DETAILED
SOLUTIONS

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ESE 2024 : Prelims Exam
CLASSROOM TEST SERIES

GS & ENGINEERING APTITUDE

Test 1

Section A : Reasoning & Aptitude **[All Topics]**
Section B : Engineering Mathematics **[All Topics]**

ANSWER KEY

1.	**(d)**	**11.**	**(b)**	**21.**	**(d)**	**31.**	**(a)**	**41.**	**(a)**
2.	**(b)**	**12.**	**(a)**	**22.**	**(c)**	**32.**	**(b)**	**42.**	**(a)**
3.	**(a)**	**13.**	**(c)**	**23.**	**(a)**	**33.**	**(d)**	**43.**	**(a)**
4.	**(a)**	**14.**	**(c)**	**24.**	**(c)**	**34.**	**(a)**	**44.**	**(a)**
5.	**(d)**	**15.**	**(a)**	**25.**	**(a)**	**35.**	**(d)**	**45.**	**(b)**
6.	**(c)**	**16.**	**(d)**	**26.**	**(c)**	**36.**	**(b)**	**46.**	**(c)**
7.	**(a)**	**17.**	**(c)**	**27.**	**(b)**	**37.**	**(c)**	**47.**	**(b)**
8.	**(c)**	**18.**	**(c)**	**28.**	**(b)**	**38.**	**(d)**	**48.**	**(a)**
9.	**(d)**	**19.**	**(b)**	**29.**	**(c)**	**39.**	**(a)**	**49.**	**(b)**
10.	**(a)**	**20.**	**(d)**	**30.**	**(c)**	**40.**	**(c)**	**50.**	**(b)**

DETAILED EXPLANATIONS

1. **(d)**

Let $x = 6q + 3$

Then, $x^2 = (6q + 3)^2 = 36q^2 + 36q + 9$

$= 6(6q^2 + 6q + 1) + 3$

Thus, when x^2 is divided by 6, then remainder = 3.

2. **(b)**

L.C.M. of 5, 6, 7, 8 = 840

∴ Required number is the form $840k + 3$

Least value of k for which $(840k + 3)$ is divisible by 9 is $k = 2$.

∴ Required number $= 840 \times 2 + 3 = 1683$

3. **(a)**

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{10+8+12}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}$$

$$= \frac{30}{3/4} = 40 \text{ km/hour}$$

4. **(a)**

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} = \frac{4\times5\times6+5\times6+2\times6+2\times3}{1\times2\times3\times4\times5\times6}$$

$$= \frac{120+30+12+6}{720} = \frac{168}{720}$$

$$= \frac{7}{30}$$

5. **(d)**

Let the number of papers be x.

Then, $63x + 20 + 2 = 65x$

$2x = 22$

$x = 11$

6. **(c)**

Let the numbers be x, y and z.

Then, $x + y = 45$

$y + z = 55$

$3x + z = 90$

$y = 45 - x, z = 55 - y$

$z = 55 - (45 - x)$

$z = 10 + x$

∴ $3x + 10 + x = 90$

or $x = 20$

$$y = 45 - 20 = 25$$

and $$z = 10 + 20 = 30$$

$\therefore$ $$\text{Third number} = 30$$

7. (a)

When a person sells two similar items (same selling price) one at a gain of say, x% and the other at a loss of x%, then seller always incurs a loss given by;

$$\%\text{ loss} = \frac{x^2}{100}$$

$\therefore$ $$\text{Loss}\% = \frac{5^2}{100} = 0.25\%\text{ loss}$$

8. (c)

Let the initial investments of A and B be $3x$ and $5x$.

$$A : B : C = (3x \times 12) : (5x \times 12) : (5x \times 6)$$
$$= 36 : 60 : 30$$
$$= 6 : 10 : 5$$

9. (d)

Originally, let there be x men

Less men, more days (Indirect proportion)

$\therefore$ $$(x - 10) : x :: 100 : 110$$
$$(x - 10) \times 110 = x \times 100$$
$$10x = 1100$$
$$x = 110$$

10. (a)

Let $$1\text{ woman's 1 day's work} = x$$

$$\text{Then, 1 man's 1 day's work} = \frac{x}{2}$$

$$\text{and 1 child's 1 day's work} = \frac{x}{4}$$

So, $$\frac{3x}{2} + 4x + \frac{6x}{4} = \frac{1}{7}$$
$$\frac{6x + 16x + 6x}{4} = \frac{1}{7}$$
$$\frac{28x}{4} = \frac{1}{7}$$
$$x = \frac{1}{7} \times \frac{4}{28} = \frac{1}{49}$$

$\therefore$ 1 woman alone can complete the work in 49 days.

