CLASS TEST

Sl. : 01-JP-EC-19072023

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ELECTRONICS ENGINEERING

Communication Systems

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 The samples of a sinusoidal message signal $m(t) = \cos(2\pi f_m t)$ V are applied to a delta modulator, whose step-size is 2 V. The minimum sampling rate required to eliminate the slope-overload distortion is approximately

(a) f_m (b) $2f_m$
(c) $3.14f_m$ (d) $6.28f_m$

Q.2 If the quality factor, (Q) of a tuned circuit varies as $10 < Q < 100$, then which of the following specifies the correct relation between resonant frequency f_c and bandwidth B?

(a) $0.01 < \frac{B}{f_c} < 0.1$ (b) $0.1 < \frac{B}{f_c} < 1$
(c) $0.001 < \frac{B}{f_c} < 0.1$ (d) $0.01 < \frac{B}{f_c} < 0.02$

Q.3 Two signals $X_1(t) = 2\cos 2000\,\pi t$ and $X_2(t) = 4\cos 5000\,\pi t$ are sampled with the sampling frequency of 50 kHz and then transmitted over the same channel. Then, the value of the guard band in the sampled signal spectrum is equal to

(a) 45 kHz (b) 40 kHz
(c) 50 kHz (d) 42 kHz

Q.4 Which of the following relation w.r.t autocorrelation function is correct?

(a) $R_{XX}(\tau) \le R_{XX}(0)$
(b) $R_{XX}(2) > R_{XX}(1)$
(c) $R_{XX}(-5) < R_{XX}(6)$
(d) $R_{XX}(-\tau) \ne R_{XX}(\tau)$

Q.5 An analog signal of 2 kHz frequency is sampled at 5 times the Nyquist Rate and each sample is coded with n bits. If the bit duration is 5 μsec, then the value of n is

(a) 8 (b) 9
(c) 10 (d) 12

Q.6 A Discrete Memoryless System, X has four symbols x_1, x_2, x_3, x_4 with their respective probabilities, $P(x_1) = 0.4$, $P(x_2) = 0.3$, $P(x_3) = 0.2$, $P(x_4) = 0.1$.

The ratio of the amount of information contained in the message $x_1\, x_2\, x_1\, x_3$ and $x_4\, x_3\, x_3\, x_2$ is

(a) 0.691 (b) 0.725
(c) 0.826 (d) 0.991

Q.7 Consider a binary symmetric channel (BSC) with a crossover probability of 0.20. If two such BSCs are cascaded, then the crossover probability of the resultant BSC will be

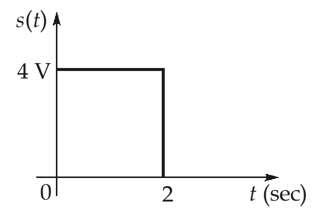
(a) 0.25 (b) 0.32
(c) 0.40 (d) 0.50

Q.8 A discrete memoryless source emits 4 symbols with probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \,\&\, \frac{1}{8}$.

The entropy of the source is equal to

(a) 1.75 bits/symbol
(b) 2.25 bits/symbol
(c) 3.42 bits/symbol
(d) 4.45 bits/symbol

Q.9 Consider the signal shown below:



This signal is passed through an AWGN channel with two-sided noise power spectral density of $\frac{N_0}{2}$ and received by a filter matched to $s(t)$. The maximum signal-to-noise ratio possible at the output of the filter is

(a) $\frac{8}{N_0}$

(b) $\frac{16}{N_0}$

(c) $\frac{32}{N_0}$

(d) $\frac{64}{N_0}$

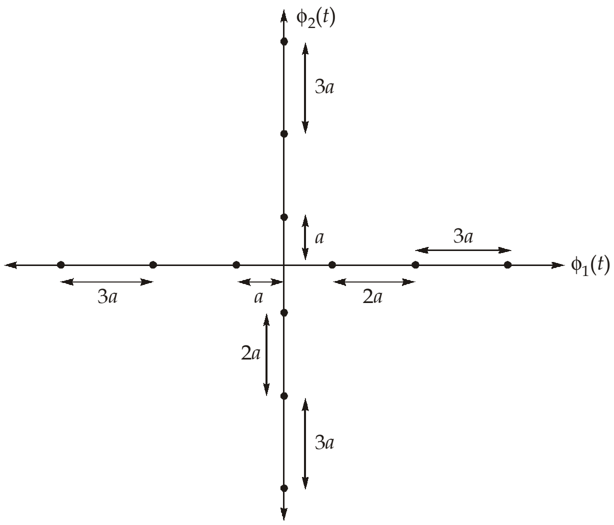
Q.10 A BPSK signal, with equiprobable bits, is transmitted through an AWGN channel and received by a correlator receiver. The two-sided power spectral density of the channel noise is 1 nW/Hz and the average bit energy transmitted is 10 μJ. If there is no phase mismatch between the carrier signals used in the transmitter and receiver, then the probability of error of the system will be

(a) $Q(10)$ (b) $Q(10\sqrt{2})$

(c) $Q(100)$ (d) $Q(100\sqrt{2})$

Q. No. 11 to Q. No. 30 carry 2 marks each

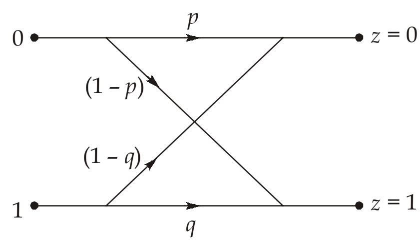
Q.11 Consider the following constellation diagram shown below. If the value of a is 2 and the two-sided noise power spectral density $\frac{N_0}{2} = 4$ W/Hz, then the probability of error in the given constellation diagram in terms of Q-function is,



(a) $Q(\sqrt{0.25})$ (b) $Q(\sqrt{0.5})$

(c) $Q(\sqrt{2})$ (d) $Q(\sqrt{3})$

Q.12 In the given channel it is given $p\left(\frac{z=0}{1}\right) = 0.25$ and $p(z = 0) = 0.40$ then the value of crossover probabilities are (Given source emits both 0 and 1 with equal probability.)



