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# NETWORK THEORY

EC-EE

Date of Test : 28/03/2024

## ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (c)  | 13. (c) | 19. (b) | 25. (b) |
| 2. (c) | 8. (a)  | 14. (d) | 20. (d) | 26. (a) |
| 3. (c) | 9. (b)  | 15. (c) | 21. (a) | 27. (c) |
| 4. (c) | 10. (c) | 16. (c) | 22. (b) | 28. (b) |
| 5. (a) | 11. (b) | 17. (a) | 23. (d) | 29. (a) |
| 6. (c) | 12. (d) | 18. (b) | 24. (c) | 30. (a) |

## DETAILED EXPLANATIONS

1. (b)

The circuit will act as an ideal current source if impedance is infinite

$$\therefore Z = \frac{(j\omega L) \left( \frac{1}{j\omega C} \right)}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{-\omega^2 LC + 1}$$

Now, put  $Z = \infty$

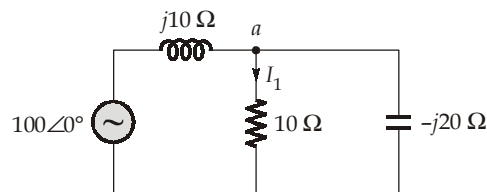
$$\Rightarrow 1 - \omega^2 LC = 0$$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{10 \times 10^{-3} \times 25 \times 10^{-6}}} = \frac{1}{5 \times 10^{-4}}$$

$$\omega = 2 \text{ k rad/sec}$$

2. (c)



Applying nodal analysis at node  $a$ ,

$$\frac{V_a}{10} + \frac{V_a}{-j20} + \frac{V_a - 100\angle 0^\circ}{j10} = 0$$

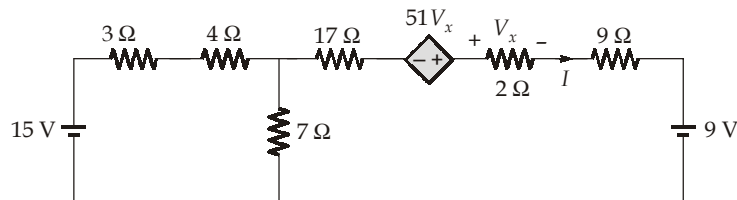
$$\Rightarrow V_a = \frac{200}{1+2j}$$

$$I_1 = \frac{V_a}{10} = \frac{20}{1+2j} = 8.94\angle -63.44^\circ \text{ A}$$

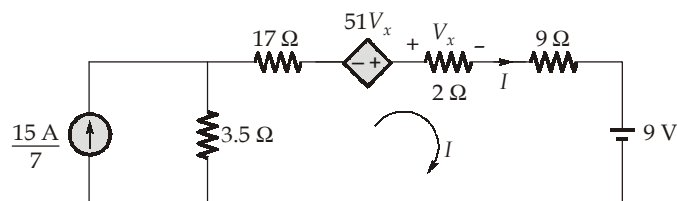
3. (c)

Applying source transformation:

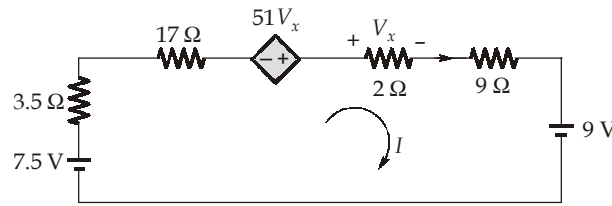
Step 1:



Step 2:



Step 3:



Applying KVL in the loop, we get:

$$7.5 + 51V_x - 9 - 31.5I = 0$$

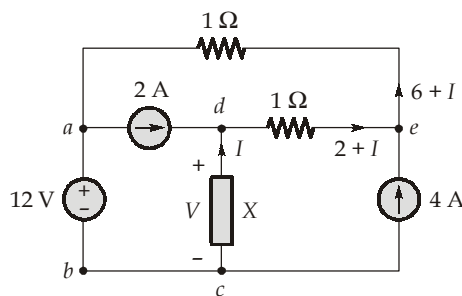
$$7.5 - 9 = 31.5I - 102I$$

$$-1.5 = -70.5I$$

$$I = 21.28 \text{ mA}$$

$$\{\because V_x = 2I\}$$

4. (c)



Applying KVL in loop abcde,

$$-12 + V - (2 + I) - (6 + I) = 0$$

$$-12 + V - 2 - I - 6 - I = 0$$

$$V = 2I + 20$$

⇒

$$A = 2 \Omega ; B = 20 \text{ V}$$

5. (a)

$$X_L = \omega L = 2500 \times 16 \times 10^{-3} = 40 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2500 \times 10 \times 10^{-6}} = 40 \Omega$$

∴

$$X_L = X_C$$

∴ The tank circuit will be open circuited.

