CLASS TEST

S.No. : 01 SKCS_ABCDFGHI_26082023

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DISCRETE MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 26/08/2023

ANSWER KEY

1.	(a)	7.	(d)	13.	(b)	19.	(a)	25.	(d)
2.	(d)	8.	(b)	14.	(a)	20.	(d)	26.	(b)
3.	(b)	9.	(a)	15.	(b)	21.	(c)	27.	(a)
4.	(a)	10.	(d)	16.	(c)	22.	(b)	28.	(a)
5.	(d)	11.	(d)	17.	(d)	23.	(c)	29.	(b)
6.	(c)	12.	(d)	18.	(b)	24.	(d)	30.	(d)

DETAILED EXPLANATIONS

1. (a)

The subset of a countable set is always countable.

2. (d)

Empty set ϕ satisfies all properties except reflexive property. Hence not an equivalence relation. A reflexive relation satisfies both symmetric and antisymmetric properties. Hence (b) is false.

The relation "divides" is not symmetric because 1 divides 2, but 2 does not divide 1.

Union of two transitive relations need not be transitive relation. Hence union need not be equivalence relation.

3. (b)

A partition of a set *S* is a collection of disjoint non-empty subsets of *S* that have *S* as their union. For partitionn in (b) {10} and {10, 20, 30, 41} are not disjoint and hence is not correct partition.

4. (a)

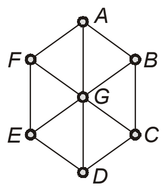
Group properties are Closure, Associativity, Existence of Inverse for every element, Identity element. Commutativity is not required for a mathematical structure to become a group.

5. (d)

$$A = \{\{\}, \{x\}\}$$
$$A = \{p, q\} \text{ [Assume } p = \{\}, q = \{x\}]$$
$$P(A) = \{\{\}, \{p\}, \{q\}, \{p, q\}\}$$
$$= \{\{\}, \{\{\}\}, \{\{x\}\}, \{\{\}, \{x\}\}\}$$

6. (c)

Consider a wheel graph of 7 vertices.



But the chromatic number of graph is 3.

Color 1 for *G*

Color 2 for *A*, *E*, *C*

Color 3 for *F*, *B*, *D*

All other statements are true.

7. (d)

- A lattice is bounded iff the lattice has a greatest and a least element.
  ∴ A finite lattice is always bounded.
- Complemented lattice is defined only for bounded lattice. A bounded lattice is complemented iff atleast one complement of every element exist in lattice. An element should one or more complements.
- A complemented lattice is distributive iff every element has a unique complement.

8. (b)

G has 4 vertices

$$\text{Maximum \# of edges} = \frac{4(4-1)}{2} = 6 \text{ Edges}$$

$$2 * 2 + 1 + 3 = 2|E|$$
$$\Rightarrow \quad 4 + 1 + 3 = 2|E|$$
$$\Rightarrow \quad |E| = 4$$

G has 4 edges

$\bar{G}$ has ${}^4C_2 - 4 = 6 - 4 = 2$ Edges

With 4 vertices and 2 edges, the graph is always disconnected.

9. (a)

$$d * c = d * (a * b)\text{[Given, } c = a * b]$$
$$= (d * a) * b$$

[Associative holds in semigroup]

$$= b * b\text{[Given, } b * b = a]$$
$$= a$$

10. (d)

To make a connected graph atleast $(n - 1)$ edges required. To make it disconnected graph should contain at most $(n - 2)$ edges. The graph has m edges, to be make it disconnected at most $n - 2$ edge must be deleted. So, $m - (n - 2) = m - n + 2$ edges.

$\therefore$ $(m - n + 2)$ edges deletion always guarantee that any graph will become-disconnected.

11. (d)

In complete graph of 'n' vertices all vertices will have $(n - 1)$ degree.

$\therefore$ Minimum degree = Maximum degree = 8 for K_9.

In complete bipartite graph with $K_{m,n}$, the size of $K_{m,n}$ is $m \times n$.

$\therefore$ In $K_{2,7}$ we will have $2 \times 7 = 14$ edges.

