

# GATE

## MADE EASY WORKBOOK 2025



**Detailed Explanations of  
Try Yourself Questions**

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**Instrumentation Engineering**  
Control Systems and Process Control



# 1

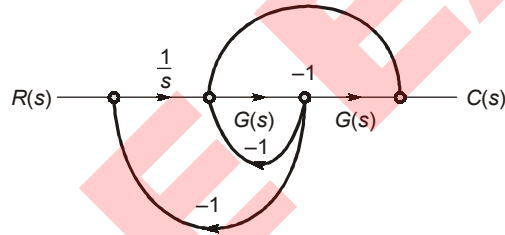
## Mathematical Models and Block Diagram



### Detailed Explanation of Try Yourself Questions

**T1. (b)**

Drawing SFG of the above



Here,  $P_1 = \frac{G^2(s)}{s}$  ;  $L_2 = -G(s)$  ;  $L_1 = -G^2(s)$  ;  $L_3 = -\frac{G(s)}{s}$

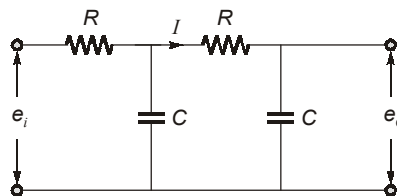
$$\frac{C(s)}{R(s)} = \frac{\frac{G^2(s)}{s}}{1 - \left( -G^2(s) - \frac{G(s)}{s} - G(s) \right)}$$

$$= \frac{G^2(s)}{s + sG^2(s) + G(s) + sG(s)}$$

Put  $G(s) = s$ ,

$$\frac{C(s)}{R(s)} = \frac{s^2}{s + s^3 + s + s^2} = \frac{s^2}{s^3 + s^2 + 2s} = \frac{s}{s^2 + s + 2}$$

**T2. Sol.**



$$E_o(s) = \frac{1}{sC} I(s) \quad \dots(i)$$

$$I(s) = \frac{E_i(s)}{\left[ R + \frac{\left( R + \frac{1}{sC} \right) \times \frac{1}{sC}}{\left( R + \frac{1}{sC} + \frac{1}{sC} \right)} \right]} \times \frac{1}{\left( R + \frac{1}{sC} + \frac{1}{sC} \right)}$$

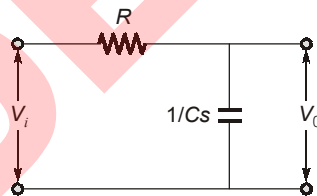
(Using current division rule)

$$= \frac{E_i(s)}{\frac{R + \frac{1}{sC}}{sCR + 2} + R} \times \frac{1}{sCR + 2} = \frac{E_i(s)}{\left( R + \frac{1}{sC} \right) + R(sCR + 2)}$$

$$E_o(s) = \frac{\frac{1}{sC} \times E_i(s)}{\frac{(1 + RSC) + SCR(SCR + 2)}{SC}}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{S^2 C^2 R^2 + 3 SCR + 1} = \frac{1}{S^2 T^2 + 3ST + 1}$$

**T3. (b)**



$$\frac{V_o}{V_i} = \frac{1/Cs}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1}$$

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# 2

## Time Response Analysis



### Detailed Explanation of Try Yourself Questions

**T1. Sol.**

$$G(s) = \frac{K}{s(s+p)}$$

Now, the closed loop system

$$T(s) = \frac{K}{s^2 + sp + K}$$

∴ Comparing it with standard equation

$$K = \omega_n^2$$

$$2\xi\omega_n = p$$

$$t_s = \frac{4}{\xi\omega_n}$$

⇒

$$\xi\omega_n = 1$$

∴

$$p = 2$$

now,

$$\frac{-\pi\xi}{e^{\sqrt{1-\xi^2}}} = 0.1$$

$$\xi = 0.537$$

∴

$$\omega_n = \frac{p}{2\xi} = 1.69$$

∴

$$K = \omega_n^2 = 2.85$$

**T2. Sol.**

Taking Laplace transform we get

$$X(s) = \frac{1}{(s^2 + 6s + 5)} \cdot 12 \left[ \frac{1}{s} - \frac{1}{(s+2)} \right]$$

$$X(s) = \frac{12}{s(s+5)(s+1)} \cdot 12 \left[ \frac{1}{s} - \frac{1}{(s+2)} \right]$$

Now, using final value theorem

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

∴

$$\begin{aligned} X(s) &= \lim_{s \rightarrow 0} \left[ \frac{12}{(s+5)(s+1)} - \frac{12s}{(s+5)(s+1)(s+3)} \right] \\ &= \frac{12}{5} = 2.4 \end{aligned}$$

