CLASS TEST

S.No.: 01JP_ECEE_16102023

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EC | EE

Control Systems

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

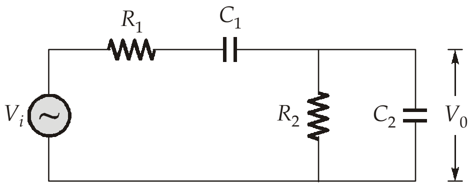
1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 For the system shown in below figure, what is the transfer function $\frac{V_0(s)}{V_i(s)}$?

(Given $R_1 = R_2 = C_1 = C_2 = 1$ unit)



(a) $\frac{V_0(s)}{V_i(s)} = \frac{s}{s^2+3s+1}$

(b) $\frac{V_0(s)}{V_i(s)} = \frac{1}{s^2+3s+1}$

(c) $\frac{V_0(s)}{V_i(s)} = \frac{s^2}{s^2+2s+1}$

(d) $\frac{V_0(s)}{V_i(s)} = \frac{s+1}{s^2+2s+1}$

Q.2 The damping ratio ξ, natural frequency ω_n and damping frequency ω_d of a system with transfer function,

$$\frac{Y(s)}{R(s)} = \frac{64}{s^2+8s+64}$$

(a) $\xi = 0.5;\ \omega_n = 8;\ \omega_d = 6.93$
(b) $\xi = 1,\ \omega_n = 8;\ \omega_d = 0$
(c) $\xi = 0.5;\ \omega_n = 64;\ \omega_d = 55.42$
(d) $\xi = 0.8;\ \omega_n = 8;\ \omega_d = 6.93$

Q.3 What is the impulse response $y(t)$ of a system with transfer function

$$\frac{Y(s)}{R(s)} = \frac{1}{(s+1)(s+10)}.$$

(a) $y(t) = \frac{1}{9}e^{-t} + \frac{1}{9}e^{-10t}$

(b) $y(t) = \frac{1}{9}e^{-t} - \frac{1}{9}e^{-10t}$

(c) $y(t) = e^{-t} - e^{-10t}$
(d) $y(t) = e^{-t} + e^{-10t}$

Q.4 The forward transfer function of a unity feedback system is $G(s) = \frac{K(s^2+1)}{(s+1)(s+2)}$. The system is stable for:

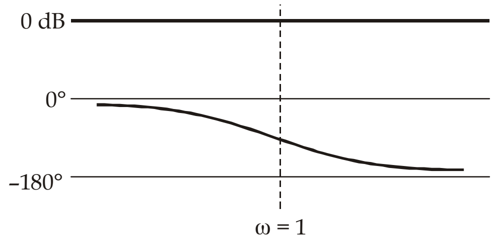
(a) $K < -1$ (b) $K > -1$
(c) $K < -2$ (d) $K > -2$

Q.5 The phase margin of a system with the open loop transfer function,

$$G(s)H(s) = \frac{(1-s)}{(1+s)(3+s)} \text{ is}$$

(a) ∞ (b) 90°
(c) 68.3° (d) 0

Q.6 For the Bode plot shown in figure, the transfer function is,

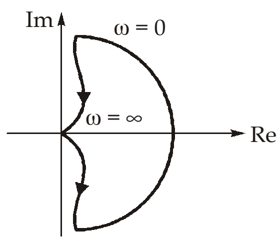


(a) $\frac{1-s}{1+s}$ (b) $\frac{1}{(1+s)^2}$

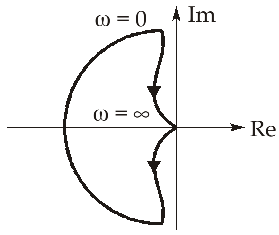
(c) $\frac{1}{s^2}$ (d) $\frac{1}{s(s+1)}$

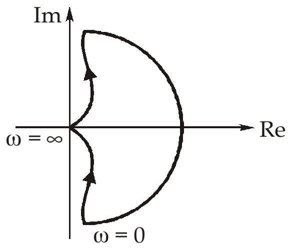
Q.7 The OLTF of a feedback control system is $G(s)H(s) = \frac{-1}{2s(1-20s)}$, the Nyquist plot of the system is

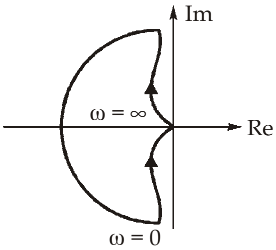
(a)



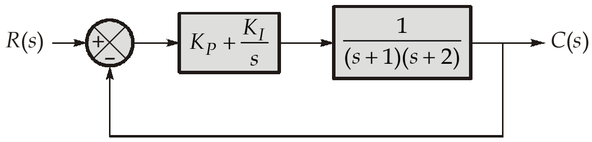
(b)



(c) 

(d) 

Q.8 A stable closed loop control system with PI controller is shown below,



The suitable values of controller parameters are

(a) $K_P > 0$ and $K_I > \frac{K_P}{3} - 3$

(b) $K_I > 0$ and $K_P > \left(\frac{K_I}{3} - 2\right)$

(c) $K_I > \frac{K_P}{3} - 2$ (or) $K_P > 0$

(d) $K_I > 0, K_P > 0$

Q.9 The state model of a system is given as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_1 \end{bmatrix}$$

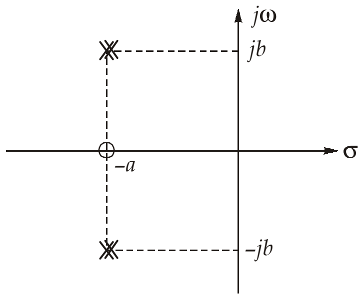
$$x^T(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The solution of homogenous equation is given by,

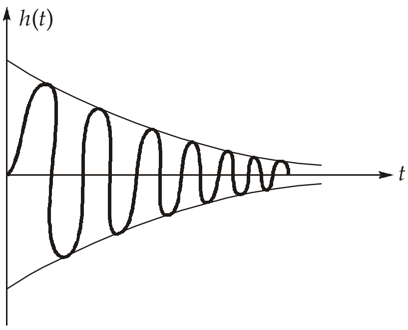
(a) $x(t) = \begin{bmatrix} e^t \\ 1 \end{bmatrix}$ (b) $x(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$

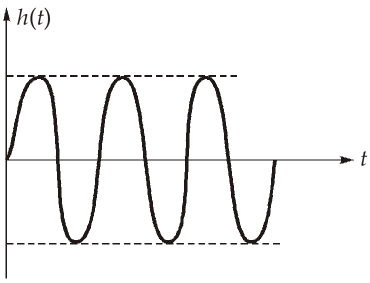
(c) $x(t) = \begin{bmatrix} 0 \\ e^t \end{bmatrix}$ (d) $x(t) = \begin{bmatrix} e^t \\ e^t \end{bmatrix}$

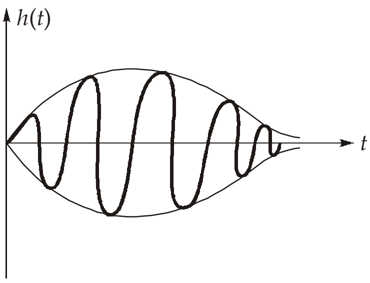
Q.10 A system shown below has multiple poles and a zero as shown below.