So, to complete the work in 7 days, number of women required $= \frac{49}{7} = 7$

11. (b)

The area of the triangle formed by joining the middle points of the sides of a triangle is equal to one-fourth area of the given triangle.

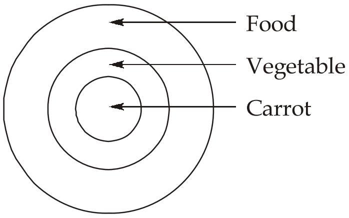
$$\text{Let area of } 15^{\text{th}}\ \Delta = A$$

$$\text{Then area of } 18^{\text{th}}\ \Delta = \frac{A}{64}$$

$$\text{So, required ratio} = \frac{64}{1} = 64$$

12. (a)

All carrot are vegetable and all vegetable are food.



13. (c)

$$\text{Total population, } P = 5000$$

Let the number of males be M

$$\text{Number of females} = 5000 - \text{M}$$

$$\text{New number of males} = M \times \left(1 + \frac{10}{100}\right) = 1.1\ \text{M}$$

$$\text{New number of females} = (5000 - M)\left(1 + \frac{15}{100}\right) = (5000 - M)1.15$$

$$\text{Total population} = 1.1\ \text{M} + (5000 - \text{M})(1.15) = 5600$$

$$5750 + (1.1 - 1.15)\text{M} = 5600$$

$$(1.15 - 1.1)\text{M} = 5750 - 5600$$

$$0.05\ \text{M} = 150$$

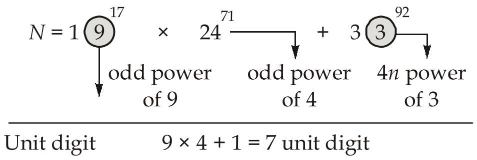
$$\text{M} = \frac{150}{0.05} = \frac{30}{0.01} = 3000$$

14. (c)

Difference between CI and SI for two years $(\text{CI-SI})_{2y} = P\left(\frac{R}{100}\right)^2$ (which is interest on first year interest)

$$(\text{CI-SI})_{2y} = 400\left(\frac{12}{100}\right)^2 = \text{Rs. } 5.76$$

15. (a)



16. (d)

$$|x-6| = 11 \quad \begin{rotate}\end{rotate}$$

$$|x-6| = 11 \qquad \left.\begin{array}{ll} \Rightarrow & x-6=11, \quad x=17 \\ \text{or} & x-6=-11, \quad x=-5 \end{array}\right\}$$

$$|2y-12| = 8 \qquad \left.\begin{array}{l} 2y-12=8, \quad y=10 \\ 2y-12=-8, \quad y=2 \end{array}\right\}$$

Possible values of $\frac{y}{x}$ are

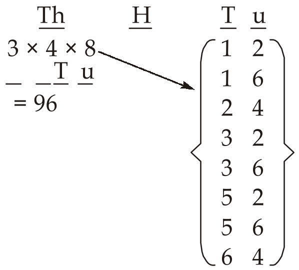
$\frac{10}{17}, \frac{10}{-5}, \frac{2}{17}, \frac{2}{-5}$, hence minimum value is $\frac{10}{-5} = -2$

17. (c)

Series follows (–3 ÷ 2); so, 46 is the odd number.

18. (c)

For a number to be divisible by 4 its last two digit have to be divisible by 4.



[T] and [u] can be filled in given 8 ways.

[H] can be filled in now only 4 ways as two digits are already occupied in [*T*] and [*u*].

[Th] can be filled in remaining 3 ways.

Total possible numbers = 3 x 4 x 8 = 96

19. (b)

5! onwards every factorial will be a multiple of 120. Hence will be divisible by 24. So, remainder will be due to

$$1 \times 1! + 3 \times 3! = 19$$

20. (d)

$S = \log_2 2 + \log_2 2^3 + \log_2 2^5 .... \log_2 2^{13} = 1 + 3 + 5 + 7.....+13 = 49$

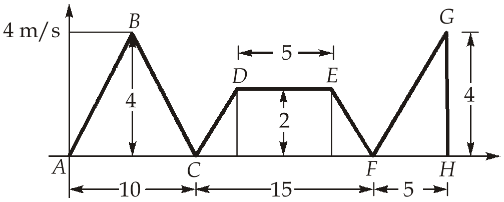
(Sum of odd numbers = n^2)

