI\Delta}{L^3}$ (d) $\frac{15EI\Delta}{L^3}$

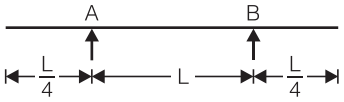
EXERCISE : 14

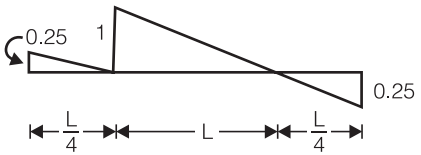
For beam shown in figure-I, an influence line diagram is shown in figure-II. This refers to

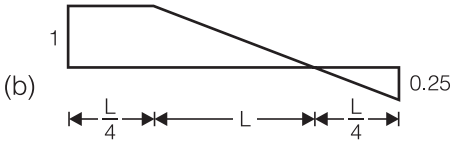
(a) reaction R_A
(b) shear force at support D
(c) BM at support B
(d) shear force at section XX

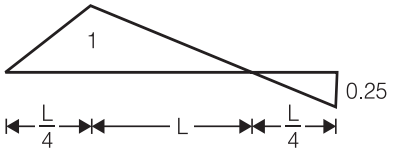
EXERCISE : 15

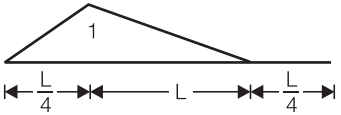
A beam with cantilevered ends is shown in the given figure. Which one of the following diagrams represent the influence line diagram for shear force at section just to the right of the support A?



(a)


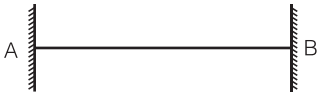
(b)


(c)


(d)


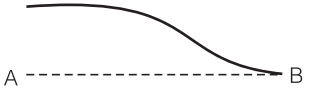
EXERCISE : 16

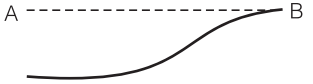
The influence line diagram for the support moment at A of the fixed beam AB of constant EI is



(a)


(b)


(c)


(d)


ANSWER

1. (a)	2. (a)	3. (a)	4. (d)
5. (b)	6. (d)	7. (a)	8. (c)
9. (a)	10. (a)	11. (a)	12. (b)
13. (d)	14. (d)	15. (a)	16. (a)