Hence, the current flowing in the circuit will be zero.

6. (c)

Bandwidth,  $B = \omega_2 - \omega_1 = 101 - 99 = 2 \text{ k rad/sec}$ 

$$B = \frac{1}{RC} = 2 \times 10^3$$

$$C = \frac{1}{100 \times 10^3 \times 2 \times 10^3} = 5 \text{ nF}$$

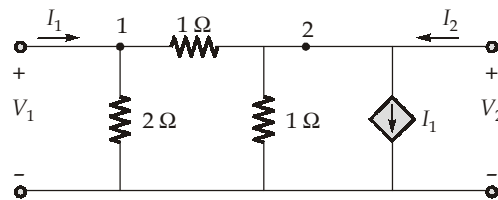
$$\omega_1 = \omega_0 - \frac{B}{2}$$

$$\Rightarrow \omega_0 = \omega_1 + \frac{B}{2} = 99 + 1 = 100 \text{ k rad/sec}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow LC = \frac{1}{\omega_0^2} \Rightarrow L = \frac{1}{10^{10} \times 5 \times 10^{-9}} = 20 \text{ mH}$$

7. (c)



Applying nodal analysis at node 1:

$$\frac{V_1}{2} + \frac{V_1 - V_2}{1} = I_1$$

$$I_1 = \frac{3}{2}V_1 - V_2 \quad \dots(i)$$

Applying nodal analysis at node 2:

$$\frac{V_2}{1} + \frac{V_2 - V_1}{1} = -I_1 + I_2$$

$$\Rightarrow I_2 = \frac{1}{2}V_1 + V_2 \quad \dots(ii)$$

Comparing equation (i) and (ii) with general equations of Y-parameter, we get

$$[Y] = \begin{bmatrix} \frac{3}{2} & -1 \\ \frac{1}{2} & 1 \end{bmatrix}$$

8. (a)

$$I(s) = \frac{I_0}{s^2}; \quad I_L(s) = \left[ \frac{5}{s^2} - \frac{1}{s} \right] + \frac{1}{s+5}$$

$$I_L(s) = I(s) \left[ \frac{1}{1+sL} \right]$$

$$\left[ \frac{5}{s^2} - \frac{1}{s} \right] + \frac{1}{s+5} = \frac{I_0}{s^2(1+sL)}$$

$$\frac{5-s}{s^2} + \frac{1}{s+5} = \frac{I_0}{s^2[1+sL]}$$

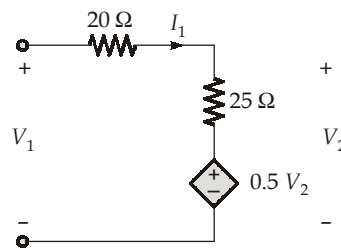
$$\frac{(25)}{s^2(s+5)} = \frac{I_0/L}{s^2\left(s + \frac{1}{L}\right)}$$

$$\Rightarrow L = \frac{1}{5} = 0.2 \text{ H}; \quad \frac{I_0}{L} = 25$$

$$\Rightarrow I_0 = 25L = 5 \text{ A}$$

9. (b)

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

If  $I_2 = 0$ , then the circuit becomes:

Applying KVL:

$$V_1 - 20I_1 - 25I_1 - 0.5V_2 = 0$$

$$V_1 = 45I_1 + 0.5V_2 \quad \dots(1)$$

$$V_2 = 25I_1 + 0.5V_2$$

$$V_2 = 50I_1 \quad \dots(2)$$

solving (1) and (2):

$$Z_{11} = \frac{V_1}{I_1} = 70 \Omega$$

10. (c)

$$V_c(j\omega) = V_i(j\omega) \left[ \frac{\frac{-j}{\omega C}}{2 \times 10^3 + \frac{-j}{\omega C}} \right]$$

$$V_c(j\omega) = V_i(j\omega) \left[ \frac{1}{1 + j2 \times 10^3 \times \omega \times C} \right]$$