**T3. Sol.**

To find the impulse response let us difference the response.

$$c'(t) = 12 e^{-10t} - 12e^{-60t}$$

taking inverse laplace transform we get

$$C'(s) = \frac{600}{(s+10)(s+60)}$$

$$C'(s) = \frac{600}{s^2 + 70s + 600}$$

∴  $c'(s)$  is the impulse response thus comparing it with the standard equation.

$$2\xi\omega_n = 70$$

$$\omega_n = \sqrt{600}$$

$$\xi = 1.428 \approx 1.43$$

∴

**T4. Sol.**

Since real part of the given second order equation is at  $-0.602$  thus they can be considered as dominant poles.

Thus

$$t_p = \frac{\pi}{\omega_d}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$\omega_n = \sqrt{2.829} = 1.6819$$

and

$$2\xi\omega_n = 1.204$$

$$\xi = \frac{1.204}{2 \times 1.6819} = 0.3579$$

∴

$$\omega_d = 1.6819 \sqrt{1 - \xi^2}$$

$$\omega_d = 1.577$$

∴

$$t_p = 1.999 \approx 2$$



# 3

## Stability



### Detailed Explanation of Try Yourself Questions

**T1. (d)**

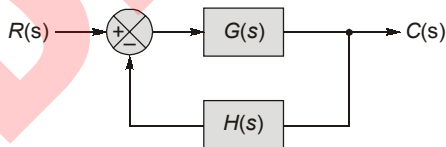
**T2. (b)**

With negative feedback, the system stability will increase. In open loop system.



The gain of the system is  $G(s)$ .

Where as in closed loop system



The closed loop gain of the systems  $\frac{G(s)H(s)}{1+G(s)H(s)}$  hence it is divided by  $1 + G(s) H(s)$ , in closed loop system with negative feedback gain decreases.

**T3. (d)**

The correct sequence of steps needed to improve system stability is to use negative feedback, reduce gain and insert deviation action.

**T4. Sol.**

$$1 + G(s) = 0$$

$$\Rightarrow s\tau_1 \left[ 1 + s(\tau_1 + \tau_2) + s^2 \tau_1 \tau_2 \right] + K = 0$$

$$s^3 \tau_1^2 \tau_2 + s^2 \tau_1 (\tau_1 + \tau_2) + s\tau_1 + K = 0$$

Using R-H criteria

$$\begin{array}{r} s^3 \\ s^2 \\ s^1 \\ s^0 \end{array} \begin{array}{l} \tau_1^2 \tau_2 \\ \tau_1(\tau_1 + \tau_2) \\ \frac{[\tau_1^2(\tau_1 + \tau_2) - K\tau_1^2 \tau_2]}{\tau_1(\tau_1 + \tau_2)} \\ K \end{array} \begin{array}{l} \tau_1 \\ K \end{array}$$

$$\Rightarrow K > 0; \tau_1 > 0; \tau_2 > 0$$

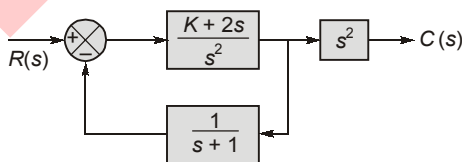
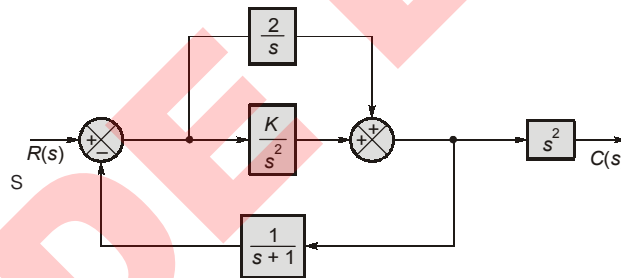
Also, 
$$\frac{\tau_1(\tau_1 + \tau_2) - K\tau_2}{(\tau_1 + \tau_2)} > 0$$

$$K \tau_2 \tau_1 < \tau_1(\tau_1 + \tau_2)$$

$$\Rightarrow K < \left(1 + \frac{\tau_1}{\tau_2}\right)$$

$$0 < K < \left(1 + \frac{\tau_1}{\tau_2}\right); [\tau_1 > 0 \text{ and } \tau_2 > 0 \text{ and this is the only possible case.}]$$

**T5. Sol.**



$$R(s) \rightarrow \frac{(s+1)(K+2s)}{(s+1)(s^2)+2s+K} \rightarrow s^2 \rightarrow C(s)$$

$$R(s) \rightarrow \frac{s^2(s+1)(2s+K)}{(s+1)s^2+2s+K} \rightarrow c(s)$$

$$\frac{C(s)}{R(s)} = \frac{s^2(s+1)(2s+K)}{s^2(s+1)+2s+K}$$

at

$$K = 2$$
$$\frac{C(s)}{R(s)} = \frac{s^2(s+1)^2}{(s^2+2)(s+1)}$$

Thus poles at  $\pm j\sqrt{2}$  and one at  $-1$ .