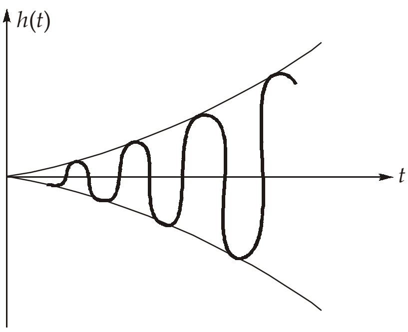


The impulse response of the system is

(a) 

(b) 

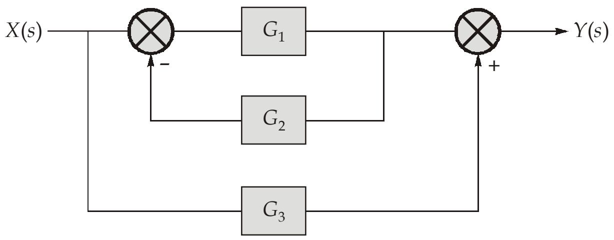
(c) 

(d) 

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 The equivalent transfer function of the system shown in block diagrams is:

$$G_1 = \frac{1}{s+1},\ G_2 = \frac{1}{s+4} \text{ and } G_3 = \frac{s+2}{s+5}$$



(a) $\dfrac{s^3+4s^2+18s+20}{s^3+5s^2+10s+14}$

(b) $\dfrac{s^3+8s^2+24s+30}{s^3+10s^2+30s+25}$

(c) $\dfrac{s^3+4s^2+18s+20}{s^3+10s^2+30s+25}$

(d) $\dfrac{s^3+8s^2+24s+30}{s^2+5s+10s+14}$

Q.12 A control system is represented by the given below differential equation,

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = x(t)$$

has all initial conditions zero. If an input $x(t) = 8\,u(t)$, then output $y(t)$ is

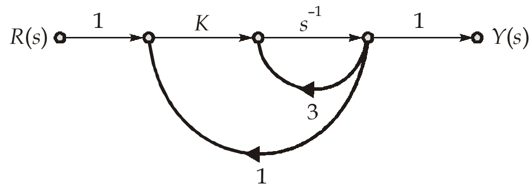
(a) $\left[1 - \frac{1}{4}e^{-4t} + \frac{1}{2}e^{-t}\right]u(t)$

(b) $\left[1 + \frac{1}{4}e^{-4t} + \frac{1}{3}e^{-t}\right]u(t)$

(c) $\left[1 + \frac{1}{8}e^{-4t} - \frac{1}{3}e^{-t}\right]u(t)$

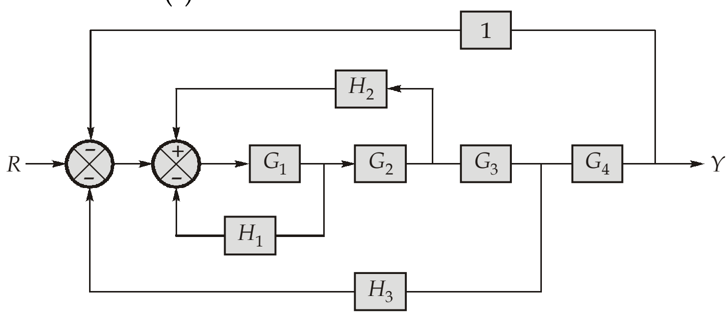
(d) $2\left[1 + \frac{1}{3}e^{-4t} - \frac{4}{3}e^{-t}\right]u(t)$

Q.13 The system shown in the figure remains stable when



(a) $K < -1$ (b) $-1 < K < 1$
(c) $1 < K < 3$ (d) $K < -3$

Q.14 Find the transfer function $\dfrac{Y(s)}{R(s)}$ for the system with following block diagram:



(a) $\dfrac{Y(s)}{R(s)} = \dfrac{G_1G_2G_3G_4}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$

(b) $\dfrac{Y(s)}{R(s)} = \dfrac{G_1G_2G_3G_4}{1-G_1H_1+G_1G_2H_2+G_1G_2G_3H_3}$

(c) $\dfrac{Y(s)}{R(s)} = \dfrac{G_1G_2G_3G_4}{1+G_1H_1+G_1G_2H_2+G_1G_2G_3H_3}$

(d) $\dfrac{Y(s)}{R(s)} = \dfrac{G_1G_2G_3G_4}{1-G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$

Q.15 The closed loop transfer function of a system is,

$$T(s) = \frac{s^3 + 4s^2 + 8s + 16}{s^5 + 3s^4 + 5s^2 + 4s + 3}$$

The number of poles in the right half plane and in left half plane are

(a) 3, 2 (b) 1, 4
(c) 2, 3 (d) 4, 1

Q.16 Consider a unity feedback system with forward path transfer function $G(s) = \frac{K(s+3)(s+5)}{(s-2)(s-4)}$. The range of K to ensure stability is

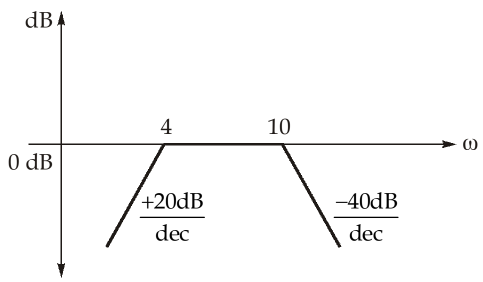
(a) $K > \frac{3}{4}$

(b) $K < -1 \text{ (or)} K > \frac{3}{4}$

(c) $K < -1$

(d) $-1 < K < \frac{3}{4}$

Q.17 For the Bode plot shown below the transfer function is,



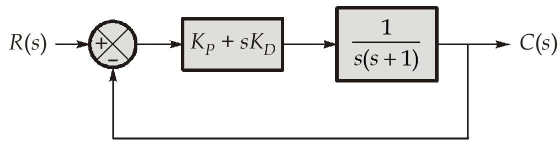
(a) $\frac{100s}{(s+4)(s+10)^2}$

(b) $\frac{100(s+4)}{s(s+10)^2}$

(c) $\frac{100}{(s+4)(s+10)}$

(d) $\frac{100}{s^2(s+4)(s+10)}$

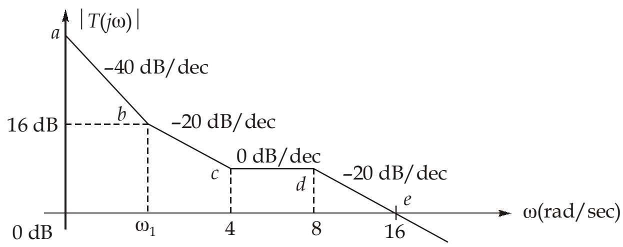
Q.18 The block diagram of a control system is shown below,