Also,

$$V_0(j\omega) = \left[ \frac{30k}{15k + 30k} \right] A V_c(j\omega)$$

$$V_0(j\omega) = \frac{2A}{3} V_c(j\omega)$$

$$\therefore \frac{V_0(j\omega)}{V_i(j\omega)} = \frac{2A/3}{1 + j2 \times 10^3 \times C \times \omega}$$

On comparing,  $A = 6$  and  $C = 5 \mu F$ .

11. (b)

$$P = I_{\text{rms}} V_{\text{rms}} \cos\theta$$

$$\Rightarrow 1000 \text{ W} = I_{\text{rms}} V_{\text{rms}} \times 0.8$$

$$\Rightarrow I_{\text{rms}} V_{\text{rms}} = \frac{1000}{0.8} = 1250$$

$$\Rightarrow \frac{V_{\text{rms}}^2}{I_{\text{rms}} \times V_{\text{rms}}} = \frac{(200)^2}{1250} = |Z|$$

$$\therefore |Z| = 32 \Omega$$

$\therefore$  Power factor is leading,  $\theta < 0^\circ$

$$\angle Z = \theta = -\cos^{-1} 0.8 = -36.86^\circ$$

$$\therefore Z = |Z| \angle Z = 32 \angle -36.86^\circ = 25.6 - j19.2 \Omega$$

12. (d)

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$= \frac{100 \angle 0^\circ}{6 + j15 - j7} = \frac{100}{6 + j8} = 6 - j8$$

Complex power supplied by the source,

$$S = V_{\text{rms}} I_{\text{rms}}^* = 100(6 + j8)$$

$$S = 600 + j800 \text{ VA} = 1000 \angle 53.13^\circ$$

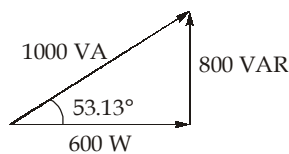
As we know that,

$$S = P + jQ$$

$$\therefore P = 600 \text{ W}, \quad Q = 800 \text{ VAR}$$

and power factor angle,

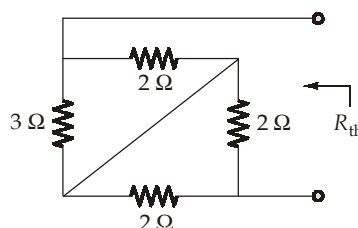
$$\theta = 53.13^\circ$$



13. (c)

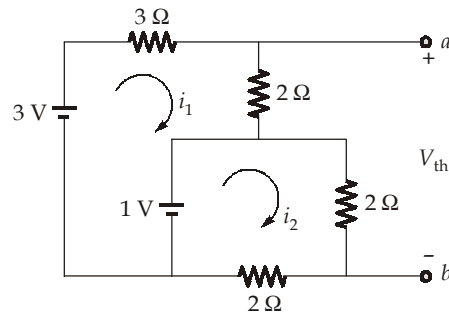
Using Thevenin's theorem taking  $1 \Omega$  as load:

- Thevenin's equivalent resistance,  $R_{\text{th}}$ :



$$R_{\text{th}} = (3 \parallel 2) + (2 \parallel 2) = 1.2 + 1 = 2.2 \Omega$$

- **Thevenin's Equivalent Voltage,  $V_{th}$ :**  
Using source transformation, we get

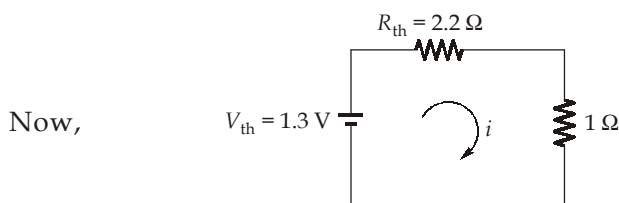


$$i_1 = \frac{3-1}{5} = 0.4 \text{ A}; \quad i_2 = \frac{1}{4} = 0.25 \text{ A}$$

