CLASS TEST

S.No.: 01JP_CE_EM_260324

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CIVIL ENGINEERING

ENGINEERING MATHEMATICS

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 At the point $x = 1$, the function

$$f(x) = \begin{cases} x^3 - 1; & 1 < x < \infty \\ x - 1; & -\infty < x \le 1 \end{cases} \text{ is}$$

(a) Continuous and differentiable
(b) Continuous and not differentiable
(c) Discontinuous and differentiable
(d) Discontinuous and not differentiable

Q.2 For a function $f(x)$, a table is given below

x	0	1	2	3	4	5
$f(x)$	2	1	0.4	0.2	0.1176	0.077

The value of $\int_0^5 f(x) \cdot dx$ by Trapezoidal's rule is ________.

(a) 2.25 (b) 2.50
(c) 2.75 (d) 3.00

Q.3 Which of the following represents the LU decomposition of the given matrix. (Using Crout's method)

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix}$$

(a) $L = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & -9 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 \\ 6 & -9 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$

(c) $L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & 0 \\ 6 & 9 \end{bmatrix}$

Q.4 Let x be a random variable with probability density function defined as,

$$f(x) = \begin{cases} 0.1 & \text{for } |x| \le 1 \\ 0.4 & \text{for } 1 < |x| \le 2 \\ 0 & \text{else} \end{cases}$$

then $P(-1 \le x \le 1)$ is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Q.5 Find the value of $\lim_{x \to 0} \frac{\log x}{\cot x}$

(a) 0 (b) 1
(c) ∞ (d) 0.5

Q.6 $\int_{-\pi}^{\pi/2} \cos(x) \cos(\sin(x)) dx$

(a) 1 (b) 0
(c) cos 1 (d) sin 1

Q.7 The general solution of the differential equation $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$ is

(a) $\sin y = C(1 - e^x)$
(b) $\cos y = C(1 - e^x)$
(c) $\cot y = C(1 - e^x)$
(d) $\tan y = C(1 - e^x)$

Q.8 The particular integral of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 8 \text{ is}$$

(a) $\frac{x^3}{3} + 8x$ (b) $\frac{x^3}{3} + 6x$

(c) $\frac{x^3}{3} + 4x^2$ (d) $\frac{x^3}{3} + 4x^2 + x$

Q.9 If $y = e^{\sin^{-1} x}$ and $z = e^{-\cos^{-1} x}$, then the value of $\left.\frac{d^2y}{dz^2}\right|_{x=1/\sqrt{2}}$ will be

(a) 0 (b) $\frac{1}{\ln 2}$

(c) $\frac{1}{(\ln 2)^2}$ (d) $\frac{1}{2}$

Q.10 The order and degree of differential equation of family of curves $y = e^x (A\cos x + B\sin x)$, are respectively

(a) 1 and 1 (b) 2 and 1
(c) 2 and 2 (d) 1 and 2

Q.No. 11 to Q.No. 30 carry 2 marks each

Q.11 If $\lim_{x\to 1}\dfrac{x^x - x}{x-1-\log x} = A$, then A is ________.

(a) 0
(b) 1
(c) 2
(d) Limit does not exists

Q.12 Evaluate $\int \dfrac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

(a) $-\cos\sqrt{x}+c$ (b) $-2\cos(x)^{3/2}+c$
(c) $-2\sin\sqrt{x}+c$ (d) $-2\cos\sqrt{x}+c$

Q.13 What is the value of $\int_0^{\pi/2}\log(\tan x)\,dx$?

