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Strength of Materials

MECHANICAL ENGINEERING

Date of Test : 15/09/2023

ANSWER KEY

1.	(c)	7.	(a)	13.	(a)	19.	(d)	25.	(b)
2.	(d)	8.	(b)	14.	(c)	20.	(d)	26.	(d)
3.	(c)	9.	(a)	15.	(b)	21.	(b)	27.	(d)
4.	(c)	10.	(b)	16.	(b)	22.	(b)	28.	(c)
5.	(a)	11.	(a)	17.	(c)	23.	(a)	29.	(a)
6.	(d)	12.	(b)	18.	(d)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (c)

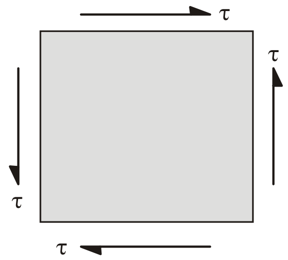
Deflection of a simply supported beam is proportional to,

$$\delta \propto \frac{PL^3}{EI}$$

$\therefore$ Increasing I, decreasing L or P will reduce deflection.

2. (d)

The stress system for an element on the surface is



All normal stresses are zero.

Stress on an inclined plane is given by,

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta - \tau \cos 2\theta$$

Now at $\theta = 45°$,

$\therefore$ $\sigma_n = \tau \sin 90° = \tau$ and $\sigma_t - \tau \cos 90° = 0$

$\therefore$ 45° plane is the principal plane as shear stress on it is zero. Value of maximum principal stress is 'τ'.

3. (c)

A ductile material fails through a cup and cone type of failure.

4. (c)

The angle of twist in both the shafts will be equal. Therefore,

$$\theta_{AB} = \theta_{BC}$$

$$\Rightarrow \quad \left(\frac{TL}{JG}\right)_{AB} = \left(\frac{TL}{JG}\right)_{BC}$$

$$\therefore \quad \frac{T_A L_A}{J_A G_A} = \frac{T_C L_C}{J_C G_C}$$

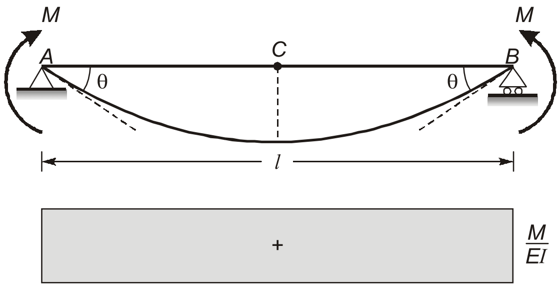
$$L_A = L_C,\ G_A = G_C$$

$$\therefore \quad T_C = \frac{J_C}{J_A} T_A = \left(\frac{D_C}{D_A}\right)^4 T_A$$

$$D_C = 2d,\ D_A = d$$

$$\therefore \quad T_C = 16\ T_A$$

5. (a)



$$\theta_C - \theta_A = \text{Area of } \frac{M}{EI} \text{ diagram}$$

$$0 - \theta_A = +\frac{M}{EI} \times \frac{l}{2} = \frac{Ml}{2EI}$$

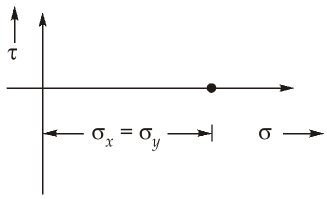
$$\theta_A = -\frac{Ml}{2EI}$$

$$\theta_A = \frac{Ml}{2EI} \text{ (Anti-clockwise)}$$

$$\because \quad \frac{Ml}{EI} = 2\theta_A = 2\theta$$

6. (d)

When normal stresses are equal (same magnitude) and of same nature, then Mohr's circle will be reduced to a point.



Ex. Hydrostatic pressure, internal contact pressure in hollow shafts due to press-fitting of bearing.