The control specification are such that the damping ratio of the closed loop system is 0.6 and damped frequency, ω_d is 10 rad/sec. The parameters of *PD* controller K_p and K_D are

(a) 156.25, 14
(b) 277.78, 19
(c) 2, 4
(d) 11, 100

Q.19 The asymptotic log-magnitude curve for open loop transfer function is sketched below,



Open loop transfer function is

(a) $T(s) = \frac{10(s+8)(s+4)}{s^2(s+1.268)}$

(b) $T(s) = \frac{16(s+1.268)(s+4)}{s^2(s+8)}$

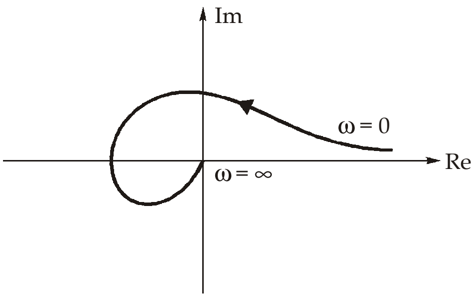
(c) $T(s) = \frac{10(s+1.268)(s+8)}{s^2(s+4)}$

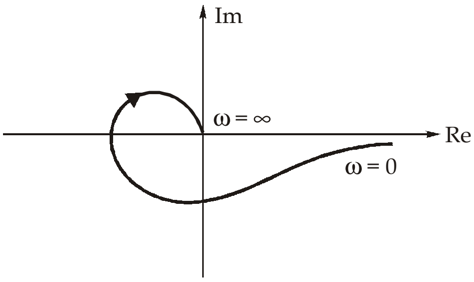
(d) $T(s) = \frac{8(s+1.268)(s+8)}{s^2(s+4)}$

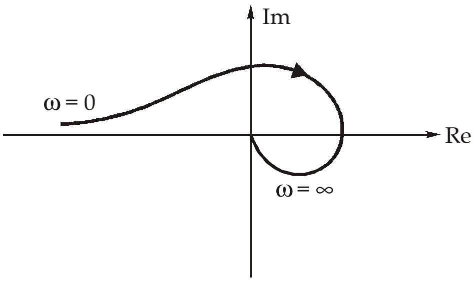
Q.20 Consider a control system having open-loop transfer function of,

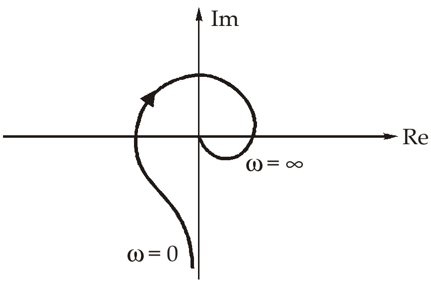
$$G(s)H(s) = \frac{1}{s^2(s-a)(s-b)(s-c)}$$

If a, b, c are positive, then the polar plot of the system will be

(a)


(b)


(c)


(d)


Q.21 A control system is represented by following state space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

If two such systems are connected in parallel then overall transfer function of the combined system is

(a) $\dfrac{4s+21}{(s+4)(s+5)}$ (b) $\dfrac{4s+36}{(s+4)(s+5)}$

(c) $\dfrac{2(s+9)}{(s+4)(s+5)}$ (d) $\dfrac{2(4s+21)}{(s+4)(s+5)}$

Q.22 If the transfer function of a control system is given as below:

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3+4s^2+5s+3}$$

The matrix $[A]$ and $[B]$ in state variable representation of the system will be

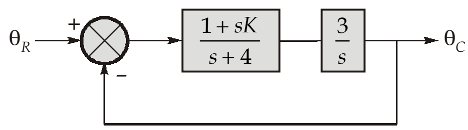
(a) $[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -5 & -3 \end{bmatrix}$; $[B] = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -5 & -4 \end{bmatrix}$; $[B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(c) $[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -10 & -6 \end{bmatrix}$; $[B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(d) $[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -10 & -18 \end{bmatrix}$; $[B] = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

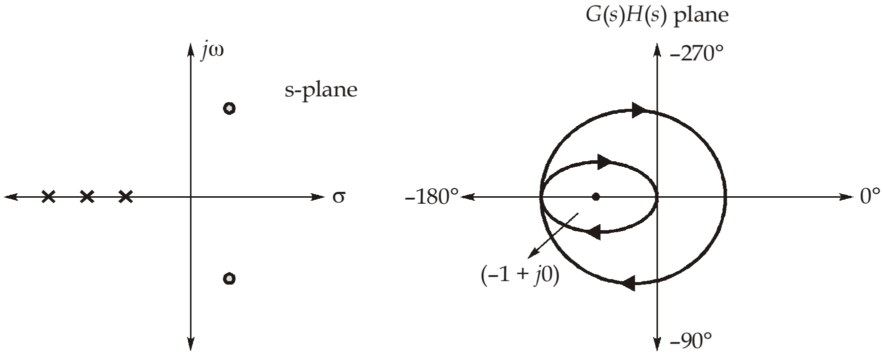
Q.23 A control system with damping ratio $\xi = \sqrt{3}$ is represented by the block diagram which employs proportional plus error rate control. The value of error rate constant K when unit step is given as input to system will be



