CLASS TEST

S.No.: 01JP_EC+EE_EM_260324

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EC-EE

ENGINEERING MATHEMATICS

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 Identify the correct statement from the following
(a) Commutative property not holds for addition of matrices
(b) Associative property not holds for addition of matrices
(c) Commutative property not holds for multiplication of matrices
(d) None of the above

Q.2 The density function of repairing a machine is given by $f(x) = \frac{1}{2}e^{\frac{-x}{2}}$, where '$x$' is repair time in hours. The probability that the repair time is more than 2 hours is
(a) 0.368 (b) 0.482
(c) 0.518 (d) 0.632

Q.3 The values of a and b for which the following system has no solutions are respectively

$$\begin{aligned} ax + y + 2z &= 0 \\ x + 2y + z &= b \\ 2x + y + az &= 0 \end{aligned}$$

(a) $a = -1, b = 0$ (b) $a = 2, b = 0$
(c) $a = -1, b \neq 0$ (d) $a = 2, b \neq 0$

Q.4 Let X be an exponential random variable with rate parameter λ. Then find variance of X.
(a) $\frac{1}{\lambda}$ (b) $\frac{1}{\lambda^2}$
(c) $\frac{2}{\lambda}$ (d) $\frac{2}{\lambda^2}$

Q.5 The rank of matrix, $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is ________.
(a) 1 (b) 2
(c) 0 (d) 3

Q.6 The following system of homogeneous equations:

$$\begin{aligned} 2x + y + 2z &= 0 \\ x + y + 3z &= 0 \\ 4x + 3y + bz &= 0 \end{aligned}$$

has non-trivial solution, then the value of 'b' is ________.
(a) 2 (b) 8
(c) 16 (d) 32

Q.7 Mean and variance of the random variable x having binomial distribution are 4 and 2 respectively then $P(x = 1)$ is
(a) 1/4 (b) 1/16
(c) 1/8 (d) 1/32

Q.8 At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1; & 1 < x < \infty \\ x - 1; & -\infty < x \leq 1 \end{cases}$ is
(a) Continuous and differentiable
(b) Continuous and not differentiable
(c) Discontinuous and differentiable
(d) Discontinuous and not differentiable

Q.9 Consider the following function.

$$f(x) = \sqrt{36 - 4x^2}$$

Find the points at which f has absolute minimum and absolute maximum respectively.
(a) $x = 0, x = 6$ (b) $x = 6, x = 0$
(c) $x = 0, x = 3$ (d) $x = 3, x = 0$

Q.10 Which of the following represents the LU decomposition of the given matrix. (Using Crout's method)

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & -9 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 \\ 6 & -9 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 0 \\ 6 & 9 \end{bmatrix}$

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 If the proportion of handicapped people in a large population is 0.006, then what is the probability that there will be atmost one handicapped person in a randomly chosen group of 500 people? Use Poisson approximation to compute the probability.

(a) $3e^{-4}$ (b) $2e^{-3}$
(c) $4e^{-3}$ (d) $3e^{-2}$

Q.12 E[X] = 1 and Var[X] = 2, which one of the following is not correct?

(a) E[6 X] = 6 (b) Var[6 X] = 72
(c) E[1 – X] = 0 (d) Var[1 – X] = 3

Q.13 Which of the following is/are true?

S_1 **:** Maximum number of distinct eigen values of matrix A less than size of matrix A.

S_2 **:** Product of eigen values of matrix A is equal to Sum of diagonal elements of A.

S_3 **:** The characteristic roots (eigen values) of Hermitian matrix are real.

(a) Only S_2 and S_3 (b) Only S_1 and S_2
(c) Only S_3 (d) Only S_1

Q.14 Find an eigen vector corresponding to largest eigen value of matrix A =

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$

Q.15 Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at ________.

(a) 5 (b) 6
(c) 7 (d) 8

Q.16 The value of $\int_{-2}^{2} \left|1 - x^4\right| dx$ is ________.

(a) 10 (b) 12
(c) 14 (d) 16

Q.17 Consider the given matrix:

$$\begin{vmatrix} 6 & 3 & 7 \\ 32 & 13 & 37 \\ 10 & 4 & 11 \end{vmatrix}$$

The value of the determinant of the above matrix is ________.

(a) 8 (b) 10
(c) 12 (d) 14

Q.18 The absolute maximum value of the function $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ is ________.

(a) 2 (b) 4
(c) 6 (d) 8

Q.19 If $\lim\limits_{x \to 1} \dfrac{x^x - x}{x - 1 - \log x} = A$, then A is ________.