7. (a)

$$\tan 2\theta_{p_1} = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$\theta_{p_1} = \frac{1}{2}\tan^{-1}\left(\frac{2\times(-5)}{8-5}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{-10}{3}\right) = \frac{1}{2}\times -73.3 = -36.65°$$

and $\theta_{p_2} = \theta_{p_1} + 90 = -36.65 + 90 = 53.349°$

8. (b)

We know that,

$$E = \frac{9KG}{3K+G}$$

$$\Rightarrow \quad 3KE + EG = 9KG$$

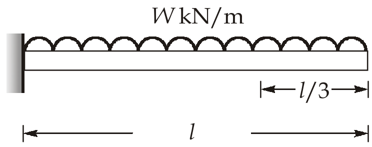
$$\Rightarrow \quad 9KG - 3KE = EG$$

$$\Rightarrow \quad K(9G - 3E) = GE$$

$$\Rightarrow \quad K = \frac{GE}{9G-3E} = \frac{120\times 42}{9\times 42 - 3\times 120}$$

$$= 280 \text{ GPa}$$

9. (a)



Maximum bending will be at fixed end,

i.e. $$M = (Wl)\times\frac{l}{2} = \frac{Wl^2}{2}$$

Bending moment at $\frac{l}{3}$ distance from free end

$$\Rightarrow \quad M' = \left(\frac{Wl}{3}\right)\times\left(\frac{l}{6}\right) = \frac{Wl^2}{18}$$

$$M' = \frac{2}{9}\times\frac{Wl^2}{2} = \frac{M}{9} = 0.1111 = 11.11\% \text{ of M}$$

10. (b)

The displacement of point 'C'

$$\delta_C = (\delta_{AB})_{self\ weight} + (\delta_{AB})_{weight\ of\ BC} + (\delta_{BC})_{Self\ weight}$$

$$= \left.\frac{WL}{2AE}\right|_{AB} + \left.\frac{P_{ext}L}{AE}\right|_{AB} + \left.\frac{WL}{2AE}\right|_{BC}$$

From given data,

$$W_{BC} = W_{AC} - W_{AB}$$

$$= 3W - W = 2W$$

$$W_{AB} = W$$

$$(P_{ext})_B = 2W$$

So, $$\delta_C = \frac{WL}{2AE} + \frac{2WL}{AE} + \frac{(2W)\times L}{2(2A)E}$$

$$\delta_C = \frac{WL}{AE}\left[\frac{2+8+2}{4}\right] = \frac{12WL}{4AE}$$

$$\delta_C = \frac{3WL}{AE}$$

11. (a)

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$\frac{dV}{V} = \frac{r^3 - (r-\Delta r)^3}{r^3} = 1 - \left\{1 - \frac{\Delta r}{r}\right\}^3 = 1 - \left\{1 - \frac{\frac{0.55}{1000}}{2.5}\right\}^3 = 6.5985 \times 10^{-4}$$

$$K = \frac{250}{6.5985 \times 10^{-4}} = 378,871.2243 \text{ MPa}$$
$$= 378.871 \text{ GPa}$$

12. (b)

$$M = 80x - 64(x-1) \; \forall \; x \in (1, 4)$$

At centre, $x = 4$ m

$$M = (80 \times 4) - 64(3) = 128 \text{ kNm}$$

13. (a)

Given, $\sigma_1 = 100$ MPa, $\sigma_2 = 50$ MPa,
$\sigma_3 = 25$ MPa, $S_{yt} = 220$ MPa,

For maximum shear strain energy theory,

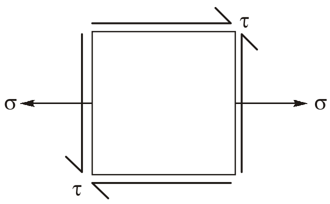
$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \leq 2\left(\frac{S_{yt}}{N}\right)^2$$ [Where, N = factor of safety]

$$(100-50)^2 + (50-25)^2 + (25-100)^2 = 2\left(\frac{220}{N}\right)^2$$

After solving,

$\because$ Factor of safety, $N = 3.326 \sim \mathbf{3.33}$

14. (c)



State of stress, $\sigma = \frac{P}{2\pi rt}$ (Area of cross-section for a thin tube = $2\pi rt = \pi dt$)

$$\sigma = \text{Simple tensile stress} = \frac{10 \times 10^3}{\pi \times 25 \times 1.6} = 79.6 \text{ N/mm}^2$$