12. (d)

The operation is not commutative as $p * q \neq q * p$

$q * p = p$ and $p * q = r$

The operation is not associative as $p * (q * r) \neq (p * q) * r$

LHS $p * r = s$

RHS $r * r = p$

13. (b)

Consider choice (b) : $(\forall x(A(x) \Rightarrow B(x))) \Rightarrow ((\forall x A(x)) \Rightarrow (\forall x B(x)))$

Let the LHS of this implication be true

This means that

$$A_1 \to B_1$$
$$A_2 \to B_2$$
$$\vdots$$
$$A_n \to B_n$$

Now we need to check if the RHS is also true. The RHS is $((\forall x A(x)) \Rightarrow (\forall x B(x)))$

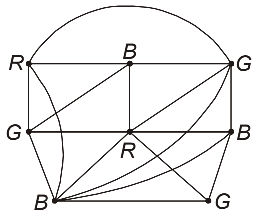
To check this let us take the LHS of this as true i.e. take $\forall x\, A(x)$ to be true. This means that $(A_1, A_2, ...A_n)$ is taken to be true. Now A_1 along with $A_1 \to B_1$ will imply that B_1 is true. Similarly A_2 along with $A_2 \to B_2$ will imply that B_2 is true. And so on...

Therefore $(B_1, B_2, ...B_n)$ all true.

i.e. $\forall x B(x)$ is true. Therefore the statement (b) is a valid predicate statement.

14. (a)
R is reflexive: Since $(a, b)\ R\ (a, b)$ for all elements (a, b) because $a = a$ and $b = b$ are always true.
R is symmetric: Since $(a, b)\ R\ (c, d)$ and $a = c$ or $b = d$ which can be written as $c = a$ or $d = b$.
So, $(a, b)\ R\ (a, b)$ is true.
R is not antisymmetric: Since $(1, 2)\ R\ (1, 3)$ and $1 = 1$ or $2 = 3$ true b/c $1 = 1$.
So $(1, 3)\ R\ (1, 2)$ but here $2 \neq 3$ so $(1, 2) \neq (1, 3)$.
So, only statement 1 and 2 are correct.

15. (b)
$d(i)$ = Degree of node i. $d(A) = 4$, $d(B) = 4$, $d(C) = 5$ $d(D) = 4$, $d(E) = 3$, $d(F) = 6$, $d(G) = 4$, $d(H) = 6$ using Welsh-powell's algorithm.



Chromatic number = 4.

16. (c)

$$X = Y \times Z \Rightarrow |X| = kn$$
$$W = P(x) \Rightarrow |W| = 2^{kn}$$
$$f : X \to W$$

$$\# \text{ functions} = |W|^{|X|} = \left(2^{kn}\right)^{kn} = 2^{(kn)^2}$$

So option (c) is correct.

17. (d)

$$f : N \to Z$$
$$f(0) = f(2) = 3$$

$\Rightarrow$ f is not injective
Clearly f is not surjective, all numbers in Z do not have preimages in N (example: 0 has no preimage)
f is function which is not injective and not surjective.

18. (b)
Total number of subset of 5 element = ${}^{25}C_5$

$$= \frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1}$$
$$= 23 \times 22 \times 21 \times 5 = 53130$$

T be a subset contain no odd number = ${}^{12}C_5$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

So number of subset with atleast 1 odd number

$$T \subseteq S = {}^{25}C_5 - {}^{12}C_5$$
$$= 53130 - 792 = 52338$$

19. (a)
$\forall s\, P(s) \wedge \exists t\, Q(t)$
$\forall s\, P(s)$: Square of every integer is always ≥ 0, so $\forall\, s\, P(s)$ is true.
$\exists t\, Q(t)$: there exists a solution i.e., $t = 2, 3$ for $t^2 - 5t + 6 = 0$ (Hence $Q(s)$ is true)
So $\exists t Q(t)$ is also true.
$\therefore \quad \forall s\, P(s) \wedge \exists t\, Q(t)$ is true.
So option (a) is correct.

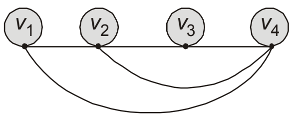
20. (d)
To check function is one-to-one:
$\Rightarrow \quad f(x_1) = f(x_2)$
$\Rightarrow \quad f(x) = x^2 + 1$
$\Rightarrow \quad x_1^2 + 1 = x_2^2 + 1$
$\Rightarrow \quad x_1 = \pm x_1$ here x_1 has to images so, it is not one-to-one function.
To check function is onto:
$$y = x^2 + 1$$
$$x = \sqrt{y - 2}$$
So, range = $|y|$ for $y \geq 1 \neq z$ so, it is not onto.