(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{2}$

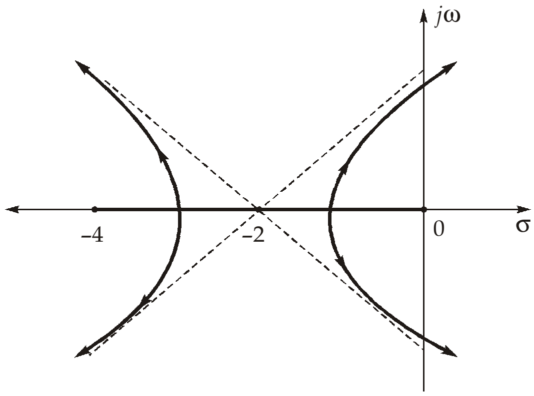
(c) $\dfrac{1}{4}$ (d) $\dfrac{2}{3}$

Q.24 The pole zero map and the Nyquist plot of the open-loop transfer function $G(s)\ H(s)$ of a feed back system are shown below, for this,

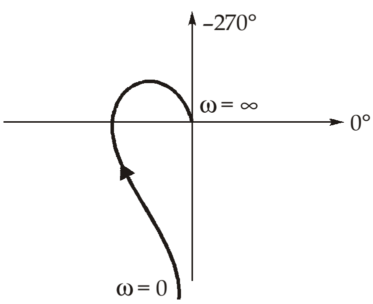


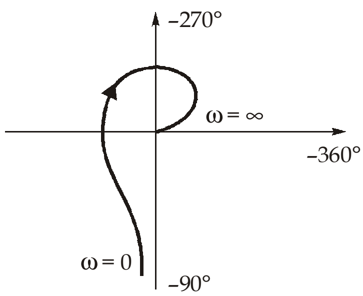
(a) Both open-loop and closed-loop systems are stable.
(b) Open-loop system is stable but closed-loop system is unstable.
(c) Open-loop system is unstable but closed-loop system is stable.
(d) Both open-loop and closed-loop systems are unstable.

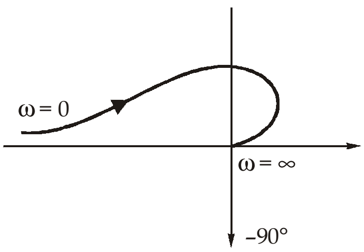
Q.25 The root locus diagram of a control system is shown below,

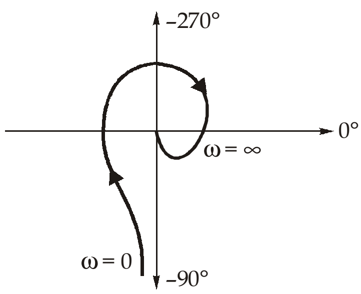


The polar plot of the same system is

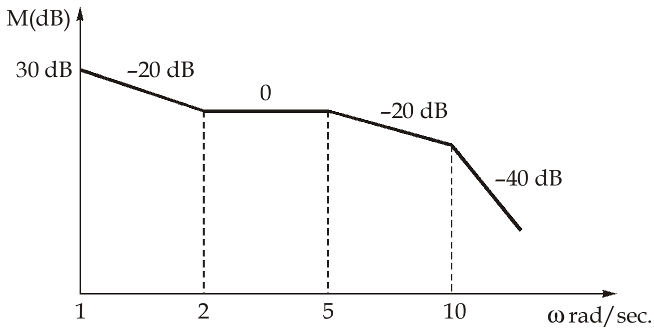
(a)


(b)


(c)


(d)


Q.26 The open-loop transfer function of the system is,



(a) $\dfrac{30\left(\frac{s}{5}+1\right)}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{2}+1\right)}$

(b) $\dfrac{30\left(\frac{s}{2}+1\right)}{s^2\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$

(c) $\dfrac{1.65(0.2s+1)}{s(0.5s+1)(0.1s+1)}$

(d) $\dfrac{31.62(0.5s+1)}{s(0.2s+1)(0.1s+1)}$

Q.27 If $G(s)H(s) = \dfrac{10}{(s+1)(s+20)}$ then the gain margin and phase margin of the system are respectively.

(a) $\frac{1}{2}, \infty$

(b) 0, 180°

(c) ∞, ∞

(d) ∞, 180°

Q.28 The state space equation of the system is given by

$$\dot{x} = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix} x$$

and the initial state value is $x(0) = [4 \;\; -4]^T$. The zero input response of the system will be

(a) $\begin{bmatrix} 4e^{-4t} \\ -4e^{-3t} \end{bmatrix}$

(b) $\begin{bmatrix} -4e^{-4t} \\ -4e^{-3t} \end{bmatrix}$

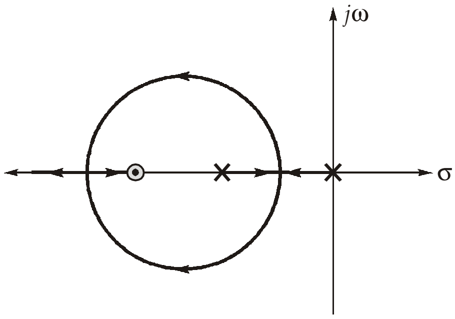
(c) $\begin{bmatrix} 4e^{-4t} \\ 4e^{-3t} \end{bmatrix}$

(d) $\begin{bmatrix} -4e^{-4t} \\ 4e^{-3t} \end{bmatrix}$

Q.29 The open-loop transfer function of a system is

$$G(s)H(s) = \frac{K(s+3)}{s(s+2)}$$

The root locus of the system consists of a circle as shown in the figure below. The equation of circle is



(a) $(\sigma + 4)^2 + \omega^2 = 4$
(b) $(\sigma - 3)^2 + \omega^2 = 9$
(c) $(\sigma + 3)^2 + \omega^2 = 3$
(d) $(\sigma - 4)^2 + \omega^2 = \sqrt{4}$

Q.30 Consider a unity gain closed-loop transfer function with forward path gain,

$$G(s) = \frac{K(s+1)}{s^3 + 0.5s^2 + 3s + 1}$$

If the system is producing undamped oscillations, then value of K and corresponding frequency of oscillations are respectively

(a) 2.5 and 1 rad/s
(b) 1 and 2 rad/s
(c) 1 and 2.5 rad/s
(d) 2 and 1 rad/s

■■■■

CLASS TEST

S.No.: 01JP_ECEE_16102023

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CONTROL SYSTEMS

EC | EE

Date of Test : 16/10/2023

ANSWER KEY

1. (a)	7. (b)	13. (d)	19. (b)	25. (b)
2. (a)	8. (b)	14. (a)	20. (a)	26. (d)
3. (b)	9. (b)	15. (c)	21. (d)	27. (c)
4. (b)	10. (c)	16. (b)	22. (b)	28. (a)
5. (a)	11. (b)	17. (a)	23. (d)	29. (c)
6. (a)	12. (d)	18. (a)	24. (b)	30. (b)

DETAILED EXPLANATIONS

1. (a)

After taking Laplace transform, the series combination of R_1 and C_1 gives: $R_1 + \dfrac{1}{sC_1}$.

