CLASS TEST

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ENGINEERING MECHANICS

CIVIL ENGINEERING

Date of Test : 03/03/2024

ANSWER KEY

1.	(b)	6.	(a)	11.	(c)	16.	(d)	21.	(c)
2.	(c)	7.	(d)	12.	(d)	17.	(d)	22.	(d)
3.	(c)	8.	(b)	13.	(d)	18.	(c)	23.	(b)
4.	(a)	9.	(d)	14.	(c)	19.	(a)	24.	(b)
5.	(d)	10.	(a)	15.	(d)	20.	(b)	25.	(d)

DETAILED EXPLANATIONS

1. (b)

Given data: Constant angular acceleration, $\alpha = 2.5\ \text{rad/s}^2$

Initial velocity, $w_o = 1$ rad/sec

$t = 6$ secs

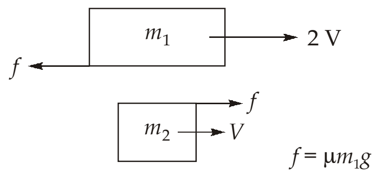
Now, $$\theta = w_o t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \quad \theta = 1 \times 6 + \frac{1}{2} \times 2.5 \times 6^2$$

$$\Rightarrow \quad \theta = 6 + 45 = 51 \text{ radians}$$

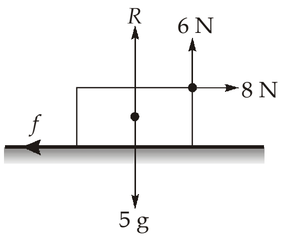
$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{51}{2\pi} = 8.12$$

2. (c)



3. (c)

Consider the free body system as shown.



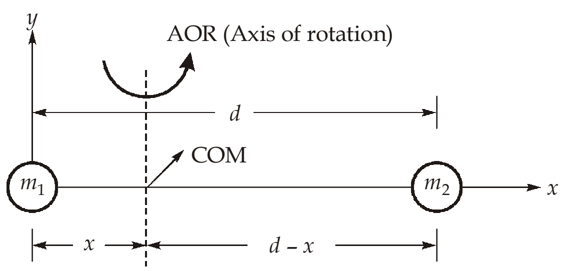
Since the system is at rest,

$$\Sigma F_V = 0$$

$$\Rightarrow \quad R + 6 = 50 \qquad [\because \ g = 10\ \text{ms}^{-2}]$$

$$\Rightarrow \quad R = 44 \text{ N}$$

4. (a)



Centre of mass calculation:

$$(m_1 + m_2)x = m_1 \times 0 + m_2 \times d$$

$$\Rightarrow \quad x = \frac{m_2 d}{m_1 + m_2}$$

$$\therefore \quad d - x = \frac{m_1 d}{m_1 + m_2}$$

$$\therefore \quad I = m_1 x^2 + m_2 (d - x)^2$$

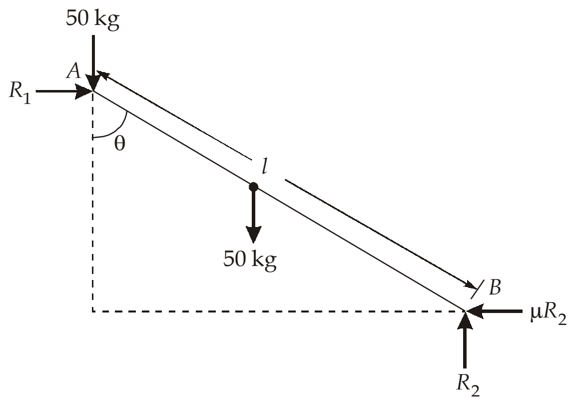
$$= \frac{m_1 \times (m_2 d)^2}{(m_1 + m_2)^2} + m_2 \times \left(\frac{m_1 d}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2^2 d^2 + m_2 m_1^2 d^2}{(m_1 + m_2)^2} = \frac{m_1 m_2 d^2 [m_2 + m_1]}{(m_1 + m_2)^2}$$

$$= \frac{m_1 m_2}{m_1 + m_2} \times d^2$$

5. (d)

FBD when man is on the top of the ladder,



$$\Sigma F_x = 0 \Rightarrow R_1 = \mu R_2$$

$$\Sigma F_y = 0 \Rightarrow 50 + 50 = R_2$$

$$\Rightarrow \quad R_2 = 100 \text{ kg}$$

$$\Rightarrow \quad R_1 = \mu R_2 = 0.25 \times 100 = 25 \text{ kg}$$

$\Sigma M_B = 0$

$$\Rightarrow \quad R_1\, l \cos\theta = 50 \times l \sin\theta + 50 \times 0.5 \times l \times \sin\theta$$

$$\Rightarrow \quad 25 \cos\theta = 75 \sin\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{3} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\therefore \quad x = \frac{1}{3}$$