Now, $J = \frac{\pi D^3 t}{8}$ for a thin tube

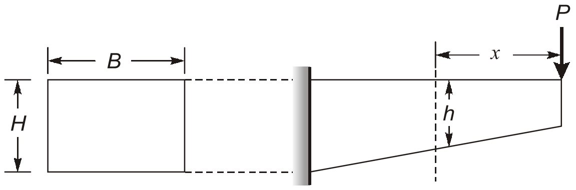
$$\therefore \quad \tau = \frac{T}{J}r = \frac{8T}{\pi D^3 t} \times \frac{D}{2} = \frac{4T}{\pi D^2 t} = \frac{4 \times 23.5 \times 10^3}{\pi \times 25^2 \times 1.6} = 29.94 \text{ N/mm}^2$$

Principal stresses, $\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 39.8 \pm \sqrt{39.8^2 + 29.94^2}$

$$= 39.8 \pm 49.8 = 89.6 \text{ N/mm}^2 \text{ and } -10 \text{ N/mm}^2$$

$\therefore$ Maximum principal stress = 89.6 MPa

15. (b)



Stress at support, $\sigma_1 = \frac{M}{Z} = \frac{P \times L}{\left(\frac{B \times H^3}{12}\right)} \times \frac{H}{2} = \frac{6PL}{BH^2}$

Stress at distance x, $\sigma_2 = \frac{M}{Z} = \frac{6P \cdot x}{B \cdot h^2}$

Equating, $\sigma_1 = \sigma_2$

$$\frac{6PL}{BH^2} = \frac{6P \cdot x}{B \cdot h^2}$$

$$h = \sqrt{\frac{x}{L}} \cdot H$$

16. (b)

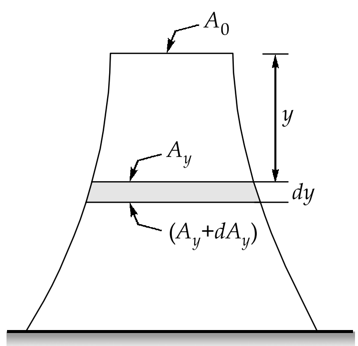
Effective length of column $(L_e) = \frac{L}{\sqrt{2}}$

Least radius of gyration $(k) = \sqrt{\frac{I_{min.}}{A}} = \left(\frac{\pi D^4}{64} \times \frac{4}{\pi D^2}\right)^{1/2} = \frac{D}{4}$

Slenderness ratio $(s) = \frac{L_e}{k} = \frac{\left(\frac{L}{\sqrt{2}}\right)}{\left(\frac{D}{4}\right)}$

$$s = 2\sqrt{2}\frac{L}{D}$$

17. (c)



Let us consider a cross-sectional area A_y at a distance x from top and having thickness dy (as shown)

Force acting due to self weight at section dy

$$\Rightarrow \quad W_y = \int_0^y \rho g A_y dy$$

Now, taking force balance at the section 'dy'

$$\Rightarrow \quad \sigma A_y + \int \rho g A_y dy = \sigma(A_y + dA_y) \qquad [\because \sigma = \text{Constant for uniform strength beam}]$$

$$\Rightarrow \quad \sigma A_y + \int \rho g A_y dy = \sigma A_y + \sigma dA_y$$

$$\Rightarrow \quad \int_0^y \rho g dy = \int_{A_0}^{Ay} \frac{\sigma dA_y}{A_y}$$

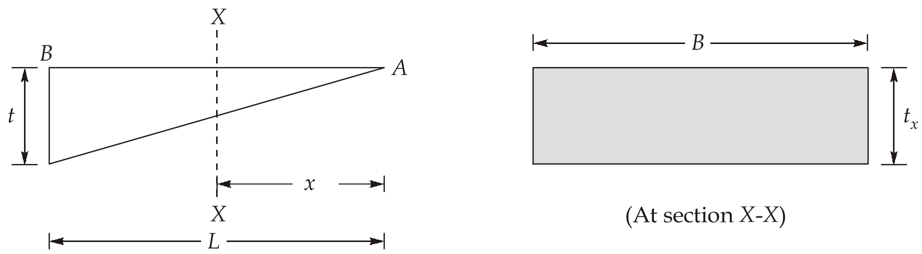
$$\Rightarrow \quad \rho g y = \sigma \log\left[\frac{A_y}{A_0}\right]$$

$$\Rightarrow \quad A_y = A_0 e^{(\rho g y/\sigma)}$$

18. (d)

The cross-sectional area varies linearly with distance from the free end.