(a) 0.25, 0.45
(b) 0.5, 0.5
(c) 0.55, 0.75
(d) 0.5, 0.35

Q.13 The peak-to-peak value of an sinusoidal signal is 4 V. If this signal is full-wave rectified and then fed to the input of an 8-bit PCM coder. Then the value of signal to quantization Noise ratio is

(a) 49.8 dB (b) 48 dB

(c) 55.94 dB (d) 54 dB

Q.14 X and Y are two independent random variables with zero mean and unit variance. Another random variable Z is defined as $Z = 2X + Y$. Then mean and variance of Z are respectively

(a) 0, 1 (b) 1, 0

(c) 0, 4 (d) 0, 5

Q.15 White noise is fed as input to a bandpass filter having passband frequencies from 2 kHz to 4 kHz. The noise power (in mW) at the filter output if two-sided noise power spectral density $\frac{N_o}{2} = 2\ \mu\text{W/Hz}$ is,

(a) 4 mW (b) 6 mW

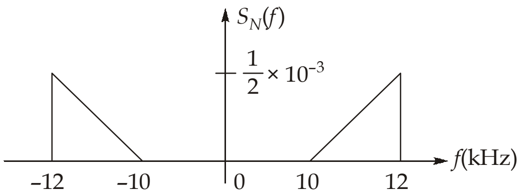
(c) 8 mW (d) 10 mW

Q.16 If the incoming frequency is in the range 540 kHz to 1600 kHz, intermediate frequency is given as 455 kHz, then the ratio of maximum local oscillator frequency to the minimum local oscillator frequency in a AM superheterodyne receiver is

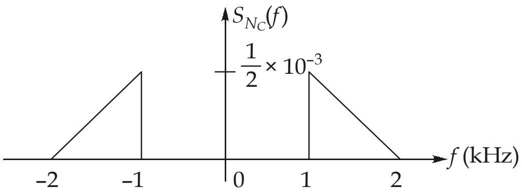
(a) 2.065 (b) 3.725

(c) 4.525 (d) 5.609

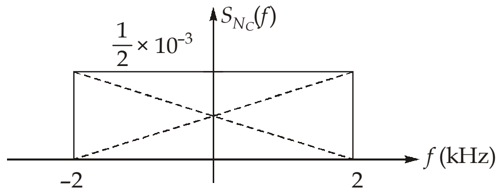
Q.17 For the narrowband noise power spectrum $S_N(f)$ shown in figure for f_c = 10 kHz, the inphase component noise power spectrum is



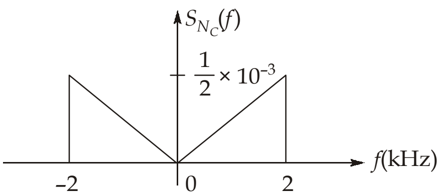
(a)



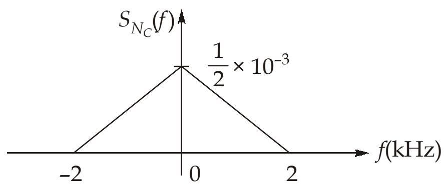
(b)



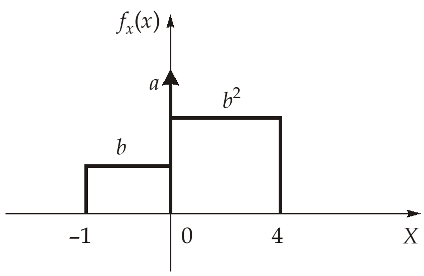
(c)



(d)

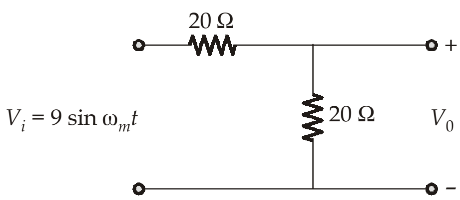


Q.18 If X is equally likely to take both positive and negative values, then the magnitude of impulse present at x = 0 i.e. 'a' value is equal to



(a) 0.2
(b) 0.5
(c) 0.8
(d) 1.0

Q.19 A sinusoidal signal $9\sin\omega_m t$ is given as input to a 9-bit PCM coder. If this sinusoidal signal is passed through a resistive network shown below and the output of the resistive network is given as input to the same PCM coder, assume the step size is not changed. Then the value of signal to quantization noise ratio is



(a) 49.8 dB
(b) 52.4 dB
(c) 60.1 dB
(d) 72.5 dB

Q.20 If an FM receiver is to be operated in the frequency range 90 MHz to 104 MHz, the minimum value of intermediate frequency (f_{IF}) to reject the image frequencies is
(a) 6 MHz
(b) 7 MHz
(c) 8 MHz
(d) 9 MHz

Q.21 A source emits zeroes twice as likely as one's. If it transmits 5 bits what is the probability that at least two bits are zeroes?
(a) 0.625 (b) 0.725
(c) 0.954 (d) 1.125

Q.22 An angle modulated signal is given by,
$S(t) = \cos[2\pi f_c t + 4\sin(4000\pi t) + 3\cos(4000\,\pi t)]$
If f_c = 100 kHz, then the maximum value of the instantaneous frequency of this signal is
(a) 107 kHz
(b) 110 kHz
(c) 114 kHz
(d) 120 kHz