Similarly, the parallel combination of R_2 and C_2 gives: $\dfrac{\dfrac{R_2}{sC_2}}{R_2 + \dfrac{1}{sC_2}}$

Substituting, $R_1 = R_2 = C_1 = C_2 = 1$ makes the above combinations $1+\dfrac{1}{s}$ and $\dfrac{\dfrac{1}{s}}{1+\dfrac{1}{s}}$

On applying the voltage divider rule, we get

$$V_0(s) = \frac{\dfrac{\dfrac{1}{s}}{1+\dfrac{1}{s}}}{1+\dfrac{1}{s}+\dfrac{\dfrac{1}{s}}{1+\dfrac{1}{s}}} V_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{\dfrac{1}{s+1}}{\dfrac{s(s+1)+(s+1)+s}{s(s+1)}}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{s}{s^2+3s+1}$$

2. (a)

The general form of transfer function of a 2nd order system is given by,

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

On comparing:

$$\omega_n^2 = 64$$

Natural frequency, $\omega_n = 8$ rad/sec

$$2\,\xi\omega_n = 8$$

Damping ratio, $\xi = \dfrac{8}{16} = 0.5$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 8\sqrt{1-0.5^2}$$

$$\omega_d = 6.93 \text{ rad/sec}$$

3. (b)

For impulse response, $R(s) = 1$

The impulse response is given by,

$$y(t) = L^{-1}\left\{\frac{1}{(s+1)(s+10)}\right\}$$

$$y(t) = L^{-1}\left\{\frac{1}{9(s+1)} - \frac{1}{9(s+10)}\right\}$$

$$y(t) = \frac{1}{9}e^{-t} - \frac{1}{9}e^{-10t}$$

4. (b)

Close loop transfer function of the system is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K(s^2+1)}{(K+1)s^2+3s+K+2}$$

$\therefore$ the characteristic equation is given by

$$(K + 1)s^2 + 3s + K + 2 = 0$$

For the system to be stable all the coefficient should be of same sign

$\therefore$ $K + 1 > 0 = K > -1$

and $K + 2 > 0 = K > -2$

$\therefore$ The system is stable for $K > -1$.

5. (a)

$$G(s)H(s) = \frac{(1-s)}{(1+s)(3+s)}$$

So, $$|G(j\omega)H(j\omega)| = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\sqrt{9+\omega^2}} = \frac{1}{\sqrt{9+\omega^2}}$$

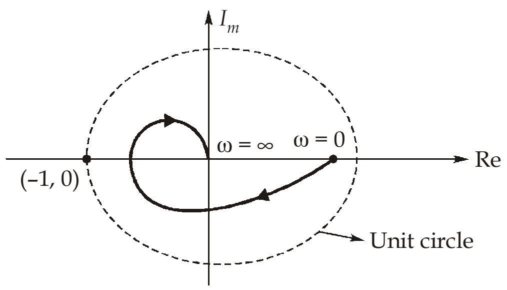
Phase angle is,

$$\phi = -2\tan^{-1}\omega - \tan^{-1}\frac{\omega}{3}$$

At $\omega = 0$; $|G(j\omega)H(j\omega)| = \frac{1}{3}$; $\phi = 0°$

At $\omega = 4$; $|G(j\omega)H(j\omega)| = \frac{1}{5}$; $\phi = -205°$

At $\omega = \infty$; $|G(j\omega)H(j\omega)| = 0$; $\phi = -270°$



Polar plot does not cross unit circle, for all values of ω (0 to ∞), magnitude will be less than unity. In this case, gain crossover frequency does not exist and phase margin becomes ∞.

6. (a)

The gain of function is 1, given system is all pass system, Hence option (a) is correct answer.

We can cross check:

$$GH(s) = \frac{1-s}{1+s}$$

$$GH(j\omega) = \frac{1-j\omega}{1+j\omega}$$

$$\angle GH(j\omega) = -\tan^{-1}(\omega) - \tan^{-1}(\omega) = -2\tan^{-1}(\omega)$$

By varying ω: $0 \to \infty$

The phase varies from 0° to –180° as shown in the plot.

7. (b)

$$\text{OLTF is } G(s)H(s) = \frac{-1}{2s(1-20s)}$$

(or) $$G(j\omega)H(j\omega) = \frac{-1}{2j\omega(1-20j\omega)}$$

$$M = |G(j\omega)H(j\omega)| = \frac{1}{2\omega\sqrt{1+400\omega^2}}$$

$$\phi = 180° - 90° - \tan^{-1}\left(\frac{-20\omega}{1}\right)$$

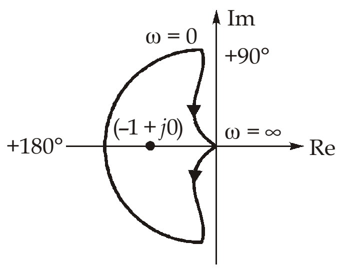
$$= 90° + \tan^{-1}(20\,\omega)$$

At $\omega = 0$; $M = \infty$; $\phi = 90°$

At $\omega = 0.1$; $M = 2.24$; $\phi = 153.43°$

At $\omega = \infty$; $M = 0$; $\phi = 180°$

We get the Nyquist plot as,



8. (b)

The characteristic equation is,

$$1 + G(s)H(s) = 0$$

$$1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{1}{(s+1)(s+2)}\right) = 0$$

(or) $$s^3 + 3s^2 + (2 + K_p)s + K_I = 0$$

Forming Routh Array:

$$\begin{array}{c|cc} s^3 & 1 & 2+K_P \\ s^2 & 3 & K_I \\ s^1 & \dfrac{6+3K_P-K_I}{3} & \\ s^0 & K_I & \end{array}$$

For stable system, $K_I > 0$

and $K_P > \dfrac{K_I}{3} - 2$

9. (b)

$$x(t) = \phi(t)\, x(0)$$

Where, $\phi(t)$ = state transition matrix

Given, $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s-1)(s-1)}\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{s-1} & 0 \\ \dfrac{1}{(s-1)^2} & \dfrac{1}{(s-1)} \end{bmatrix}$$

$$L^{-1}[sI - A]^{-1} = \phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

$$\therefore \quad x(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

10. (c)

The transfer function of the system is,

$$H(s) = \frac{(s+a)}{[(s-(-a-jb))\,(s-(-a+jb))]^2}$$

$$= \frac{s+a}{[(s+a)^2+b^2]^2} = \frac{1}{2}\cdot\frac{2(s+a)}{[(s+a)^2+b^2]^2}$$