Applying KVL in outer loop:

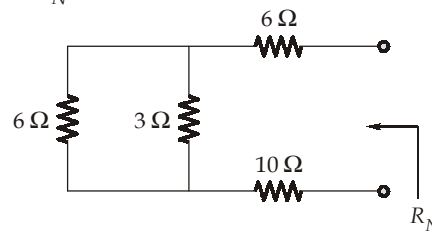
$$V_{th} + 3i_1 - 3 + 2i_2 = 0$$

$$\therefore V_{th} = 1.3 \text{ V}$$



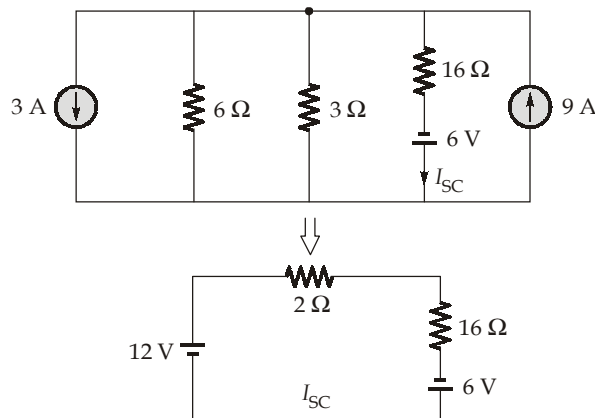
$$i = \frac{1.3}{2.2+1} = 0.406 \text{ A}$$

14. (d)  
Norton equivalent resistance  $R_N$ :



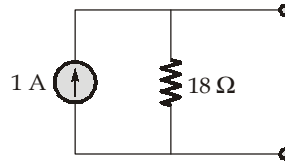
$\therefore R_N = 18 \Omega$

Norton current,  $I_{SC}$ :



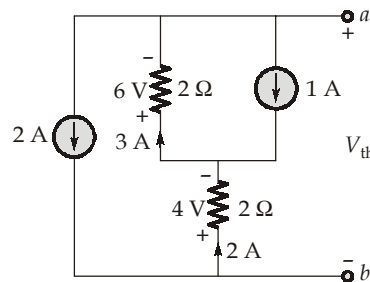
$$I_{SC} = \frac{12+6}{18} = 1 \text{ A}$$

∴ Norton equivalent circuit will be



15. (c)

The Thevenin's voltage across  $ab$  is



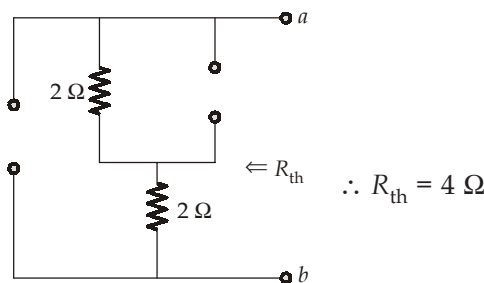
$$+V_{th} + 6 \text{ V} + 4 \text{ V} = 0$$

$$V_{th} = -10 \text{ V}$$

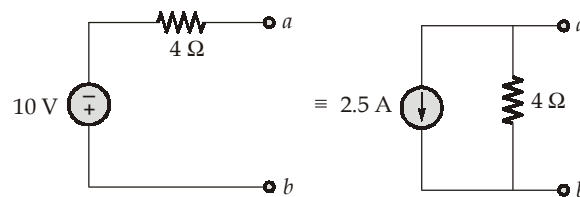
∴

Thevenin's resistance,  $R_{th}$ :

by open circuiting all independent current sources,

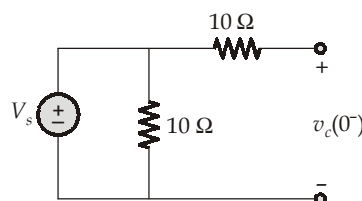


∴ The Norton's equivalent circuit is



16. (c)

