CLASS TEST

S.No.: 01SP_ABCDEFG_200424

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 20/04/2024

ANSWER KEY

1.	(b)	7.	(d)	13.	(d)	19.	(d)	25.	(b)
2.	(c)	8.	(a)	14.	(a)	20.	(a)	26.	(c)
3.	(c)	9.	(a)	15.	(a)	21.	(c)	27.	(a)
4.	(a)	10.	(b)	16.	(d)	22.	(d)	28.	(b)
5.	(a)	11.	(c)	17.	(d)	23.	(c)	29.	(d)
6.	(d)	12.	(d)	18.	(d)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (b)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x-1) = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(x^3-1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And $Lf'(1) = 2$, $Rf'(1) = 1$

$\Rightarrow f$ is not differentiable at $x = 1$

2. (c)

$$\int_0^5 f(x)\cdot dx = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{1}{2}\left[(2+0.077)+2(1+0.4+0.2+0.1176)\right]$$

$$\simeq 2.75$$

3. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$l_{11} = 2$ $\qquad$ $l_{11}u_{12} = 4$

$u_{12} = \frac{4}{2} = 2$

$l_{21} = 6$ $\qquad$ $l_{21}u_{12} + l_{22} = 3$

$6 \times 2 + l_{22} = 3$

$l_{22} = 3 - 12$

$l_{22} = -9$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

4. (a)

$$P(-1 \le x \le 1) = \int_{-1}^{1}(0.1)dx$$

$$= 2\times\frac{1}{10} = \frac{1}{5}$$

5. (a)

$$\lim_{x \to 0} \frac{\log x}{\cot x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{x}}{-\operatorname{cosec}^2 x} = -\lim_{x \to 0} \frac{\sin^2 x}{x} \qquad \left(\text{from } \frac{\infty}{\infty}\right)$$

$$= -\lim_{x \to 0} \frac{2 \sin x \cos x}{1} = 0 \qquad \left(\text{from } \frac{0}{0}\right)$$

6. (d)

$$u = \sin x$$

$$du = \cos x \, dx$$

$$x = \frac{\pi}{2} \Rightarrow u = \sin \frac{\pi}{2} = 1$$

$$x = -\pi \Rightarrow u = \sin(-\pi) = 0$$

$$\int_{-\pi}^{\pi/2} \cos(x) \cos(\sin(x)) dx = \int_0^1 \cos u \, du$$

$$= |\sin u|_0^1$$

$$= (\sin 1) - \sin (0) = \sin 1$$

7. (d)

$$\frac{e^x}{(1 - e^x)} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

Integrating on both sides, we get,

$$-\ln(1 - e^x) + \ln(\tan y) = C_1$$

$$\ln\left(\frac{\tan y}{(1 - e^x)}\right) = C_1$$

$$\frac{\tan y}{(1 - e^x)} = e^{C_1} = C$$

$$\tan y = C(1 - e^x)$$

8. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$PI = \frac{x^2 + 2x + 8}{D(1 + D)}$$

$$= \frac{1}{D}(1 + D)^{-1}(x^2 + 2x + 8) = \frac{1}{D}(1 - D + D^2 - D^3 + \ldots)(x^2 + 2x + 8)$$

$$= \frac{1}{D}(x^2 + 2x + 8 - 2x - 2 + 2) = \frac{1}{D}(x^2 + 8) = \frac{x^3}{3} + 8x$$

9. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

10. (b)

We have $y = e^x\,(A\cos x + B\sin x)$

$$y' = e^x\,(A\cos x + B\sin x) + e^x\,(-A\sin x + B\cos x)$$

$$= y + e^x\,[-A\sin x + B\cos x]$$

$$y'' = y' + e^x\,(-A\sin x + B\cos x) + e^x\,(-A\cos x - B\sin x)$$

$$= y' + y' - y - y = 2y' - 2y$$

$\Rightarrow$ Order = 2

Degree = 1

11. (c)

$$\lim_{x\to 1}\frac{x^x - x}{x - 1 - \log x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}}$$

Let, $y = x^x$

$$\log y = x\log x$$

$\therefore$
$$\frac{1}{y}\frac{dy}{dx} = x\cdot\frac{1}{x} + 1\cdot\log x$$

or
$$\frac{d}{dx}(x^x) = x^x(1 + \log x)$$

$$= \lim_{x\to 1}\frac{x^x(1+\log x) - 1}{1 - \frac{1}{x}} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{\frac{d}{dx}(x^x)\cdot(1+\log x) + x^x\left(\frac{1}{x}\right) - 0}{\frac{1}{x^2}}$$

$$= \lim_{x\to 1}\frac{x^x(1+\log x)^2 + x^x\left(\frac{1}{x}\right)}{x^{-2}} = \frac{1(1+0)^2 + 1\cdot 1}{1} = 2$$