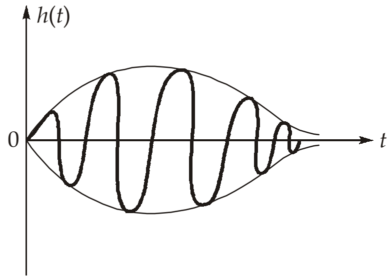
$$H(s) = \frac{1}{2}\left[-\frac{dF(s)}{ds}\right]$$

where, $$F(s) = \frac{1}{(s+a)^2+b^2} \xleftarrow{\text{I.L.T.}} f(t) = \frac{1}{b}e^{-at}\sin(bt)$$

$$-\frac{d}{ds}F(s) \longleftrightarrow t\,f(t)$$

$$h(t) = \frac{t}{2}f(t) = \frac{t}{2b}e^{-at}\sin(bt)$$

$$h(t) = K\,te^{-at}\sin(bt)$$

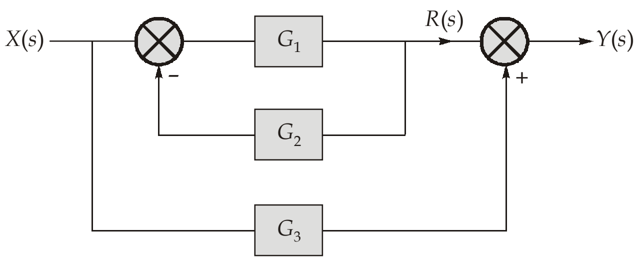


11. **(b)**

For given block diagram.

In inner loop feedback we can write,

$$\frac{R(s)}{X(s)} = \frac{G_1(s)}{1+G_1(s)G_2(s)}$$



∴ Overall transfer function,

$$Y(s) = X(s)\,R(s) + G_3(s)\,X(s)$$

$$\frac{Y(s)}{X(s)} = R(s) + G_3(s)$$

$$= \frac{\frac{1}{s+1}}{1+\frac{1}{(s+1)}\frac{1}{(s+4)}} = \frac{(s+4)}{(s+1)(s+4)+1} = \frac{s+4}{s^2+5s+4+1}$$

$$\frac{s+4}{s^2+5s+5} + \frac{s+2}{s+5} \Rightarrow \frac{(s+4)(s+5)+(s+2)(s^2+5s+5)}{(s^2+5s+5)(s+5)}$$

$$= \frac{s^2+9s+20+s^3+5s^2+5s+2s^2+10s+10}{s^3+5s^2+5s^2+25s+5s+25} =$$

$$\frac{s^3+8s^2+24s+30}{s^3+10s^2+30s+25}$$

12. (d)

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y = x(t)$$

$$s^2Y(s) + 5sY(s) + 4Y(s) = X(s)$$

$$x(t) = 8u(t) \quad \text{or} \quad X(s) = \frac{8}{s}$$

$$(s^2 + 5s + 4)\, Y(s) = \frac{8}{s}$$

$$Y(s) = \frac{8}{s(s^2+5s+4)} = \frac{A}{s}+\frac{B}{s+4}+\frac{C}{s+1}$$

$$Y(s) = \frac{2}{s}-\frac{8}{3(s+1)}+\frac{2}{3(s+4)}$$

$$y(t) = 2\left[1+\frac{1}{3}e^{-4t}-\frac{4}{3}e^{-t}\right]u(t)$$

13. (d)

$$\frac{Y(s)}{R(s)} = \frac{K/s}{1-\left(\frac{3}{s}+\frac{K}{s}\right)} = \frac{K}{s-(3+K)}$$

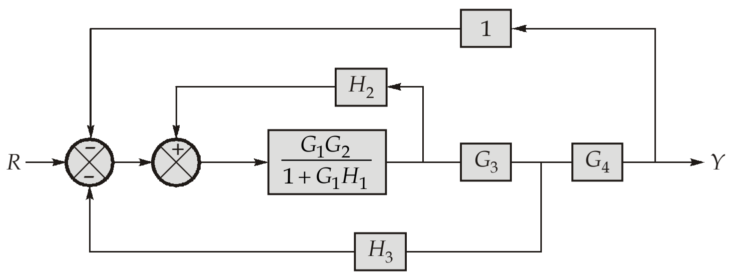
For system to be stable,

$$3 + K < 0$$

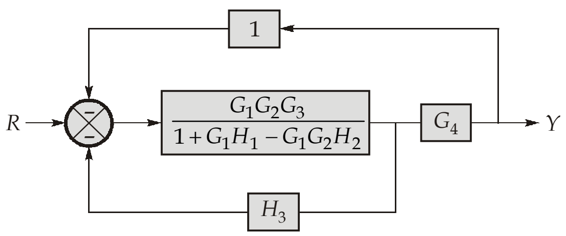
$$K < -3$$

14. (a)

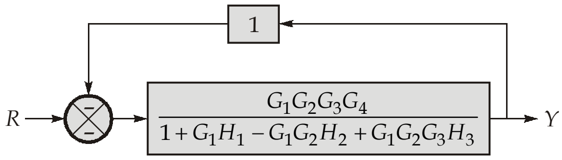
G_1 and H_1 form a negative feedback loop and that loop is in cascade with G_2. The block diagram can be simplified as:



Now $\dfrac{G_1G_2}{1+G_1H_1}$ forms a positive feedback loop with H_2 and that loop is in cascade with G_3. The block diagram can be further simplified as:



Now there is another negative feedback loop with H_3 and cascade with G_4. Block diagram becomes



$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_1H_1-G_1G_2H_2+G_1G_2G_3H_3+G_1G_2G_3G_4}$$

15. (c)

The Routh table for the characteristic equation corresponding to the given transfer function is:

s^5	1	0	4
s^4	3	5	3
s^3	–1.67	3	
s^2	10.4	3	
s^1	3.48		
s^0	3		

There are two sign changes in the fist column of Routh table. So there are 2 poles in RHP. Order of the characteristic equation is 5 = number of poles

$\therefore$ No. of poles in LHP = 5 – 2 = 3

16. (b)

Closed loop transfer function is,

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K(s+3)(s+5)}{(K+1)s^2+(8K-6)s+(15K+8)}$$

For stability all coefficients should be positive or all negative

Case-I: All coefficient are positive,

$$K+1>0 \Rightarrow K>-1$$
$$8K-6>0 \Rightarrow K>0.75$$
$$15K+8>0 \Rightarrow K>\frac{-8}{15}$$