Alternative method:

$$S_{AP} = \frac{n}{2}[1^{\text{st}}\text{ term} + \text{last term}]$$

$$\therefore \quad S = \frac{7}{2}[1+13] = 49$$

21. **(d)**



$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{\text{Area under } (s-t) \text{ graph}}{\text{Total time}}$$

$$\text{Total area} = Ar(\Delta ABC) + \text{Ar (Trapezium } CDEF) + \text{Ar } (\Delta FGH)$$

$$= \frac{1}{2}\times 10\times 4+\frac{1}{2}(15+5)\times 2+\frac{1}{2}\times 5\times 4$$

$$= 50 \text{ m}$$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{50}{30} = 1.66 \text{ m/sec.}$$

22. **(c)**

Different types of rectangles are:

1 × 1, 1 × 2, 1 × 3, 1 × 4, 1 × 5, 1 × 6 → 6 types

2 × 2, 2 × 3, 2 × 4, 2 × 5, 2 × 6 → 5 types

3 × 3, 3 × 4, 3 × 5, 3 × 6 → 4 types

4 × 4, 4 × 5, 4 × 6 → 3 types

Total different types of rectangles are 3 + 4 + 5 + 6 = 18

Note: Remember (1 × 2) and (2 × 1) is regarded same type of rectangle.

23. **(a)**

Let the work lasted for t days

Raman's 2 days work + Ashok's $(t - 3)$ days work + Satish's t days work = Total work done

$$\frac{2}{8}+\frac{t-3}{16}+\frac{t}{32} = 1$$

$$\frac{1}{4}+\frac{t-3}{16}+\frac{t}{32} = 1$$

$$\frac{8+2(t-3)+t}{32} = 1$$

$$3t + 2 = 32$$

$$3t = 30$$
$$t = 10 \text{ days}$$

24. (c)

Quantity of pure alcohol in 400 ml solution = $\frac{15}{100} \times 400 = 60$ ml

Quantity of water in solution = 340 ml

According to the question

Let amount of alcohol to be added be x

$$\frac{60+x}{400+x} = 0.32$$
$$x = 100 \text{ ml}$$

25. (a)

Probability of getting atleast 6 heads = Probability of getting 2 tails + Prob. of getting 1 tail + Prob. of getting no tail

$$= {}^8C_2 \times \frac{1}{256} + {}^8C_1 \times \frac{1}{256} + {}^8C_0 \times \frac{1}{256} = \frac{37}{256}.$$

26. (c)

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = 4^3\left(\frac{d^3y}{dx^3}\right)^3$$

∴ Order and degree of this equation, are both 3.

⇒ Option (c) is correct.

27. (b)

Let
$$I = \int_0^1\int_0^x (x+y)\,dydx$$
$$= \int_0^1 \left[xy + \frac{y^2}{2}\right]_0^x dx = \int_0^1 \left(x^2 + \frac{x^2}{2}\right) dx$$
$$= \int_0^1 \frac{3x^2}{2} dx = \frac{3}{2}\left[\frac{x^3}{3}\right]_0^1$$
$$= \frac{1}{2} = 0.5 \text{ sq.units}$$

28. (b)

∵
$$u = (x^2 + y^2 + z^2)^{1/2}$$
$$\frac{\partial u}{\partial x} = \frac{2x}{2(x^2+y^2+z^2)^{1/2}} = \frac{x}{u}$$

$$\frac{\partial^2 u}{\partial x^2} = -\frac{1}{u^2}\times\frac{x}{u}\times x+\frac{1}{u} = \frac{y^2+z^2}{u^3}$$

Similarly, $$\frac{\partial^2 u}{\partial y^2} = \frac{x^2+z^2}{u^3}$$ and

$$\frac{\partial^2 u}{\partial z^2} = \frac{y^2+x^2}{u^3}$$

$$\therefore \quad \frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} = \frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{3/2}} = \frac{2}{(x^2+y^2+z^2)^{1/2}} = \frac{2}{u}$$

⇒ Option (b) is correct.

29. (c)

$$\text{PI} = \frac{1}{D^3+1}\cos(2x-1) \qquad (\text{Put } D^2 = -2^2 = -4)$$

$$= \frac{1}{D(-4)+1}\cos(2x-1) = \frac{1+4D}{(1-4D)(1+4D)}\cos(2x-1)$$

$$= 1+4D\times\frac{1}{1-16D^2}\cos(2x-1)$$

Put $D^2 = -2^2 = -4$

$$= (1+4D)\times\frac{1}{1-16(-4)}\cos(2x-1)$$

$$= \frac{1}{65}\left[\cos(2x-1)+4D\cos(2x-1)\right]$$

$$= \frac{1}{65}\left[\cos(2x-1)-8\sin(2x-1)\right]$$

⇒ Option (c) is correct.

