CLASS TEST

S.No.: 01SK_ECEE_ABCDEF_30324

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CONTROL SYSTEM

EC-EE

Date of Test : 30/03/2024

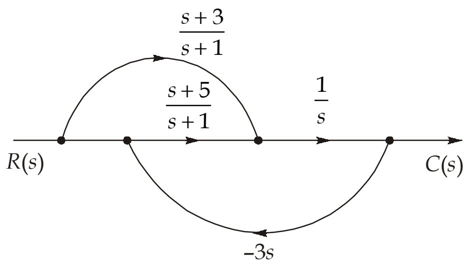
ANSWER KEY

1.	(a)	7.	(b)	13.	(c)	19.	(a)	25.	(a)
2.	(d)	8.	(b)	14.	(c)	20.	(c)	26.	(d)
3.	(c)	9.	(d)	15.	(c)	21.	(d)	27.	(d)
4.	(b)	10.	(b)	16.	(c)	22.	(d)	28.	(d)
5.	(d)	11.	(a)	17.	(d)	23.	(d)	29.	(b)
6.	(d)	12.	(d)	18.	(b)	24.	(d)	30.	(b)

DETAILED EXPLANATIONS

1. (a)

Signal flow graph of the system is



Mason's gain formula,

$$\frac{C(s)}{R(s)} = \frac{P_k\Delta_k}{\Delta}$$

Where, P_k = forward path gain

Δ_k = 1 – (sum of individual loops) + (sum of two non touching loops)....

$$P_1 = \left(\frac{s+5}{s+1}\right)\cdot\frac{1}{s}$$

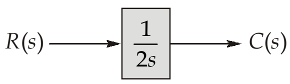
$$P_2 = \left(\frac{s+3}{s+1}\right)\cdot\frac{1}{s}$$

Loops: $$L_1 = \left(\frac{s+5}{s+1}\right)\left(\frac{1}{s}\right)(-3s)$$

$$\frac{C(s)}{R(s)} = \frac{\left(\frac{s+5}{s+1}\right)\times\frac{1}{s}+\left(\frac{s+3}{s+1}\right)\times\frac{1}{s}}{1+\left(\frac{s+5}{s+1}\right)\times\frac{1}{s}\times(3s)} = \frac{s+5+s+3}{s[s+1+3s+15]}$$

$$= \frac{2s+8}{s(4s+16)} = \frac{1}{2s}$$

Thus system can be represented as



It is an integrator with gain = 0.5

2. (d)

It is desirable to remove the effect of disturbance on response. So $\frac{C(s)}{D(s)}$ ratio can be calculated as follows:

$$\frac{C(s)}{D(s)} = \frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

Response due to disturbance,

$$C(s) = D(s) \cdot \frac{G_3(1+G_1H_2)}{1+G_1G_2G_3H_1+G_1H_2}$$

$C(s)$ should be zero for $D(s) \neq 0$

$\therefore \quad G_1H_2 = -1$

3. (c)

$$G(s) = \frac{1}{(Ts+1)}$$

Given that, $R(s) = 1$

$$C(s) = R(s).G(s)$$

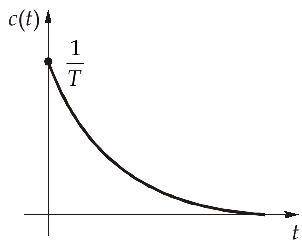
$$C(s) = \frac{1}{(Ts+1)} = \frac{1}{T\left(s+\frac{1}{T}\right)}$$

Taking inverse Laplace transform of above equation, we get

$$C(t) = \frac{1}{T}e^{-t/T}$$

At, $t = 0, \quad C(t) = \frac{1}{T}$

$t = \infty, \quad C(t) = 0$



4. (b)

State transition matrix, $\phi(t) = e^{At}$

$\phi(0) = I$

From property of state transmission matrix at $t = 0$,

$\phi(0) = I$

$$= \begin{bmatrix} 1+0 & 0 \\ -0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Option (b) is correct

5. (d)