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# 4

## Root Locus Technique



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

Characteristic equation is given as

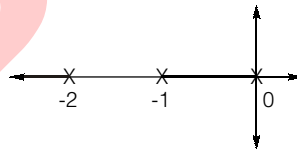
$$1 + G(s)H(s) = 0$$

On comparing this characteristic equation with the equation given in problem, we have

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

$P =$  Number of open loop poles  $= 3 =$  number of branches on root locus

$Z = 0 =$  Number of branches terminating at zeros.



**Angle of Asymptotes:** The  $P - Z$  branches terminating at infinity will go along certain straight lines.

$$\text{Number of asymptotes} = P - Z$$

$$= 3 - 0 = 3$$

$$\theta = \frac{180^\circ(2q+1)}{P-Z}$$

$$q = 0, 1, 2 \dots$$

$$\theta_1 = \frac{180 \times (2 \times 0 + 1)}{3} = 60^\circ$$

$$\theta_2 = \frac{180^\circ(2 \times 1 + 1)}{3} = 180^\circ$$

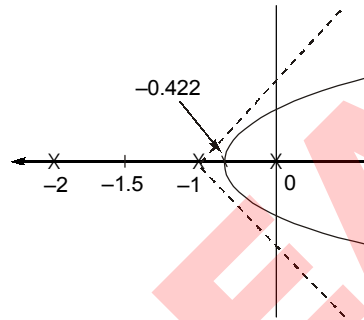
$$\theta_3 = \frac{180^\circ(2 \times 2 + 1)}{3} = 300^\circ$$

**Centroid:** It is the intersection point of the asymptotes on the real axis. It may or may not be a part of root locus.

$$\begin{aligned}\text{Centroid} &= \frac{\sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeros}}{P - Z} \\ &= \frac{0 - 1 - 2}{3} = -1\end{aligned}$$

Centroid  $\rightarrow (-1, 0)$

**Break-away or break-in points:** These are those points on whose multiple roots of the characteristic equation occur.



$$\begin{aligned}s(s^2 + 3s + 2) + K &= 0 \\ K &= -(s^3 + 3s^2 + 2s) \\ \frac{dK}{ds} &= -(3s^2 + 6s + 2) = 0 \\ s &= -0.422, -1.577\end{aligned}$$

Now verify the valid break-away point

$$\begin{aligned}K &= 0.234 \text{ (valid) at } s = -0.422 \\ K &= \text{negative (not valid) at } s = -1.577\end{aligned}$$

### T2. Sol.

Poles are at

and

$$\begin{aligned}OLTF &= \frac{K}{s(s^2 + 4s + 8)} \\ s_1 &= 0 \\ s_{2,3} &= \frac{-4 \pm \sqrt{16 - 32}}{2} = -2 \pm j2\end{aligned}$$

There are 3 poles and no zero with root loci, all terminating at infinity.

$\phi = \frac{(2q+1)180^\circ}{P-Z} = 60^\circ, 180^\circ, 300^\circ$  for  $q = 0, 1, 2$  (angle of asymptotes)

$$\text{Centroid} = \frac{0 - 2 + j2 - 2 - j2}{3} = \frac{-4}{3} = -1.33$$

$$1 + \frac{K}{s(s^2 + 4s + 8)} = 0$$

$$\Rightarrow K = -s(s^2 + 4s + 8) = -(s^3 + 4s^2 + 8s)$$

for break away points,  $\frac{dK}{ds} = 0$

$$\Rightarrow \frac{dK}{ds} = -(3s^2 + 8s + 8)$$

$$s_{1,2} = -\frac{8 \pm \sqrt{64 - 4 \times 8 \times 3}}{2 \times 3}$$

$$= -\frac{8 \pm \sqrt{64 - 96}}{6} = -1.33 \pm j0.943$$

As  $\frac{dK}{ds}$  is imaginary, there is no breakaway point from the real axis.

**Imaginary axis crossing:**

$$\begin{aligned} \text{Characteristic equation} &= s(s^2 + 4s + 8) + K \\ &= s^3 + 4s^2 + 8s + K = 0 \end{aligned}$$

$s^3$	1	8
$s^2$	4	$K$
$s^1$	$\frac{32-K}{4}$	
$s^0$	$K$	

**From Routh-Hurwitz Criteria:**

For  $K = 32$ , the system is marginally stable and beyond  $K = 32$  the system becomes unstable.

