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# ENGINEERING MATHEMATICS

EC & EE

Date of Test : 02/05/2023

## ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (b)  | 13. (b) | 19. (a) | 25. (d) |
| 2. (a) | 8. (a)  | 14. (a) | 20. (c) | 26. (b) |
| 3. (a) | 9. (a)  | 15. (b) | 21. (a) | 27. (d) |
| 4. (b) | 10. (b) | 16. (b) | 22. (a) | 28. (c) |
| 5. (a) | 11. (d) | 17. (a) | 23. (b) | 29. (c) |
| 6. (d) | 12. (c) | 18. (a) | 24. (c) | 30. (b) |

## DETAILED EXPLANATIONS

1. (b)

Let  $e^x = p$ 

$$e^x dx = dp$$

$$I = \int_0^{\infty} \frac{dx}{e^x + e^{-x}} = \int_0^{\infty} \frac{e^x dx}{e^{2x} + 1}$$

$$= \int_{p=e^0=1}^{p=e^{\infty}=\infty} \frac{dp}{p^2 + 1} \Rightarrow \left( \tan^{-1} p \right)_1^{\infty}$$

$$= \tan^{-1} \infty - \tan^{-1} 1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

2. (a)

Eigen value of A are,  $\lambda_1, \lambda_2, \lambda_3$ 

$$|A| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3$$

Eigen value of  $A^{-1}$  is  $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$ 

$$\frac{1}{\lambda_1} = 1 \Rightarrow \lambda_1 = 1$$

$$\frac{1}{\lambda_2} = 2 \Rightarrow \lambda_2 = \frac{1}{2}$$

$$\frac{1}{\lambda_3} = 5 \Rightarrow \lambda_3 = \frac{1}{5}$$

$$\lambda_1 \lambda_2 \lambda_3 = (1) \left( \frac{1}{2} \right) \left( \frac{1}{5} \right) = \frac{1}{10} = 0.1$$

$$|A| = 0.1$$

3. (a)

Diverge of curl of a vector is always zero.

4. (b)

$$2x + y + 2z = 0$$

$$x + y + 3z = 0$$

$$4x + 3y + z = 0$$

$$[A : B] = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 4R_1$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & -11 & 0 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - R_2,$$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

Rank of  $[A : B] = 3$

Rank of  $[A] = 3 = \text{Rank of } [A : B] = \text{number of unknowns}$

So, unique solution exists

**5. (a)**

$$\begin{aligned} I &= \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right) dx \\ &= \int_0^{\pi/2} [\log(\sin x) dx - \log(\cos x) dx] \qquad \left[ \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right] \\ &= \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx - \int_0^{\pi/2} \log(\cos x) dx \\ &= \int_0^{\pi/2} \log(\cos x) dx - \int_0^{\pi/2} \log(\cos x) dx \\ I &= 0 \end{aligned}$$

**6. (d)**

For function to be differentiable i.e. continuous  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$

$$\begin{aligned} f(0^-) &= \lim_{x \rightarrow 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)} \\ &= \lim_{x \rightarrow 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3} \\ f(0^+) &= \lim_{x \rightarrow 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)} \\ &= \lim_{x \rightarrow 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2} \end{aligned}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get,  $p = -\frac{5}{3}$

**7. (b)**

We have

$$\begin{aligned} y &= e^x (A \cos x + B \sin x) \\ y' &= e^x (A \cos x + B \sin x) + e^x (-A \sin x + B \cos x) \\ &= y + e^x [-A \sin x + B \cos x] \\ y'' &= y' + e^x (-A \sin x + B \cos x) + e^x (-A \cos x - B \sin x) \end{aligned}$$

$$= y' + y' - y - y$$

$$= 2y' - 2y$$

$$\Rightarrow \text{Order} = 2$$

$$\text{Degree} = 1$$

8. (a)

$$\nabla \cdot \vec{F} = 0$$

[For solenoidal vector]

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

$$\text{From here, } a = 2$$

9. (a)

Greatest rate of increase of  $\phi$  is magnitude of directional derivative at that point.