(a) 0
(b) 1
(c) 2
(d) Limit does not exists

Q.20 The value of integral $\displaystyle\int_1^3 \frac{|x-2|}{x} dx$ is ________.

(a) $2\ln\frac{4}{3}$ (b) $2\ln\frac{3}{4}$

(c) $\ln\frac{4}{3}$ (d) $4\ln\frac{2}{3}$

Q.21 Evaluate $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

(a) $-\cos\sqrt{x} + c$ (b) $-2\cos(x)^{3/2} + c$

(c) $-2\sin\sqrt{x} + c$ (d) $-2\cos\sqrt{x} + c$

Q.22 Which of the following represents the LU decomposition of the given matrix. (Using Doolittle method)

$$A = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} U = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$

(b) $L = \begin{bmatrix} 25 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 0.7 \end{bmatrix} U = \begin{bmatrix} 1 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} U = \begin{bmatrix} 25 & -5 & 1 \\ 0 & 4.8 & 1.56 \\ 0 & 0 & -0.7 \end{bmatrix}$

(d) $L = \begin{bmatrix} 25 & 0 & 0 \\ 2.56 & -4.8 & 0 \\ 5.76 & 3.5 & 0.7 \end{bmatrix} U = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & -1.56 \\ 0 & 0 & 1 \end{bmatrix}$

Q.23 What is the value of $\int_0^{\pi/2} \log(\tan x)\,dx$?

(a) $-2\pi \log 2$ (b) $-\pi \log 2$
(c) 1 (d) 0

Q.24 Suppose we have 2 bags. Bag 1 contains 3 red and 7 green balls. Bag 2 contains 4 red and 8 green balls. A person tosses a coin and if it is heads goes to bag 1 and draws a ball. If it is tails, he goes to bag 2 and draws a ball. Given that the ball draw is red, then what is probability that it came from bag 1?

(a) 0.500 (b) 0.250
(c) 0.317 (d) 0.288

25. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 will be at ________.

(a) 2 (b) 4
(c) 5 (d) 6

Q.26 Consider the system of linear equations given below:

$$-2x + y + z = l$$
$$x - 2y + z = m$$
$$x + y - 2z = n$$

If $l + m + n = 0$, then the system of equations has

(a) No solution
(b) Trivial solutions
(c) Unique solution
(d) Infinitely many solutions

Q.27 A matrix A = $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ has three linearly independent eigen vectors X_1, X_2, X_3 corresponding to the three eigen values 1, 2 and 3 respectively. Which of the following is correct?

(a) X_1 and X_3 are orthogonal
(b) X_2 and X_3 are orthogonal
(c) X_1 and X_2 are orthogonal
(d) None of these

Q.28 A function is defined by $f(x) = 2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 3$. Which one of the following statements is true about this function?

(a) function is decreasing for (–2, –1).
(b) function has a minima for $x = -1$.
(c) function has a maxima for $x = 2$.
(d) function is decreasing for (–1, 2).

Q.29 The value of $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sec^2 x - 2\tan x}{1 + \cos 4x}$ is ________.

(a) 0.5 (b) 0.25
(c) 1 (d) 2

Q.30 How many different value of x exist for the following equation:

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

(a) 2 (b) 3
(c) 1 (d) 4

■■■■

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ENGINEERING MATHEMATICS

EC-EE

Date of Test : 26/03/2024

ANSWER KEY

1.	(c)	7.	(d)	13.	(c)	19.	(c)	25.	(c)
2.	(a)	8.	(b)	14.	(a)	20.	(a)	26.	(d)
3.	(c)	9.	(d)	15.	(a)	21.	(d)	27.	(d)
4.	(b)	10.	(c)	16.	(b)	22.	(a)	28.	(d)
5.	(b)	11.	(c)	17.	(b)	23.	(d)	29.	(a)
6.	(b)	12.	(d)	18.	(b)	24.	(c]	30.	(a)

DETAILED EXPLANATIONS

1. **(c)**

Commutative for multiplication of matrices does not hold.

$$AB \neq BA$$

2. **(a)**

$$\text{Probability} = \int_2^{\infty} f(x)dx$$

$$= \int_2^{\infty}\left[\frac{1}{2}e^{\frac{-x}{2}}\right]dx = \left[-e^{\frac{-x}{2}}\right]_2^{\infty} = e^{-1} = 0.368$$

3. **(c)**

The matrix formed by the coefficients is $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$\therefore$ $D = 0$ for $a = 2$ or $a = -1$

(A) If $D \neq 0$, then the system will have unique solution because the rank of matrix will be 3.