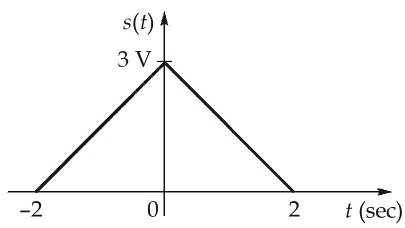
Q.23 The message signal applied to an FM modulator is $m(t) = A_m \cos(2\pi f_m t)$. In an experiment conducted with f_m = 2 kHz and increasing A_m (starting from zero) it is found that the carrier component in FM signal is reduced to zero for the first time when A_m = 4 volt. The frequency sensitivity of the FM modulator is

Assume Bessel coefficient, $J_o(x) = 0$ for x = 2.41, 5.52, 8.65 and 11.8.
(a) 1.2 kHz/V
(b) 1.5 kHz/V
(c) 2.5 kHz/V
(d) 3.5 kHz/V

Q.24 Consider a DMS X with four symbols x_1, x_2, x_3 and x_4 with the probabilities 0.125, 0.25, 0.125 and 0.5 respectively. The symbols are encoded using Huffman coding. The redundancy of the code is
(a) 0 (b) 1
(c) 2 (d) 3

Q.25 A 1 Mbps data is to be transmitted through a baseband channel, whose bandwidth is 600 kHz. If the raised cosine pulse shaping is used for baseband modelling of the data, then the maximum allowed value of the roll-off factor of the filter will be
(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.4

Q.26 Consider the signal shown in the figure below:



If the signal $s(t)$ is applied to its matched filter, then the peak value of the filter output signal will be equal to
(a) 10 V (b) 11 V
(c) 12 V (d) 14 V

Q.27 The probability density functions of two independent random variables X and Y are given by,

$$f_X(x) = ae^{-ax}u(x) \text{ and } f_Y(y) = be^{-by}\,u(y)$$

Where a, b are positive real constants and $u(\cdot)$ represents the unit step function. The probability density function of the random variable $Z = X + Y$ will be

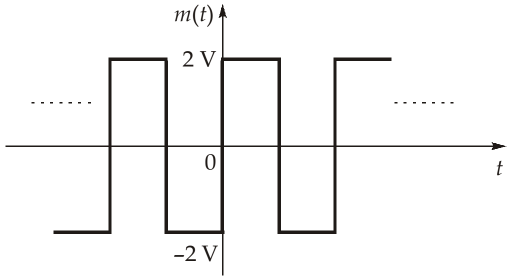
(a) $\dfrac{ab}{(b-a)}\left[e^{-bz} - e^{-az}\right]u(z)$

(b) $\dfrac{ab}{(b-a)}\left[e^{-az} - e^{-bz}\right]u(z)$

(c) $\dfrac{ab}{(a+b)}\left[e^{-az} - e^{-bz}\right]u(z)$

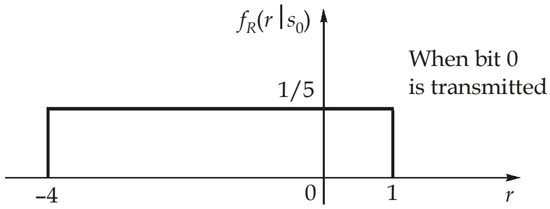
(d) $\dfrac{ab}{(a+b)}e^{-(a+b)z}u(z)$

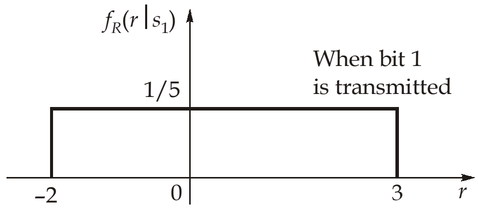
Q.28 The periodic message signal, shown in the figure below, is applied to a modulator to modulate the amplitude of a sinusoidal carrier signal. If the amplitude sensitivity of the modulator is 0.25 V^{-1}, then the transmission efficiency achieved by the resultant modulated signal will be



(a) 11.11%
(b) 20%
(c) 33.33%
(d) 50%

Q.29 In a digital communication system, bits 0 and 1 are transmitted through a channel with probabilities $\frac{2}{3}$ and $\frac{1}{3}$ respectively. The receiver decides for either 0 or 1 based on the received value R. The conditional probability density functions of R are as follows:

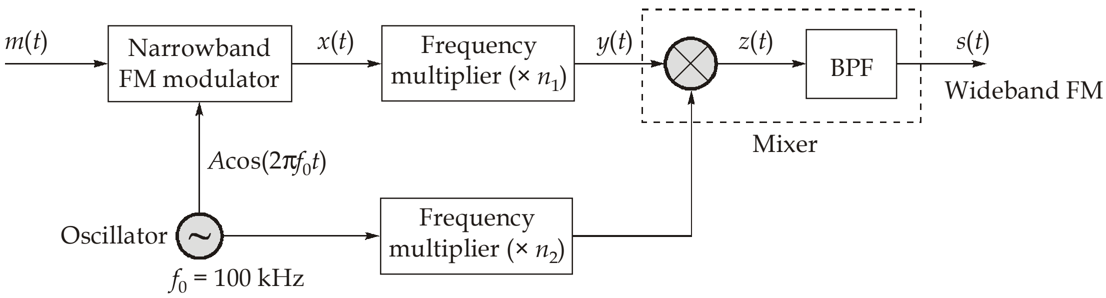




Decision is made in favour of 0, if $r < r_{th}$ and in favour of 1, if $r > r_{th}$. If the threshold value "r_{th}" is decided in an optimum way using maximum a posteriori (MAP) criteria, then the value of "r_{th}" is equal to