Characteristic equation of system is,

$$1 + G(s)\,H(s) = 0$$

$$1+\frac{K}{s(s+1)(s+3)} = 0$$

$$s^3 + 4s^2 + 3s + K = 0$$

Routh-Hurwitz method,

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	0
s^0	K	

According to Routh-Hurwitz criteria, for a stable system, first column of Routh array should have positive sign

$$\frac{12-K}{4} > 0$$

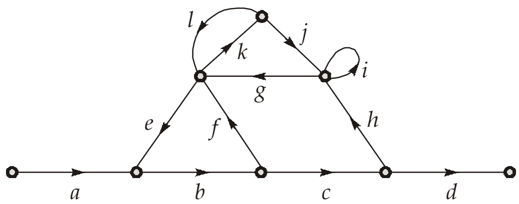
and
$$K < 12$$
$$K > 0$$

Common range is $0 < K < 12$

6. **(d)**

Since the transfer function has one pole on the R.H.S. thus the system is unstable. So, the final value theorem is not applicable in this case. The output will be unbounded.

7. **(b)**



$L_1 = bfe$, $L_2 = bchge$, $L_3 = i$, $L_4 = kjg$ and $L_5 = kl$.

8. **(b)**

Transfer function of proportional integral controller is

$$T(s) = K_p + \frac{K_I}{s} = \frac{K_I + sK_p}{s}$$

Initial slope = –20 dB/dec, due to pole at origin
Final slope = 0 dB/dec, due to finite zero.

9. **(d)**

Solution of homogeneous equation is given by,

$$x(t) = \phi(t)\, x(0)$$
$\phi(t)$ = State transition matrix
$x(0)$ = Initial conditions of system

Given : $$[A] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & -1 \\ 0 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s-1} & \frac{1}{(s-1)^2} \\ 0 & \frac{1}{s-1} \end{bmatrix}$$

$$\phi(t) = L^{-1}[sI - A]^{-1} = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$$

$$x(t) = \phi(t)\, x(0) = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$

10. (b)

$$\text{C.L.T.F. } T(s) = \frac{4}{s^2+4}$$

$$\text{Characteristic equation} = s^2 + 4 = 0$$

On comparing, $s^2 + 2\xi\omega_n s + \omega_n^2 = 0, \quad \xi = 0$

So, $\quad$ Resonant peak $(M_r) = \dfrac{1}{2\xi\sqrt{1-\xi^2}} = \infty$

11. (a)

Response of the system in Laplace form is,

$$Y(s) = X_2(s) \qquad \text{...(i)}$$

For zero state response,

$$X(s) = \phi(s)\ BR(s) \qquad \text{...(ii)}$$

Where, $\quad \phi(s) = [sI - A]^{-1}$

$$R(s) = \frac{1}{s^2}$$

State space representation is

$$\dot{x}(t) = A\ x(t) + Br(t)$$

$$y(t) = C\ x(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{\begin{bmatrix} s+1 & 1 \\ -2 & s \end{bmatrix}}{s^2+s+2}$$

From equation (ii), we get

$$X(s) = \frac{1}{s^2+s+2}\begin{bmatrix} s+1 & 1 \\ -2 & s \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+s+2} \\ \frac{s}{s^2+s+2} \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{s^2} \end{bmatrix}$$

$$X(s) = \begin{bmatrix} \frac{1}{s^2(s^2+s+2)} \\ \frac{1}{s(s^2+s+2)} \end{bmatrix}$$

From equation (i), we get

$$Y(s) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s^2(s^2+s+2)} \\ \frac{1}{s(s^2+s+2)} \end{bmatrix}$$

$$Y(s) = \frac{1}{s(s^2+s+2)}$$

12. **(d)**

At phase crossover frequency,

$$\angle G(j\omega)H(j\omega) = -180°$$

Given,

$$G(s)\,H(s) = \frac{(9+0.5s)(s+2)}{s(s+3)}$$

$$G(j\omega)\,H(j\omega) = \frac{(9+j0.5\omega)(j\omega+2)}{j\omega(3+j\omega)}$$

$$\angle G(j\omega)\,H(j\omega) = \tan^{-1}\left(\frac{0.5\omega}{9}\right)+\tan^{-1}\left(\frac{\omega}{2}\right)-90°-\tan^{-1}\left(\frac{\omega}{3}\right)$$