12. (d)

Let $u = \sqrt{x}$

Then $du = \dfrac{1}{2\sqrt{x}}dx$

$\therefore \quad dx = du \cdot 2\sqrt{x}$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\,du = 2\int \sin u\,du$$

$$= -2\cos\sqrt{x} + c$$

13. (d)

$$I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right)dx$$

$$= \int_0^{\pi/2} [\log(\sin x)dx - \log(\cos x)dx]$$

$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx - \int_0^{\pi/2} \log(\cos x)dx$$

$$I = 0$$

14. (a)

$$|\lambda - AI| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2$$

$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$

$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$

$\lambda = 0, \lambda = 1$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$
$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$
$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$
$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

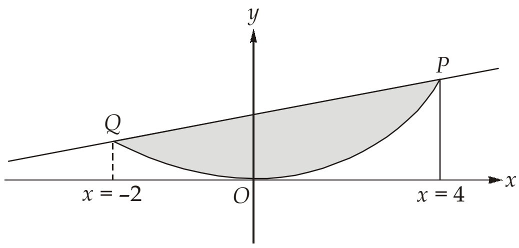
$\therefore$ $$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

15. (a)

Parabola given : $x^2 = 4y$ ...(i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$
$$\Rightarrow \quad x^2 = 2(x+4)$$
$$\Rightarrow \quad x^2 - 2x - 8 = 0$$
$$\Rightarrow \quad x^2 - 4x + 2x - 8 = 0$$
$$\Rightarrow \quad x(x-4) + 2(x-4) = 0$$
$$\Rightarrow \quad x = 4, -2$$



$$\text{Required area} = POQ$$
$$= \int_{-2}^{4} y\,dx \text{ from straight line} - \int_{-2}^{4} y\,dx \text{ from parabola}$$
$$= \int_{-2}^{4} \left(\frac{x+4}{2}\right) - \int_{-2}^{4} \frac{x^2}{4} dx$$
$$= \frac{1}{2}\left|\frac{x^2}{2} + 4x\right|_{-2}^{4} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-2}^{4}$$
$$= \frac{1}{2}\{8 + 16 - (-6)\} - \frac{1}{12}(64 + 8)$$
$$= \frac{1}{2} \times 30 - \frac{1}{12} \times 72 = 15 - 6 = 9$$

16. (d)

$$AX = B$$

Augmented matrix, $$[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

$\Rightarrow$ Infinitely many solutions are possible.

17. (d)

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$X_1 = c_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_3 = 0$$

$$2x_1 + 2x_2 + x_3 = 0$$

$$X_2 = c_2\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = \frac{-1}{2}x_3$$

$$X_3 = c_3\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Since, $X_1^T X_2 \neq 0$

$X_2^T X_3 \neq 0$

$X_3^T X_1 \neq 0$

None of the above is correct.

18. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$

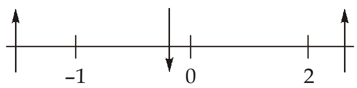
For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.

Critical point (–1, 2) draw plot on line graph:

Since $0 \in (-1, 2)$ and $f'(0) = 6 \times 0^2 - 6 \times 0 - 12 = -12 < 0$



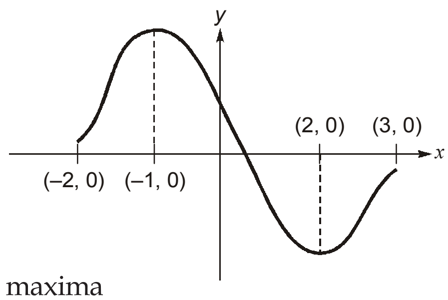
The function is decreasing between –1 and 2.

19. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$



The function has maxima at $x = -1$ and minima at $x = 2$.

The function is decreasing between –1 and 2.

20. (a)

$$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x\, dx + \tan y\, dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$
$$\log \frac{1}{\cos x \cos y} = C_1$$
$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

21. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

22. (d)

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -6xy - 6y$$

$$dv = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$= (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy$$

$$v = 3x^2y + 6xy - y^3 + C$$

23. (c)

If the two curves intersects, then at point of intersection,

$$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$$

$$x^3 + 2x^2 + 5x - 7 = 0$$

$$f(x) = x^3 + 2x^2 + 5x - 7$$

$$f(0) = 0 + 0 + 0 - 7 = -7 < 0$$

$$f(1) = 1 + 2 + 5 - 7 = 1 > 0$$

$\Rightarrow$ One root lies between 0 and 1. Let us assume 1 as initial value.