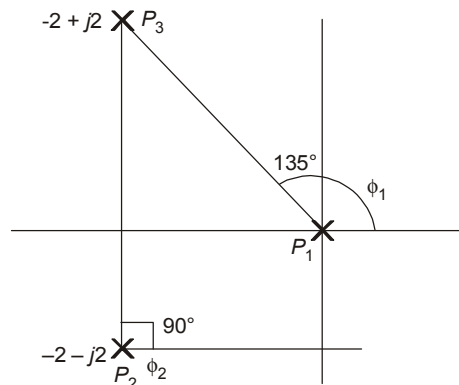
Hence,

$$4s^2 + K = 4s^2 + 32$$

$$s = j2\sqrt{2} = j\omega$$

$$\omega = 2\sqrt{2} = 2.83$$

The root locus cuts the imaginary axis at  $\pm j2.83$ .



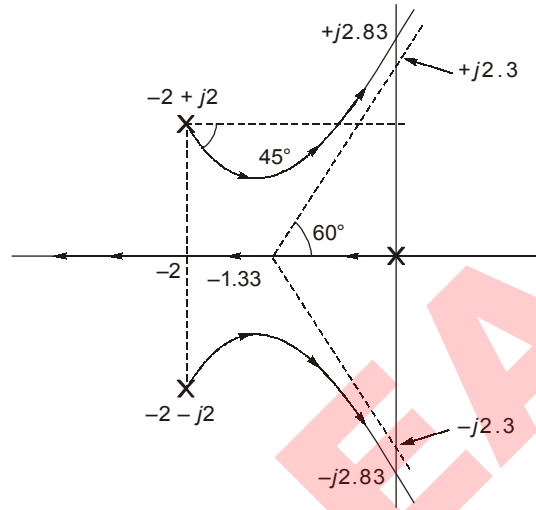
$$\phi_1 = 180^\circ - \tan^{-1}\left(\frac{2}{2}\right) = 135^\circ$$

$$\begin{aligned}\phi_2 &= 90^\circ \\ \Sigma\phi_p &= \phi_1 + \phi_2 = 135^\circ + 90^\circ = 225^\circ \\ \phi &= \Sigma\phi_z - \Sigma\phi_p = 0 - 225^\circ = -225^\circ\end{aligned}$$

Angle of departure,

$$\phi_D = 180 + \phi = 180 - 225^\circ = -45^\circ$$

Root locus of the given system:



### T3. Sol.

$$G(s)H(s) = \frac{K}{s(s+1)(s+4)}$$

**Step-1** Number of open loop poles ;

$$P = 3$$

Number of open loop zeros ;  $Z = 0$

Number of branches terminating at infinity

$$= P - Z = 3$$

**Step-2** Angle of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z} \quad \text{where } q = 0, 1, 2$$

$$\theta_1 = \frac{180^\circ}{3} = 60^\circ$$

$$\theta_2 = \frac{3 \times 180^\circ}{3} = 180^\circ$$

$$\theta_3 = \frac{5 \times 180^\circ}{3} = 300^\circ$$

**Step-3**

$$\text{Centroid} = \frac{\Sigma \text{ real part of open loop poles} - \Sigma \text{ real part of open loop zeros}}{P - Z}$$

$$= \frac{(-1-4) - (0)}{3-0} = -\frac{5}{3}$$

**Step-4** Break away point

$$K + s(s^2 + 5s + 4) = 0$$

$$K = -s^3 - 5s^2 - 4s$$

$$\frac{dK}{ds} = -3s^2 - 10s - 4 = 0$$

$$3s^2 + 10s + 4 = 0$$

$$\Rightarrow s_1, s_2 = -0.4648, -2.8685$$

Valid break-away point will be  $-0.4648$

**(i)** Routh array table

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 5 & K \\ s^1 & \frac{20-K}{5} & 0 \\ s^0 & 1 & \end{array}$$

For system to be stable

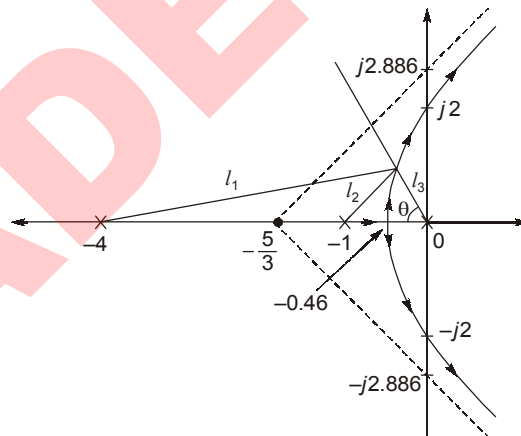
$$20 - K > 0 \Rightarrow K < 20$$

For system to be marginally stable.