At  $t = 0^-$





$$v_c(0^-) = 3 + 12e^0 = 15 \text{ V}$$

 $\therefore$ 

$$V_s = 15 \text{ V}$$

 At  $t = \infty$ 

$$v_c(\infty) = 3 \text{ V}$$

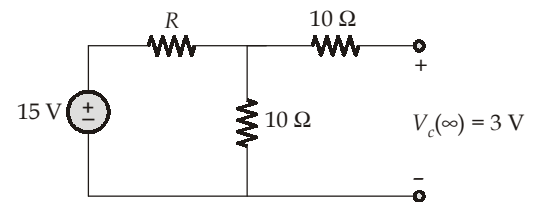
 $\therefore$ 

$$3 = \frac{15 \times 10}{10 + R}$$

 $\Rightarrow$ 

$$R = 40 \ \Omega$$

$$R_{\text{eq}} = (40 \parallel 10) + 10 = 18 \ \Omega$$



$$\text{Time constant, } \tau = R_{\text{eq}} C = 18C = \frac{1}{5.56}$$

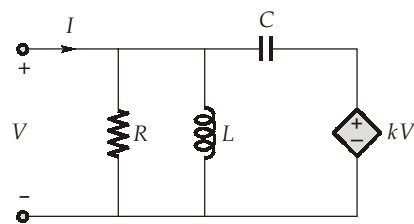
 $\Rightarrow$ 

$$C = \frac{1}{18 \times 5.56}$$

$$C \simeq 10 \text{ mF}$$

17. (a)

The equivalent impedance of the network can be found as:



$$R_{\text{eq}} = \frac{V}{I}$$

$$\therefore \frac{V}{R} + \frac{V}{jX_L} + \frac{V - kV}{-jX_C} = I$$

$$\frac{I}{V} = \left[ \frac{1}{R} + \frac{1}{j\omega L} + j\omega C(1 - k) \right]$$

$$Y(j\omega) = \frac{1}{R} + j \left[ \omega C(1 - k) - \frac{1}{\omega L} \right]$$

At resonance, imaginary part of input admittance becomes zero.

$$\therefore \omega_0 = \frac{1}{\sqrt{LC(1 - k)}}$$

For a parallel RLC circuits, quality factor is

$$Q = \frac{R}{\omega_0 L} = \frac{R \sqrt{LC(1 - k)}}{L} = R \sqrt{\frac{C(1 - k)}{L}}$$

18. (b)

$$v_0(t) = 2e^{-t}$$

Taking Laplace transform,  $V_0(s) = \frac{2}{s+1}$

$$V_0(s) = Z(s) I(s)$$

∴

$$Z(s) = V_0(s)$$

When  $i(t)$  is a pulse i.e.

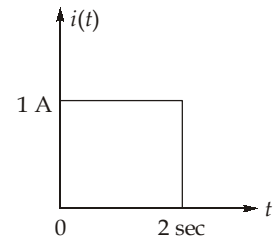
$$i(t) = u(t) - u(t - 2)$$

$$\{\because i(t) = \delta(t)\}$$

$$I(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$$

⇒

$$V_0(s) = \frac{2}{s+1} \left[ \frac{1}{s} - \frac{e^{-2s}}{s} \right]$$



$$V_0(s) = \frac{2}{s(s+1)} - \frac{2e^{-2s}}{s(s+1)}$$

Taking inverse Laplace transform:

$$v_0(t) = 2[u(t) - e^{-t}u(t)] - 2[u(t - 2) - e^{-(t-2)}u(t - 2)]$$

At  $t = 3$  sec,

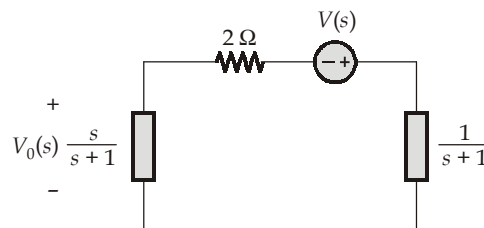
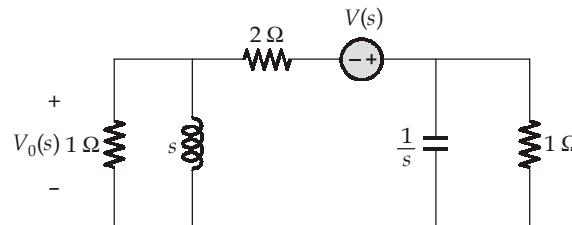
$$v_0(t) = 2[1 - e^{-3} - 1 + e^{-1}]$$

$$= 2[e^{-1} - e^{-3}]$$

$$= 0.636 \text{ V}$$

19. (b)