6. (a)

Centroid, $\bar{x} = \int \frac{x dA}{A}$

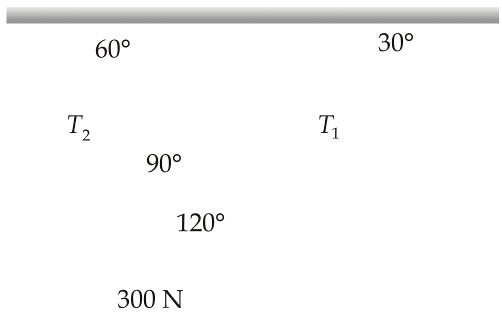
$$A = \int_0^1 (1 - x^2) dx = \left[x - \frac{x^3}{3}\right]_0^1 = \left(1 - \frac{1}{3}\right) - (0 - 0) = \frac{2}{3} \text{ unit}$$

$$\bar{x} = \int_0^1 \frac{x(1-x^2)dx}{2/3} \qquad [dA = ydx = (1 - x^2)dx]$$

$$\Rightarrow \qquad \bar{x} = \frac{3}{2}\int_0^1 (x - x^3)dx = \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1\right] = \frac{3}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$\Rightarrow \qquad \bar{x} = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8} \text{ unit}$$

7. (d)

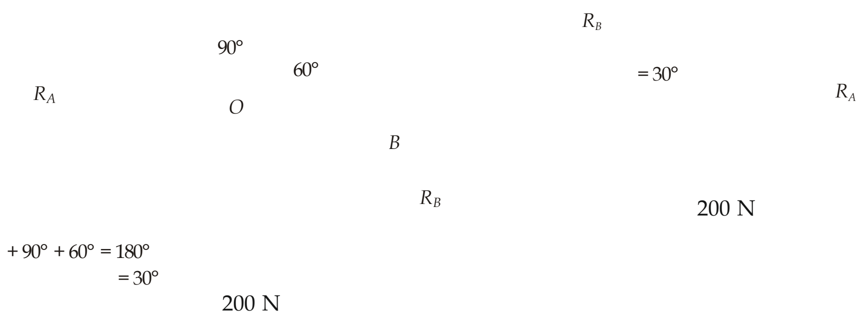


Using Lami's Theorem

$$\frac{T_2}{\sin 120°} = \frac{T_1}{\sin\{360° - (90° + 120°)\}}$$

$$\Rightarrow \qquad \frac{T_1}{T_2} = \frac{\sin 150°}{\sin 120°} = 0.577$$

8. (b)



Using Lami's theorem

$$\frac{R_A}{\sin 120°} = \frac{200}{\sin 150°} = \frac{R_B}{\sin 90°}$$

$$\therefore \qquad R_A = 200 \times \frac{\sin 120°}{\sin 150°} = 346.4 \text{ N}$$

$$R_B = \frac{200 \times \sin 90°}{\sin 150°} = 400\,\text{N}$$

$$\therefore \quad R_A + R_B = 400 + 346.4$$
$$= 746.41 \text{ N} \simeq 746.4 \text{ N}$$

9. (d)

$$I_P = I_x + I_y = \frac{bd^3}{12} + \frac{db^3}{12}$$
$$= \frac{bd}{12}\left(b^2 + d^2\right)$$
$$= \frac{2 \times 5}{12}\left(2^2 + 5^2\right) = 24.167 \text{ cm}^4$$

10. (a)

$$\text{Radial acceleration, } a_r = \frac{V^2}{R} = \frac{(40)^2}{1000} = 1.6 \text{ m/s}^2$$

$$\text{Total acceleration, } a = 2 \text{ m/s}^2$$

$\therefore$ Maximum deceleration with speed can be decreased is

Tangential acceleration,
$$a_t = \sqrt{a^2 - a_r^2} = \sqrt{(2)^2 - (1.6)^2}$$
$$= \sqrt{4 - 2.56} = \sqrt{1.44} = 1.2 \text{ m/s}^2$$

11. (c)

For perfectly elastic collision $e = 1.0$

12. (d)

Using energy principle,

Initial energy = Final energy + Work done by air resistance

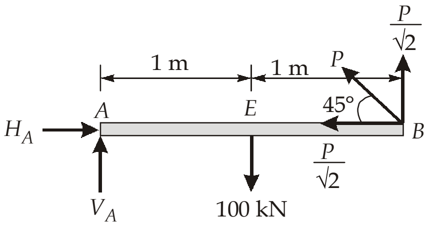
$$\Rightarrow \quad mgh_1 = mgh_2 + \frac{1}{2}mv^2 + W_{air}$$

$$\therefore \quad W_{air} = 5 \times 10 \times 20 - \frac{1}{2} \times 5 \times 10^2 = 750 \text{ J} \qquad [\because \; h_2 = 0]$$