$$K = 20$$

$$A(s) = 5s^2 + 20 = 0$$

$$\Rightarrow s = \pm 2j$$

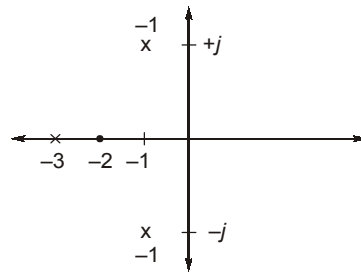


**(ii)** To find  $K$ ,

$$\theta = \cos^{-1} \xi = \cos^{-1}(0.34) = 70.123^\circ$$

$$K = l_1 l_2 l_3$$

$$\text{Gain margin (GM)} = \frac{K(\text{Marginal stability})}{K(\text{desired})}$$

**T4. Sol.**

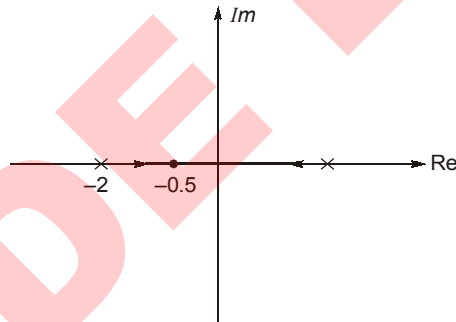
$$\begin{aligned}\phi_d &= 180 - (\phi_p - \phi_z) \\ &= 180 - \left( 180 + \tan^{-1}\left(\frac{1}{2}\right) + 90^\circ - 225^\circ \right) \\ &= 108.4^\circ\end{aligned}$$

**T5. Sol.**

Given that

$$G(s) = \frac{K}{(s+2)(s-1)}$$

Using root locus method, the break point can be



obtain as

$$\Rightarrow 1 + G(s) = 0$$

$$1 + \frac{K}{(s+2)(s-1)} = 0$$

$$\text{or } K = -(s+2)(s-1)$$

$$\frac{dK}{ds} = -2s - 1 = 0$$

$$\text{or } s = -0.5$$

To have, both the poles at the same directions

$$|G(s)|_{s=-0.5} = 1$$

$$K = 2.25$$



# 5

## Frequency Response Analysis



### Detailed Explanation of Try Yourself Questions

**T1. Sol.**

Given,

$$G(s) = \frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

$(1 + as)$  is addition of zero to the transfer function whose contribution in slope = +20 dB/decade or -6 dB/octave.

$(1 + bs)$  is addition of pole to the transfer function whose contribution in slope = -20 dB/decade or -6 dB/octave

Observing the change in the slope at different corner frequencies, we conclude that

$$a = \frac{1}{4} \text{ rad/s and } b = \frac{1}{24} \text{ rad/s}$$

From

$$\omega = 0.01 \text{ rad/s to } \omega = 8 \text{ rad/s,}$$

$$\text{slope} = -20 \text{ dB/decade}$$

Let the vertical length in dB be  $y$

$$\therefore -20 = \left( \frac{0 - y}{\log 8 - \log 0.01} \right)$$

$$\text{or, } -20 = \frac{y}{\log 8 + 2}$$

or,

$$y = 58 \text{ dB}$$

Applying  
we have:

$$y = mx + C \text{ at } \omega = 0.01 \text{ rad/s,}$$

$$58 = -20 \log 0.01 + C$$

or,

$$C = 58 - 40 = 18$$

Now,

$$C = 20 \log K$$

or,

$$\log K = \frac{18}{20} = 0.9$$

$$\therefore K = \log^{-1}(0.9) = (10)^{0.9} = 7.94$$

$$\therefore \frac{a}{bK} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{24}\right) \times 7.94}$$

$$= \frac{24}{4 \times 7.94} = 0.755$$

$$\therefore \frac{a}{bK} = 0.755$$

**T2. Sol.**

$$\text{OLTF} = G(s) = \frac{1}{(s+2)^2}$$

For unity feedback system,

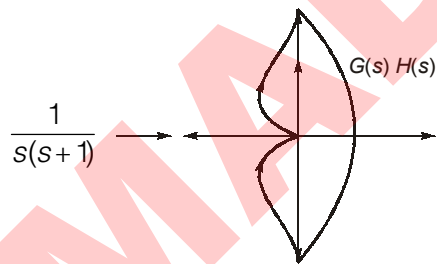
$$H(s) = 1$$

$$\therefore \text{CLTF} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{(s+2)^2}}{1 + \frac{1}{(s+2)^2}}$$