(a) 1
(b) 2
(c) 3
(d) 4

Q.30 Consider the Armstrong FM modulator shown in the figure below:



The narrowband FM signal has a maximum angular deviation of 0.10 radians in order to keep distortion under control. The message signal $m(t)$ has a bandwidth of 15 kHz, the oscillator frequency is 100 kHz and the mixer circuit is used for the up-conversion. If the wideband FM signal $s(t)$ has a carrier frequency of f_c = 104 MHz and a maximum frequency deviation of Δf_{max} = 75 kHz, then the multiplication factor n_2 will be

(a) 50
(b) 104
(c) 990
(d) 1040

■■■■

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COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 19/07/2023

ANSWER KEY

1.	(c)	7.	(b)	13.	(c)	19.	(a)	25.	(b)
2.	(a)	8.	(a)	14.	(d)	20.	(b)	26.	(c)
3.	(a)	9.	(d)	15.	(c)	21.	(c)	27.	(b)
4.	(a)	10.	(c)	16.	(a)	22.	(b)	28.	(b)
5.	(c)	11.	(b)	17.	(c)	23.	(a)	29.	(a)
6.	(a)	12.	(a)	18.	(b)	24.	(a)	30.	(c)

Detailed Explanations

1. (c)

The condition required to eliminate the slope-overload distortion is,

$$\frac{\Delta}{T_s} \geq \left|\frac{dm(t)}{dt}\right|_{\max} = \left|2\pi f_m \sin(2\pi f_m t)\right|_{\max}$$

$$2f_s \geq 2\pi f_m$$

$$f_s \geq \pi f_m \approx 3.14\, f_m$$

$$f_{s\,(\min)} = 3.14\, f_m$$

2. (a)

$$\text{Quality factor, } Q = \frac{f_c}{B}$$

$$\text{Range of } Q,\ 10 < Q < 100$$

$$10 < \frac{f_c}{B} < 100$$

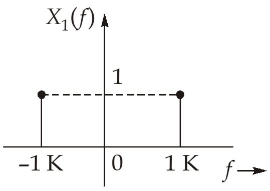
$$\frac{f_c}{B} > 10 \qquad \frac{B}{f_c} < 0.1$$

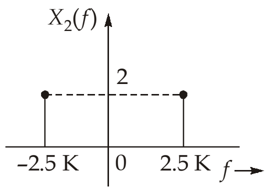
$$\frac{f_c}{B} < 100 \qquad \frac{B}{f_c} > 0.01$$

$$0.01 < \frac{B}{f_c} < 0.1$$

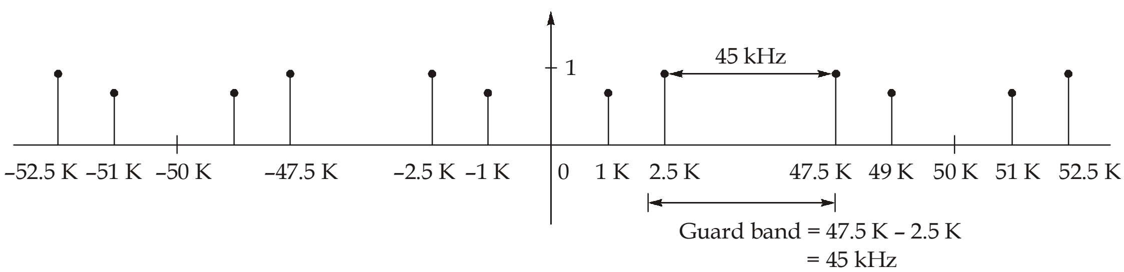
3. (a)

Before sampling





After sampling at a frequency of f_s = 50 kHz, $X_s(f)_{n=-\infty}^{\infty} = \Sigma X_1(f - nf_s)_{n=-\infty}^{\infty} + \Sigma X_2(f - nf_s)$



4. (a)

Autocorrelation function has maximum value at $\tau = 0$.

$\therefore$ $$R_{XX}(0) \geq R_{XX}(\tau)$$

Autocorrelation function is even function.

$\therefore$ $$R_{XX}(-\tau) = R_{XX}(\tau)$$

Value of autocorrelation function decreases as τ increases.

5. (c)

$$\text{Bit duration, } T_b = \frac{1}{R_b}$$

$$R_b = nf_s$$

$$f_s = 5 \times 2 \times 2 = 20 \text{ kHz}$$

$$T_b = \frac{1}{n \times 20 \times 10^3}$$

$$5 \times 10^{-6} = \frac{1}{n \times 20 \times 10^3}$$

$$n = \frac{10^3}{20 \times 5}$$

$$n = 10$$

6. (a)

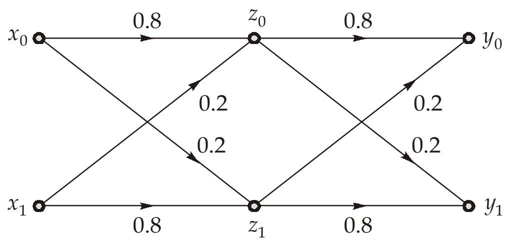
$$P(x_1\ x_2\ x_1\ x_3) = (0.4)(0.3)(0.4)(0.2)$$
$$= 0.0096$$

$\therefore$ $$\text{Information, } I(x_1\ x_2\ x_1\ x_3) = -\log_2(0.0096) = 6.7 \text{ bits}$$

$$P(x_4\ x_3\ x_3\ x_2) = (0.1)(0.2)(0.2)(0.3)$$
$$= 0.0012$$
$$\mathrm{I}(x_4\ x_3\ x_3\ x_2) = -\log_2(0.0012)$$
$$= 9.7 \text{ bits}$$