13. (d)

14. (c)

FBD of beam AB,



$$\Sigma M_A = 0$$

$$\Rightarrow \quad -\frac{P}{\sqrt{2}} \times 2 + 100 \times 1 = 0$$

$$\Rightarrow \quad P = 70.71 \text{ kN}$$

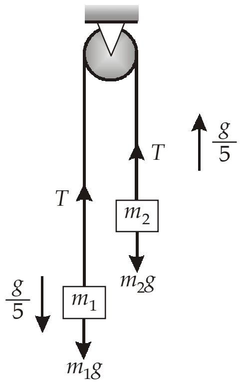
15. (d)

$$\text{Time } (t) = \frac{L}{v} = \frac{12}{5} = 2.4 \text{ sec}$$

$$\text{Height } (h) = \frac{1}{2} g t^2 \qquad [\because \ u = 0]$$

$$= \frac{1}{2} \times 9.81 \times 2.4^2 = 28.25 \text{ m}$$

16. (d)



$$m_1g - T = m_1a = \frac{m_1g}{5} \qquad \text{...(i)}$$

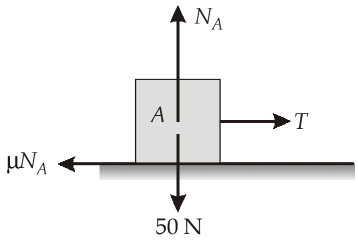
$$T - m_2g = m_2a = m_2\frac{g}{5} \qquad \text{...(ii)}$$

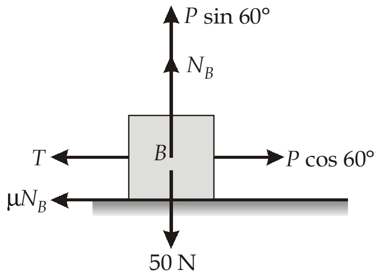
From (i) and (ii), we get

$$\frac{m_1}{m_2} = \frac{6}{4} = 1.5$$

17. (d)

FBD of block *A* and *B* :





$$\text{Normal reaction at } A,\ N_A = 50 \text{ N}$$

$$\text{Normal reaction at } B,\ N_B = 50 - P\sin 60°$$

$$T = \mu N_A \quad \text{...(i)}$$

$$T + \mu N_B = P\cos 60° \quad \text{...(ii)}$$

From (i)

$$T = 0.3 \times 50 = 15 \text{ N} \quad \text{...(iii)}$$

Substituting (iii) in (ii),

$$T + (0.3) \times [50 - P\sin 60°] = P\cos 60°$$

$$\Rightarrow \quad 15 + 0.3 \times 50 = P \times 0.3\sin 60° + P\cos 60°$$

$$\Rightarrow \quad 30 = P[\cos 60° + 0.3\sin 60°]$$

$$\Rightarrow \quad P = \frac{30}{0.5 + 0.2598}$$

$$\Rightarrow \quad P = 39.48 \text{ N}$$

18. (c)

Centroid from base,

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{d^2 \times \frac{d}{2} - \frac{\pi}{8} d^2 \times \frac{2d}{3\pi}}{d^2 - \frac{\pi d^2}{8}}$$

$$= \frac{5 \times 8d}{12(8 - \pi)} = \frac{10d}{3(8 - \pi)}$$

19. (a)

Applying conservation of angular momentum,

$$I\omega = I'\omega'$$

$$\Rightarrow \quad MR^2 \times \omega = \left(MR^2 + 2mR^2\right)\omega'$$

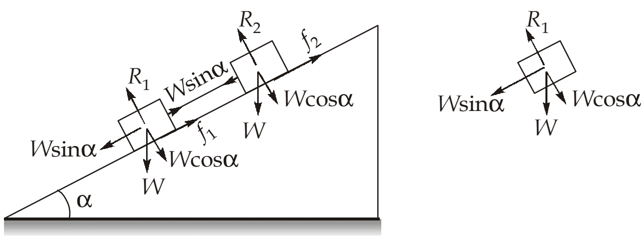
$$\Rightarrow \quad 5 \times (0.3)^2 \times 15 = \left(5 \times 0.3^2 + 2 \times 0.1 \times 0.3^2\right) \times \omega'$$

$$\Rightarrow \quad \omega' = 14.42 \text{ rad/s}$$

20. (b)