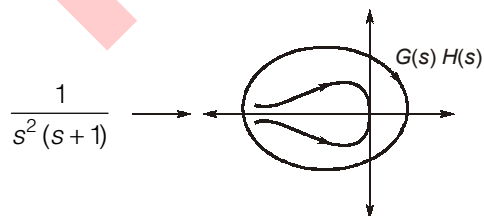
$$= \frac{1}{s^2 + 4s + 5}$$

$\therefore$  Close loop poles will be the roots of  $s^2 + 4s + 5 = 0$

i.e.  $s = -2 + j$  and  $-2 - j$

**T3. (b)**

After adding pole at origin



So, nyquist plot of a system will rotate by  $90^\circ$  in clockwise direction.



**T4. Sol.**

For gain margin we have to find

$$G(s)H(s) = \frac{0.75}{s(1+s)(1+0.5s)}$$

Phase over frequency

$$-180^\circ = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(0.5\omega)$$

$$-90^\circ = \tan^{-1}(\omega) + \tan^{-1}(0.5\omega)$$

$$\frac{1.5\omega}{1-0.5\omega^2} = \tan(90^\circ)$$

$$0.5\omega^2 = 1$$

$$\omega = \sqrt{2}$$

$$\therefore |G(j\omega)H(j\omega)| = \frac{0.75}{\omega\sqrt{1+\omega^2}\sqrt{1+0.25\omega^2}} = \frac{0.75}{\sqrt{2}\sqrt{1+2}\sqrt{1+0.5}} = \frac{1}{4}$$

$$\therefore \text{Gain margin} = 20 \log \frac{1}{|G(j\omega)H(j\omega)|} = 20 \log 4 = 12 \text{ dB}$$

**T5. Sol.**

$$-90^\circ - \tan^{-1}(2\omega) - \tan^{-1}(3\omega) = -180^\circ$$

$$\tan^{-1}(2\omega) + \tan^{-1}(3\omega) = 90^\circ$$

$$\frac{5\omega}{1-6\omega^2} = \tan(90^\circ)$$

$$\therefore 1 - 6\omega^2 = 0$$

$$\omega = \frac{1}{\sqrt{6}} = 0.41$$

**T6. Sol.**

The Bode plot is of type zero system thus steady state error

$$e_{ss} = \frac{1}{1+K_p}$$

Where

$K_p$  = propotional error constant

$K_p = 40 \text{ db}$

or

$K_p = 100$

$$\therefore e_{ss} = \frac{1}{1+100} = \frac{1}{101} = 0.009$$



# 6

## Controllers and Compensators



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

The given compensator represents phase lead compensator having maximum phase

$$\phi = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

Here,

$$\alpha = \frac{R_2}{R_1 + R_2} = \frac{(1/2)\Omega}{1 + (1/2)\Omega} = 0.333$$

$\therefore$

$$\phi = \sin^{-1}\left(\frac{1-0.333}{1+0.333}\right) = \sin^{-1}\left(\frac{0.667}{1.333}\right) = \sin^{-1}(0.5) = 30^\circ$$

#### T2. (a)

The effect of addition of a zero to a transfer function is providing a phase lead.

#### T3. Sol.

$$G(s) = \frac{\left(1 + \frac{s}{4}\right)}{\left(1 + \frac{s}{25}\right)}$$

Comparing it with the standard transfer function of phase lead compensator

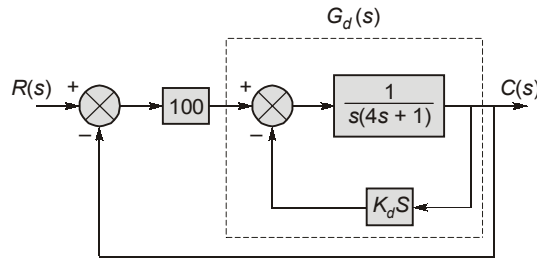
$$G(s) = \frac{\alpha(1 + Ts)}{(1 + \alpha Ts)}$$

$$T = \frac{1}{4}, \quad \alpha T = \frac{1}{25}$$

Now, frequency  $\omega_m$  occurs at

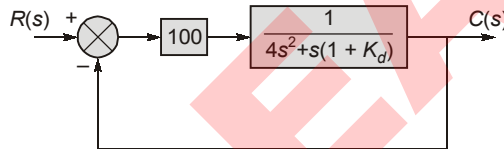
$$= \sqrt{\frac{1}{\alpha T} \cdot \frac{1}{T}} = \sqrt{25 \times 4} = 10 \text{ rad/sec.}$$

