CLASS TEST

S.No.: 01SK_ABC_28022024

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ENGINEERING MATHEMATICS

EC-EE

Date of Test : 28/02/2024

ANSWER KEY

1.	(c)	7.	(d)	13.	(c)	19.	(c)	25.	(c)
2.	(a)	8.	(b)	14.	(a)	20.	(a)	26.	(d)
3.	(c)	9.	(d)	15.	(a)	21.	(d)	27.	(d)
4.	(b)	10.	(c)	16.	(b)	22.	(a)	28.	(d)
5.	(b)	11.	(c)	17.	(b)	23.	(d)	29.	(a)
6.	(b)	12.	(d)	18.	(b)	24.	(c]	30.	(a)

DETAILED EXPLANATIONS

1. (c)

Commutative for multiplication of matrices does not hold.

$$AB \neq BA$$

2. (a)

$$\text{Probability} = \int_2^{\infty} f(x)dx$$

$$= \int_2^{\infty}\left[\frac{1}{2}e^{\frac{-x}{2}}\right]dx = \left[-e^{\frac{-x}{2}}\right]_2^{\infty} = e^{-1} = 0.368$$

3. (c)

The matrix formed by the coefficients is $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$\therefore$ $D = 0$ for $a = 2$ or $a = -1$

(A) If $D \neq 0$, then the system will have unique solution because the rank of matrix will be 3.

(B) If $a = 2$, the matrix formed by the coefficients is $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.

Considering 'z' as side unknown.

The characteristic determinant will be $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.

The system will have infinite solutions when $a = 2$.

(C) If $a = -1$, the matrix formed by the coefficients is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.

Considering 'z' as side unknown.

The characteristic matrix is $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is $3b$.

The system will have no solution if $b \neq 0$

$\therefore$ For $a = -1$ and $b \neq 0$, the system will have no solution.

4. (b)

Probability density function:

$$f(x) = \lambda \cdot e^{-\lambda x}, x > 0$$

$$E(X) = \int_0^\infty x \cdot f(x) \cdot dx$$

$$= \int_0^\infty x\lambda \cdot e^{-\lambda x} \cdot dx = \frac{1}{\lambda}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2 \cdot f(x) \cdot dx$$

$$= \int_0^\infty x^2 \cdot \lambda \cdot e^{-\lambda x} \cdot dx = \frac{2}{\lambda^2}$$

$$\Rightarrow \quad \text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$$

5. (b)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - 2(R_1) \text{ and } R_3 \leftarrow R_3 - R_1$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - R_2$$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank of matrix is 2.

6. (b)

For a non-trivial solution of homogeneous system of equations,

$$|A| = 0$$

where
$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix}$$

$$\Rightarrow \quad 2\begin{vmatrix} 1 & 3 \\ 3 & b \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 4 & b \end{vmatrix} + 2\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad b = 8$$

7. (d)

Given,

$$np = 4$$
$$npq = 2$$
$$q = \frac{1}{2}, p = \frac{1}{2}, n = 8$$
$$P(x = 1) = {}^nC_1\, p^1\, q^{n-1}$$
$$= {}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 = \frac{8}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$
$$= 0.03125$$

8. (b)

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x^3 - 1) = 0$$

Also $f(1) = 0$

Thus $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow$ f is continuous at $x = 1$

And $Lf'(1) = 2$, $Rf'(1) = 1$

$\Rightarrow f$ is not differentiable at $x = 1$

9. (d)

$$f(x) = \sqrt{36 - 4x^2}$$

At $x = 0$, $f(x) = 6$

If $x \neq 0 \Rightarrow f(x) < 6$

$\therefore$ f has absolute maximum at $x = 0$

At $x = 3$, $f(x) = 0$

$\therefore$ f has absolute minimum at $x = 3$.

10. (c)

Using Crout's method

$$A = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 2 & 4 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$l_{11} = 2 \qquad l_{11} u_{12} = 4$$
$$u_{12} = \frac{4}{2} = 2$$
$$l_{21} = 6 \qquad l_{21} u_{12} + l_{22} = 3$$
$$6 \times 2 + l_{22} = 3$$

$$l_{22} = 3 - 12$$
$$l_{22} = -9$$

So, LU decomposition of given matrix is

$$L = \begin{bmatrix} 2 & 0 \\ 6 & -9 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Note: Candidates can use options to solve such questions.