(a) $-2\pi\log 2$ (b) $-\pi\log 2$
(c) 1 (d) 0

Q.14 Find an eigen vector corresponding to largest eigen value of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}$

Q.15 The area of the segment made by the parabola $x^2 = 4y$ by the line $x - 2y + 4 = 0$ is

(a) 9 (b) 12
(c) 16 (d) 24

Q.16 Consider the system of linear equations given below:

$-2x + y + z = l$
$x - 2y + z = m$
$x + y - 2z = n$

If $l + m + n = 0$, then the system of equations has

(a) No solution
(b) Trivial solutions
(c) Unique solution
(d) Infinitely many solutions

Q.17 A matrix A = $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ has three linearly independent eigen vectors X_1, X_2, X_3 corresponding to the three eigen values 1, 2 and 3 respectively. Which of the following is correct?

(a) X_1 and X_3 are orthogonal
(b) X_2 and X_3 are orthogonal
(c) X_1 and X_2 are orthogonal
(d) None of these

Q.18 A function is defined by $f(x) = 2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 3$. Which one of the following statements is true about this function?

(a) function is decreasing for (–2, –1).
(b) function has a minima for $x = -1$.
(c) function has a maxima for $x = 2$.
(d) function is decreasing for (–1, 2).

Q.19 A function is defined by $f(x) = 2x^3 - 3x^2 - 12x + 5$ for $-2 \le x \le 3$. Which one of the following statements is true about this function?

(a) function is decreasing for (–2, –1).
(b) function has a minima for $x = -1$.
(c) function has a maxima for $x = 2$.
(d) function is decreasing for (–1, 2).

Q.20 The equation of the curve passing through the point $\left(0, \frac{\pi}{3}\right)$ satisfies the following differential equation is $\sin x \cos y\,dx + \cos x \sin y\,dy = 0$

(a) $\cos x \cos y = \frac{1}{2}$ (b) $\sin x \cos y = 0$

(c) $\cos x \cos y = \frac{\sqrt{3}}{2}$ (d) $\sin x \sin y = 0$

Q.21 Which one of the following statements is true about the differential equation given below?

$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

(a) The equation is exact.
(b) The equation is not exact and can be made exact by multiplying with $\frac{1}{x}$.
(c) The equation is not exact and can be made exact by multiplying with y.
(d) The equation is not exact and can be made exact by multiplying with $\frac{1}{y}$.

Q.22 The real part of an analytic function is $x^3 - 3xy^2 + 3x^2 - 3y^2$, then the imaginary part of the function will be

(a) $3xy^2 + 6xy + x^2 + C$
(b) $3x^2y + 2xy + y^2 + C$
(c) $8y^2 - 3xy^2 + 3y^2 - 3x^2 + C$
(d) $3x^2y + 6xy - y^3 + C$

Q.23 The point of intersection of the curves $3x^3 + 2x^2 + 8x - 5 = 0$ and $2x^3 + 3x + 2 = 0$, is calculated by using Newton- Rapson's method. The value of x at intersection correct upto 2 decimal points is approximately

(a) 1.21 (b) 2.62
(c) 0.91 (d) 3.82

Q.24 A and B throw alternatively a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A starts the game, then the probability that B wins the game is

(a) $\frac{5}{6}$ (b) $\frac{31}{61}$
(c) $\frac{30}{61}$ (d) $\frac{36}{71}$

Q.25 The values of 'a' and 'b' such that the surface $ax^2 - byz = (a + 2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, –1, 2), are respectively

(a) $a = 2, b = 1$ (b) $a = 2.5, b = 1$
(c) $a = 3, b = 1$ (d) $a = 4, b = 1$

Q.26 An urn A contains 2 white and 4 black balls. Another urn B contains 5 white and 7 black balls. A ball is transferred from urn A to urn B, then a ball is drawn from urn B. The probability, that the drawn ball is white, is

(a) $\frac{2}{13}$ (b) $\frac{10}{39}$
(c) $\frac{16}{39}$ (d) $\frac{12}{39}$

Q.27 The probability density function of a continuous random variable is given by,

$$f(x) = \begin{cases} x & ; \quad 0 \le x \le 1 \\ 2 - x & ; \quad 1 \le x \le 2 \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

The mean value of the random variable is

(a) 1 (b) 1.5
(c) 1.67 (d) 0

Q.28 A curve given by $x^2 + 4y^2 = 36$ is revolved around x axis. The volume of solid generated is