$\therefore$ $$\text{Ratio} = \frac{6.7}{9.7} = 0.691$$

7. (b)



Crossover probability of overall channel = $P(y_0|x_1) = P(y_1|x_0)$

$$P(y_0|x_1) = P(y_0|z_0)\ P(z_0|x_1) + P(y_0|z_1)\ P(z_1|x_1)$$
$$= (0.80 \times 0.20) + (0.20 \times 0.80) = 0.32$$

So, the crossover probability of the resultant BSC = 0.32

8. (a)

$$H(x) = -\sum_{i=0}^{3} P(x_i)\log_2 P[x_i]$$

$$= -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{1}{8} \times 2\log_2\left(\frac{1}{8}\right)\right]$$

$$= \frac{1}{2}+\frac{2}{4}+\frac{6}{8}$$

$$H(x) = 1.75 \text{ bits/symbol}$$

9. (d)

For matched filter,

$$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

$$E_s = \text{Energy of the signal } s(t)$$

$$= \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^2 (4)^2 dt = 32$$

So, $$(\text{SNR})_{\max} = \frac{2(32)}{N_0} = \frac{64}{N_0}$$

10. (c)

For coherent BPSK,

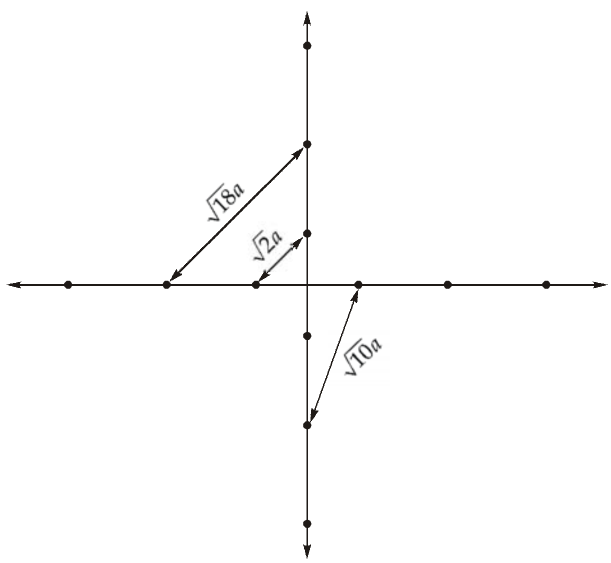
$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{(N_0/2)}}\right)$$

$$= Q\left(\sqrt{\frac{10\times10^{-6}}{1\times10^{-9}}}\right) = Q\left(\sqrt{10^4}\right) = Q(100)$$

11. (b)

Probability of error in terms of Q-function is given by,

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right)$$



$$d_{\min} = \sqrt{2}a = 2\sqrt{2}$$

$$\frac{N_0}{2} = 4 \text{ W/Hz}$$

$$N_0 = 8 \text{ W/Hz}$$

$$P_e = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right) = Q\left(\sqrt{\frac{(2\sqrt{2})^2}{2\times 8}}\right)$$

$$P_e = Q\left(\sqrt{\frac{8}{2\times 8}}\right)$$

$$P_e = Q(\sqrt{0.5})$$

12. (a)

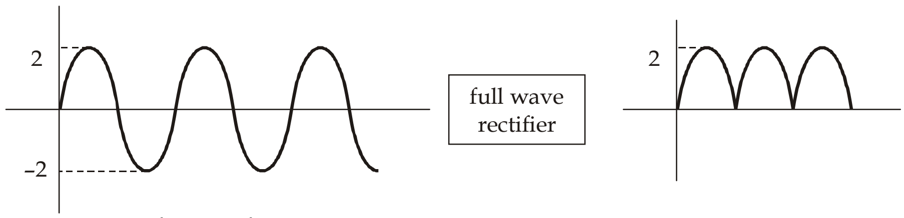
$$p(0/1) = 0.25$$
$$p(0/1) = 1 - q$$
$$1 - q = 0.25$$
$$q = 0.75$$

$$p(z = 0) = 0.4 = p(0)p\left(\frac{0}{0}\right) + p(1)p\left(\frac{0}{1}\right)$$

$$0.4 = 0.5p + 0.5 \times 0.25$$
$$0.4 = 0.5p + 0.125$$
$$0.5p = 0.275; \quad p = 0.55$$

Crossover probabilities $\rightarrow$ $(1 - p) = 0.45$ and $(1 - q) = 0.25$

13. (c)



Before rectification

RMS value of signal remains same after rectification but dynamic range decreases therefore step size decreases and Noise power decreases.

$$\text{Signal power} = \frac{A^2}{2} = \frac{2^2}{2} = 2$$

$$\text{Quantization Noise Power} = \frac{\Delta^2}{12} \quad \text{where } \Delta = \text{step size}$$

$$\Delta = \frac{2-0}{2^n} = \frac{2}{2^8} = \frac{1}{2^7}$$

$$\frac{S}{N} = \frac{2}{1}\cdot 2^{14} \times 12 = 393216 = 55.94 \text{ dB}$$

14. (d)

$$Z = 2X + Y$$
$$E[Z] = E[2X + Y]$$
$$= 2E[X] + E[Y]$$
$$E[X] = 0, \quad E[Y] = 0$$