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

$$\text{Greatest rate of increase} = \sqrt{1^2 + 6^2} = \sqrt{37} = 6.08$$

10. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1 P_2 P_3 = \frac{15}{50} \times \frac{14}{49} \times \frac{13}{48} = \frac{13}{560}$$

11. (d)

$$D^2 + 7D + 12 = 0$$

$$(D + 3)(D + 4) = 0$$

$$D = -3, -4$$

$$y = C_1 e^{-3x} + C_2 e^{-4x}$$

$$y(0) = C_1 + C_2 = 1$$

$$y'(0) = -3C_1 - 4C_2 = 0$$

$$\Rightarrow -3C_1 - 4C_2 = 0$$

$$3C_1 + 3C_2 = 3$$

$$C_2 = -3$$

$$C_1 = 4$$

$$y(x) = 4e^{-3x} - 3e^{-4x}$$

12. (c)

For a diagonal matrix

$$\lambda_1, \lambda_2 = a, b$$

$$\lambda_1 = a$$

$$\lambda_2 = b$$

$$ab = 25$$

we know,

$$AM \geq GM$$

$$\frac{\lambda_1 + \lambda_2}{2} \geq \sqrt{\lambda_1 \lambda_2} = \sqrt{ab} = 5$$

$$\lambda_1 + \lambda_2 \geq 10$$

$$\min(\lambda_1 + \lambda_2) = 10$$

13. (b)

$$\text{If } A^T = A^{-1} \Rightarrow A \cdot A^T = I$$

$$\Rightarrow \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix} \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{3}{5}x + \frac{4}{5} \times \frac{3}{5} = 0$$

$$\Rightarrow x = \frac{-4}{5}$$

14. (a)

$$\left[ \begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array} \right] = \left[ \begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right]$$

$$= \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3/2 & -8 \\ 0 & 1 & 0 & 0 & 1/2 & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -3/2 & -8 \\ 0 & 1/2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

15. (b)

$$\int_0^y \cos t^2 dt = \int_0^{x^2} \frac{\sin t dt}{t}$$

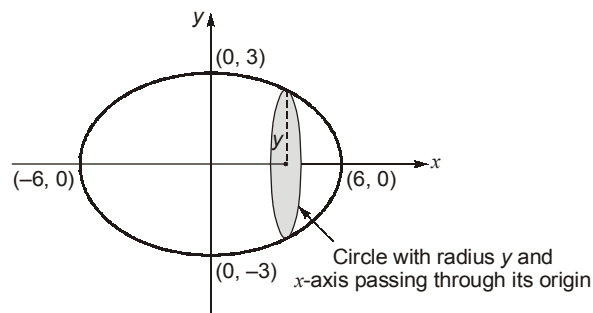
Differentiating both sides w.r.t  $x$ 

$$\frac{d}{dy} \left( \int_0^y \cos t^2 dt \right) \cdot \frac{dy}{dx} = \frac{d}{dx^2} \left( \int_0^{x^2} \frac{\sin t dt}{t} \right) \cdot \frac{dx^2}{dx}$$

$$\cos y^2 \cdot \frac{dy}{dx} = \frac{\sin x^2}{x^2} \cdot 2x$$

$$\frac{dy}{dx} = \frac{2 \sin x^2}{x \cdot \cos y^2}$$

16. (b)



$$\begin{aligned} \text{Volume generated} &= \int_{-6}^6 \pi y^2 dx = \int_{-6}^6 \pi \left( \frac{36 - x^2}{4} \right) dx \\ &= \frac{\pi \times 2}{4} \int_0^6 (36 - x^2) dx = \frac{\pi}{2} \left[ 36x - \frac{x^3}{3} \right]_0^6 \\ &= 72\pi \text{ unit}^3 \end{aligned}$$

17. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2 + 4)} = 96 \frac{1}{4D^2 \left( 1 + \frac{D^2}{4} \right)} x^2 = \frac{96}{4} \left[ \frac{\left( 1 - \frac{D^2}{4} \right) x^2}{D^2} \right]$$

$$= 24 \frac{\left( x^2 - \frac{1}{2} \right)}{D^2}$$

$$PI = 24 \left[ \frac{x^4}{4 \times 3} - \frac{x^2}{4} \right] = 2x^2(x^2 - 3)$$

$$PI|_{x=2} = 2 \times 2^2(4 - 3) = 8$$

18. (a)