(B) If $a = 2$, the matrix formed by the coefficients is $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.

Considering 'z' as side unknown.

The characteristic determinant will be $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.

The system will have infinite solutions when $a = 2$.

(C) If $a = -1$, the matrix formed by the coefficients is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.

Considering 'z' as side unknown.

The characteristic matrix is $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is $3b$.

The system will have no solution if $b \neq 0$

$\therefore$ For $a = -1$ and $b \neq 0$, the system will have no solution.

4. (b)

Probability density function:

$$f(x) = \lambda \cdot e^{-\lambda x},\ x > 0$$

$$E(X) = \int_0^\infty x \cdot f(x) \cdot dx$$

$$= \int_0^\infty x\lambda \cdot e^{-\lambda x} \cdot dx = \frac{1}{\lambda}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= \int_0^\infty x^2 \cdot \lambda \cdot e^{-\lambda x} \cdot dx = \frac{2}{\lambda^2}$$

$$\Rightarrow \quad \text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

5. (b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2(R_1)$ and $R_3 \leftarrow R_3 - R_1$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_2$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank of matrix is 2.

6. (b)

For a non-trivial solution of homogeneous system of equations,

$$|A| = 0$$

where
$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix}$$

$$\Rightarrow \quad 2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad b = 8$$

7. (d)

Given,
$$np = 4$$
$$npq = 2$$
$$q = \frac{1}{2}, p = \frac{1}{2}, n = 8$$
$$P(x = 1) = {}^nC_1\, p^1\, q^{n-1}$$
$$= {}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 = \frac{8}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$
$$= 0.03125$$

8. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x-1) = 0$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And $Lf'(1) = 2$, $Rf'(1) = 1$

$\Rightarrow f$ is not differentiable at $x = 1$

9. (d)

$$f(x) = \sqrt{36 - 4x^2}$$

At $x = 0$, $f(x) = 6$

If $x \neq 0 \Rightarrow f(x) < 6$

$\therefore$ f has absolute maximum at $x = 0$

At $x = 3$, $f(x) = 0$

$\therefore$ f has absolute minimum at $x = 3$.

10. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$l_{11} = 2$ $\qquad$ $l_{11}\, u_{12} = 4$

$$u_{12} = \frac{4}{2} = 2$$

$l_{21} = 6$ $\qquad$ $l_{21}\, u_{12} + l_{22} = 3$

$$6 \times 2 + l_{22} = 3$$

$$l_{22} = 3 - 12$$
$$l_{22} = -9$$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

11. (c)

$$P(x = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}, \text{ where } \lambda = np$$
$$\lambda = 500 \times 0.006 = 3$$
$$P(x \le 1) = P(x = 0) + P(x = 1)$$
$$= \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$
$$= e^{-3} + e^{-3} \cdot 3 = 4e^{-3}$$

12. (d)

$$E[6X] = 6.E[X] = 6$$
$$Var[6X] = 6^2\,Var[X] = 36 \times 2 = 72$$
$$E[1 - X] = 1 + (-1)\,E[X] = 1 - 1 = 0$$
$$Var[1 - X] = (-1)^2\,Var[X] = Var[X] = 2$$
$$\therefore \quad Var[1 - X] \neq 3$$

13. (c)

Maximum number of distinct eigen values = Size of matrix A.

$\therefore$ S_1 is False.

Sum of eigen values = Sum of diagonal elements.

$\therefore$ S_2 is False.

14. (a)

$$|\lambda - AI| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2$$
$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$

$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

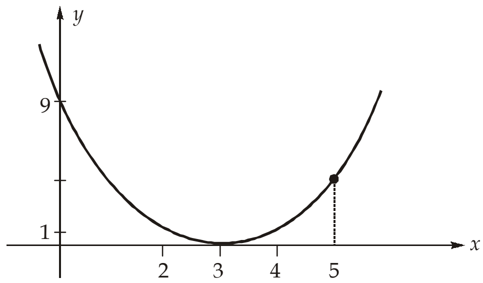
$$\therefore \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

15. (a)

$$y = x^2 - 6x + 9 = (x - 3)^2$$

$$y(2) = 1$$

$$y(5) = 4$$



$\therefore$ Maximum value of y over the interval 2 to 5 will be at $x = 5$.