21. (c)
Not (there exist a student who has written a GATE in every stream)
$$\sim \left[\exists x\, S(x) \wedge \forall_y \text{GATE}(x, y)\right]$$
$$= \sim \exists x\, \forall y \left[S(x) \wedge \text{GATE}(x, y)\right]$$
$$= \forall x \exists y \left[\sim S(x) \vee \sim \text{GATE}(x, y)\right]$$
$\therefore$ Option (c) is correct.

22. (b)
Let (a) $n = 2$;

edge = 1

(b) $n = 4$



edges = 5
So option (b) is correct.

23. (c)
S_1 :The maximum number of edges in graph is given by
$$n - k \leq e \leq \frac{(n-k)(n-k+1)}{2}$$
For $k = 2$ we get $\frac{(n-1)(n-2)}{2}$ when a graph has disconnected into two components.
$\therefore \quad S_1$ is true.
S_2 :If G is a forest, then each connected component is a tree. Each of the tree contain $n - 1$ edges.
$(n_1 - 1) + (n_2 - 1) + (n_3 - 1)$ ..., upto k times.
So, total number of edges = $(n_1 + n_2 + n_3 \ldots) - k = n - k$.

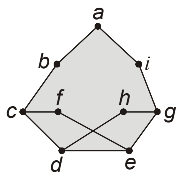
24. (d)

- $R = \{\langle x, y\rangle | x \equiv y \bmod m\}$ when $x = y$ then $x \equiv x \bmod m$ is always reflexive.
- $R = \{\langle x, y\rangle | x \equiv y \bmod m\}$ $(x - y) \bmod m$ is always equal to $(y - x) \bmod m$. So relation is always symmetric.
- $R = \{\langle x, y\rangle | x \equiv y \bmod m\}$ if $(x - y) \bmod m$ is always equal to $(y - z) \bmod m$. Which is equal to $(x - z) \bmod m$ so relation is always transitive.

The given relation $R = \{\langle x, y\rangle | x \equiv y \bmod m\}$ is equivalence relation (reflexive, symmetric and transitive).

25. (d)

f–c–b–a–i–g–h–d–e–f



It covers all vertices in cycle.
So option (d) is correct.

26. (b)

$A \oplus B$ is the symmetric difference i.e.

$$A \oplus B = (A \cup B) - (A \cap B)$$
$$C = \{1, 2, 3, 4, 5, 8, 12\} - \{1, 8\}$$
$$C = \{2, 3, 4, 5, 12\}$$
$$|C| = 5$$

27. (a)

The total number of ways of choosing 6 squares out of 8 is $^8C_6 = 28$.
But out of these, 2 possibilities need to be removed.
One being the upper row being empty.
Second being the lower row being empty.
Both being empty at the same time is not a possibility.
$\therefore$ 28 – 2 = 26 ways are there.

28. (a)

Number of chits = $^{10}C_5 = 252$

Using Pigeon hole principle, $\left\lfloor \frac{252-1}{6} \right\rfloor + 1 = 42$

$\therefore$ Atleast 42 chits will be in same box.

29. (b)

R^1 is nothing but R itself.
Now, $R^2 = R \cdot R$ i.e. composite of R with R.
If $(a, b) \in R$, then $(a, c) \in R^2$ iff $(b, c) \in R$.
This composite of relations
$R^2 = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$
$R^3 = \{(1, 1), (2, 1), (3, 1) (4, 1)\}$
$P = \{(1, 1) (2, 1), (3, 2), (4, 1), (4, 2), (4, 3), (3, 1)\}$
$\therefore$ Cardinality of $P = 7$.

30. (d)

(Z, +) is both group and Abelian group, as it satisfies commutative property and inverse element is $-a \; \forall a \in Z$.

(Z, –) is never semigroup, because subtraction operation is not associative and hence cannot be monoid too.

(Z, X) is monoid but not group, because inverse does not exist i.e. for any integer 'a' it's inverse is $1/a$ which is a rational number.

Hence it is monoid only.

■■■■