$\therefore$ $K > 0.75$

Case-II : All coefficients are negative:

$$K+1<0 \Rightarrow K<-1$$
$$8K-6<0 \Rightarrow K<0.75$$
$$15K+8<0 \Rightarrow K<\frac{-8}{15}$$

$\therefore$ $K < -1$

So, for stability either $K > 0.75$ or $K < -1$

17. (a)

Transfer function is,

$$G(s)H(s) = \frac{Ks}{\left(\frac{s}{4}+1\right)\left(\frac{s}{10}+1\right)^2} = \frac{400Ks}{(s+4)(s+10)^2}$$

Magnitude of initial plot is given by,

$$M = 20 \log \omega + 20 \log K$$

and $$M = 0 \text{ dB at } \omega = 4$$

$$0 = 20 \log_{10} 4 + 20 \log K$$

(or) $$20\log\frac{1}{K} = 20 \log 4$$

(or) $$\frac{1}{K} = 4$$

(or) $$K = \frac{1}{4}$$

Hence, $$G(s)H(s) = \frac{100s}{(s+4)(s+10)^2}$$

18. (a)

Damped frequency, $$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$\Rightarrow$ $$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{10}{\sqrt{1-(0.6)^2}}$$

The desired second order C.E. is,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 2(0.6)(12.5)s + (12.5)^2 = 0$$

$$s^2 + 15s + 156.25 = 0$$

The C.E. of given system is,

$$1 + (K_p + sK_D)\left(\frac{1}{s(s+1)}\right) = 0$$

(or) $$s^2 + (1 + K_D)s + K_p = 0$$

Hence, $$K_p = 156.25$$

and $$1 + K_p = 15$$

$\Rightarrow$ $$K_D = 14$$

19. (b)

From the above Bode plot,

For section *de*, slope is –20 dB/dec

$\therefore$ $$-20 = \frac{y-0}{\log 8 - \log 16}$$

$$y = 6.02 \text{ dB}$$

Now, for section *bc*, slope is –20 dB/dec

$$\therefore \qquad -20 = \frac{16-6.02}{\log\omega_1 - \log 4}$$

$$\omega_1 = 1.268 \text{ rad/sec}$$

To find value of gain K

$$y = mx + c$$

$$16 = -40 \log 1.268 + 20 \log K$$

$$K = 10.14$$

From all the result, transfer function is,

$$T(s) = \frac{10.14\left(\frac{s}{1.268}+1\right)\left(\frac{s}{4}+1\right)}{s^2\left(\frac{s}{8}+1\right)}$$

$$T(s) = \frac{16(s+1.268)(s+4)}{s^2(s+8)}$$

20. (a)

Given, $$G(s)\,H(s) = \frac{1}{s^2(s-a)\,(s-b)\,(s-c)}$$

Put $s = j\omega$, $$G(j\omega)\ H(j\omega) = \frac{1}{(j\omega)^2(j\omega-a)\,(j\omega-b)\,(j\omega-c)}$$

$$|G(j\omega)\,H(j\omega)| = \frac{1}{\omega^2\sqrt{\omega^2+a^2}\cdot\sqrt{\omega^2+b^2}\cdot\sqrt{\omega^2+c^2}}$$

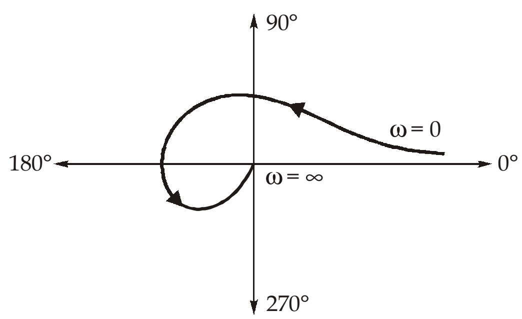
when, $\omega = 0$ Magnitude = ∞; Phase = $0°$

$\omega = \infty$ Magnitude = 0; Phase = $270°$

$$\angle G(j\omega)\ H(j\omega) = -\left[180° + \left(180° - \tan^{-1}\left(\frac{\omega}{a}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{b}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{c}\right)\right)\right]$$

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{\omega}{b}\right) + \tan^{-1}\left(\frac{\omega}{c}\right)$$

Note: Since a, b and c are positive values, for $\omega = 1$, phase of $G(j\omega)\ H(j\omega)$ will be a positive quantity. So, the polar plot will start into the first quadrant.



21. (d)

The transfer function from steady state model can be written as

$$T(s) = C[sI - A]^{-1} B$$

$$[sI - A]^{-1} = \frac{\text{adj}[sI - A]}{|sI - A|}$$

$$[sI - A] = \begin{bmatrix} s+5 & -1 \\ 0 & s+4 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+5)(s+4)}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} s+4 & 1 \\ 0 & s+5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$T(s) = \frac{1}{(s+5)(s+4)}\begin{bmatrix} 1 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ s+5 \end{bmatrix}$$

$$T(s) = \frac{1+4(s+5)}{(s+4)(s+5)} = \frac{4s+21}{(s+4)(s+5)}$$

If two systems are connected in parallel
Overall transfer function,

$$T'(s) = T(s) + T(s) = \frac{2(4s+21)}{(s+4)(s+5)}$$

22. (b)

For the given system we can write,

$$Y(s)\,[s^3 + 4s^2 + 5s + 3] = U(s)[1] \qquad \text{...(i)}$$

Let, $x_1 = y$

So, $x_2 = \dot{x}_1 = \dot{y}$ ...(ii)

$x_3 = \dot{x}_2 = \ddot{y}$ ...(iii)

The equation (i) can be written as

$$\dot{x}_3 + 4x_3 + 5x_2 + 3x_1 = u$$

i.e. $\dot{x}_3 = -3x_1 - 5x_2 - 4x_3 + u$ ...(iv)

Using equation (ii), (iii) and (iv) we can write in state variable form as below,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -5 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

So, $$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -5 & -4 \end{bmatrix}$$

and $$[B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Hence option (b) is correct.

23. (d)

The closed-loop transfer function for given system,

$$\frac{G(s)}{1+G(s)} = \frac{\frac{3(1+sK)}{s(s+4)}}{1+\frac{3(1+sK)}{s(s+4)}} = \frac{3(1+sK)}{s(s+4)+3(1+sK)}$$

$$= \frac{3(1+sK)}{s^2+4s+3(1+sK)} = \frac{3+3sK}{s^2+(4+3K)s+3}$$

Comparing the transfer function with standard second order transfer function.

$$\omega_n^2 = 3$$

$$\Rightarrow \quad \omega_n = \sqrt{3}\,\text{rad/sec}$$

$$2\xi\omega_n = 4 + 3K$$

$$2\sqrt{3}\xi = 3K + 4$$

$$\frac{2\sqrt{3}\xi - 4}{3} = K$$

Given, $\xi = \sqrt{3}$, $\quad K = \frac{6-4}{3} = \frac{2}{3}$

24. (b)

From plot –1

Number of poles on the right side of s-plane = 0

$$P = 0$$

Open-loop system is stable.