Therefore, $E[Z] = 0$

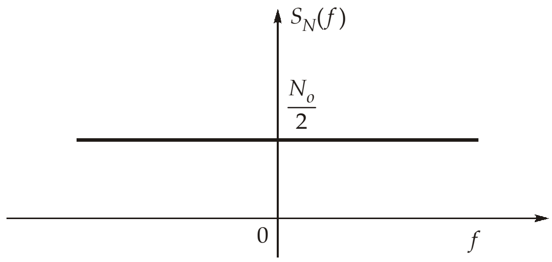
$$\text{Var}(Z) = E[Z^2] - E[Z]^2$$
$$= E[(2X+Y)^2] - 0$$
$$= E[4X^2 + Y^2 + 4XY]$$
$$= 4E[X^2] + E[Y^2] + 4E[XY]$$
$$E[X^2] = \text{Var}[X] \text{ as } E[X] = 0$$
$$E[Y^2] = \text{Var}[Y] \text{ as } E[Y] = 0$$
$$E[XY] = E[X]\,E[Y] \text{ as } X \text{ and } Y \text{ are independent}$$

Therefore, $\text{Var}(Z) = 4 \times 1 + 1 + 4 \times 0$

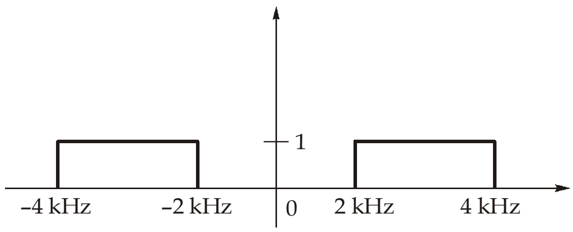
$$\text{Var}(Z) = 5$$

15. (c)

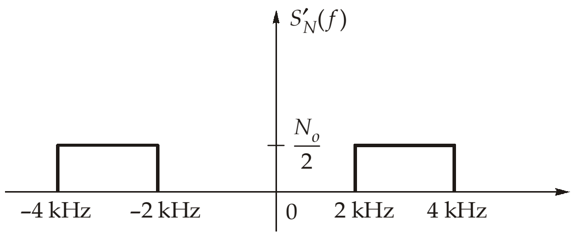
Power spectral density of white noise



Frequency response of filter



Noise after passing through the filter



Output noise power = Area under power spectral density curve

$$= 2 \times \frac{N_o}{2} \times 2 \times 10^3 = 2 \times 4 \times 10^{-6} \times 10^3$$
$$= 8 \times 10^{-3} \text{ W} = 8 \text{ mW}$$

16. (a)

Local oscillation frequency, $f_{LO} = f_s + f_{IF}$

$$\text{Maximum } f_{LO} = f_{s_{max}} + f_{IF}$$

$$f_{LO_{max}} = 1600 + 455 = 2055 \text{ kHz}$$

Minimum local oscillation frequency, $f_{LO_{min}} = f_{s_{min}} + f_{IF}$

$$f_{LO_{min}} = 540 + 455 = 995 \text{ kHz}$$

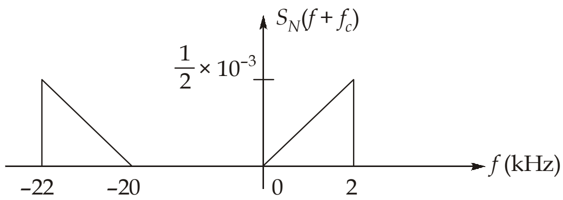
$$\frac{f_{LO_{max}}}{f_{LO_{min}}} = \frac{2055}{995} = 2.065$$

17. (c)

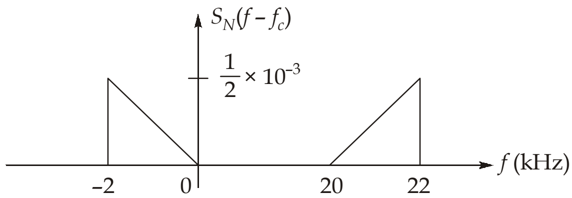
$$S_{N_C}(f) = \begin{cases} S_N(f-f_c)+S_N(f+f_c), & -B \le f \le B \\ 0 & \text{else} \end{cases}$$

$$f_c = 10 \text{ kHz},$$

Plotting $S_N(f + f_c)$



Plotting $S_N(f - f_c)$



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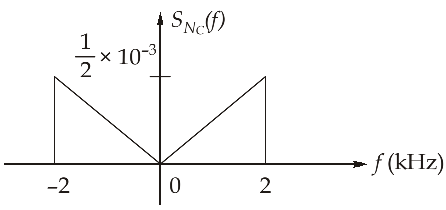


18. (b)

If X is equally likely to take both positive and negative values then,

$$P(X < 0) = P(X > 0)$$

$$P(X) = \text{Area under curve}$$

$$P(X < 0) = b \times 1 = b$$

$$P(X > 0) = b^2 \times 4 = 4b^2$$

$$b = 4b^2$$

$$b = \frac{1}{4}$$

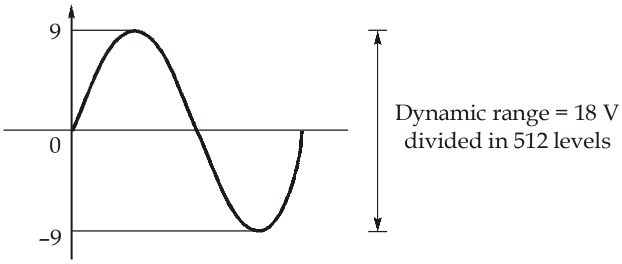
Also, Area of PDF curve = 1

$$b + 4b^2 + a = 1$$

$$\frac{1}{4}+4\times\frac{1}{16}+a = 1$$

$$a = 1-\frac{1}{2}=0.5$$

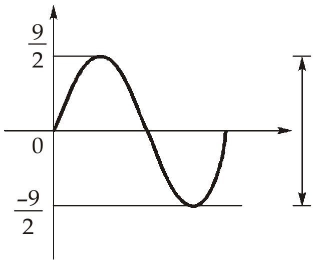
19. (a)