**T4. Sol.**



Converting the blocked portion into simplified form as

$$G_d(s) = \frac{1}{s(4s+1)} \cdot \frac{1}{1 + \frac{sK_d}{s(4s+1)}} = \frac{1}{4s^2 + s(1+K_d)}$$



Now,

Now, simplifying the above block diagram as

$$\begin{aligned} G(s) &= \frac{100}{4s^2 + s(1+K_d)} \cdot \frac{1}{1 + \frac{100}{4s^2 + s(1+K_d)}} \\ &= \frac{100}{4s^2 + s(1+K_d) + 100} \\ &= \frac{25}{s^2 + \frac{s(1+K_d)}{4} + 25} \end{aligned}$$

Comparing it with standard equation as

$$\omega_n = 5 \text{ rad/sec.}$$

$$2\xi\omega_n = \left(\frac{1+K_d}{4}\right) \quad \text{Given } \xi = 0.5$$

$$5 = \frac{1+K_d}{4}$$

⇒

$$K_d = 19$$



# 7

## State Space Analysis



### Detailed Explanation of Try Yourself Questions

**T1. (b)**

**T2. Sol.**

Given

$$\dot{x}(t) = Ax(t), \quad x(0) = x_0$$

Taking the Laplace transform

$$sX(s) - x(0) = AX(s)$$

$$[sI - A] X(s) = x(0)$$

$$X(s) = [sI - A]^{-1} x(0)$$

$$x(t) = \mathcal{L}^{-1}[sI - A]^{-1} x(0) \quad \dots(i)$$

Conditions given are

For

$$x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$$

For

$$x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix}$$

Using the linearity property in equation (i)

$$K_1 x_1(t) = \mathcal{L}^{-1}[sI - A]^{-1} x_1(0) K_1$$

$$K_2 x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1} x_2(0) K_2$$

Using the linearity property as

$$K_1 x_1(t) + K_2 x_2(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$

$$[K_1 x_1(0) + K_2 x_2(0)]$$

...(ii)

Also

$$X_3(s) = [sI - A]^{-1} x_3(0)$$

So,

$$K_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + K_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} K_1 + 0K_2 \\ -K_1 + K_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{aligned} K_1 &= 3 \\ K_2 &= 8 \end{aligned}$$

So, from equation (ii), we get  $x(t)$

$$\begin{aligned} x(t) &= K_1 x_1(t) + K_2 x_2(t) \\ &= 3 \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + 8 \begin{bmatrix} e^{-t} - e^{-2t} \\ -e^{-t} + 2e^{-2t} \end{bmatrix} \\ &= \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix} \end{aligned}$$

**T3. Sol.**

Given

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$[sI - A] = s^2$$

$$\phi(t) = \mathcal{L}^{-1}[sI - A]^{-1}$$

$$= \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$$

$$= \mathcal{L}^{-1} \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

**T4. (b)**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, C = [1 \ 1 \ 1]$$

Check for controllability:

$$Q_c = [B : AB : A^2B]$$

$$= \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

For controllable

$$|Q_c| \neq 0$$

Here,

$$|Q_c| = 4(0) = 0 \therefore \text{Uncontrollable.}$$

Check for observability:

$$Q_o = [C^T : A^T C^T : A^{2T} C^T]$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 4 \end{bmatrix}$$

For observable

$$|Q_o| \neq 0$$

Here

$$|Q_o| = 1 \therefore \text{Observable.}$$

**T5. Sol.**

$$\text{Characteristic equation} = |(sI - A)^{-1}|$$

$$= \begin{bmatrix} 2 & -1 \\ 3 & s+5 \end{bmatrix}^{-1}$$

$$= s(s+5) + 3$$

$$= s^2 + 5s + 3$$

■■■■



### Detailed Explanation of Try Yourself Questions

#### T1. Sol.

The open loop transfer function for the primary loop is given by

$$G(s)_{\text{primary}} = K_2 \frac{1}{(s+1)} \cdot \frac{8}{(s+2)(s+4)}$$

Phase cross-over frequency for primary loop is given by

$$-\tan^{-1} \omega_{pc} - \tan^{-1} \frac{\omega_{pc}}{2} - \tan^{-1} \frac{\omega_{pc}}{4} = -180^\circ$$

$$\tan^{-1} \omega_{pc} + \tan^{-1} \frac{\omega_{pc}}{2} + \tan^{-1} \frac{\omega_{pc}}{4} = 180^\circ$$