Let assume that the maximum thickness of beam = t.



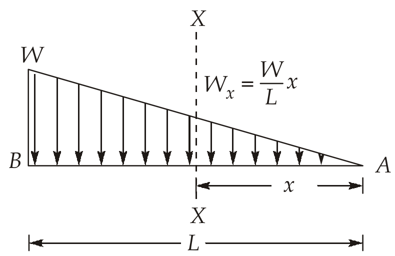
(At section X-X)

$$I_x = \frac{b(t_x)^3}{12} = \frac{b\left(\frac{t}{L}x\right)^3}{12}$$

$$\Rightarrow \quad I_x = \frac{b(t)^3}{12} \times \frac{x^3}{L^3} \qquad \ldots(i)$$

$$\Rightarrow \quad I_x \propto x^3$$

Bending moment at section (X-X)

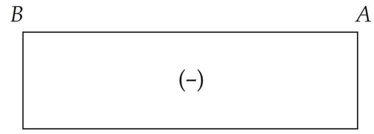


$$\Rightarrow \quad M_x = -\frac{W}{L}x \times \left(\frac{1}{2} \times x\right) \times \left(\frac{x}{3}\right)$$

$$= -\frac{W}{6L}x^3$$

The ratio of $\left(\frac{M}{EI}\right)_x = \frac{K_1 x^3}{K_2 x^3} = \text{Constant}$ (Independent of x)

Hence, $\left(\frac{M}{EI}\right)_x$ diagram $\Rightarrow$



19. (d)

Effective equivalent length,

$$L_e = \frac{L}{3}$$

As,
$$P_e = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 EI}{(L/3)^2} = \frac{9\pi^2 EI}{L^2}$$

20. (d)

Actual column is having end constraints as fixed and free but it was considered as fixed and hinged column.

$$(L_{eq})_{actual} = 2L$$

$$\Rightarrow \quad (P_e)_{actual} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{4L} \quad \ldots(i)$$

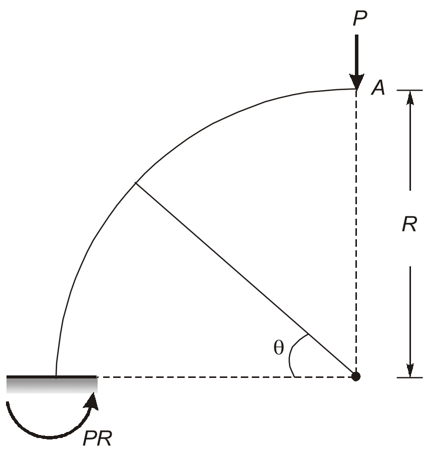
$$(P_e)_{assumed} = \frac{\pi^2 EI}{\left(L/\sqrt{2}\right)^2} \qquad \because (L_{eq}) = \frac{L}{\sqrt{2}}$$

$$= \frac{2\pi^2 EI}{L^2}$$

$$\%\text{error} = \frac{\text{Error}}{(P_e)_{Actual}} = \frac{\frac{\pi^2 EI}{L^2}\left[2 - \frac{1}{4}\right]}{\frac{\pi^2 EI}{L^2} \times \frac{1}{4}}$$

$$= 700\%$$

21. (b)



$$M = PR\cos\theta$$

$$\frac{\partial M}{\partial P} = R\cos\theta$$

Now,
$$\delta_v = \frac{\partial U}{\partial P} \qquad \text{(where } U = \text{strain energy)}$$

$$\frac{\partial U}{\partial P} = \int_0^{\pi/2} \frac{M\times\left(\frac{\partial M}{\partial P}\right)Rd\theta}{EI}$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}\cos^2\theta d\theta$$

$$\delta_v = \frac{\pi PR^3}{4EI}$$

22. (b)

$$\text{Column I} = P_{cr} = \frac{2\pi^2 EI}{h_1^2}$$

$$\text{Column II} = P_{cr} = \frac{4\pi^2}{3}\frac{EI}{h_2^2}$$

$$\frac{2\pi^2 EI}{h_1^2} = \frac{4\pi^2 EI}{3h_2^2}$$

$$\frac{h_2}{h_1} = \sqrt{\frac{2}{3}} = 0.82$$

23. (a)