(a) 64π unit^3 (b) 72π unit^3
(c) 144π unit^3 (d) 48π unit^3

Q.29 Consider the differential equation given below:

$$\frac{dy}{dx} + yf'(x) = f(x) \cdot f'(x)$$

Here $f(x)$ is purely a function of x. The solution of the equation is

(a) $ye^{f(x)} = f(x)\,[e^{f(x)} + 1] + c$
(b) $ye^{f(x)} = e^{f(x)} + f(x) + c$
(c) $\log\,[y + f(x)] + f(x) = 0$
(d) $\log\,[1 + y - f(x)] + f(x) = c$

Q.30 The particular integral of the differential equation $D^2(D^2 + 4)y = 96x^2$ for $x = 2$ will be

(a) 8 (b) 5
(c) 9 (d) 2

■■■■

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 26/03/2024

ANSWER KEY

1. (b)	7. (d)	13. (d)	19. (d)	25. (b)
2. (c)	8. (a)	14. (a)	20. (a)	26. (c)
3. (c)	9. (a)	15. (a)	21. (c)	27. (a)
4. (a)	10. (b)	16. (d)	22. (d)	28. (b)
5. (a)	11. (c)	17. (d)	23. (c)	29. (d)
6. (d)	12. (d)	18. (d)	24. (b)	30. (a)

DETAILED EXPLANATIONS

1. (b)

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x-1) = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^3-1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And Lf′(1) = 2, Rf′(1) = 1

$\Rightarrow f$ is not differentiable at $x = 1$

2. (c)

$$\int_0^5 f(x)\cdot dx = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$= \frac{1}{2}\left[(2+0.077)+2(1+0.4+0.2+0.1176)\right]$$

$$\simeq 2.75$$

3. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$l_{11} = 2$ $\quad$ $l_{11}u_{12} = 4$

$u_{12} = \frac{4}{2} = 2$

$l_{21} = 6$ $\quad$ $l_{21}u_{12} + l_{22} = 3$

$6 \times 2 + l_{22} = 3$

$l_{22} = 3 - 12$

$l_{22} = -9$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

4. (a)

$$P(-1 \le x \le 1) = \int_{-1}^{1} (0.1)\,dx$$

$$= 2\times\frac{1}{10} = \frac{1}{5}$$

5. (a)

$$\lim_{x\to 0}\frac{\log x}{\cot x}$$

$$= \lim_{x\to 0}\frac{\frac{1}{x}}{-\operatorname{cosec}^2 x} = -\lim_{x\to 0}\frac{\sin^2 x}{x} \qquad \left(\text{from}\frac{\infty}{\infty}\right)$$

$$= -\lim_{x\to 0}\frac{2\sin x\cos x}{1} = 0 \qquad \left(\text{from}\frac{0}{0}\right)$$

6. (d)

$$u = \sin x$$

$$du = \cos x\, dx$$

$$x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$$

$$x = -\pi \Rightarrow u = \sin(-\pi) = 0$$

$$\int_{-\pi}^{\pi/2}\cos(x)\cos(\sin(x))dx = \int_0^1 \cos u\, du$$

$$= \left|\sin u\right|_0^1$$

$$= (\sin 1) - \sin(0) = \sin 1$$

7. (d)

$$\frac{e^x}{(1-e^x)}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating on both sides, we get,

$$-\ln(1-e^x) + \ln(\tan y) = C_1$$

$$\ln\left(\frac{\tan y}{(1-e^x)}\right) = C_1$$

$$\frac{\tan y}{(1-e^x)} = e^{C_1} = C$$

$$\tan y = C(1-e^x)$$

8. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$PI = \frac{x^2+2x+8}{D(1+D)}$$

$$= \frac{1}{D}(1+D)^{-1}(x^2+2x+8) = \frac{1}{D}(1-D+D^2-D^3+\ldots)(x^2+2x+8)$$

$$= \frac{1}{D}(x^2+2x+8-2x-2+2) = \frac{1}{D}(x^2+8) = \frac{x^3}{3}+8x$$

9. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

10. (b)