Now output of resistive network = $\frac{1}{2}V_i = \frac{9}{2}\sin\omega_m t$

Dynamic range = 9 V

Dynamic ranges reduces to $\frac{1}{2}$



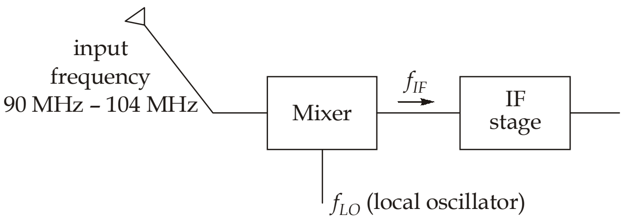
Step size remains same,

$\therefore$ No. of levels $\frac{1}{2}\times 512 = 256$

As Dynamic range reduces, $n = 8$

$$\frac{S}{N} = 1.76 + 6n$$
$$= 1.76 + 6 \times 8$$
$$= 49.8 \text{ dB}$$

20. (b)



$$f_{LO} = f_s + f_{IF}$$
$$f_{si} \text{ (image frequency)} = f_s + 2f_{IF}$$

When $f_s = 90$ MHz

$$f_{LO} = f_s + f_{IF},\ f_{si} = f_s + 2f_{IF}$$

We have to select f_{IF} such that f_{si} falls outside the tuned range.

$$f_{si} \geq 104 \text{ MHz}$$
$$104 = 90 + 2f_{IF}$$
$$f_{IF_{\min}} = 7 \text{ MHz}$$

21. (c)

Probability of transmitting zero, $P(0) = \frac{2}{3}$

Probability of transmitting one, $P(1) = 1 - \frac{2}{3} = \frac{1}{3}$

$$
\begin{aligned}
P\text{ (at least two bits are zeroes)} &= 1 - P(\text{no bit is zero}) \\
&\quad - P\text{ (one bit is zero).} \\
&= 1 - {}^5C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^5 - {}^5C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^4 \\
&= 1 - \frac{1}{3^5} - \frac{10}{3^5} \\
&= 1 - \frac{11}{243} = 0.954
\end{aligned}
$$

22. (b)

The angle of the modulated signal $s(t)$ can be given as,

$$\theta(t) = 2\pi f_c t + 4\sin(4000\pi t) + 3\cos(4000\pi t)$$

The instantaneous frequency of the modulated signal can be given as,

$$f_i = \frac{1}{2\pi}\frac{d[\theta(t)]}{dt}$$

$$
\begin{aligned}
f_i &= f_c + \frac{1}{2\pi}[4 \times 4000\pi\cos 4000\pi t + 3 \times 4000\pi[-\sin 4000\pi t]] \\
&= f_c + [8000\cos(4000\pi t) - 6000\sin(4000\pi t)] \\
&= f_c + 2000 \times 5[\cos(4000\pi t + \alpha)] \text{ where } \alpha = \tan^{-1}\left(\frac{3}{4}\right)
\end{aligned}
$$

$$
\begin{aligned}
f_{i(\max)} &= f_c + 2000 \times 5 \\
&= 100\text{ kHz} + 10\text{ kHz} \\
f_{i(\max)} &= 110\text{ kHz}
\end{aligned}
$$

23. (a)

The carrier component of the FM signal will be zero when $J_0(\beta) = 0$.
We know $J_0(\beta) = 0$ for $\beta = 2.41, 5.52, 8.65, 11.8$ ......
So, when $A_m = 4$ V, the corresponding modulation index is $\beta = 2.41$.

$$\beta = 2.41$$

$$\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

$$k_f = \frac{\beta f_m}{A_m} = \frac{2.41 \times 2 \times 10^3}{4}$$

$$k_f = 1.205\text{ kHz/V}$$

24. (a)

x_4	0.5 (0)	0.5 (0)	0.5 (0)
x_2	0.25 (10)	0.25 (10)	0.5 (1)
x_3	0.125 (110)	0.25 (11)	
x_1	0.125 (111)		

$$L = \sum_{i=1}^{4} P(x_i)n_i$$

$$= (0.5 \times 1) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3)$$

$$= 0.5 + 0.5 + 0.75$$

$$L = 1.75 \text{ bits/symbol}$$

$$H = -\sum_{i=1}^{4} P(x_i)\log_2 P(x_i)$$

$$H = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{4}\log_2\left(\frac{1}{4}\right) + \frac{2}{8}\log_2\left(\frac{1}{8}\right)\right]$$

$$H = 1.75 \text{ bits/symbol}$$

$$\eta = \frac{H}{L} = \frac{1.75}{1.75} = 1$$

$$\text{Redundancy } \gamma = 1 - \eta = 1 - 1 = 0$$

25. (b)

Bandwidth of the baseband signal with raised cosine pulse shaping will be,

$$(\text{BW})_{\text{signal}} = \frac{R_b}{2}(1+\alpha) = \frac{1000}{2}(1+\alpha) = 500(1+\alpha)\text{ kHz}$$

For proper transmission of the data,

$$(\text{BW})_{\text{signal}} \le (\text{BW})_{\text{channel}}$$

$$500(1+\alpha) \le 600$$

$$(1+\alpha) \le 1.20$$

$$\alpha \le 0.20$$

$$\alpha_{\text{max}} = 0.20$$

26. (c)

$$s(t) \longrightarrow \boxed{\text{Filter matched to } s(t)} \longrightarrow y(t)$$

For a matched filter, peak value of the output will be numerically equal to the energy of the input signal.