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^2 kx dx + \int_2^4 2k dx + \int_4^6 (-kx + 6k) dx = 1$$

$$\left. \frac{kx^2}{2} \right|_0^2 + 2kx \Big|_2^4 + \left. \left( -\frac{kx^2}{2} + 6kx \right) \right|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4 - 2) - \frac{k}{2}(6^2 - 4^2) + 6k(6 - 4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x) dx = \int_0^2 \frac{1}{8} x^2 dx + \int_2^4 \frac{1}{4} x dx + \int_4^6 \left( -\frac{1}{8} x^2 + \frac{3}{4} x \right) dx$$

$$= \left. \frac{1}{8} \frac{x^3}{3} \right|_0^2 + \left. \frac{1}{4} \frac{x^2}{2} \right|_2^4 - \left. \frac{1}{8} \frac{x^3}{3} \right|_4^6 + \left. \frac{3}{4} \frac{x^2}{2} \right|_4^6$$

$$= \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

19. (a)

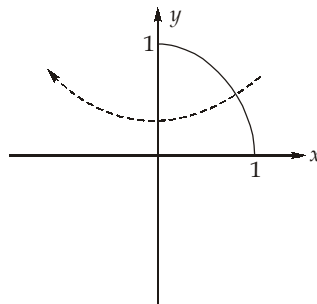
$$x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$$

Just by seeing, we can know that it represents a circle in  $x - y$  plane, given by

$$x^2 + y^2 = 1$$

Given  $0 \leq k \leq 1$ , which gives  $0 \leq x \leq 1$ ;  $0 \leq y \leq 1$

or  $0 \leq \frac{\pi k}{2} \leq \frac{\pi}{2}$



So we get a quarter circle, when this is rotated with respect to  $y$ -axis by  $360$  degree, it creates a hemisphere of radius  $1$ .

Surface area of hemisphere,

$$A_S = 2\pi r^2$$

$$= 2\pi (1)^2 = 2\pi$$

20. (c)

$$\begin{aligned}
 f(y) &= y^2 e^{-y} \\
 f'(y) &= y^2 (-e^{-y}) + e^{-y} \times 2y \\
 &= e^{-y} (2y - y^2)
 \end{aligned}$$

Putting  $f'(y) = 0$ 

$$e^{-y} (2y - y^2) = 0$$

$$e^{-y} y (2 - y) = 0$$

 $y = 0$  or  $y = 2$  are the stationary points

$$\begin{aligned}
 \text{Now, } f''(y) &= e^{-y} (2 - 2y) + (2y - y^2) (-e^{-y}) \\
 &= e^{-y} (2 - 2y - 2y + y^2) \\
 &= e^{-y} (y^2 - 4y + 2)
 \end{aligned}$$

$$\text{At } y = 0, \quad f''(0) = e^{-0} (0 - 0 + 2) = 2$$

Since  $f''(0) = 2$  is  $> 0$  at  $y = 0$  we have a minima

$$\begin{aligned}
 \text{Now, at } y = 2 \quad f''(2) &= e^{-2} (2^2 - 4 \times 2 + 2) \\
 &= e^{-2} (4 - 8 + 2) \\
 &= -2e^{-2} < 0
 \end{aligned}$$

 $\therefore$  At  $y = 2$  we have a maxima.

21. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by  $\cos x \cos y$ , we get ,

$$\tan x dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\log \frac{1}{\cos x \cos y} = C_1$$

$$\cos x \cos y = C$$

Since it passes through  $\left(0, \frac{\pi}{3}\right)$ 

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

 $\Rightarrow$  The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$



**22. (a)**

To obtain maximum value of  $f(x)$ , first  $f'(x)$  should be equated to zero.

$$\Rightarrow f'(x) = 6x^2 - 6x - 36 = 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$\therefore f'(x) = 0$$

at  $x = 3$  and  $-2$

Now,  $f''(x) = 12x - 6$

$$f''(3) = 30 > 0$$

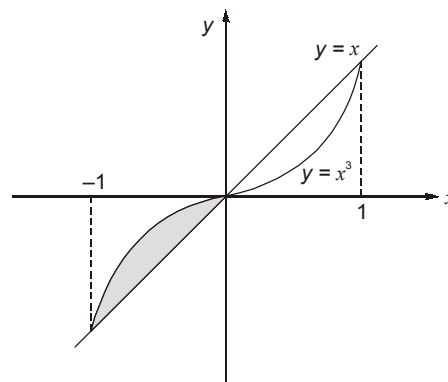
at  $x = 3$ , there is local minima

and  $f''(2) = -30 < 0$

$\therefore$  at  $x = -2$ , a local maxima is observed.