From plot –2

Number of encirclement about (–1, 0) is = 2 in clockwise

$$N = -2$$

$$N = P - Z$$

$$-2 = 0 - Z$$

$$Z = 2$$

Two closed loop poles on the right side of s-plane. Close-loop system is unstable.

25. (b)

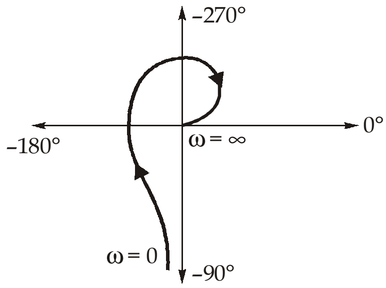
From the figure, the transfer function of the system is,

$$G(s)\,H(s) = \frac{K}{s(s+2)^2(s+4)}$$

$$|G(j\omega)H(j\omega)| = \frac{K}{\omega(4+\omega^2)\sqrt{16+\omega^2}}$$

$$\angle G(j\omega)\,H(j\omega) = -90° - 2\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{4}\right)$$

ω	M	ϕ
0	∞	–90°
⋮	⋮	⋮
∞	0	–360°



26. (d)

$$\text{T.F.} = \frac{K\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$$

To obtain the value of K,

$$30 \text{ dB} = 20 \log K - 20\, r \log(1) \qquad [\because\ r = 1]$$

$$1.5 = \log K$$

$$K = 10^{1.5} = 31.62$$

$$\text{T.F.} = \frac{31.62(0.5s+1)}{s(0.2s+1)\,(0.1s+1)}$$

27. (c)

$$\angle G(j\omega)\,H(j\omega) = \angle\frac{10}{(j\omega+1)\,(j\omega+20)} = -180°$$

$$= -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{20}\right) = -180°$$

$$-\tan^{-1}\left(\frac{\omega + \frac{\omega}{20}}{1 - \frac{\omega^2}{20}}\right) = -180°$$

$$\frac{\omega + \frac{\omega}{20}}{1 - \frac{\omega^2}{20}} = 0$$

The above expression to become zero $1 - \frac{\omega^2}{20} = \infty$.

$$\therefore \quad \omega_{pc} = \infty$$

$$|G(j\omega)H(j\omega)| = \frac{10}{\sqrt{1+\omega^2}\sqrt{(20)^2+\omega^2}}$$

$$|G(j\omega)H(j\omega)|_{\omega_{pc}=\infty} = 0$$

$$\text{Gain margin} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega_{pc}}} = \infty$$

$\therefore$ Gain crossover frequency does not exist.

$\therefore$ Phase margin is = ∞

28. (a)

For given state model,

$$A = \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s+4 & 0 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{|sI - A|}\text{Adj}[sI - A] = \frac{1}{(s+3)(s+4)}\begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

So zero input response of the given system will be

$$x(t) = e^{At} \cdot x(0)$$

$$= \begin{bmatrix} e^{-4t} & 0 \\ 0 & e^{-3t} \end{bmatrix}\begin{bmatrix} 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 4e^{-4t} \\ -4e^{-3t} \end{bmatrix}$$

29. **(c)**

Method-I

For root locus point,

$$\angle G(s)\ H(s) = 180^\circ$$

$\therefore$ Substituting, $s = (\sigma + j\omega)$ we get,

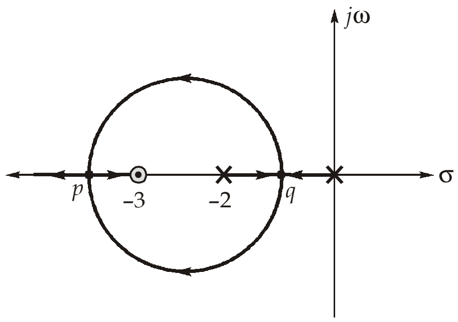
$$G(\sigma + j\omega)\ H(\sigma + j\omega) = \frac{K(\sigma + 3 + j\omega)}{(\sigma + j\omega)(\sigma + 2 + j\omega)}$$

$$\tan^{-1}\left(\frac{\omega}{\sigma+3}\right) - \tan^{-1}\left(\frac{\omega}{\sigma}\right) = 180^\circ + \tan^{-1}\left(\frac{\omega}{\sigma+2}\right)$$

$$\frac{\dfrac{\omega}{\sigma+3} - \dfrac{\omega}{\sigma}}{1 + \left(\dfrac{\omega}{\sigma+3}\right)\left(\dfrac{\omega}{\sigma}\right)} = \frac{\omega}{\sigma+2}$$

$$\therefore \quad (\sigma + 3)^2 + \omega^2 = 3$$

Method-II:



p and q are breakaway and breakin points, to obtain them we have to perform, $\dfrac{dK}{ds} = 0.$

$$K = -\frac{s(s+2)}{s+3}$$

$$\frac{dK}{ds} = \frac{d}{ds}\left[-\frac{s(s+2)}{s+3}\right] = 0$$

$$s^2 + 6s + 6 = 0$$

$$q = -1.268 \quad \text{to} \quad p = -4.732$$

As we know two points on the diameter, center of the circle is (–3, 0) and radius is 1.732.

Equation of the circle is $(\sigma + 3)^2 + \omega^2 = (\sqrt{3})^2$.

30. **(b)**

The characteristic equation is,

$$q(s) = s^3 + 0.5s^2 + (K+3)\,s + (K+1) = 0$$

$$\begin{array}{c|cc} s^3 & 1 & K+3 \\ s^2 & 0.5 & K+1 \\ s^1 & (3+K)-2(K+1) & 0 \\ s^0 & (K+1) & \end{array}$$

For a system to oscillate a row should become zero.

$\therefore$ $$K + 3 - 2K - 2 = 0$$

$$K = 1$$

Given system is third order system $(s + a)\,(s^2 + bs + c) = 0$

For a marginally stable system, $\xi = 0$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + \omega_n^2 = 0$$

Take the coefficients of s^2 row.

$$0.5s^2 + (K+1) = 0$$

$$0.5s^2 + 2 = 0$$

$$s = \pm j2$$

$$\omega = 2 \text{ rad/s}$$

■■■■