So, $$|y(t)|_{\text{max}} = \int_{-\infty}^{\infty} |s(t)|^2\, dt$$

$$s(t) = \begin{cases} \left(3 - \frac{3}{2}|t|\right)\text{V}\ ; & 0 \le |t| \le 2 \\ 0\ ; & \text{otherwise} \end{cases}$$

So, $$|y(t)|_{max} = 2\int_0^2 \left(3-\frac{3}{2}t\right)^2 dt$$

$$= \frac{9}{2}\int_0^2 (t^2+4-4t)\,dt$$

$$= \frac{9}{2}\left[\frac{t^3}{3}+4t-2t^2\right]_0^2 = 12 \text{ V}$$

27. (b)

$$f_Z(z) = f_X(z) * f_Y(z)$$
$$f_X(z) = ae^{-az}\,u(z)$$
$$f_Y(z) = be^{-bz}\,u(z)$$

$$L\{f_X(z)\} = \frac{a}{s+a} \quad \text{and} \quad L\{f_Y(z)\} = \frac{b}{s+b}$$

$$f_Z(z) = L^{-1}\left\{\frac{ab}{(s+a)(s+b)}\right\} = L^{-1}\left\{\frac{ab}{(b-a)}\left[\frac{1}{(s+a)}-\frac{1}{(s+b)}\right]\right\}$$

$$= \frac{ab}{(b-a)}\left[e^{-az}-e^{-bz}\right]u(z)$$

28. (b)

The transmission efficiency of an AM signal can be given by,

$$\eta = \frac{k_a^2 P_m}{1+k_a^2 P_m}$$

Here, k_a = amplitude sensitivity of the modulator
= 0.25 V^{-1}

P_m = Power of the message signal

For the given message signal,

$$P_m = A^2 = (2)^2 = 4$$

So, $$\eta = \frac{(0.25)^2(4)}{1+(0.25)^2(4)} = \frac{0.25}{1+0.25} = \frac{1}{5} = 0.20 \text{ (or) } 20\%$$

29. (a)

The rule to decide an optimum threshold value using MAP criteria is as follows:

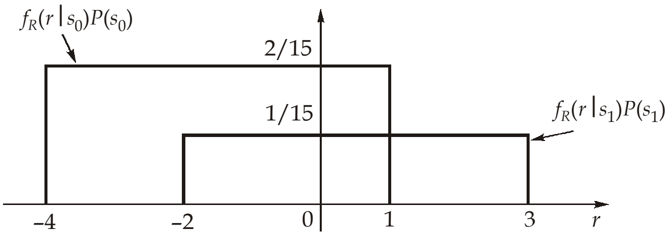
$$f_R(r\,|\,s_0)P(s_0) \underset{H_1}{\overset{H_0}{\gtrless}} f_R(r\,|\,s_1)P(s_1)$$

The above expression says that,

- Decision is made in favour of "0", if $f_R(r\,|\,s_0)\,P(s_0)$ is greater than $f_R(r\,|\,s_1)\,P(s_1)$.
- Decision is made in favour of "1", if $f_R(r\,|\,s_1)\,P(s_1)$ is greater than $f_R(r\,|\,s_0)\,P(s_0)$.

Given that $P(s_0) = \frac{2}{3}$ and $P(s_1) = \frac{1}{3}$.

The optimum threshold can be decided by using MAP criteria, by plotting the functions $f_R(r\,|\,s_1)P(s_0)$ and $f_R(r\,|\,s_1)P(s_1)$ as follows:



It is clear from the above diagram that,
For $r < 1$, $f_R(r\,|\,s_0)P(s_0) > f_R(r\,|\,s_1)P(s_1)$ and for $r > 1$, $f_R(r\,|\,s_1)P(s_1) > f_R(r\,|\,s_0)P(s_0)$.
So, the optimum threshold value is, $r_{th} = 1$.

30. (c)

The output of the narrowband FM modulator can be given by,

$$x(t) = A\cos[2\pi f_0 t + \phi(t)]\ ;\ |\phi(t)|_{\max} = 0.10 \text{ radians}$$

The signal at the output of upper frequency multiplier can be given by,

$$y(t) = A\cos[2\pi n_1 f_0 t + n_1\phi(t)]$$

After mixing $y(t)$ with the output signal of the lower frequency multiplier, we get,

$$z(t) = A^2\cos[2\pi n_1 f_0 t + n_1\phi(t)]\cos[2\pi n_2 f_0 t]$$

$$= \frac{A^2}{2}\cos[2\pi(n_1+n_2)f_0 t + n_1\phi(t)] + \frac{A^2}{2}\cos[2\pi(n_1-n_2)f_0 t + n_1\phi(t)]$$

It is given that the mixer is designed for up-conversion. So, the signal $s(t)$ can be given by,

$$s(t) = \frac{A^2}{2}\cos[2\pi(n_1+n_2)f_0 t + n_1\phi(t)] \quad \text{...(i)}$$

It is given that, $f_c = 104$ MHz and $\Delta f_{\max} = 75$ kHz for $s(t)$.
So, the modulation index of the wideband signal $s(t)$ will be,

$$\beta = \frac{\Delta f_{\max}}{f_{m(\max)}} = n_1|\phi(t)|_{\max}$$

$$n_1(0.10) = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

$$n_1 = \frac{5}{0.10} = 50$$

$$f_c = (n_1 + n_2)f_0 = 104 \text{ MHz}$$

$$(n_1 + n_2) \times 100 = 104 \times 1000$$

$$n_2 = 1040 - n_1 = 1040 - 50 = 990$$

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