16. (b)

$$I = \int_{-2}^{2} \left|1 - x^4\right| dx$$

The given function is an even function i.e., $f(x) = f(-x)$

$$\Rightarrow \quad I = 2\int_{0}^{2} \left|1 - x^4\right| dx$$

$$= 2\left\{\int_0^1 (1 - x^4)\, dx + \int_1^2 (x^4 - 1) dx\right\}$$

$$= 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^2\right\} = 12$$

17. (b)

$6(13 \times 11 - 4 \times 37) - 3(32 \times 11 - 10 \times 37) + 7\,(32 \times 4 - 10 \times 13)$

$$= -30 + 54 - 14 = 10$$

18. (b)

$$\frac{\partial f}{\partial x} = 2 - 2x \qquad \frac{\partial f}{\partial y} = 2 - 2y$$

$$r = \frac{\partial^2 f}{\partial x^2} = -2 \qquad t = \frac{\partial^2 f}{\partial y^2} = -2, \qquad s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

Finding stationary points,

$$\frac{\partial f}{\partial x} = 2 - 2x = 0$$

$\Rightarrow x = 1$

$$\frac{\partial f}{\partial y} = 2 - 2y = 0$$

$\Rightarrow y = 1$

At the stationary point (1, 1)

$$rt - s^2 = (-2)(-2) - 0 = 4 > 0$$

So, $f(x, y)$ is maxima at (1, 1)

Maximum value of $f(x, y) = 2 + 2 + 2 - 1 - 1 = 4$

19. (c)

$$\lim_{x\to 1} \frac{x^x - x}{x - 1 - \log x} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 1} \frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}}$$

Let, $y = x^x$

$\log y = x \log x$

$\therefore \quad \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log x$

or $\quad \frac{d}{dx}(x^x) = x^x(1 + \log x)$

$$= \lim_{x\to 1} \frac{x^x(1 + \log x) - 1}{1 - \frac{1}{x}} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 1} \frac{\frac{d}{dx}(x^x) \cdot (1 + \log x) + x^x\left(\frac{1}{x}\right) - 0}{\frac{1}{x^2}}$$

$$= \lim_{x\to 1} \frac{x^x(1 + \log x)^2 + x^x\left(\frac{1}{x}\right)}{x^{-2}} = \frac{1(1+0)^2 + 1 \cdot 1}{1} = 2$$

20. (a)

$$|x-2| = \begin{cases} -(x-2)\,; & x<2 \\ (x-2)\,; & x>2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x}dx = \int_1^2 \frac{-(x-2)}{x}dx + \int_2^3 \frac{x-2}{x}dx$$

$$= \int_1^2\left(-1+\frac{2}{x}\right)dx + \int_2^3\left(1-\frac{2}{x}\right)dx$$

$$= [-x]_1^2 + [2\ln x]_1^2 + [x]_2^3 - 2[\ln x]_2^3$$

$$= -(2-1) + 2\ln 2 - 2\ln\frac{3}{2} + (3-2)$$

$$= 2\ln 2 - 2\ln\frac{3}{2}$$

$$= 2\ln\frac{2}{\frac{3}{2}} = 2\ln\frac{4}{3}$$

21. (d)

Let $\quad u = \sqrt{x}$

Then $\quad du = \dfrac{1}{2\sqrt{x}}dx$

$\therefore \quad dx = du\cdot 2\sqrt{x}$

$$\int\frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int\frac{\sin u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\int \sin u\,du$$

$$= -2\cos\sqrt{x} + c$$

22. (a)

Using Doolittle method:

$$A = LU$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = 25,\ u_{12} = 5,\ u_{13} = 1$

$$u_{11}\,l_{21} = 64$$

$$l_{21} = \frac{64}{25} = 2.56$$

$$l_{21}u_{12} + u_{22} = 8$$

$$2.56\,u_{12} + u_{22} = 8$$

$$u_{22} = -4.8$$

$$u_{13}\,l_{21} + u_{23} = 1$$

$$u_{23} = -1.56$$
$$l_{31}\, u_{11} = 144$$
$$l_{31} = \frac{144}{25} = 5.76$$
$$l_{31}u_{12} + l_{32}u_{22} = 12$$
$$(5.76 * 5) + (u_{22}\, l_{32}) = 12$$
$$l_{32} = 3.5$$
$$l_{31}\, u_{13} + l_{32}\, u_{23} + u_{33} = 1$$
$$u_{33} = 0.7$$