30. (c)

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$$

$$\vec{a}\times\vec{b} = |\vec{a}|\cdot|\vec{b}|\sin\theta\,\hat{n}$$

$$|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2\cdot|\vec{b}|^2(\sin^2\theta+\cos^2\theta) = |\vec{a}|^2\cdot|\vec{b}|^2$$

31. (a)

Comparing with general partial differential equation,

$$A\frac{\partial^2 f}{\partial x^2}+B\frac{\partial^2 f}{\partial x\partial t}+C\frac{\partial^2 f}{\partial t^2}+D\frac{\partial f}{\partial x}+E\frac{\partial f}{\partial t}+F=0$$

$A = 1$, $B = 0$, $C = 0$, $E = -1$

Now, since, $B^2 - 4AC = 0$

∴ Given partial differential equation is parabolic.

⇒ Option (a) is correct.

32. **(b)**

The total area under the curve for any pdf is always equal to 1, hence

$$\int_{-\infty}^{\infty} f_x(x)\,dx = 1$$

$$\int_0^{\infty} \frac{e^{-2x}}{k}\,dx = 1$$

$$\frac{1}{k}\left(\frac{e^{-2x}}{(-2)}\right)_0^{\infty} = 1$$

$$-\frac{1}{2k}\left(e^{-2x}\right)_0^{\infty} = 1$$

$$-\frac{1}{2k}(0-1) = 1$$

∴ $k = 0.5$

33. **(d)**

Rank of matrix $A = 2$

⇒ $|A| = 0$

⇒ $\mu(6 + 4) + 1(-4) = 0$

⇒ $10\mu = 4$

$\mu = 2/5$

34. **(a)**

The product of all eigenvalues of a matrix is equal to the determinant of the matrix.

Since A is a singular matrix,

∴ $|A| = 0$

Hence, $\lambda_1 \lambda_2 \lambda_3 = 0$

At least one eigen value = 0

Given that $\lambda_1 = 1$

$\lambda_2 = 3$

∴ $\lambda_3 = 0$

35. **(d)**

The auxiliary equation for differential equation is given by

$$D^3 - 3D + 2 = 0$$

$$(D - 1)(D^2 + D - 2) = 0$$

$$(D - 1)^2 (D + 2) = 0$$

$$D = 1, 1, -2$$

⇒ $CF = (C_1 + C_2x)\,e^x + C_3e^{-2x}$

36. (b)

$$|z| = \left|\left(z-\frac{4}{z}\right)+\frac{4}{z}\right| \le \left|z-\frac{4}{z}\right|+\left|\frac{4}{z}\right|$$

$$\Rightarrow \quad |z| \le 2+\frac{4}{|z|}$$

$$\Rightarrow \quad |z|^2 - 2|z| - 4 \le 0$$

$$\left[|z|-\left(\sqrt{5}+1\right)\right]\left[|z|-\left(1-\sqrt{5}\right)\right] \le 0$$

$$1-\sqrt{5} \le |z| \le \sqrt{5}+1$$

37. (c)

The characteristic equation of the matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$

$$(-5 - \lambda)(-\lambda) + 6 = 0$$

$$\Rightarrow \quad \lambda^2 + 5\lambda + 6 = 0$$

By Cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation.

$$\therefore \quad A^2 + 5A + 6I = 0$$

$$A^2 = -5A - 6I$$

$$A^3 + 5A^2 + 6A = 0$$

$$A^3 = -5(-5A - 6I) - 6A$$

$$= 19A + 30I$$

38. (d)

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{\partial F_z}{\partial z}$$

$$= y^2 + 2x^2z - 6yz$$

At (1, –1, 1), $\vec{\nabla}\cdot\vec{F} = 9$

39. (a)

By Cauchy's integral formula

$$\int_C \frac{z}{(z-1)^3}dz = \frac{2\pi i}{2!}\frac{d^2}{dz^2}f(z)\bigg|_{z=1}$$

where $f(z) = z$, $f'(z) = 1$, $f''(z) = 0$

So, $$\int_C \frac{z}{(z-1)^3}dz = 0$$

40. **(c)**

$$\begin{aligned} \frac{dy}{dx} &= e^{x+y} + x^2 e^{x^3+y} \\ \frac{dy}{dx} &= e^y\left[e^x + x^2 e^{x^3}\right] \\ \int \frac{1}{e^y} dy &= \int\left(e^x + x^2 e^{x^3}\right) dx \\ -e^{-y} &= e^x + \int x^2 e^{x^3} dx + c \end{aligned}$$