Applying source transformation and transforming the circuit into s-domain.



$$V_0(s) = V(s) \frac{s}{3(s+1)}$$

$$V_0(j\omega) = V(j\omega) \frac{j\omega}{3(j\omega + 1)}$$

$$= 30 \angle 0^\circ \times \frac{1 \angle 90^\circ}{3\sqrt{2} \angle 45^\circ}$$

$$\{\because \omega = 1 \text{ rad/sec}\}$$

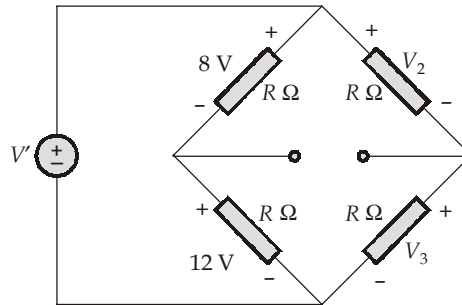
$$V_0(j\omega) = 5\sqrt{2}\angle 45^\circ$$

$\therefore$

$$v_0(t) = 5\sqrt{2} \cos(t + 45^\circ) \text{ V}$$

20. (d)

Under bridge balance condition, (since current through  $6 \Omega$  resistor is zero).



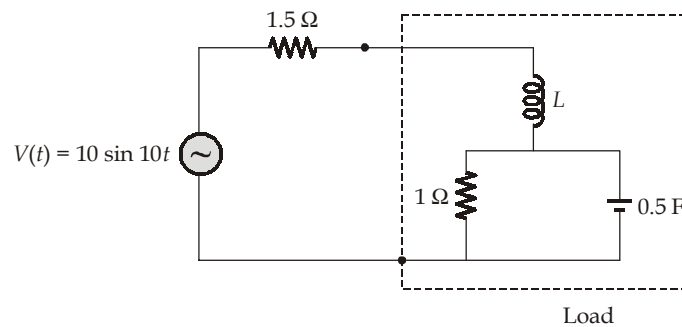
$$V' = 8 + 12 = V_2 + V_3$$

$\Rightarrow$

$$V_2 + V_3 = 20 \text{ V}$$

Option (d) satisfies the above condition.

21. (a)



The maximum power is transferred at the frequency at which the load is resistive and it is equal to  $1.5 \Omega$  i.e., the load is resistive means the imaginary part of the load is equal to zero.

$$\begin{aligned} Z_{\text{load}} &= \frac{1 \times \frac{2}{s}}{1 + \frac{2}{s}} + Ls = \frac{2}{s+2} + Ls \\ &= \frac{2(s-2)}{s^2-4} + Ls \end{aligned}$$

Put  $s = j\omega$

$$\begin{aligned} Z_{\text{load}} &= \frac{2(j\omega-2)}{-\omega^2-4} + j\omega L \\ &= \frac{2(j10-2)}{-104} + j10L \end{aligned}$$

$$Z_{\text{load}} = \frac{4}{104} + j\left(10L - \frac{20}{104}\right)$$

equating imaginary part to zero.

$$10L = \frac{20}{104}$$

$$\therefore L = \frac{20}{10 \times 10^4} = 19.23 \text{ mH}$$

22. (b)

The reactive power in the circuit is

$$Q \propto \sin \theta$$

If  $Q$  is positive then angle of impedance ( $\theta$ ) is positive which implies that current phasor is lagging voltage phasor i.e., load is inductive.

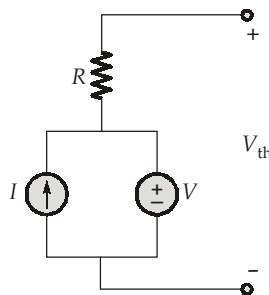
$$Z = \frac{V \angle \theta_V}{I \angle \theta_I} = \frac{V}{I} \angle \theta_V - \theta_I$$

$$\theta_V > \theta_I$$

Hence, an inductive load has lagging power factor.