So, LU decomposition is

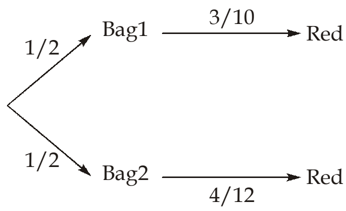
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

23. (d)

$$I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right) dx$$
$$= \int_0^{\pi/2} [\log(\sin x)dx - \log(\cos x)dx]$$
$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx - \int_0^{\pi/2} \log(\cos x)dx$$
$$I = 0$$

24. (c]

The tree diagram for above problem, is shown below:

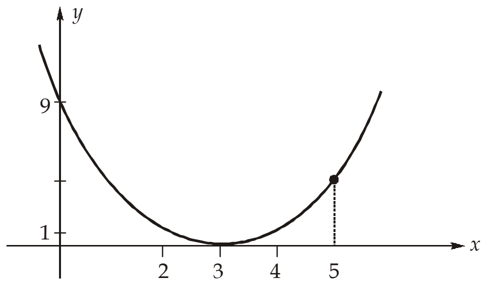


$$P\,(\text{bag1}\,|\,\text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})}$$
$$= \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{3}{20}}{\frac{3}{20} + \frac{1}{6}} = 0.317$$

25. (c)

$$y = x^2 - 6x + 9 = (x-3)^2$$
$$y(2) = 1$$

$$y(5) = 4$$



$\therefore$ maximum value of y over the interval 2 to 5 will be at $x = 5$.

26. **(d)**

$$AX = B$$

Augmented matrix, $[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$

$R_3 \to R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

$\Rightarrow$ Infinitely many solutions are possible.

27. **(d)**

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$X_1 = c_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_3 = 0$$
$$2x_1 + 2x_2 + x_3 = 0$$

$$X_2 = c_2\begin{bmatrix}-2\\1\\2\end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix}-2 & 0 & -1\\1 & -1 & 1\\2 & 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = \frac{-1}{2}x_3$$

$$X_3 = c_3\begin{bmatrix}-1\\1\\2\end{bmatrix}$$

Since, $X_1{}^T X_2 \neq 0$

$$X_2{}^T X_3 \neq 0$$

$$X_3{}^T X_1 \neq 0$$

None of the above is correct.

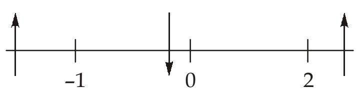
28. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x+1)(x-2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \quad \Rightarrow \quad \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \quad \Rightarrow \quad \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.
Critical point $(-1, 2)$ draw plot on line graph:
Since $0 \in (-1, 2)$ and $f'(0) = 6 \times 0^2 - 6 \times 0 - 12 = -12 < 0$



The function is decreasing between -1 and 2.

29. (a)

Given,

$$\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2\tan x}{1 + \cos 4x} \quad : \quad \frac{0}{0} \text{ Form}$$

Applying L′ hospital rule

$$= \lim_{x \to \frac{\pi}{4}} \frac{\frac{d}{dx}(\sec^2 x - 2\tan x)}{\frac{d}{dx}(1+\cos 4x)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(2\sec x \cdot \sec x \tan x - 2\sec^2 x)}{-4\sin 4x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x(\tan x - 1)}{-2\sin 4x} \quad : \frac{0}{0} \text{ Form}$$

Applying L′ hospital's rule

$$= \lim_{x \to \frac{\pi}{4}} \frac{2\sec x \cdot \sec x \cdot \tan x(\tan x - 1) + \sec^2 x \sec^2 x}{-8\cos 4x}$$

$$= \frac{2.2.1(1-1)+2.2}{-8(-1)} = \frac{1}{2}$$

30. (a)

Operating $R_3 - (R_1 + R_2)$ we get

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0 \qquad \text{(Operating } R_2 - R_1 \text{ and } R_1 + R_3\text{)}$$

or $$\begin{vmatrix} x+2 & 2x+4 & 6x+12 \\ x+1 & x+1 & x+1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

or $$(x+1)(x+2)\begin{vmatrix} 1 & 2 & 6 \\ 1 & 1 & 1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

To bring one more zero in C_1, operate $R_1 - R_2$.

$$\therefore \quad (x+1)(x+2)\begin{vmatrix} 0 & 1 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

Now expand by C_1.

$\therefore \quad -(x + 1)(x + 2)(3x + 8 - 5) = 0$ or $-3(x + 1)(x + 2)(x + 1) = 0$

Thus, $x = -1, -1, -2.$

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