**23. (b)**

Point of inter-section of the two curves are  $x = 0, 1, -1$



$$\text{Area} = \int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx = \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{0 - (-1)^4}{4} - \frac{0 - (-1)^2}{2} + \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

**24. (c)**

$$\frac{d^2y}{dx^2} = y$$

$$\Rightarrow D^2y = y \quad (\because d/dx = D)$$

$$(D^2 - 1)y = 0$$

$$D^2 - 1 = 0$$

$$D = \pm 1$$

$$y = C_1 e^x + C_2 e^{-x}$$

Given point passes through origin

$$\Rightarrow 0 = C_1 + C_2$$

$$C_1 = -C_2 \quad \dots(i)$$

Also, point passes through  $(\ln 2, 3/4)$

$$\Rightarrow \frac{3}{4} = C_1 e^{\ln 2} + C_2 e^{-\ln 2}$$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$$\Rightarrow C_2 + 4C_1 = 1.5 \quad \dots(ii)$$

From (i)  $C_1 = -C_2$ , putting in (ii), we get

$$\Rightarrow -3C_2 = 1.5$$

$$C_2 = -0.5$$

$$\therefore C_1 = 0.5$$

$$\Rightarrow y = 0.5(e^x - e^{-x})$$

$$y = \frac{e^x - e^{-x}}{2}$$

25. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima,  $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

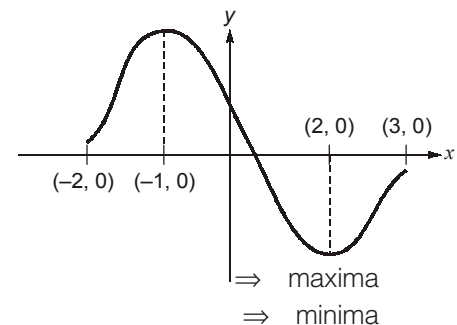
$$(x + 1)(x - 2) = 0$$

$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0$$

$$f''(2) = 24 - 6 = 18 > 0$$



The function has maxima at  $x = -1$  and minima at  $x = 2$ .  
 The function is decreasing between  $-1$  and  $2$ .

26. (b)

$z$  varies from 0 to  $\frac{x^2 + y^2}{4}$ ;  $y$  varies from 0 to  $\sqrt{16 - x^2}$ ;  $x$  varies from 0 to 4.

$$\text{Volume} = \iiint dx dy dz = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\frac{x^2+y^2}{4}} dz dy dx$$

$$= \frac{1}{4} \int_0^4 \int_0^{\sqrt{16-x^2}} (x^2 + y^2) dy dx = \frac{1}{4} \int_0^4 \left( x^2 y + \frac{y^3}{3} \right) \Big|_0^{\sqrt{16-x^2}} dx$$

$$= \frac{1}{4} \int_0^4 \left( x^2 \sqrt{16-x^2} + \frac{(\sqrt{16-x^2})^3}{3} \right) dx$$

Let,  $x = 4 \sin \theta$   $x \rightarrow 0$  to  $4$

$$dx = 4 \cos \theta d\theta \quad \theta \rightarrow 0 \text{ to } \frac{\pi}{2}$$

$$\text{Volume} = \frac{1}{4} \left[ 4^4 \int_0^{\pi/2} \sin^2 \theta \cos^2 \theta d\theta + \frac{4^4}{3} \int_0^{\pi/2} \cos^4 \theta d\theta \right]$$

$$= \frac{1}{4} \left[ 4^4 \times \frac{\frac{3}{2} \times \frac{3}{2}}{2 \times \frac{6}{2}} + \frac{4^4}{3} \times \frac{\frac{5}{2} \times \frac{3}{2}}{2 \times \frac{6}{2}} \right]$$

$$= \frac{1}{4} \left[ 4^4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!} \pi + \frac{4^4}{3} \times \frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!} \times \pi \right]$$