At $\omega = \omega_{pc}$, $\angle G(j\omega)H(j\omega) = -180°$

$$-180° + 90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right)+\tan^{-1}\left(\frac{\omega}{2}\right)-\tan^{-1}\left(\frac{\omega}{3}\right)$$

$$-90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right)+\tan^{-1}\left(\frac{\frac{\omega}{2}-\frac{\omega}{3}}{1+\frac{\omega^2}{6}}\right)$$

$$-90° = \tan^{-1}\left(\frac{0.5\omega}{9}\right)+\tan^{-1}\left(\frac{\omega}{(6+\omega^2)}\right)$$

$$-90° = \tan^{-1}\left(\frac{\frac{0.5\omega}{9}+\frac{\omega}{6+\omega^2}}{1-\frac{0.5\omega^2}{9(6+\omega^2)}}\right)$$

$$\Rightarrow \quad \left(\frac{\frac{0.5\omega}{9}+\frac{\omega}{6+\omega^2}}{1-\frac{0.5\omega^2}{9(6+\omega^2)}}\right) = -\infty$$

$$1 = \frac{0.5\omega^2}{54+9\omega^2}$$

$$9\,\omega^2 - 0.5\,\omega^2 + 54 = 0$$

$$\omega = \sqrt{-6.35}$$

Frequency can not be imaginary. Thus, phase crossover frequency does not exist.
Alteratively, for second order system, Bode phase plot never crosses –180° axis. Thus phase crossover frequency is infinite.

13. (c)

From time response curve,

$$t_s = \frac{4}{\xi\omega_n} \qquad \text{(For 2\% tolerance band)}$$

$$4 = \frac{4}{\xi\omega_n}$$

$$\xi\omega_n = 1$$

and

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$0.09 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\ln(0.09) = -\frac{\pi\xi}{\sqrt{1-\xi^2}}$$

$$(2.41)^2(1-\xi^2) = \pi^2\xi^2$$

$$(2.41)^2 = \xi^2(\pi^2 + (2.41)^2)$$

$$\sqrt{\frac{(2.41)^2}{\pi^2+(2.41)^2}} = \xi$$

$$\text{Damping ratio, } \xi = 0.61$$

$$\omega_n = \frac{1}{0.61} = 1.64$$

Standard second order transfer function:

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$T(s) = \frac{2.7}{s^2+2s+2.7}$$

14. **(c)**

At resonant frequency, resonant peak is achieved.

$$M_r = 1.6,$$
$$\omega_r = 4 \text{ rad/sec}$$

M_r and ω_r are given by

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

Now,

$$1.6 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$4\xi^4 - 4\xi^2 + \left(\frac{10}{16}\right)^2 = 0$$

$$\xi = 0.94, 0.33 \qquad (\omega_r \text{ to exist } 1 - 2\xi^2 > 0; \xi \text{ should be less than } \frac{1}{\sqrt{2}})$$

$$\xi = 0.33 \text{ is acceptable}$$

and

$$\omega_n = \frac{4}{\sqrt{1-2(0.33)^2}}$$

$$\omega_n = 4.53 \text{ rad/sec}$$

Settling time for 2% tolerance band is,

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.33\times 4.53} = 2.7 \text{ sec}$$

15. **(c)**

$$\text{The system matrix } A = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$$

The state equation with stable variable feedback is

$$\dot{x} = (A - BK)x + Br$$

$$(A - BK) = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} K_1 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -20-K_1 & -9-K_2 \end{bmatrix}$$

The desired characteristic equation is

$$s^2 + 2s + 5 = 0 \qquad \text{...(i)}$$

The characteristic equation using state variable feedback is

$$|sI - (A - BK)| = 0$$

$$\begin{vmatrix} s & -1 \\ 20+K_1 & s+9+K_2 \end{vmatrix} = 0$$

$$s^2 + 9s + K_2 s + 20 + K_1 = 0$$

$$s^2 + (9 + K_2)s + (20 + K_1) = 0 \qquad \text{...(ii)}$$

Comparing (i) and (ii),

$$K_2 = -7$$
$$K_1 = -15$$
$$K = [-15 \quad -7]$$

16. (c)