$$\gamma_{xy} = 2\varepsilon_{45^\circ} - (\varepsilon_0 + \varepsilon_{90^\circ})$$

$$= 2\times 200 - (-500 + 300)$$

$$\gamma_{xy} = 600\ \mu\text{m/m}$$

$$\varepsilon_1, \varepsilon_2 = \left(\frac{\varepsilon_x+\varepsilon_y}{2}\right) \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{-500+300}{2} \pm \sqrt{\left(\frac{-500-300}{2}\right)^2 + 300^2}$$

$$\varepsilon_{1,2} = -100 \pm 500$$

$$\varepsilon_1 = -600\ \mu\text{m/m}$$

$$\varepsilon_2 = 400\ \mu\text{m/m}$$

$$\sigma_1 = \frac{E}{1-\mu^2}[\varepsilon_1 + \mu\varepsilon_2]$$

$$= \frac{200\times 10^3}{(1-0.3^2)}[-600 + 0.3\times 400]$$

$$\sigma_1 = -105.49 \simeq -105\ \text{MPa}$$

24. (a)

$$\sigma_h = \frac{pd}{2t \times \eta_{LJ}} = \frac{6 \times 150}{2 \times 12.5 \times 0.8} = 45 \text{ MPa}$$

$$\sigma_l = \frac{pd}{4t \times \eta_{CJ}} = \frac{6 \times 150}{4 \times 12.5 \times 0.9} = 20 \text{ MPa}$$

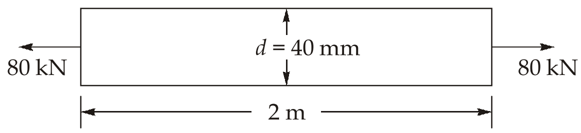
$$\frac{\delta d}{d} = \frac{1}{E}(\sigma_h - \mu\sigma_L) = \frac{1}{200 \times 10^3}(45 - 0.25 \times 20)$$

$$\frac{\delta d}{d} = 0.2 \times 10^{-3}$$

$$\delta d = 0.2 \times 150 \times 10^{-3} \text{ mm}$$

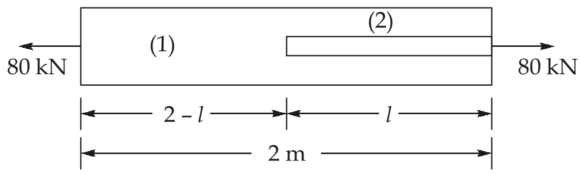
$$\delta d = 0.03 \text{ mm}$$

25. (b)



Elongation of the rod, $\delta l_1 = \frac{PL}{AE} = \frac{80 \times 1000 \times 2000}{\frac{\pi}{4} \times 40^2 \times 2 \times 10^5} = 0.6366$ mm

Let the rod be bored to a length of l meters.



Elongation of the rod after boring,

$$\delta l_2 = 1.2 \times \delta l_1 = 1.2 \times 0.6366 = 0.7639 \text{ mm}$$

$$\delta l_2 = \frac{Pl}{A_2 E} + \frac{P(2-l)}{A_1 E}$$

$$= \frac{80000}{2 \times 10^5}\left[\frac{l \times 1000}{\frac{\pi}{4} \times \left(40^2 - 20^2\right)} + \frac{(2-l) \times 1000}{\frac{\pi}{4} \times 40^2}\right]$$

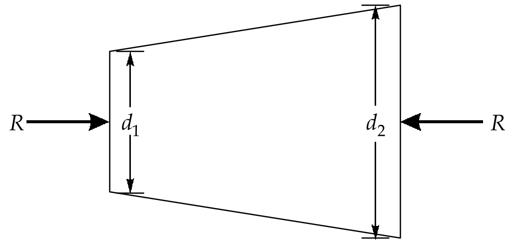
$$= \frac{80000}{2 \times 10^5}[1.061l + (2-l)0.795]$$

$$= \frac{80000}{2 \times 10^5}[1.59 + 0.266l] = 0.7639$$

$$l = 1.202 \text{ m}$$

26. (d)