We have
$$y = e^x (A\cos x + B\sin x)$$

$$y' = e^x (A\cos x + B\sin x) + e^x (-A\sin x + B\cos x)$$

$$= y + e^x [-A\sin x + B\cos x]$$

$$y'' = y' + e^x (-A\sin x + B\cos x) + e^x (-A\cos x - B\sin x)$$

$$= y' + y' - y - y = 2y' - 2y$$

$\Rightarrow$ Order = 2

Degree = 1

11. (c)

$$\lim_{x\to 1}\frac{x^x - x}{x - 1 - \log x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}}$$

Let,
$$y = x^x$$

$$\log y = x\log x$$

$\therefore$
$$\frac{1}{y}\frac{dy}{dx} = x\cdot\frac{1}{x} + 1\cdot\log x$$

or
$$\frac{d}{dx}(x^x) = x^x(1 + \log x)$$

$$= \lim_{x\to 1}\frac{x^x(1+\log x) - 1}{1 - \frac{1}{x}} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 1}\frac{\frac{d}{dx}(x^x)\cdot(1+\log x) + x^x\left(\frac{1}{x}\right) - 0}{\frac{1}{x^2}}$$

$$= \lim_{x\to 1}\frac{x^x(1+\log x)^2 + x^x\left(\frac{1}{x}\right)}{x^{-2}} = \frac{1(1+0)^2 + 1\cdot 1}{1} = 2$$

12. (d)

Let $$u = \sqrt{x}$$

Then $$du = \frac{1}{2\sqrt{x}}dx$$

$$\therefore \quad dx = du \cdot 2\sqrt{x}$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\,du = 2\int \sin u\,du$$

$$= -2\cos\sqrt{x} + c$$

13. (d)

$$I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right)dx$$

$$= \int_0^{\pi/2} [\log(\sin x)dx - \log(\cos x)dx]$$

$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx - \int_0^{\pi/2} \log(\cos x)dx$$

$$I = 0$$

14. (a)

$$|\lambda - AI| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2$$

$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$

$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$

$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$

$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

$\therefore$ $$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

15. (a)

Parabola given : $x^2 = 4y$ ...(i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

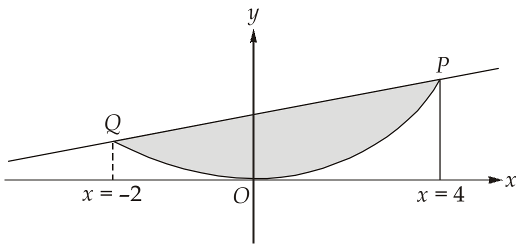
$$\Rightarrow x^2 = 2(x + 4)$$

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow x(x - 4) + 2(x - 4) = 0$$

$$\Rightarrow x = 4, -2$$



Required area $= POQ$

$$= \int_{-2}^{4} y\,dx \text{ from straight line} - \int_{-2}^{4} y\,dx \text{ from parabola}$$

$$= \int_{-2}^{4} \left(\frac{x+4}{2}\right) - \int_{-2}^{4} \frac{x^2}{4} dx$$

$$= \frac{1}{2}\left|\frac{x^2}{2} + 4x\right|_{-2}^{4} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-2}^{4}$$

$$= \frac{1}{2}\{8 + 16 - (-6)\} - \frac{1}{12}(64 + 8)$$

$$= \frac{1}{2} \times 30 - \frac{1}{12} \times 72 = 15 - 6 = 9$$

16. (d)

$$AX = B$$

Augmented matrix, $$[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$$

$R_3 \to R_3 + R_2 + R_1$:

$$|A:B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

$$\text{Rank of } [A : B] = 2$$

$$\text{Rank of } [A] = \text{Rank of } [A : B] = 2 < 3 \text{ (Number of variables)}$$

$\Rightarrow$ Infinitely many solutions are possible.