11. (c)

$$P(x = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}, \text{ where } \lambda = np$$
$$\lambda = 500 \times 0.006 = 3$$
$$P(x \le 1) = P(x = 0) + P(x = 1)$$
$$= \frac{e^{-\lambda} \cdot \lambda^0}{0!} + \frac{e^{-\lambda} \cdot \lambda^1}{1!}$$
$$= e^{-3} + e^{-3} \cdot 3 = 4e^{-3}$$

12. (d)

$$E[6X] = 6.E[X] = 6$$
$$Var[6X] = 6^2 \, Var[X] = 36 \times 2 = 72$$
$$E[1 - X] = 1 + (-1) \, E[X] = 1 - 1 = 0$$
$$Var[1 - X] = (-1)^2 \, Var[X] = Var[X] = 2$$
$$\therefore \quad Var[1 - X] \neq 3$$

13. (c)

Maximum number of distinct eigen values = Size of matrix A.

$\therefore$ S_1 is False.

Sum of eigen values = Sum of diagonal elements.

$\therefore$ S_2 is False.

14. (a)

$$|\lambda - AI| = (1 - \lambda)(\lambda^2 - 2) + (2 - \lambda) - \lambda = -\lambda^3 + \lambda^2$$
$$\Rightarrow \quad -\lambda^3 + \lambda^2 = 0$$
$$\Rightarrow \quad -\lambda^2(\lambda - 1) = 0$$
$$\lambda = 0, \lambda = 1$$

The largest eigen value is 1

$$A - I = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_1 \leftrightarrow R_2}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}_{R_3 \leftarrow R_3 + R_1}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}_{R_3 \leftarrow R_3 - R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{R_1 \leftarrow R_1 - 2R_2}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A - I]\vec{x} = 0$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = x_3,$$

$$-x_2 + x_3 = 0 \Rightarrow x_2 = x_3$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} x_3$$

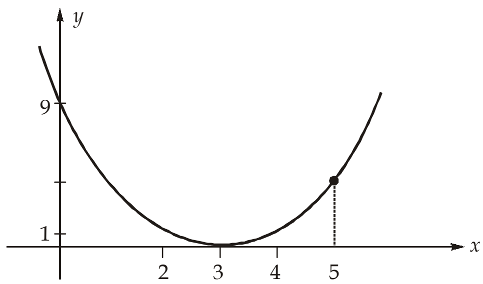
$$\therefore \quad x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ is an eigen vector.}$$

15. (a)

$$y = x^2 - 6x + 9 = (x - 3)^2$$

$$y(2) = 1$$

$$y(5) = 4$$



$\therefore$ Maximum value of y over the interval 2 to 5 will be at $x = 5$.

16. (b)

$$I = \int_{-2}^{2} |1 - x^4|\, dx$$

The given function is an even function i.e., $f(x) = f(-x)$

$$\Rightarrow \quad I = 2\int_{0}^{2} |1 - x^4|\, dx$$

$$= 2\left\{\int_0^1 (1 - x^4)\,dx + \int_1^2 (x^4 - 1)\,dx\right\}$$

$$= 2\left\{\left[x - \frac{x^5}{5}\right]_0^1 + \left[\frac{x^5}{5} - x\right]_1^2\right\} = 12$$

17. (b)

$6(13 \times 11 - 4 \times 37) - 3(32 \times 11 - 10 \times 37) + 7\,(32 \times 4 - 10 \times 13)$

$$= -30 + 54 - 14 = 10$$

18. **(b)**

$$\frac{\partial f}{\partial x} = 2 - 2x \qquad \frac{\partial f}{\partial y} = 2 - 2y$$

$$r = \frac{\partial^2 f}{\partial x^2} = -2 \qquad t = \frac{\partial^2 f}{\partial y^2} = -2, \qquad s = \frac{\partial^2 f}{\partial x \partial y} = 0$$