$$f'(x) = 3x^2 + 4x + 5$$

$$x_1 = 1 - \left.\frac{f(x)}{f'(x)}\right|_{x=1} = 1 - \frac{1^3 + 2\times 1^2 + 5\times 1 - 7}{3\times 1^2 + 4\times 1 + 5} = 0.9167$$

$$x_2 = x_1 - \left.\frac{f(x)}{f'(x)}\right|_{x=0.9167} = 0.9136$$

24. (b)

Number of ways of throwing 6 is five ⇒ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)
Number of ways of throwing 7 is six ⇒ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \frac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \frac{5}{36} = \frac{31}{36}$

Probability of throwing 7, $q_1 = \frac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \frac{6}{36} = \frac{30}{36}$

$$\text{Probability of } B \text{ winning} = p_2q_1 + p_2q_2p_2q_1 + p_2q_2p_2q_2p_2q_1 + ...$$
$$= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + ...]$$
$$= \frac{p_2q_1}{(1-p_2q_2)} = \frac{\frac{31}{36}\times\frac{6}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{31\times 6}{366} = \frac{31}{61}$$

25. (b)

$$\phi_1 = ax^2 - byz - (a+2)x$$
$$\nabla \phi_1 = [2ax - (a+2)]\hat{i} - bz\hat{j} - by\hat{k}$$
$$\nabla \phi_1(1, -1, 2) = (a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$
$$\phi_2 = 4x^2y + z^3 - 4$$
$$\nabla \phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$
$$\nabla \phi_2(1, -1, 2) = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

Since surfaces are orthogonal to each other at (1, –1, 2)

$$\nabla \phi_1 \cdot \nabla \phi_2 = 0$$
$$[(a-2)\hat{i} - 2b\hat{j} + b\hat{k}]\cdot[-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$
$$-8(a-2) - 8b + 12b = 0 \qquad \text{... (i)}$$

Also point (1, –1, 2) lies on the surface.

$$\Rightarrow \quad a \times 1 + 2b = (a+2)1$$
$$b = 1$$

Putting this in equation 1, we get,

$$-8(a-2) - 8 + 12 = 0$$
$$a - 2 = -\frac{1}{8}\times(-4) = 0.5$$
$$a = 2.5$$

26. (c)

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from B = $\frac{2}{2+4}\times\frac{6}{13} = \frac{2}{13}$

Case-II: Black ball is transferred from A to B

Probability of drawing black ball from B = $\frac{4}{2+4}\times\frac{5}{13}=\frac{10}{39}$

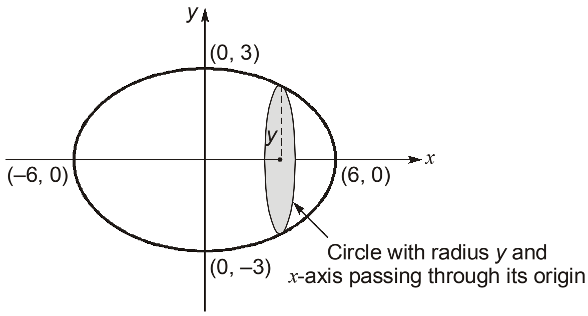
Required probability = $\frac{2}{13}+\frac{10}{39}=\frac{16}{39}$

27. (a)

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)x\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1 + \left.\left(x^2-\frac{x^3}{3}\right)\right|_1^2 = \frac{1}{3}+4-1-\frac{8-1}{3}=1$$

28. (b)



$$\text{Volume generated} = \int_{-6}^{6} \pi y^2\,dx = \int_{-6}^{6}\pi\left(\frac{36-x^2}{4}\right)dx$$

$$= \frac{\pi\times 2}{4}\int_0^6 (36-x^2)\,dx = \frac{\pi}{2}\left[36x-\frac{x^3}{3}\right]_0^6$$

$$= 72\pi$$

29. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF\cdot f(x)\cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)}\cdot f(x)\cdot f'(x)dx$$

Let $f(x) = t$

$$f'(x)\,dx = dt$$

$$y \times e^t = \int e^t\cdot t\,dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log\,[y + 1 - f(x)] + f(x) = c'$$

30. **(a)**

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2+4)} = 96\frac{1}{4D^2\left(1+\frac{D^2}{4}\right)}x^2 = \frac{96}{4}\left[\frac{\left(1-\frac{D^2}{4}\right)x^2}{D^2}\right]$$

$$= 24\frac{\left(x^2-\frac{1}{2}\right)}{D^2}$$

$$PI = 24\left[\frac{x^4}{4\times 3}-\frac{x^2}{4}\right] = 2x^2(x^2-3)$$

$$PI\big|_{x=2} = 2 \times 2^2(4-3) = 8$$

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