$$G(s) = \frac{K(s+4)}{(s+2)^2}$$

The open loop poles are,

$$s = -2, -2$$

The open loop zero is $s = -4$

$$P - Z = 2 - 1 = 1$$

So angle of asymptote is 180°

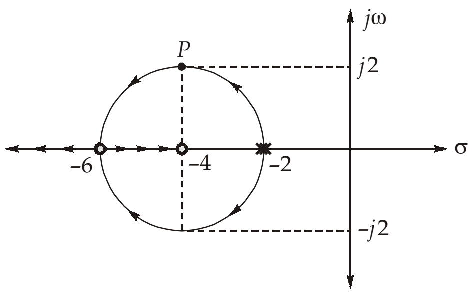
The characterisitic equation is,

$$(s+2)^2 + K(s+4) = 0$$

Put $\frac{dK}{dS} = 0$

$$\frac{dK}{dS} = \frac{(s+2)(s+6)}{(s+4)^2} = 0$$

$s = -2$ and $s = -6$ are the breakaway points.



At point P, $S = -4 + j2$,

at this point $\omega_d = 2$ rad/sec

$$\left|\frac{K(s+4)}{(s+2)^2}\right|_{s=-4+j2} = 1$$

$$\left|\frac{K(-4+j2+4)}{(-4+j2+2)^2}\right| = 1$$

$$\frac{2K}{\left(\sqrt{2^2+2^2}\right)^2} = 1$$

$$K = 4$$

17. (d)

Given, $G(s) = \frac{1}{s(1+2s)(1+4s)}$

$$G(j\omega) = \frac{1}{j\omega(1+2j\omega)(1+j4\omega)}$$

$$G(j\omega) = \frac{(1-j2\omega)(1-j4\omega)}{j\omega(1+4\omega^2)(1+16\omega^2)} = \frac{1-j6\omega-8\omega^2}{j\omega(1+4\omega^2)(1+16\omega^2)}$$

$$= \frac{-6}{(1+4\omega^2)(1+16\omega^2)} - \frac{j(1-8\omega^2)}{\omega(1+4\omega^2)(1+16\omega^2)}$$

At $\omega = 0$; $G(j\omega) = -\frac{6}{1} - \frac{j(1)}{0} = -6 - j\infty$

At $\omega = \infty$; $G(j\omega) = -\frac{6}{\infty} - \frac{j}{\infty} = -0 - j0$

plot cuts the negative real axis, when

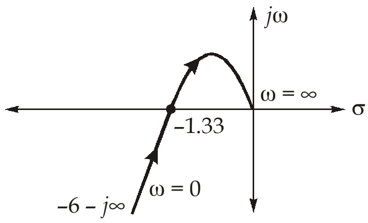
$$\text{Img}\,[G(j\omega)] = 0$$

$$1 - 8\omega^2 = 0$$

$$\omega = \frac{1}{2\sqrt{2}} \text{ rad/sec}$$

$$G(j\omega) = \frac{-6}{\left(1+4\times\frac{1}{8}\right)\left(1+16\times\frac{1}{8}\right)} = -1.33$$

From above points, polar plot can be drawn as



18. **(b)**

From the above Bode plot,

For section *de*, slope is –20 dB/dec

$$\therefore \quad -20 = \frac{y-0}{\log 8 - \log 16}$$

$$y = 6.02 \text{ dB}$$

Now, for section *bc*, slope is –20 dB/dec

$$\therefore \quad -20 = \frac{16-6.02}{\log \omega_1 - \log 4}$$

$$\omega_1 = 1.268 \text{ rad/sec}$$

To find value of gain *K*

$$y = mx + c$$

$$16 = -40 \log 1.268 + 20 \log K$$

$$K = 10.14$$

From all the result, transfer function is,

$$T(s) = \frac{10.14\left(\frac{s}{1.268}+1\right)\left(\frac{s}{4}+1\right)}{s^2\left(\frac{s}{8}+1\right)}$$

$$T(s) = \frac{16(s+1.268)(s+4)}{s^2(s+8)}$$

19. (a)

Closed loop transfer function with unity feedback

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= \frac{\frac{K}{s(s+4)}}{1+\frac{K}{s(s+4)}} = \frac{K}{s^2+4s+K}$$