$$\tan^{-1} \omega_{pc} + \tan^{-1} \left[ \frac{\frac{\omega_{pc}}{2} + \frac{\omega_{pc}}{4}}{1 - \frac{\omega_{pc}^2}{8}} \right] = 180^\circ$$

$$\tan^{-1} \omega_{pc} + \tan^{-1} \left[ \frac{6\omega_{pc}}{8 - \omega_{pc}^2} \right] = 180^\circ$$

$$\tan^{-1} \left[ \frac{\omega_{pc} + \frac{6\omega_{pc}}{8 - \omega_{pc}^2}}{1 - \frac{6\omega_{pc}^2}{8 - \omega_{pc}^2}} \right] = 180^\circ$$

$$\omega_{pc} + \frac{6\omega_{pc}}{8 - \omega_{pc}^2} = 0$$

$$1 + \frac{6}{8 - \omega_{pc}^2} = 0$$

$$\omega_{pc} = \sqrt{14}$$

We know that magnitude = 1, at  $\omega_{pc}$  (By polar plot)

$$M = K_2 \frac{1}{\sqrt{1+\omega^2}} \frac{1}{\sqrt{\omega^2+4}} \frac{8}{\sqrt{\omega^2+16}}$$

$$1 = K_2 \frac{1}{\sqrt{1+\omega_{pc}^2}} \frac{1}{\sqrt{\omega_{pc}^2+4}} \frac{8}{\sqrt{\omega_{pc}^2+16}}$$

$$1 = K_2 \frac{1}{\sqrt{15}} \frac{1}{\sqrt{18}} \frac{8}{\sqrt{30}} = \frac{8K_2}{\sqrt{8100}}$$

$$K_2 = \frac{90}{8} = 11.25$$

The open loop transfer function for the secondary loop is given by

$$G_{\text{secondary}} = K_1 \frac{1}{s+1}$$

Cross-over frequency for secondary loop

$$-\tan^{-1} \omega_{pc} = -180^\circ$$

$$\omega_{pc} = 0$$

Since there is no cross-over frequency for the secondary loop, so we can use any value of gain  $K_1$  for secondary loop.

$$\therefore \text{Upper limit of } K_1 = \infty$$

$$\text{Upper limit of } K_2 = 11.25$$

### T2. Sol.

$$[D(s) G_{ff}(s) - Y(s)] \left[ \frac{1}{s(s+1)} \right] + D(s) = Y(s)$$

$$D(s) G_{ff}(s) + (s^2 + s) D(s) = (s^2 + s + 1) Y(s)$$

$$\frac{Y(s)}{D(s)} = H(s) = \frac{G_{FF}(s) + s^2 + s}{s^2 + s + 1}$$

$$\therefore G_{ff}(s) = 1 + s \text{ [P-D controller]}$$

$$\therefore H(s) = \frac{(s^2 + 2s + 1)}{(s^2 + s + 1)}$$

$$|H(j\omega)|_{\omega=2} = \frac{5}{\sqrt{9+4}} = \frac{5}{\sqrt{13}}$$

### T3. Sol.

Using the value of  $K_u$  and  $P_u$ , Ziegler and Nichols recommended the following settings for feedback controllers

	$K_C$	$\tau_I(\text{min})$	$\tau_D(\text{min})$
P	$K_u/2$	-	-
P-1	$K_u/2.2$	$P_u/1.2$	-
P-1-D	$K_u/1.7$	$P_u/2$	$P_u/8$

Then, the Ziegler-Nichols setting for the proportional controller is

$$\frac{K_u}{2} = \frac{10}{2} = 5$$



**T4. Sol.**

Open loop transfer function is given by

$$G(s) = \left[ K_p + \frac{K_I}{s} \right] \frac{1}{(s+2)(s+10)} = \left[ \frac{K_p s + K_I}{s} \right] \frac{1}{(s+2)(s+10)}$$

Ch: equation is given by

$$\begin{aligned} 1 + G(s) &= 0 \\ s(s+2)(s+10) + K_p s + K_I &= 0 \\ s^3 + 12s^2 + (20 + K_p)s + K_I &= 0 \end{aligned}$$

By R-H criteria

$s^3$	1	$20 + K_p$
$s^2$	12	$K_I$
$s^1$	$\frac{12(20 + K_p) - K_I}{12}$	0
$s^0$	$K_I$	

For stable system,

$$\boxed{K_I > 0}$$

$$\begin{aligned} 12(20 + K_p) &\geq 0 \\ 240 + 20K_p - K_I &\geq 0 \\ 1240 + 20K_p &\geq K_I \end{aligned}$$

$$K_p \geq \frac{K_I - 240}{20}$$