Given,

$$W_1 = W_2 = W(\text{say})$$



$$R_1 = R_2 = W\cos\alpha$$

Along the plane, $0.2R_1 + 0.3R_2 = 2W\sin\alpha$

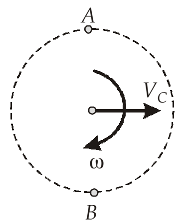
$$\tan\alpha = \frac{0.2+0.3}{2} = 0.25$$

$$\alpha = \tan^{-1}(0.25) = 14°.04°$$

21. (c)

$\because$ Velocities are in opposite directions,

$\therefore$ I will lie between A and B,



$\therefore$ $V_A = V_C + R\omega$

$V_B = R\omega - V_C$

$\therefore$ $V_C + R\omega = 5$

and $R\omega - V_C = 3$

$\therefore$ $V_C + 0.25\,\omega = 5$ ...(a)

$0.25\,\omega - V_C = 3$ ...(b)

On solving (a) and (b),

$$\omega = 16 \text{ rad/s}$$

22. (d)

Let the two forces are P kN and Q kN and angle between P and Q be ‘θ’

$\therefore$ Resultant force, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

For maximum resultant force, cosθ must be equal to 1.

$\therefore$ $R_{max} = \sqrt{(P+Q)^2} = P + Q$

$\Rightarrow$ $P + Q = 40$ ...(i)

For minimum resultant force, cos θ must be equal to (–1)

$\therefore$ $R_{min} = \sqrt{(P-Q)^2} = P - Q$

$\Rightarrow$ $P - Q = 10$ ...(ii)

Solving (i) and (ii), we get $P = 25$ kN, $Q = 15$ kN.

23. (b)

Mass of the block is m, therefore, stretch in the spring (x) is given by,

$$mg = kx$$

$$\Rightarrow \quad x = \frac{mg}{k}$$

Total mechanical energy of the system just after the blow is,

$$T_i = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad T_i = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2$$

$$\Rightarrow \quad T_i = \frac{1}{2}mv^2 + \frac{m^2g^2}{2k}$$

If the block descends through a height 'h' before coming to an instantaneous rest then the elastic potential energy becomes $\frac{1}{2}k\left(\frac{mg}{k}+h\right)^2$ and the gravitational potential energy will be $-mgh$.

$$\therefore \quad T_f = \frac{1}{2}k\left(\frac{mg}{k}+h\right)^2 - mgh$$

On applying conservation of energy, we get

$$T_i = T_f$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{m^2g^2}{2k} = \frac{1}{2}k\left(\frac{mg}{k}+h\right)^2 - mgh$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}kh^2$$

$$\Rightarrow \quad h = v\sqrt{\frac{m}{k}}$$

24. **(b)**

The free body diagrams of the two blocks are as shown:

From the consideration of equilibrium of block 1 and block 2.

For block 1:

$$W_1\cos\alpha - N_1 = 0$$

$$\Rightarrow \quad N_1 = W_1\cos\alpha$$

Also, $$W_1\sin\alpha = T + F_1 = T + \mu \cdot N_1$$

$$= T + \mu \cdot W_1\cos\alpha \qquad \text{...(i)}$$

For block 2:

$$W_2 = N_2$$

$$T_1 = \mu \cdot N_2 = \mu \cdot W_2 \qquad \text{...(ii)}$$

Put the value of T in eq. (i)

$$W_1 \sin\alpha = \mu W_2 + \mu W_1 \cos\alpha$$

$\Rightarrow$ $$W_1 \sin\alpha - \mu W_1 \cos\alpha = \mu W_2$$

Since, $$W_1 = W_2$$

$\therefore$ $$\sin\alpha - \mu \cdot \cos\alpha = \mu$$

$\Rightarrow$ $$\sin\alpha = \mu(1 + \cos\alpha)$$

$\Rightarrow$ $$2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \mu \cdot 2 \cdot \cos^2\frac{\alpha}{2}$$

$\Rightarrow$ $$\tan\frac{\alpha}{2} = \mu$$

$\Rightarrow$ $$\frac{\alpha}{2} = \tan^{-1}(\mu)$$

$\Rightarrow$ $$\alpha = 2 \times \tan^{-1}(0.3) = (16.7°) \times 2 = 33.4°$$

25. (d)

For no tipping or prevent overturning

$$Ph < \frac{Wb}{2}$$

where W is weight of block

and b is width of block

$$h < \frac{Wb}{2P} \qquad \text{...(1)}$$

and for slipping without tipping

$$P > f\text{(force of friction)}$$

$$P > \mu W \qquad \text{...(2)}$$

From (1) and (2)

$$h < \frac{b}{2\mu}$$

$\therefore$ $$h < \frac{60}{0.6}$$

$\therefore$ $$h < 100 \text{ mm}$$

Option (d) is correct.

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