Comparing $T(s)$ with standard form

$$T(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

We get, $\omega_n^2 = K$

$\Rightarrow$ $\omega_n = \sqrt{K}$

$2\xi\omega_n = 4$

Given damping ratio, $\xi = 0.5$

$\therefore$ $\omega_n = \frac{4}{2\xi} = \frac{4}{2\times(0.5)} = 4$

$K = \omega_n^2 = 16$

Peak overshoot is given by,

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-\pi(0.5)/\sqrt{1-0.25}}$$

$$= e^{-1.814} = 0.163$$

20. (c)

$$y(t) = AM\sin(2t+\phi)$$

where, $A = 2$, and $M = \left|\frac{1}{j\omega+2}\right|$

At $\omega = 2$, $M = \frac{1}{2\sqrt{2}}$

and $\phi = -\tan^{-1}\left(\frac{\omega}{2}\right) = -\tan^{-1}\left(\frac{1}{1}\right) = -\frac{\pi}{4}$

$$y(t) = \frac{1}{\sqrt{2}}\sin\left(2t-\frac{\pi}{4}\right)$$

21. (d)

Using Routh table :

s^6	1	6	10	5
s^5	1	5	5	
s^4	1	5	5	
s^3	0	0		

The Routh table construction procedure breaks down here. Since the s^3 row has all zeros. The auxiliary polynomial coefficients are given by the s^4 row. Therefore the auxiliary polynomial is

$$A(s) = s^4 + 5s^2 + 5$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s$$

Replacing the s^3 row in the Routh table with the coefficients of $\frac{dA(s)}{ds}$, we have,

s^6	1	6	10	5
s^5	1	5	5	
s^4	1	5	5	
s^3	4	10		
s^2	$\frac{20-10}{4} = 2.5$	5		
s^1	$\frac{25-20}{2.5} = 2$			
s^0	5			

Examining the first column of this table we see that there are no sign changes. Hence, there is no root lying in the RHS of s-plane.

22. (d)

Steady state gain = 1

Given, $\left.\begin{aligned} |G(j\omega)| &= \frac{1}{2} \\ \angle G(j\omega) &= -90° \end{aligned}\right\}$ at $\omega = 1$ rad/sec

In option (d), $G(j\omega)|_{\omega=1}$ $= \frac{1}{\left[\sqrt{1+1}\right]^2}\angle -45° - 45° = \frac{1}{2}\angle -90°$

23. (d)

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$\omega_n = \frac{10}{\sqrt{1-0.6^2}} = 12.5 \text{ rad/sec}$$

Desired characteristic equation of second order is

$$= s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 15s + 156.25 = 0$$

C.E. of given system is

$$1 + (K_p + sK_D)\left[\frac{1}{s(s+2)}\right] = 0$$

$$s^2 + 2s + K_p + sK_D = 0$$

$$s^2 + (2 + K_D)s + K_p = 0$$

On comparing $K_p = 156.25$

$$K_D = 15 - 2 = 13$$

24. (d)

$$G(j\omega) = \frac{1+j4\omega}{1+j2\omega}$$

At $\omega = 0$, $G(j\omega) = 1\angle 0°$
At $\omega = 2$, $G(j\omega) = 1.9553\angle 6.91°$
At $\omega = 10$, $G(j\omega) = 1.99\angle 1.43°$
At $\omega = \infty$, $G(j\omega) = 2\angle 0°$

First quadrant with clockwise direction

So, correct option is (d).

25. (a)

Number of open loop poles in R.H.S. of s-plane(P) = 1.

$$\text{C.E.,}(s + 0.5)(s - 2) + 1.25(s + 1) = 0$$

$$s^2 - 1.5s - 1 + 1.25s + 1.25 = 0$$

$$s^2 - 0.25s + 0.25 = 0$$

$$s = 0.125 \pm j0.4841 \text{[Location of closed loop poles]}$$

So, both closed loop poles lies in R.H.S. of s-plane, $Z = 2$.

Number of encirclement (N)

$$N = Z - P [\because \text{ Nyquist contour in anticlockwise direction}]$$

$$N = 2 - 1 = 1$$

N is positive for clockwise encirclement.

N is negative for anti-clockwise encirclement.

So, Nyquist plot will encircles $-1 + j0$, once in clockwise direction.