17. (d)

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$X_1 = c_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_3 = 0$$

$$2x_1 + 2x_2 + x_3 = 0$$

$$X_2 = c_2\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = \frac{-1}{2}x_3$$

$$X_3 = c_3\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

Since, $X_1{}^T X_2 \neq 0$

$$X_2{}^T X_3 \neq 0$$

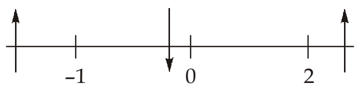
$$X_3{}^T X_1 \neq 0$$

None of the above is correct.

18. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$
For minima/maximum, $f'(x) = 0$
$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$

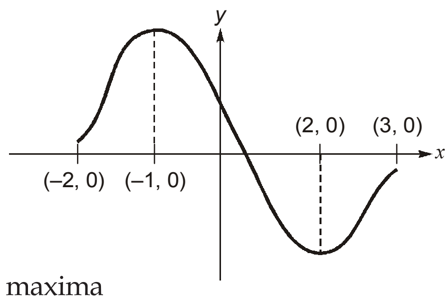
The function has maxima at $x = -1$ and minima at $x = 2$.
Critical point (–1, 2) draw plot on line graph:
Since $0 \in (-1, 2)$ and $f'(0) = 6 \times 0^2 - 6 \times 0 - 12 = -12 < 0$



The function is decreasing between –1 and 2.

19. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$
For minima/maxima, $f'(x) = 0$
$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$



The function has maxima at $x = -1$ and minima at $x = 2$.
The function is decreasing between –1 and 2.

20. (a)

$$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$$
Divide by $\cos x \cos y$, we get ,
$$\tan x\, dx + \tan y\, dy = 0$$
Integrating the equation,
$$\log \sec x + \log \sec y = C_1$$
$$\log \frac{1}{\cos x \cos y} = C_1$$
$$\cos x \cos y = C$$
Since it passes through $\left(0, \frac{\pi}{3}\right)$
$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

21. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

22. (d)

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -6xy - 6y$$

$$dv = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$= (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy$$

$$v = 3x^2y + 6xy - y^3 + C$$

23. (c)

If the two curves intersects, then at point of intersection,

$$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$$

$$x^3 + 2x^2 + 5x - 7 = 0$$

$$f(x) = x^3 + 2x^2 + 5x - 7$$

$$f(0) = 0 + 0 + 0 - 7 = -7 < 0$$

$$f(1) = 1 + 2 + 5 - 7 = 1 > 0$$

$\Rightarrow$ One root lies between 0 and 1. Let us assume 1 as initial value.

$$f'(x) = 3x^2 + 4x + 5$$

$$x_1 = 1 - \left.\frac{f(x)}{f'(x)}\right|_{x=1} = 1 - \frac{1^3 + 2\times 1^2 + 5\times 1 - 7}{3\times 1^2 + 4\times 1 + 5} = 0.9167$$

$$x_2 = x_1 - \left.\frac{f(x)}{f'(x)}\right|_{x=0.9167} = 0.9136$$

24. **(b)**

Number of ways of throwing 6 is five $\Rightarrow$ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six $\Rightarrow$ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \dfrac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \dfrac{5}{36} = \dfrac{31}{36}$

Probability of throwing 7, $q_1 = \dfrac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \dfrac{6}{36} = \dfrac{30}{36}$

$$\begin{aligned}\text{Probability of } B \text{ winning} &= p_2q_1 + p_2q_2p_2q_1 + p_2q_2p_2q_2p_2q_1 + \dots\\ &= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + \dots]\\ &= \frac{p_2q_1}{(1-p_2q_2)} = \frac{\frac{31}{36}\times\frac{6}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{31\times 6}{366} = \frac{31}{61}\end{aligned}$$