Both the ends are fixed,



$$(\delta_L)_{\text{total}} = 0$$

$$(\delta_L)_{\text{Contraction due to load}} + (\delta_L)_{\text{Expansion due to temperature}} = 0$$

$$\Rightarrow \quad -\frac{4RL}{\pi d_1 d_2 E} + \alpha\Delta TL = 0$$

$$R = \frac{\pi d_1 d_2 E}{4}\alpha\Delta T$$

$$\sigma_{max} = \frac{R}{A_{min}} = \frac{\pi d_1 d_2 E\alpha\Delta T}{4 \times \frac{\pi}{4} \times d_1^2} = \alpha\Delta TE\left(\frac{d_2}{d_1}\right)$$

$$= 12 \times 10^{-6} \times 50 \times 2 \times 10^5 \times \frac{160}{80}$$

$$= 240 \text{ MPa}$$

$$\sigma_{min} = \frac{R}{A_{min}} = \alpha\Delta TE\left(\frac{d_1}{d_2}\right)$$

$$= 12 \times 10^{-6} \times 50 \times 2 \times 10^5 \times \frac{80}{160}$$

$$= 60 \text{ MPa}$$

27. (d)

Given: $y = \frac{1}{EI}\left(2x^3 - \frac{x^4}{6} - 36x\right)$

In a simply supported beam, bending moment at both the end of beam is zero.

$$M = EI\frac{d^2y}{dx^2} = EI \times \frac{1}{EI} \times \frac{d}{dx}\left(6x^2 - \frac{4x^3}{6} - 36\right)$$

$$= 12x - \frac{12x^2}{6} - 0 = 12x - 2x^2$$

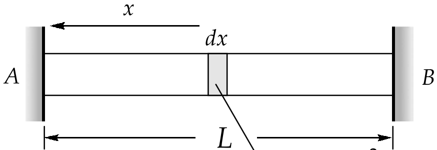
Put $M = 0$, $\quad 12x - 12x^2 = 0$

$$2x(6 - x) = 0$$

$$x = 0, x = 6$$

So, the span of the beam is 6 metres.

28. (c)



Induced strain in dx element,

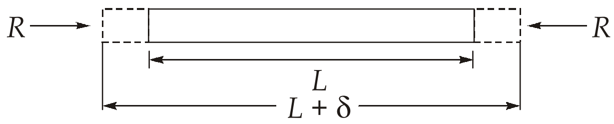
$$\frac{\delta x}{dx} = \alpha\,\Delta T$$

$$\delta x = \alpha \Delta T . dx$$

Total change in length,

$$\int_0^L \delta x = \int_0^L \alpha . \Delta T_1 \cdot \frac{x^2}{L^2} \cdot dx$$

$$\delta = \frac{\alpha \, \Delta T_1 \, L^3}{3 \, L^2} = \frac{\alpha \, \Delta T_1 L}{3}$$



As the both ends are rigid, no elongation will occur due to rigid support reactions induced R

$$\sigma_{thermal} = \frac{R}{A} = E . \frac{\delta}{L}$$

$$\sigma_{thermal} = \frac{E \, \alpha \, \Delta T_1}{3}$$

29. (a)

$$\Delta L_s = \Delta L_A$$

$$\Rightarrow \quad \left(\frac{PL}{AE}\right)_S = \left(\frac{PL}{AE}\right)_A$$

$$\frac{P_s}{P_A} = \frac{A_S E_S / L_S}{A_A E_A / L_A} = \frac{0.5 \times 200/2}{2 \times 100/1} = 0.25$$

30. (b)

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$= \frac{(3P)^2 L}{6 \times 2AE} + \frac{(-2P)^2 L}{2 \times 2AE} + \frac{P^2 L}{3 \times 2AE}$$

$$= \frac{9P^2 L}{12AE} + \frac{4P^2 L}{4AE} + \frac{P^2 L}{6AE}$$

$$= \frac{3P^2 L}{4AE} + \frac{P^2 L}{AE} + \frac{P^2 L}{6AE}$$

$$= \frac{(9 + 12 + 2)P^2 L}{12AE} = \frac{23P^2 L}{12AE}$$

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