26. (d)

The location of the poles are given by, $-\xi\omega_n \pm j\omega_d$ ...(i)

where, ξ = damping ratio

ω_n = natural frequency of oscillation

ω_d = damped frequency of oscillation

Using maximum peak overshoot, the value of ξ can be obtained as

$$e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.15$$

$$\frac{\xi}{\sqrt{1-\xi^2}} = 0.604$$

Squaring both the sides,

$$\xi^2 = 0.364(1 - \xi^2)$$

or $$\xi^2 = \frac{0.364}{1.364} = 0.267$$

or $$\xi = 0.517$$ ...(ii)

now, peak time, $$\tau_p = \frac{\pi}{\omega_d} = 3$$

or $$\omega_d = \frac{\pi}{3} = 1.047 \text{ rad/sec}$$

$\because$ $$\omega_d = \omega_n\sqrt{1-\xi^2} \quad \text{...(iii)}$$

$\therefore$ From equation (ii) and (iii), we have

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{1.047}{\sqrt{1-0.517^2}}$$

$$\omega_n = 1.223 \text{ rad/sec} \quad \text{...(iv)}$$

$\therefore$ Location of poles are,

$$P = -\xi\omega_n \pm j\omega_d$$
$$= -(0.517 \times 1.223) \pm j1.047$$
$$= -0.632 \pm j1.047$$

27. (d)

Steady state error,

$$e_{ss} = \lim_{s\to 0} sE(s)$$
$$= \lim_{s\to 0} s \times \frac{R(s)}{1+G(s)H(s)}$$
$$= \lim_{s\to 0} \frac{s\times\left(2+\frac{5}{s}\right)\times\frac{1}{s}}{1+\frac{K}{s(s+3)}}$$
$$= \lim_{s\to 0} \frac{\frac{(2s+5)}{s}\times s(s+3)}{(s^2+3s+K)}$$
$$= \lim_{s\to 0} \frac{(2s+5)(s+3)}{s^2+3s+K}$$
$$2.75 = \frac{15}{K}$$

or $$K = \frac{15}{2.75} = 5.45$$

28. (d)

For any point to lie on the root locus the angle condition must be satisfied.

$$\angle G(s)H(s)\big|_{s=(-1+j2)} = \pm 180°$$

$\therefore$ $$G(s)H(s)\big|_{s=(-1+j2)} = \frac{K(-1+j2+1)}{(-1+j2+9)(-1+j2+3)} = \frac{K(j2)}{(8+j2)(2+j2)}$$

$\therefore$ $$\angle G(s)H(s)\big|_{s=-1+j2} = 90° - \tan^{-1}\left(\frac{2}{8}\right) - \tan^{-1}(1)$$
$$= 90° - 14.036° - 45°$$
$$= 30.96°$$

$\because$ $$\angle G(s)H(s)\big|_{s=s_0} \neq \pm 180°$$

Angle condition does not satisfy.

29. (b)

The steady state error is defined by

$$e_{ss} = \lim_{s \to 0} \frac{s \times \frac{1}{s^2}}{1 + \frac{(s+\alpha)}{s} \times \frac{(s+2)}{s^2 - 1}}$$

$$= \lim_{s \to 0} \frac{(s^2 - 1)}{s(s^2 - 1) + (s+\alpha)(s+2)}$$

$$e_{ss} = -\frac{1}{2\alpha}$$

$\therefore$

$$S_{\alpha}^{e_{ss}} = \frac{\frac{\partial e_{ss}}{e_{ss}}}{\frac{\partial \alpha}{\alpha}} = \frac{\partial e_{ss}}{\partial \alpha} \times \frac{\alpha}{e_{ss}} = \frac{\partial}{\partial \alpha}\left(\frac{-1}{2\alpha}\right) \times \frac{\alpha}{-\frac{1}{2\alpha}}$$

$$= -\frac{\alpha^2}{\alpha^2} = -1$$

30. (b)

$$\phi(t) = L^{-1}[(sI - A)^{-1}]$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & -1 \\ 8 & s+6 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 6s + 8} \begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+6}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{-8}{(s+2)(s+4)} & \frac{s}{(s+2)(s+4)} \end{bmatrix}$$

$$\phi(t) = L^{-1}[(sI - A)^{-1}] = \begin{bmatrix} (2e^{-2t} - e^{-4t}) & \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right) \\ (-4e^{-2t} + 4e^{-4t}) & (-e^{-2t} + 2e^{-4t}) \end{bmatrix}$$

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