$$= \frac{1}{4} [16\pi + 16\pi] = 8\pi = 25.13 \text{ unit}^3$$

27. (d)

$$c(y+c)^2 = x^3 \quad \dots(i)$$

Differentiating, we get

$$2c(y+c) \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{2x^3}{(y+c)^2} (y+c) \frac{dy}{dx} = 3x^2 \quad \left( \because c = \frac{x^3}{(y+c)^2} \right)$$

$$\Rightarrow \frac{2x^2}{y+c} \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{2x}{3} \left( \frac{dy}{dx} \right) = y+c$$

$$\Rightarrow c = \frac{2x}{3} \left( \frac{dy}{dx} \right) - y$$

Putting this value of 'c' in equation (i)

$$\left[ \frac{2x}{3} \left( \frac{dy}{dx} \right) - y \right] \left[ \frac{2x}{3} \frac{dy}{dx} \right]^2 = x^3$$

28. (c)

Suppose  $y = \lim_{x \rightarrow \infty} \left( \frac{x+6}{x+1} \right)^{x+4}$

$$\Rightarrow y = \lim_{x \rightarrow \infty} \left[ \left( 1 + \frac{5}{x+1} \right)^{\frac{x+1}{5}} \right]^{\left[ \frac{5(x+4)}{x+1} \right]}$$

$$\Rightarrow \ln y = \lim_{x \rightarrow \infty} \frac{5(x+4)}{(x+1)} \ln \left( 1 + \frac{5}{x+1} \right)^{\frac{x+1}{5}} \quad \dots(i)$$

$\lim_{x \rightarrow \infty} \frac{5(x+4)}{(x+1)}$  is in the form of  $\frac{\infty}{\infty}$  and  $\lim_{x \rightarrow \infty} \ln \left( 1 + \frac{5}{x+1} \right)^{\frac{x+1}{5}}$  is in the form of  $0^0$ .

Calculating the limits of both terms separately

$$\lim_{x \rightarrow \infty} 5 \frac{(x+4)}{(x+1)} = \lim_{x \rightarrow \infty} 5 \frac{\left( 1 + \frac{4}{x} \right)}{\left( 1 + \frac{1}{x} \right)} = 5 \frac{(1+0)}{(1+0)}$$

$$= 5$$

We can use direct result of  $\lim_{t \rightarrow 0} (1+t)^{1/t} = e \quad \dots(ii)$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow \infty} \ln \left[ 1 + \frac{5}{x+1} \right]^{\frac{x+1}{5}} &= \ln(e) \\ &= 1 \qquad \dots(\text{iii}) \\ \therefore \ln y &= 5(1) \\ \Rightarrow y &= e^5 \end{aligned}$$

29. (c)

The matrix formed by the coefficients is  $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$$\therefore D = 0 \text{ for } a = 2 \text{ or } a = -1$$

(A) If  $D \neq 0$ , then the system will have unique solution.

(B) If  $a = 2$ , the matrix formed by the coefficients is  $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.

Considering 'z' as side unknown.

The characteristic determinant will be  $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.

The system will have infinite solutions when  $a = 2$ .

(C) If  $a = -1$ , the matrix formed by the coefficients is  $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.

Considering 'z' as side unknown.

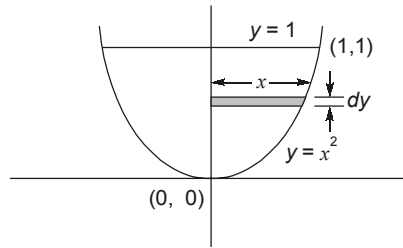
The characteristic matrix is  $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is  $3b$ .

The system will have no solution if  $b \neq 0$

$\therefore$  For  $a = -1$  and  $b \neq 0$ , the system will have no solution.

30. (b)



$y = x^2$  and  $y = 1$  intersect at  $(1, 1)$

Small disk of radius ' $x$ ' and depth ' $dy$ ' are integrated to compute the volume

$$\text{Volume} = \int_0^1 \pi x^2 dy$$

$$= \int_0^1 \pi y dy = \pi \left[ \frac{y^2}{2} \right]_0^1 = \frac{\pi}{2} \quad (\because y = x^2)$$