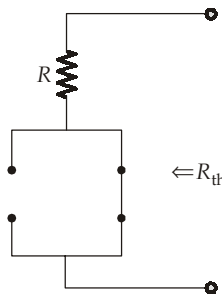
23. (d)

Given, circuit,



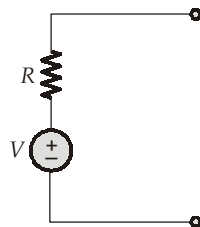
The open circuit voltage (Thevenin voltage  $V_{th}$ ) is equal to  $V$ .

For Thevenin's resistance :  $R_{th}$ , by setting all independent sources to zero, i.e., open circuit the current source and short circuit the voltage source.



$$\therefore R_{th} = R$$

The equivalent circuit is



24. (c)

In series RLC circuit at resonance,

$$\text{Current, } I_R = \frac{V_s}{R}$$

Voltage across inductor is,  $V_L = j\omega_0 LI_R = j\omega_0 L \frac{V_s}{R}$

$$V_L = jQV_s$$

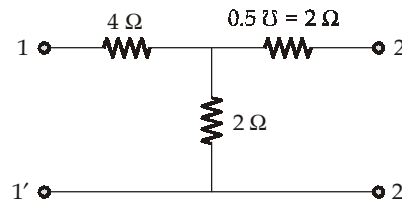
where,

$$Q = \frac{\omega_0 L}{R}$$

Since,  $Q > 1 \Rightarrow V_L > V_s$

25. (b)

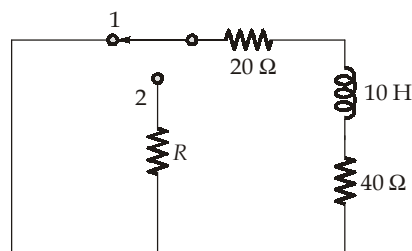
Given, two port network



$$\therefore [z] = \begin{bmatrix} 6 & 2 \\ 2 & 4 \end{bmatrix} \Omega$$

26. (a)

After the switch moves to position 1, the circuit can be drawn as below,



$$\therefore \text{time constant, } \tau = \frac{L}{R_{eq}} = \frac{10}{20 + 40} = 0.167 \text{ sec}$$

27. (c)

Given,

$$\omega_0 = 1000 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Resonant frequency,

$$\omega_0^2 = \frac{1}{LC}$$

$\Rightarrow$

$$L = \frac{1}{\omega_0^2 \times C}$$

$\therefore$

$$L = \frac{1}{10^6 \times 0.2 \times 10^{-6}} = \frac{1}{0.2} = 5 \text{ H}$$

For parallel RLC circuit, Q-factor,

$$Q = \omega_0 RC$$

$\Rightarrow$

$$R = \frac{Q}{\omega_0 C}$$

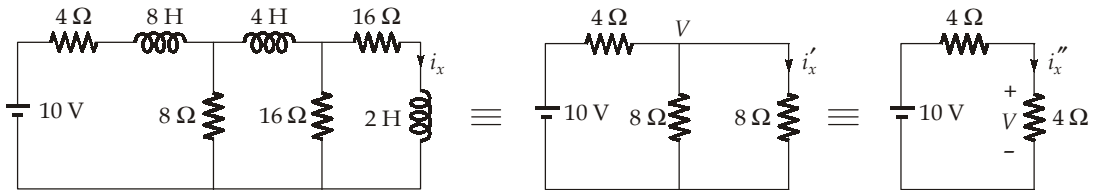
$$\therefore R = \frac{80}{10^3 \times 0.2 \times 10^{-6}} = \frac{80}{0.2 \times 10^{-3}} = 400 \text{ k}\Omega$$

$$\therefore \frac{L}{R} = \frac{5}{400 \times 10^3} = 12.5 \times 10^{-6} \text{ s}^{-1}$$

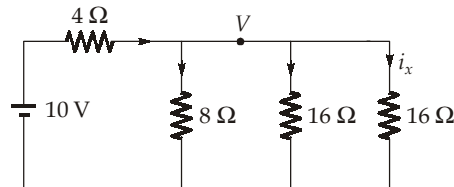
28. (b)

As the circuit has been connected for a long time. Therefore, the inductors behave like a short circuit for the dc voltage source,

$\therefore$  The circuit can be redrawn as



$$V = \frac{10}{(4+4)} \times 4 = 5 \text{ V}$$



By KCL,

$$i_x = \frac{V}{16} = \frac{5}{16} \text{ A}$$

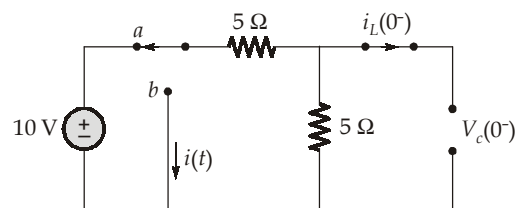
29. (a)

At  $t = 0^+$ :

The switch is in position 'a' and the independent source is connected from a long time to circuit.