25. **(b)**

$$\begin{aligned}\phi_1 &= ax^2 - byz - (a+2)x\\ \nabla\phi_1 &= [2ax-(a+2)]\hat{i} - bz\hat{j} - by\hat{k}\\ \nabla\phi_1(1,-1,2) &= (a-2)\hat{i} - 2b\hat{j} + b\hat{k}\\ \phi_2 &= 4x^2y + z^3 - 4\\ \nabla\phi_2 &= 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}\\ \nabla\phi_2(1,-1,2) &= -8\hat{i} + 4\hat{j} + 12\hat{k}\end{aligned}$$

Since surfaces are orthogonal to each other at (1, –1, 2)

$$\nabla\phi_1\cdot\nabla\phi_2 = 0$$

$$[(a-2)\hat{i} - 2b\hat{j} + b\hat{k}]\cdot[-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$

$$-8(a-2) - 8b + 12b = 0 \qquad \dots \text{(i)}$$

Also point (1, –1, 2) lies on the surface.

$$\Rightarrow \quad a\times 1 + 2b = (a+2)1$$

$$b = 1$$

Putting this in equation 1, we get,

$$\begin{aligned}-8(a-2) - 8 + 12 &= 0\\ a-2 &= -\frac{1}{8}\times(-4) = 0.5\\ a &= 2.5\end{aligned}$$

26. **(c)**

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from B = $\dfrac{2}{2+4}\times\dfrac{6}{13} = \dfrac{2}{13}$

Case-II: Black ball is transferred from A to B

$$\text{Probability of drawing black ball from } B = \frac{4}{2+4}\times\frac{5}{13}=\frac{10}{39}$$

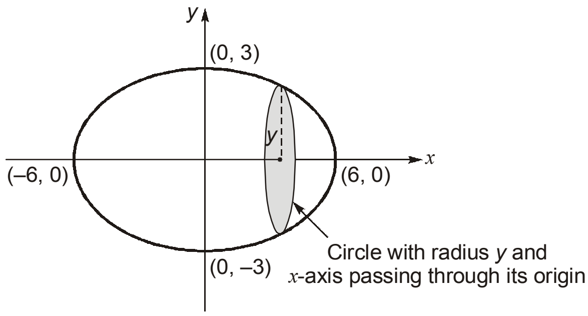
$$\text{Required probability} = \frac{2}{13}+\frac{10}{39}=\frac{16}{39}$$

27. **(a)**

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)x\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1 + \left.\left(x^2-\frac{x^3}{3}\right)\right|_1^2 = \frac{1}{3}+4-1-\frac{8-1}{3}=1$$

28. **(b)**



$$\text{Volume generated} = \int_{-6}^{6} \pi y^2\,dx = \int_{-6}^{6} \pi\left(\frac{36-x^2}{4}\right)dx$$

$$= \frac{\pi\times 2}{4}\int_0^6 (36-x^2)\,dx = \frac{\pi}{2}\left[36x-\frac{x^3}{3}\right]_0^6$$

$$= 72\pi$$

29. **(d)**

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF\cdot f(x)\cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)}\cdot f(x)\cdot f'(x)dx$$

Let

$$f(x) = t$$

$$f'(x)\,dx = dt$$

$$y \times e^t = \int e^t\cdot t\,dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y+1-t) = -t + c'$$

$$\log\,[y+1-f(x)] + f(x) = c'$$

30. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2+4)} = 96\frac{1}{4D^2\left(1+\frac{D^2}{4}\right)}x^2 = \frac{96}{4}\left[\frac{\left(1-\frac{D^2}{4}\right)x^2}{D^2}\right]$$

$$= 24\frac{\left(x^2-\frac{1}{2}\right)}{D^2}$$

$$PI = 24\left[\frac{x^4}{4\times 3}-\frac{x^2}{4}\right] = 2x^2(x^2-3)$$

$$PI|_{x=2} = 2 \times 2^2(4-3) = 8$$

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