Let $x^3 = t$

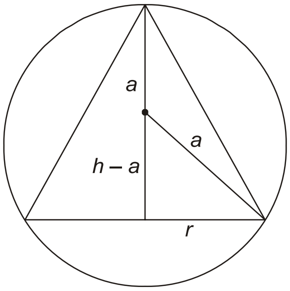
$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

$$\begin{aligned} -e^{-y} &= e^x + \int e^t \frac{dt}{3} + c \\ -e^{-y} &= e^x + \frac{1}{3} e^t + c \\ e^x + e^{-y} + \frac{e^{x^3}}{3} + c &= 0 \end{aligned}$$

41. **(a)**

From figure,



$$\begin{aligned} r^2 &= a^2 - (h - a)^2 \\ r^2 &= -h^2 + 2ah \end{aligned}$$

Now volume of cone,

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ V &= \frac{\pi}{3}[-h^3 + 2ah^2] \end{aligned}$$

For 'V' to be maximum,

$$\frac{dV}{dh} = \frac{\pi}{3}[-3h^2 + 4ah] = 0$$

$\Rightarrow \quad h = 0, \frac{4a}{3}$

$$\frac{d^2V}{dh^2} < 0 \text{ for } h = \frac{4a}{3} \text{ indicating a maxima}$$

$\therefore$ Volume is maximum with $h = \frac{4a}{3}$

42. **(a)**

$$\begin{aligned} L\,[t^2 u(t - 3)] &= e^{-3s} L\,[(t + 3)^2 u(t)] \\ &= e^{-3s} L\,[(t^2 + 6t + 9)u(t)] \end{aligned}$$

$$= e^{-3s}\left(\frac{2}{s^3}+\frac{6}{s^2}+\frac{9}{s}\right)$$

43. (a)

$$L^{-1}\left[\frac{1}{s^2-5s+6}\right] = L^{-1}\left[\frac{1}{s-3}-\frac{1}{s-2}\right]$$

$$= L^{-1}\left(\frac{1}{s-3}\right)-L^{-1}\left(\frac{1}{s-2}\right)$$

$$= (e^{3t} - e^{2t})u(t)$$

44. (a)

$$\text{Div}\,\vec{V} = \nabla\cdot\vec{V}$$

$$= \frac{\partial}{\partial x}(xy\sin z)+\frac{\partial}{\partial y}(y^2\sin x)+\frac{\partial}{\partial z}(z^2\sin xy)$$

$$= y\,\sin z + 2y\,\sin x + 2z\,\sin xy$$

Now, at the given point

$$x = 0,\ y = \frac{\pi}{2},\ z = \frac{\pi}{2}$$

We have,

$$\text{Div}\,\vec{V} = \frac{\pi}{2}\sin\frac{\pi}{2}+2\times\frac{\pi}{2}\sin 0+2\cdot\frac{\pi}{2}\cdot\sin 0$$

$$= \frac{\pi}{2}$$

45. (b)

For Newton Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

We have to find

$$x = \sqrt{28}$$

$$f(x) = x^2 - 28$$

$$f'(x) = 2x$$

$$x_1 = 5.6 - \left.\frac{x^2-28}{2x}\right|_{x_0}$$

$$= 5.6 - \frac{5.6^2-28}{2\times 5.6} = 5.30$$

46. (c)

Residue at $z = 0$ will be,

$$\text{Residue} = \underset{z\to 0}{lt}\ z\times\frac{1+e^z}{z\left(\frac{\sin z}{z}+\cos z\right)} = \frac{1+e^0}{1+1} = 1$$

47. (b)

Since A and B are independent, hence

$$P(A\cap B) = P(A)\times P(B) = 0.15\times P(B)$$

Also $$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$

$$0.45 = 0.15 + P(B) - (0.15)\times P(B)$$

$$= 0.15 + P(B)(1-0.15)$$

$$= 0.15 + 0.85P(B)$$

$\therefore$ $$0.85P(B) = 0.45 - 0.15 = 0.30$$

$\therefore$ $$P(B) = \frac{0.30}{0.85} = \frac{30}{85} = \frac{6}{17}.$$

48. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + \theta)$

$\therefore$ $$\frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta+1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)}{b\times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

49. (b)

$$\lim_{x\to 0}\frac{\ln(1+5x)}{e^{7x}-1} \qquad \left(\frac{0}{0}\text{ indeterminate form}\right)$$

Applying L' Hospital's rule

$$\lim_{x\to 0}\frac{\ln(1+5x)}{e^{7x}-1} = \lim_{x\to 0}\frac{5}{(1+5x)7e^{7x}} = \frac{5}{7}$$

50. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1 P_2 P_3 = \frac{15}{50} \times \frac{14}{49} \times \frac{13}{48} = \frac{13}{560}$$

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