Hence, the circuit is in steady state.

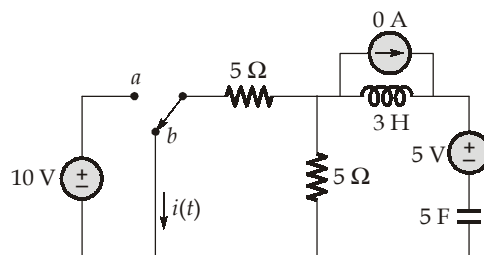
Hence, inductor and capacitor are replaced by short circuit and open circuit respectively.



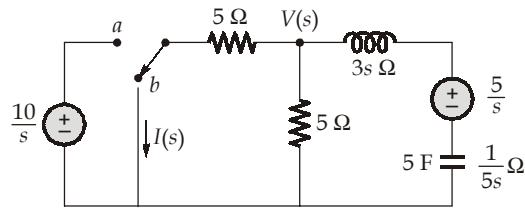
$$i_L(0^-) = 0 = i_L(0^+)$$

$$V_c(0^-) = \frac{10 \times 5}{5 + 5} = 5 \text{ V} = V_c(0^+)$$

At  $t > 0$ : The switch is moved to position b,



By using Laplace transform approach.



Let  $V(s)$  be the node voltage.

by nodal analysis:

$$\frac{V(s)}{5} + I(s) + \frac{V(s) - \frac{5}{s}}{3s + \frac{1}{5s}} = 0$$

but,

$$V(s) = 5 \times I(s)$$

$$\frac{5I(s)}{5} + I(s) + \frac{5I(s) - \frac{5}{s}}{3s + \frac{1}{5s}} = 0$$

$$2I(s) + \frac{5I(s) - \frac{5}{s}}{3s + \frac{1}{5s}} = 0$$

$$6sI(s) + \frac{2}{5s}I(s) + 5I(s) - \frac{5}{s} = 0$$

$$I(s) \left[ 5 + 6s + \frac{2}{5s} \right] = \frac{5}{s}$$

$$I(s) = \frac{\frac{5}{s}}{5 + 6s + \frac{2}{5s}}$$

$$= \frac{\frac{5}{s} \times 5s}{25s + 30s^2 + 2}$$

$\therefore$

$$I(s) = \frac{25}{30s^2 + 25s + 2}$$

$$I(s) = \frac{\frac{5}{6}}{s^2 + \frac{5}{6}s + \frac{1}{15}}$$

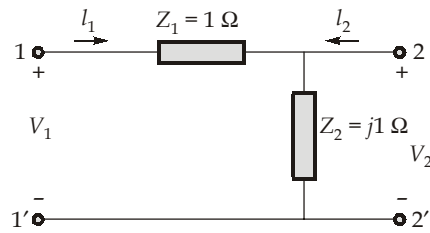
$$= \frac{\frac{5}{6}}{(s + 0.0896)(s + 0.7436)}$$

$$I(s) = \frac{1.274}{s + 0.0896} - \frac{1.274}{s + 0.7436}$$

$\therefore$

$$i(t) = 1.274(e^{-0.0896t} - e^{-0.7436t})u(t); t > 0$$

30. (a)  
Given, two port circuit is,



We know that, h-parameters for any two port circuit is defined as

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\therefore h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

$$\therefore h_{11} = z_1 = 1 \Omega \quad h_{21} = \frac{I_2}{I_1} = -1$$

$$h_{12} = \frac{V_1}{V_2} = 1 \quad h_{22} = -j1 \text{ } \Omega$$

$$[h] = \begin{bmatrix} 1 & 1 \\ -1 & -j1 \end{bmatrix}$$