Finding stationary points,

$$\frac{\partial f}{\partial x} = 2 - 2x = 0$$

$\Rightarrow x = 1$

$$\frac{\partial f}{\partial y} = 2 - 2y = 0$$

$\Rightarrow y = 1$

At the stationary point (1, 1)

$$rt - s^2 = (-2)(-2) - 0 = 4 > 0$$

So, $f(x, y)$ is maxima at (1, 1)

Maximum value of $f(x, y) = 2 + 2 + 2 - 1 - 1 = 4$

19. **(c)**

$$\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 1} \frac{\frac{d}{dx}(x^x) - 1}{1 - 0 - \frac{1}{x}}$$

Let, $y = x^x$

$\log y = x \log x$

$\therefore \quad \frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log x$

or $\quad \frac{d}{dx}(x^x) = x^x(1 + \log x)$

$$= \lim_{x \to 1} \frac{x^x(1 + \log x) - 1}{1 - \frac{1}{x}} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 1} \frac{\frac{d}{dx}(x^x) \cdot (1 + \log x) + x^x \left(\frac{1}{x}\right) - 0}{\frac{1}{x^2}}$$

$$= \lim_{x \to 1} \frac{x^x(1 + \log x)^2 + x^x \left(\frac{1}{x}\right)}{x^{-2}} = \frac{1(1+0)^2 + 1 \cdot 1}{1} = 2$$

20. (a)

$$|x-2| = \begin{cases} -(x-2)\,; & x<2 \\ (x-2)\,; & x>2 \end{cases}$$

$$\int_1^3 \frac{|x-2|}{x}dx = \int_1^2 \frac{-(x-2)}{x}dx + \int_2^3 \frac{x-2}{x}dx$$

$$= \int_1^2\left(-1+\frac{2}{x}\right)dx + \int_2^3\left(1-\frac{2}{x}\right)dx$$

$$= [-x]_1^2 + [2\ln x]_1^2 + [x]_2^3 - 2[\ln x]_2^3$$

$$= -(2-1) + 2\ln 2 - 2\ln\frac{3}{2} + (3-2)$$

$$= 2\ln 2 - 2\ln\frac{3}{2}$$

$$= 2\ln\frac{2}{\frac{3}{2}} = 2\ln\frac{4}{3}$$

21. (d)

Let $u = \sqrt{x}$

Then $du = \dfrac{1}{2\sqrt{x}}dx$

$\therefore$ $dx = du\cdot 2\sqrt{x}$

$$\int\frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int\frac{\sin u}{\sqrt{x}}\cdot 2\sqrt{x}\,du = 2\int\sin u\,du$$

$$= -2\cos\sqrt{x} + c$$

22. (a)

Using Doolittle method:

$$A = LU$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$u_{11} = 25,\ u_{12} = 5,\ u_{13} = 1$

$$u_{11}\,l_{21} = 64$$

$$l_{21} = \frac{64}{25} = 2.56$$

$$l_{21}u_{12} + u_{22} = 8$$

$$2.56\,u_{12} + u_{22} = 8$$

$$u_{22} = -4.8$$

$$u_{13}\,l_{21} + u_{23} = 1$$

$$u_{23} = -1.56$$
$$l_{31}\,u_{11} = 144$$
$$l_{31} = \frac{144}{25} = 5.76$$
$$l_{31}u_{12} + l_{32}u_{22} = 12$$
$$(5.76 * 5) + (u_{22}\,l_{32}) = 12$$
$$l_{32} = 3.5$$
$$l_{31}\,u_{13} + l_{32}\,u_{23} + u_{33} = 1$$
$$u_{33} = 0.7$$

So, LU decomposition is

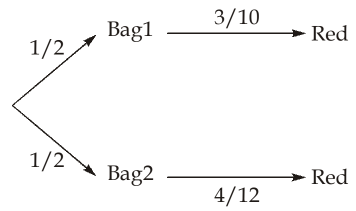
$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

23. (d)

$$I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right) dx$$
$$= \int_0^{\pi/2} [\log(\sin x)dx - \log(\cos x)dx]$$
$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx - \int_0^{\pi/2} \log(\cos x)dx$$
$$I = 0$$

24. (c]

The tree diagram for above problem, is shown below:

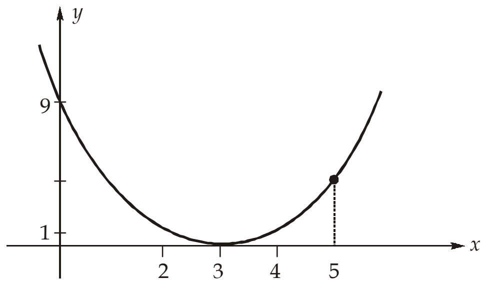


$$P\ (\text{bag1}\,|\,\text{Red}) = \frac{P(\text{bag1} \cap \text{Red})}{P(\text{Red})}$$
$$= \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{3}{20}}{\frac{3}{20} + \frac{1}{6}} = 0.317$$

25. (c)

$$y = x^2 - 6x + 9 = (x-3)^2$$
$$y(2) = 1$$

$$y(5) = 4$$



∴ maximum value of y over the interval 2 to 5 will be at $x = 5$.

26. **(d)**

$$AX = B$$

Augmented matrix, $$[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$$

$R_3 \to R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

⇒ Infinitely many solutions are possible.

27. **(d)**

For $\lambda = 1$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$X_1 = c_1\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

For $\lambda = 2$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$x_1 + x_3 = 0$$

$$2x_1 + 2x_2 + x_3 = 0$$

$$X_2 = c_2\begin{bmatrix}-2\\1\\2\end{bmatrix}$$

For $\lambda = 3$

$$\begin{bmatrix}-2 & 0 & -1\\1 & -1 & 1\\2 & 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = 0$$

$$x_1 = -x_2$$

$$x_1 = \frac{-1}{2}x_3$$

$$X_3 = c_3\begin{bmatrix}-1\\1\\2\end{bmatrix}$$

Since, $X_1{}^T X_2 \neq 0$

$$X_2{}^T X_3 \neq 0$$

$$X_3{}^T X_1 \neq 0$$

None of the above is correct.

28. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

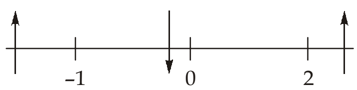
$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$

$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.
Critical point (–1, 2) draw plot on line graph:
Since $0 \in (-1, 2)$ and $f'(0) = 6 \times 0^2 - 6 \times 0 - 12 = -12 < 0$



The function is decreasing between –1 and 2.

29. (a)

Given,

$$\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2\tan x}{1 + \cos 4x} \quad : \quad \frac{0}{0} \text{ Form}$$

Applying L′ hospital rule

$$= \lim_{x\to\frac{\pi}{4}} \frac{\frac{d}{dx}(\sec^2 x - 2\tan x)}{\frac{d}{dx}(1+\cos 4x)}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{(2\sec x \cdot \sec x \tan x - 2\sec^2 x)}{-4\sin 4x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\sec^2 x(\tan x - 1)}{-2\sin 4x} \quad : \frac{0}{0} \text{ Form}$$

Applying L′ hospital's rule

$$= \lim_{x\to\frac{\pi}{4}} \frac{2\sec x \cdot \sec x \cdot \tan x(\tan x - 1) + \sec^2 x \sec^2 x}{-8\cos 4x}$$

$$= \frac{2.2.1(1-1)+2.2}{-8(-1)} = \frac{1}{2}$$

30. (a)

Operating $R_3 - (R_1 + R_2)$ we get

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0 \qquad \text{(Operating } R_2 - R_1 \text{ and } R_1 + R_3\text{)}$$

or
$$\begin{vmatrix} x+2 & 2x+4 & 6x+12 \\ x+1 & x+1 & x+1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

or
$$(x+1)(x+2)\begin{vmatrix} 1 & 2 & 6 \\ 1 & 1 & 1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

To bring one more zero in C_1, operate $R_1 - R_2$.

$$\therefore \quad (x+1)(x+2)\begin{vmatrix} 0 & 1 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & 3x+8 \end{vmatrix} = 0$$

Now expand by C_1.

$\therefore \quad -(x+1)(x+2)(3x+8-5) = 0$ or $-3(x+1)(x+2)(x+1) = 0$

Thus, $x = -1, -1, -2.$

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